Fluid Kinematics

Fluid kinematics is the branch of fluid mechanics in which the study of the different fluid flows is carried out but without the help of the causing parameters. Fluid kinematics can be understood properly with the help of the following points.

- **Fluid Kinematics** deals with the motion of fluids, such as displacement, velocity, acceleration, and other aspects. This topic is useful regarding the exam and the candidate's knowledge.
- Kinematics is the branch of classical mechanics that describes the motion of bodies and systems without consideration of the forces that cause the motion.

Types of Fluid Flows

Fluid flow can be classified into different categories based on various parameters. These parameters consist of the nature of the flow, flow patterns etc. Fluid flow may be classified under the following headings;

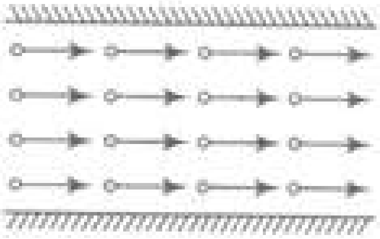
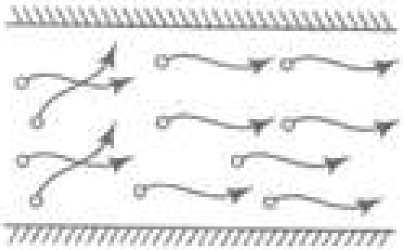
Steady & Unsteady Flow

Steady flow	Unsteady flow
The flow in which characteristics of fluid like velocity, pressure, density etc., at a point, do not change with time is called as steady flow. $\frac{\partial v}{\partial t}=0, \frac{\partial p}{\partial t}=0, \frac{\partial p}{\partial t}=0$	If velocity pressure and density changes with time then flow is unsteady flow. $\frac{\partial v}{\partial t}\neq 0, \frac{\partial p}{\partial t}\neq 0, \frac{\partial p}{\partial t}\neq 0$

Uniform & Non-uniform Flow

Uniform Flow	Non-uniform Flow
The flow in which velocity at any given time does not change with respect to distance. $\left(\frac{\partial v}{\partial s}\right)_{t=c} = 0$	In this flow, velocity at any given time changes with respect to distance. $\left(\frac{\partial v}{\partial s}\right)_{t=c} \neq 0$

Laminar & Turbulent Flow

Laminar Flow	Turbulent Flow
The flow in which the adjacent layers do not cross each other and move along well defined path. 	The flow in which adjacent layers cross each other and do not move along well defined path. 

Rotational & Irrotational Flow

Rotational Flow	Irrotational Flow
If the fluid particles flowing alone stream lines, also rotate about their own axes, then flow is rotational.	If fluid particles do not rotate about their own axes, then flow is irrotational.

Combining these, the most common flow types in Fluid Kinematics are:

- **Steady uniform flow**
  - Conditions do not change with position in the stream or with time.
  - E.g. flow of water in a pipe of constant diameter at a constant velocity.
- **Steady non-uniform flow**
  - Conditions change from point to point in the stream but do not change with time.
  - E.g. Flow in a tapering pipe with the constant velocity at the inlet.
- **Unsteady uniform flow**
  - At a given instant, the conditions at every point are the same but will change with time.
  - E.g. A pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.
- **Unsteady non-uniform flow**
  - Every flow condition may change from point to point and with time at every point.
  - E.g. Waves in a channel

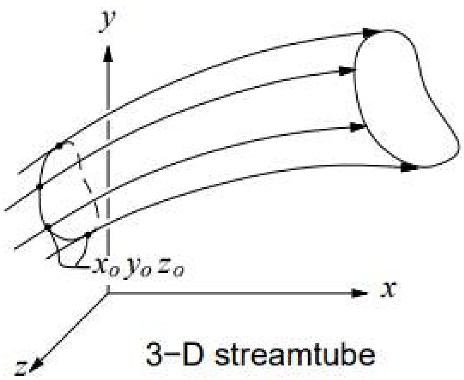
Different Flow Pattern

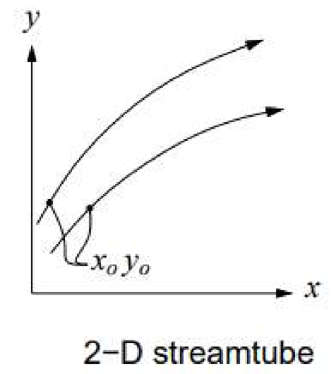
Three fluid element trajectories are defined: **Streamlines, Pathlines,** and **Streaklines**. These flow patterns depend on the motion of the fluid particle. It is explained below:

- **Pathline** is the actual path traveled by an individual fluid particle over some time period. The pathline of a fluid element A is simply the path it takes through space as a function of time. An example of a pathline is the trajectory taken by one puff of smoke carried by the steady or unsteady wind.
- **A streamline** is a line that is everywhere tangent to the velocity field. Streamlines are obtained analytically by integrating the equations defining lines tangent to the velocity field. i.e. dy/dx = v/u

where u,v, and w are the velocity components in the x, y and z directions.

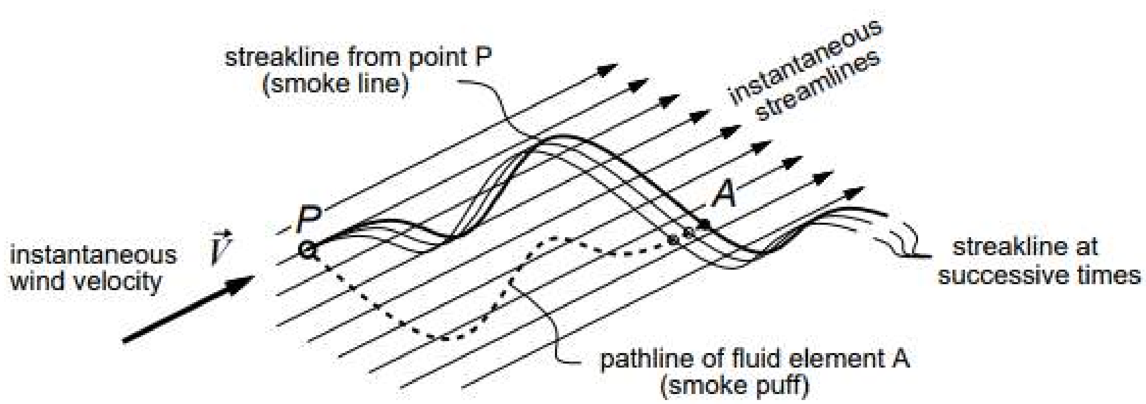
- **Streakline** is the locus of particles earlier passed through a prescribed point.
  A streakline is associated with a particular point P in space with fluid moving past it. All points which pass through this point are said to form the streakline of point P. An example of a streakline is the continuous line of smoke emitted by a chimney at point P, which will have some curved shape if the wind has a time-varying direction
- **Streamtube**: The streamlines passing through all these points form the surface of a stream-tube. Because there is no flow across the surface, each cross-section of the stream tube carries the same mass flow. So the stream tube is equivalent to a channel flow embedded in the rest of the flow field.





Note:

- The figure below illustrates **streamlines, pathlines**, and **streaklines** for the case of smoke being continuously emitted by a chimney at point P in the presence of shifting wind.
- In a steady flow, streamlines, pathlines, and streaklines all coincide.
- In this example, they would all be marked by the smoke line.



Velocity of a Fluid Particle

The velocity of a fluid particle is the distance moved by the fluid particle in units of time. The velocity of the fluid flow can be different in all directions. It is represented as u, v and w in x, y and z directions, respectively.

- The velocity of the fluid along any direction can be defined as the rate of change of displacement of the fluid along that direction.
- Let V be the resultant velocity of the fluid along any direction, and u, v and w be the velocity components in the x, y and z directions, respectively.
- Mathematically the velocity components can be written as

u = f (x, y, z, t)

w = f (x, y, z, t)

v = f (x, y, z, t)

- Let V_R is the resultant velocity at any point in a fluid flow.
- Resultant velocity $V_R = ui + vj + wk$

$V_R = \sqrt{(u^2 + v^2 + w^2)}$

Where u=dx/dt, v=dy/dt and w=dz/dt are the resultant vectors in X, Y and Z directions, respectively.

Acceleration of Fluid Particle

The acceleration of a fluid particle is the change of velocity of the fluid particle with respect to time. Acceleration can be classified into temporal acceleration and convective acceleration.

- Acceleration of a fluid element along any direction can be defined as the rate of change of velocity of the fluid along that direction.
- If a_x, a_y and a_z are the acceleration components along the x, y and z directions, it can be mathematically written as $a_x = du/dt$.

Similarly

$$a_y = \frac{\partial v}{\partial x}\frac{dx}{dt} + \frac{\partial v}{\partial y}\frac{dy}{dt} + \frac{\partial v}{\partial z}\frac{dz}{dt} + \frac{\partial v}{\partial t}$$

and $$a_z = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt} + \frac{\partial w}{\partial t}$$

But $\boldsymbol{u} = (dx/dt)$, $\mathbf{v} = (dy/dt)$ and $\mathbf{w} = (dz/dt)$.

Hence

$$\left.\begin{aligned} a_x &= \overbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}^{\text{Convective accln}} + \overbrace{\frac{\partial u}{\partial t}}^{\text{Local accln}} \\ a_y &= u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \\ a_z &= u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} \end{aligned}\right\} \text{Total accln}$$

If A is the resultant acceleration vector, it is given by

$$A = a_x i + a_y j + a_z k$$
$$= \sqrt{a_x^2 + a_y^2 + a_z^2}$$

For steady flow, the local acceleration will be zero

What is Stream Function?

The stream function is a valid mathematical function used to describe the fluid flow in the two dimensions flow. It will be explained below.

- The partial derivative of the stream function to any direction gives the velocity component at right angles to that direction. It is denoted by ψ.

$\partial\Psi/\partial x = v$, $\partial\Psi/\partial y = -u$

- The continuity equation for two-dimensional flow is

$(\partial^2\Psi/\partial y.\partial x + \partial^2\Psi/\partial x.\partial y) = 0$

Equations of Rotational Flow

Equations of rotational flow are used to describe the rotational characteristics of the flow. With the help of rotational velocity ω, the rotational characteristics of a flow can be categorized.

- As ψ satisfies the continuity equation, if ψ exists, it is a possible case of fluid flow.

- Rotational components of fluid particles are:

$\omega_z = 1/2(\partial w/\partial y - \partial v/\partial z)$

$\omega_z = 1/2(\partial u/\partial z - \partial w/\partial x)$

$\omega_z = 1/2(\partial v/\partial x - \partial u/\partial x)$

Equation of Irrotational Flow

- If $\boldsymbol{\omega_x = \omega_y = \omega_z}$, then the flow is irrotational.
- For irrotational flow, $\omega_z = 0$

$\omega_z = 1/2(\partial v/\partial x - \partial u/\partial y) = 1/2(\partial(-\partial\Psi/\partial x)/\partial x - \partial(-\partial\Psi/\partial y)/\partial y) = 1/2(\partial^2\Psi/\partial x^2 - \partial^2\Psi/\partial y^2)$

- This is the **Laplace equation** for ψ.

Note: It can be concluded that if stream function (ψ) exits, it is a possible case of fluid flow. But we can't decide whether the flow is rotational or irrotational. But if stream function ψ satisfies the Laplace equation, it is a possible case of irrotational flow. Otherwise, it is a rotational flow.

Velocity Potential Function

The velocity potential function is a mathematical expression used to represent the flow characteristics of the fluid. It can represent the flow characteristics of a 3D flow. Here different characteristics of the fluid flow are described in detail.

- It is a scalar function of space and time such that its negative derivative to any direction gives the fluid velocity in that direction. It is denoted b φ

$-\partial\Phi/\partial x = u, \partial\Phi/\partial y = -v, \partial\Phi/\partial z = -w$

We know that the continuity equation for steady flow is

$\partial u/\partial x + \partial v/\partial y = 0$

$\partial(-\partial\Phi/\partial x)/\partial x + \partial(-\partial\Phi/\partial y)/\partial y = 0$

$\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 = 0$

- If φ satisfies the Laplace equation, it is a possible case of fluid flow.

Rotational component (ω_z) can be given by

$\omega_z = 1/2(\partial v/\partial x - \partial u/\partial y) = 1/2(\partial(-\partial\Phi/\partial y)/\partial x - \partial(-\partial\Phi/\partial x)/\partial y) = 1/2(\partial^2\Phi/\partial y.\partial x - \partial^2\Phi/\partial x.\partial y)$

- It shows that φ exits. Then, the flow will be irrotational.

Relation between Stream Function and Velocity Potential Function

The stream function and velocity potential function are the mathematical functions in fluid kinematics. These terms are used to explain the different flow characteristics. Stream functions represent the two-dimensional flow, and the velocity potential function can also be used in the three-dimensional flow. These terms are explained below.

The velocity potential function can be written as

dΦ/dx = -u

dΦ/dy = -v

and stream function can be written as

dψ/dx = v

dψ/dy = -u

So, dψ/dx = dΦ/dy

dΦ/dx = dψ/dy

Stream versus Velocity Potential Function

Stream Function	Velocity Potential
$\frac{\partial \psi}{\partial x} = v$ $\frac{\partial \psi}{\partial y} = -u$ If Ψ exists then, a possible case of fluid flow. If Ψ satisfies Laplace equation then, flow is irrotational.	$-\frac{\partial \phi}{\partial x} = u$ $-\frac{\partial \phi}{\partial y} = v$ If φ exists then, flow is irrotational. If φ satisfies the Laplace equation then, a possible case of fluid flow.

Equipotential Line versus StreamLine

Equipotential Line	Stream Line
It is the line along which velocity potential φ is constant. $d\phi = \frac{d\phi}{dx} \cdot dx + \frac{\partial\phi}{\partial y} \cdot dy = 0$ $\Rightarrow \quad -u.dx - v.dy = 0$ $\Rightarrow \quad \frac{dy}{dx} = \frac{-u}{v}$	It is the line along which stream function Ψ is constant. $d\psi = \frac{\partial\psi}{\partial x} \cdot dx + \frac{\partial\psi}{\partial y} \cdot dy = 0$ $\Rightarrow \quad -v\ dx - u\ dy = 0$ $\Rightarrow \quad \frac{dy}{dx} = \frac{v}{u}$