BYJU'S
EXAM PREP

GATE/ESE

Civil Engineering

Engineering Mechanics

Important Formula Notes

IMPORTANT FORMULAS ON ENGINEERING MECHANICS

CHAPTER-1- INTRODUCTION & SYSTEM OF FORCES

1. **FORCE**

   An agent which produces or tends to produce, destroy or tends to destroy motion. It is a push or a pull.

   SI Unit: Newton (N)

   a) System of forces

   When two or more than two forces of different magnitude and direction act upon a body, they constitute a system of forces.

   i. Concurrent forces

   Two or more forces that act at the same point are called concurrent forces. Concurrent forces need not have the same direction. They act at the same point.

   ii. Collinear forces

   If concurrent forces have the same direction, they are collinear forces.

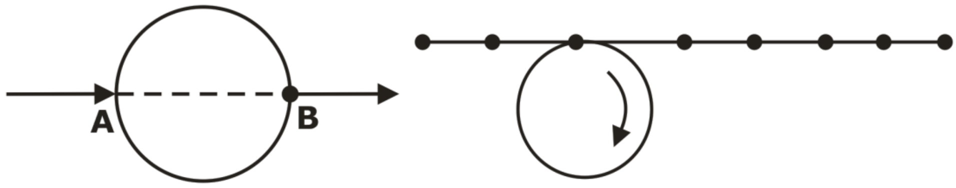
   iii. Coplanar forces

   Two or more forces whose directed arrows lie in the same plane are called coplanar forces.

- Since two concurrent forces always lie in a common plane, they are always coplanar.
- Three or more concurrent forces are not necessarily coplanar.

2. **PRINCIPLE OF TRANSMISSIBILITY**

- The state of rest or motion of a rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the applied forces.



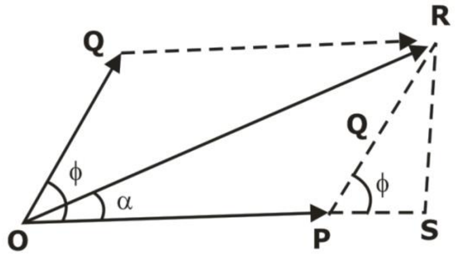
3. **RESULTANT OF FORCE**

- It is possible to find a single force that produces the same effect as many forces acting on a body.
- The single force is called resultant force, and the process of finding out the resultant force is called the composition of forces.
- The reverse of the composition of forces is called the resolution of force.

4. **PARALLELOGRAM LAW OF FORCES**

- If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram, which passes through their point of intersection, their resultant force is represented, both in magnitude and in direction, by the diagonal of the parallelogram drawn through their point of intersection.

- Let two forces, P and Q, be represented by OP and OQ, respectively, on two sides of the parallelogram.
- Now the parallelogram OPRQ is completed, and diagonal OR represents the resultant.



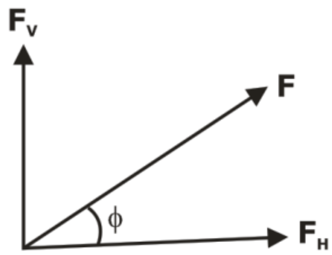
$$R = \sqrt{\left(P^2 + Q^2 + 2PQ\cos\phi\right)}$$

$$\tan\alpha = \frac{Q\sin\phi}{P + Q\cos\phi}$$

a. ANALYTICAL METHOD

ΣH = algebraic sum of all horizontal components.

ΣV = algebraic sum of all vertical components.



$$R^2 = \sqrt{\sum H^2 + \sum V^2}$$

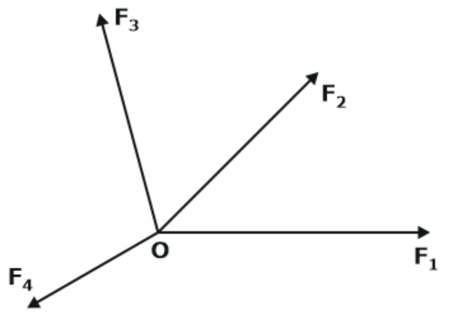
$$\tan\phi = \frac{\sum V}{\sum H}$$

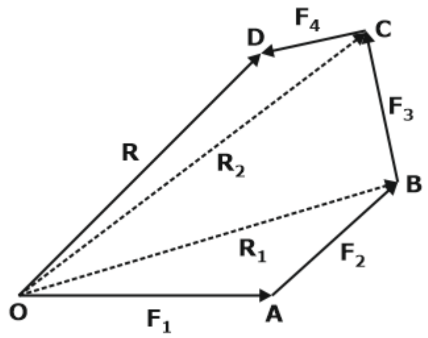
5. TRIANGLE LAW OF FORCES

Suppose two forces acting simultaneously on a body are represented in magnitude and direction by two sides of a triangle in order. In that case, the third side will represent the results of the two forces in the direction and magnitude taken in the opposite order.

6. POLYGON LAW OF FORCES

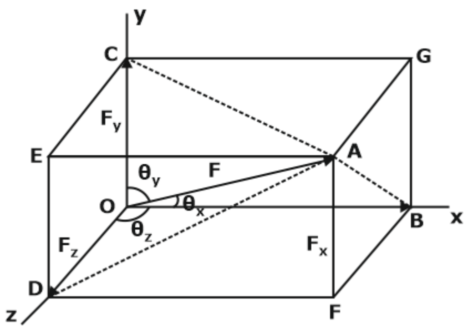
- When the forces acting on a body are more than two, the triangle law can be extended to polygon law. Polygon law states that if many coplanar concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in order, their resultant can be represented by the closing side of the polygon in magnitude and direction in the opposite order.
- Let forces F_1, F_2, F_3 and F_4 act at a point O as shown in the figure and can be represented by sides of polygon OA, AB, BC, and CD, respectively.
- Vector OD represents the resultant force $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4$ in magnitude and direction.





RESOLUTION AND RESULTANT OF FORCE IN SPACE

- Consider a force **F** acting at the origin O of the System of rectangular coordinates x, y, z as shown in Fig. Three angles define the direction of force F θ_x, θ_y , and θ_z .
- Let F_x, F_y , and F_z be the components of force F in x, y, and zdirections, respectively.



- From the triangle OAB $\cos\theta_x = \frac{OB}{OA} = \frac{F_x}{F}$, component of **F** along the x-direction $F_x = F\cos\theta_x$
- From the triangle OAC $\cos\theta_y = \frac{OC}{OA} = \frac{F_y}{F}$, component of **F** along the y-direction $F_y = F\cos\theta_x$
- From the triangle OAD $\cos\theta_x = \frac{OC}{OA} = \frac{F_z}{F}$,component of **F** along the z-direction $F_z = F\cos\theta_z$
- The cosines of θ_x, θ_y and θ_z are known as the direction cosine of the force **F**.
- The angles θ_x θ_y and θ_z are not independent. They are related as

  $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$.
- **F** in the vector form $\mathbf{F} = F_x i + F_y j + F_x k$
- The magnitude of force **F** is $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$
- Direction of force $\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right), \theta_y = \cos^{-1}\left(\frac{F_y}{F}\right)$, and $\theta_z = \cos^{-1}\left(\frac{F_z}{F}\right)$

7. MOMENTS

- It is the turning effect produced by force on the body on which it acts.
- SI Unit: Nm
- Moment M = F x L

- Where F = Force acting on the body
- L = Perpendicular distance of the point, about which the moment is determined and the line of action of the force.

Force F and position vector r in rectangular components may be written as

$$\mathbf{F} = F_x i + F_y j + F_z k$$

$$\mathbf{r} = xi + yj + zk$$

Thus,

$$\mathbf{M}_o = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} i & j & k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$M_o = i\left(yF_z - zF_y\right) - \left(xF_z - zF_x\right)j + \left(xF_y - yF_x\right)k$$

- In case of problems involving only two dimensions, the force F may be assumed to lie in the xy-plane. Carrying $z = 0$ and $F_z = 0$, we obtain

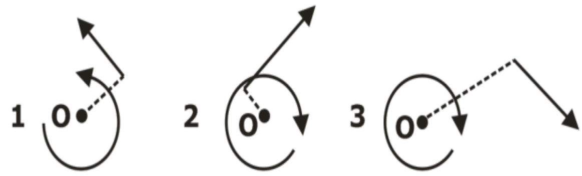
$$\mathbf{M}_o = \left(xF_y - yF_x\right)k$$

Note.

8. Just as the force tends to translate the body, the moment tends to rotate the body about the point.

9. DETERMINATION OF DIRECTIONS

- Assume perpendicular distance as the hand of a clock.
- Keep the point (one end of perpendicular distance), about which the moment is to be determined, hinged like the hinged end of hands of a clock.
- Move the other end of perpendicular distance in the direction of action of force.



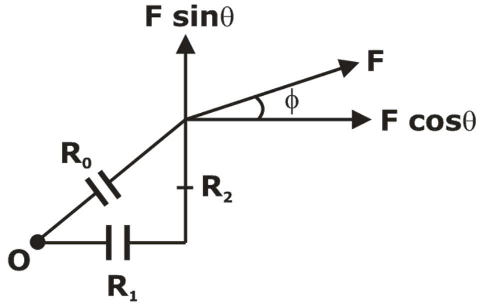
1. Anticlockwise
2. Clockwise
3. Clockwise

Note.

- **Clockwise,** whose effect is to turn, in the same direction in which the clock's hands move.
- **Anticlockwise,** whose effect is to turn, in the opposite direction in which the clock's hands move.

10. VARIGNON'S THEOREM

- The moment of a force about any point is equal to the algebraic sum of the moments of the components of that force about the same point.



$M_o = F\cos\theta \times R_2 - F\sin\theta \times R_1$

11.Couples

- The moment produced by two equal, opposite and non-collinear forces is called a couple.
- It does not produce any translation but produces only rotation.
- The resultant force of a couple is zero.
- The moment of a couple is the product of the magnitude of one of the forces and the perpendicular distance between their lines of action.

CHAPTER-2- EQUILIBRIUM OF RIGID BODY & FBD

1. CONDITIONS OF EQUILIBRIUM

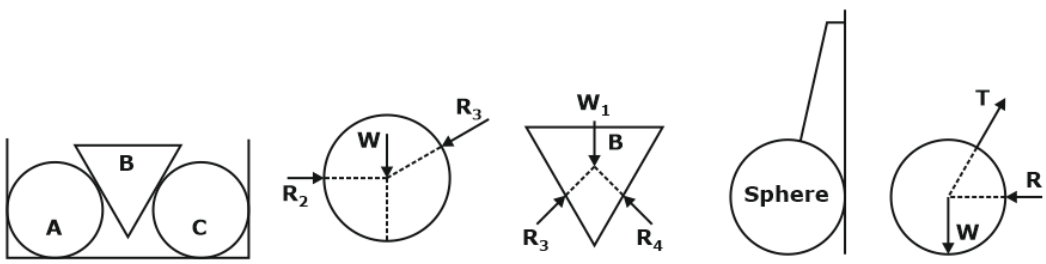
- The term equilibrium implies that either the body is at rest or it moves with a constant velocity.
- A body is said to be in static equilibrium when the resultant force must be zero, and the body must have no tendency to rotate.

i.e.$\Sigma F = 0$ means that $\Sigma H = 0$ and $\Sigma V = 0$

and $\Sigma M = 0$ about any point.

2. FREE BODY DIAGRAM

- A free-body diagram is a sketch of the body that shows the body (by itself, free of the other part of the system) and all the forces applied to it, i.e., all forces acting on the body. Fig shows free body diagrams.

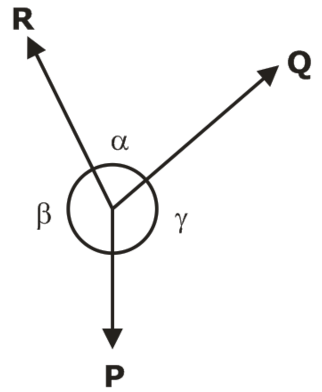


FBD of A FBD of B FBD of sphere

3. LAMI'S THEOREM

- If three coplanar forces acting at a point are in equilibrium, then each force is proportional to the smaller angle between the other two forces.

- Here P, Q & R are the three coplanar forces and α, β, γ are three angles. According to Lami's theorem $\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$



CHAPTER-3- INTERNAL FORCES IN A STRUCTURE

TRUSSES

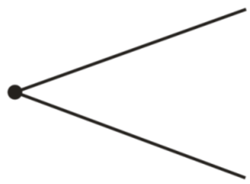
- A framework composed of straight members joined at their ends to form a structure is called a truss.

CLASSIFICATION OF TRUSS

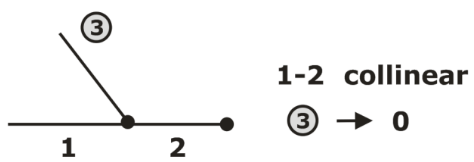
1) **EFFICIENT OR PERFECT TRUSS ,** $m = 2j-3$
- **DEFICIENT OR COLLAPSIBLE TRUSS,** $m < 2j-3$
2) **REDUNDANT TRUSS ,** $m > 2j-3$
- Where m is the number of members and j is the number of joints.

ZERO FORCE MEMBERS

- Zero force members in a truss are members which do not have any force in them. **Case 1:** At a two-member joint that is not parallel and there are no other external loads or reactions at the joint, both members are zero force members.



Case 2: At a three-member joint, if two of those members are parallel and there are no other external loads (or reaction) at the joint, then the not parallel member is a zero force member.



Methods of analysis of statically determinate trusses

i. Method of joint

$\Sigma F_x = 0$ and $\Sigma F_y = 0$

Analysis should start at joint having at least one known force and at most two unknown forces.

ii. Method of section

A section is cut such that it cuts maximum of those members in which forces are Unknown and subsequently force equilibrium and moment equilibrium equations are used to obtain the unknown forces.

CHAPTER-4- CENTRE OF MASS & MOMENT OF INERTIA

1. INTRODUCTION

- In engineering structures, the members of various cross-sections are used to withstand loads.
- The load-carrying capacity depends on the type of material used as also the cross-section of the members.

2. CENTROID

- Centroid is the point where the whole area is assumed to be concentrated.
- It is related to the shape or the geometry of the object.
- The centroid of line

$$\bar{x} = \frac{\int x dL}{L},\ \bar{y} = \frac{\int y dL}{L} \text{ and } \bar{z} = \frac{\int z dL}{L}.\ \text{where } \int dL = L$$

- The centroid of the area

$$\bar{x} = \frac{\int x dA}{\int dA},\quad \bar{y} = \frac{\int y dA}{\int dA},\quad \bar{z} = \frac{\int z dA}{\int dA},\ \text{where } \int dA = A$$

- The centroid of Volume

$$\bar{x} = \frac{\int x dV}{V},\ \bar{y} = \frac{\int y dV}{V} \text{ and } \bar{z} = \frac{\int z dV}{V} \quad \text{where } \int dV = V$$

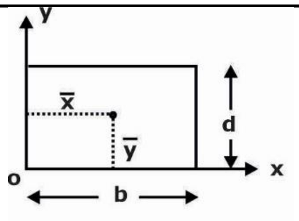
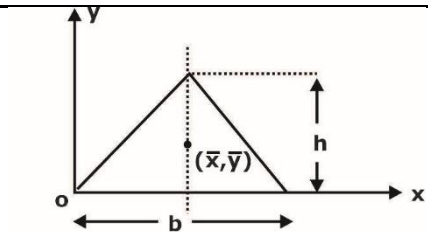
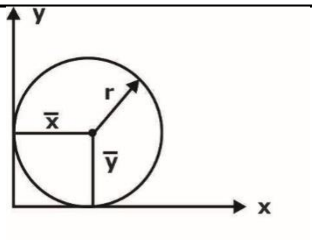
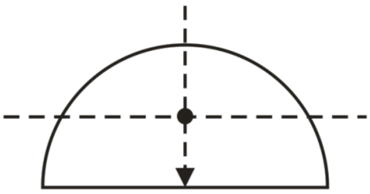
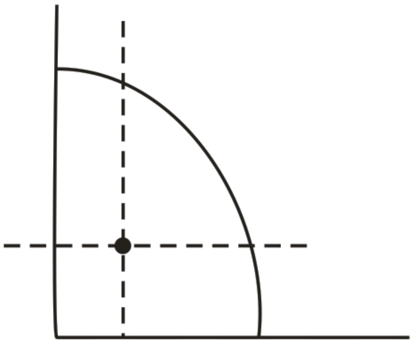
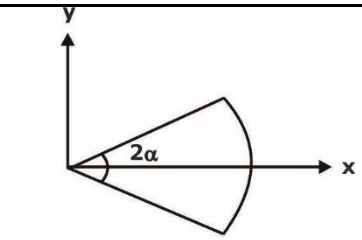
3. CENTRE OF GRAVITY

- The centre of gravity of the body or the system of particles rigidly connected together is that point where the body's weight is concentrated, and gravitational force acts through it.

$$\bar{x} = \frac{\int x dm}{\int dm},\quad \bar{y} = \frac{\int y dm}{\int dm},\quad \bar{z} = \frac{\int z dm}{\int dm},\ \text{where } \int dm = M$$

Note.

- When density is uniform, i.e. the body material is homogenous, centroid and C.G are the same, but two points will not be the same when density is not the same. C.G lies where density is more, i.e. weight is more.

SHAPE	FIGURE	$\bar{x}$	$\bar{y}$
Rectangle		$\frac{b}{2}$	$\frac{d}{2}$
Triangle		$\frac{b}{2}$	$\frac{h}{3}$
Circle		r	r
Semi-Circle		0(taking centre as origin)	$\frac{4R}{3\pi}$
Quarter of Circle		$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$
The sector of a Circle		$\frac{2R}{3\alpha}\sin\alpha$	0

4. AREA MOMENT OF INERTIA

- Moment of Inertia for an area: - This is also known as the second moment of area.

$I_{xx} = \int y^2 dA$ = second moment of area about x-axis

$I_{yy} = \int x^2 dA$ = second moment of area about the y-axis.

5. THEOREM OF PARALLEL AXIS

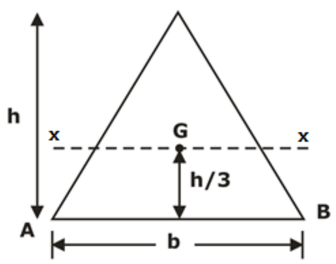
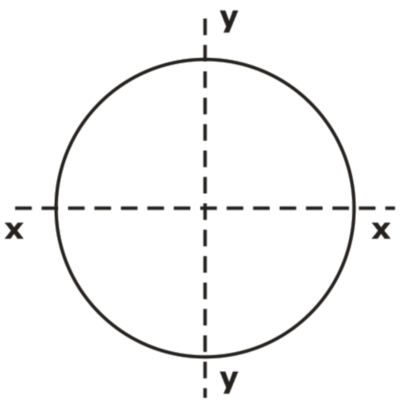
- The moment of inertia about any axis parallel to the centroidal axis at a distance (h) is equal to the sum of moment of Inertia about the centroidal axis and product of area and square of distance B/w two axis.

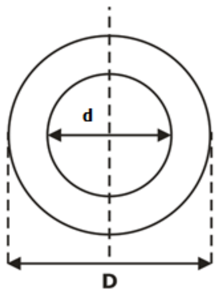
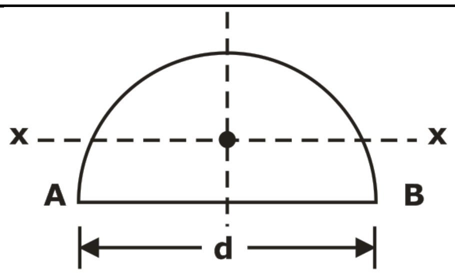
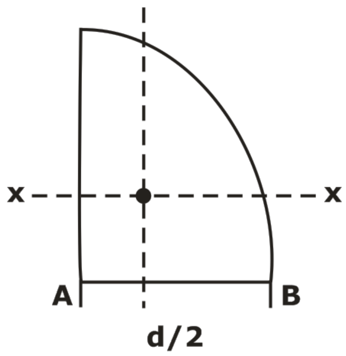
$$I_{parallelAxis} = I_{Centroid} + Ah^2$$

6. THEOREM OF PERPENDICULAR AXIS

- If the moment of Inertia about two perpendicular centroidal axis in a plane are given (I_1, I_2), then the moment of Inertial about the third axis, which is perpendicular to both mutually perpendicular axis passing through the point of intersection, is equal to the sum of moment of Inertia about two centroidal axis.

$$I_{zz} = I_{xx} + I_{yy}$$

SHAPE	AXIS	Moment of Inertia
	x-x AB	$\frac{bh^3}{36}$ $\frac{bh^3}{12}$
	x-x y-y	$\frac{\pi}{64}D^4$ $\frac{\pi}{64}D^4$

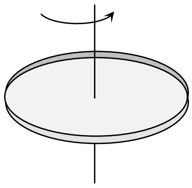
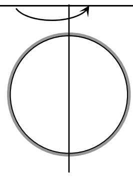
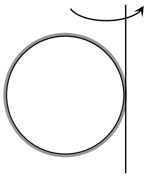
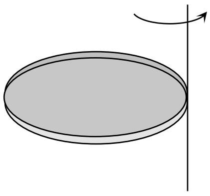
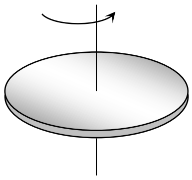
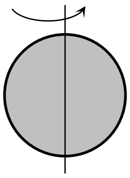
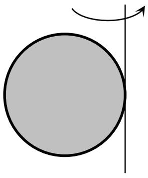
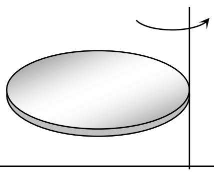
	x-x y-y	$\frac{\pi}{64}(D^4 - d^4)$ $\frac{\pi}{64}(D^4 - d^4)$
	x-x AB	$0.0068598\ d^4$ $\frac{\pi d^4}{128}$
	x-x AB	$0.00343\ d^4$ $\frac{\pi d^4}{256}$

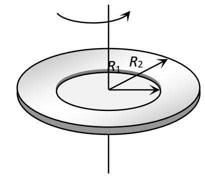
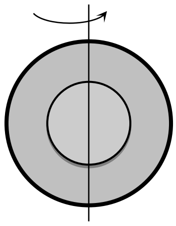
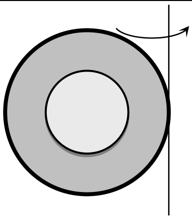
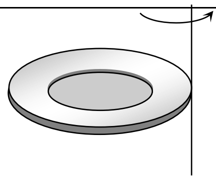
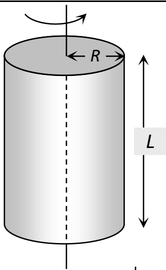
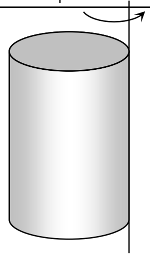
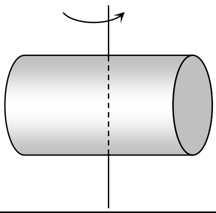
7. MASS MOMENT OF INERTIA

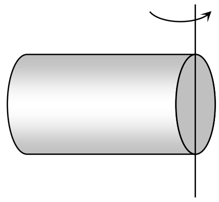
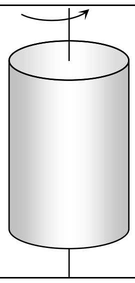
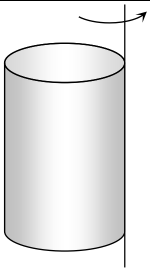
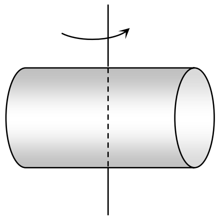
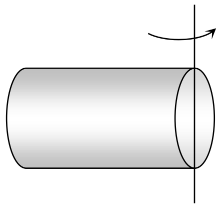
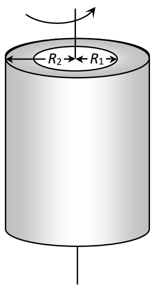
- Moment of the moment of mass is called the mass moment of inertia.
- It is also called the second moment of mass.
- The theorem of parallel axis and perpendicular axis also applies in it.

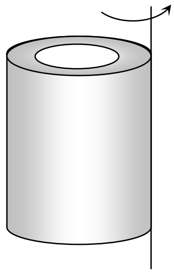
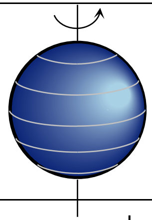
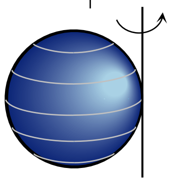
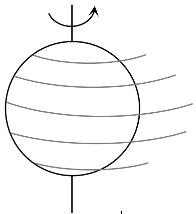
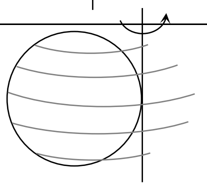
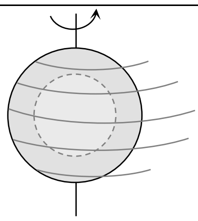
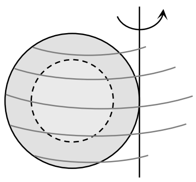
8. RADIUS OF GYRATION

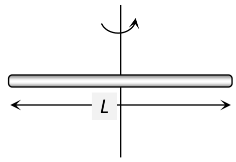
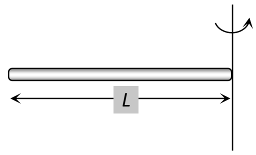
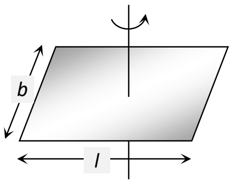
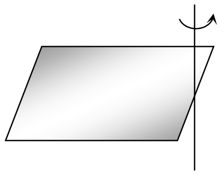
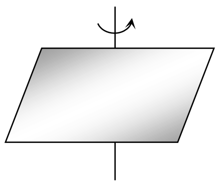
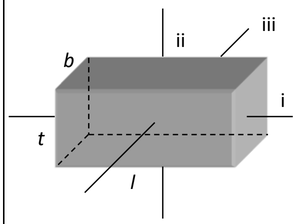
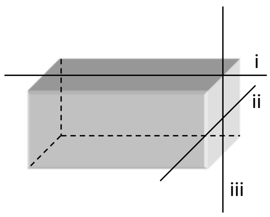
- The radius of gyration k describes how the area of a cross-section is distributed around its centroidal axis.
- If the area is concentrated far from the centroidal axis, it will have a greater value of k and a greater resistance to buckling (or bending).
- The radius of gyration is defined as: $k = \sqrt{\frac{I}{A}}$, where k is the radius of gyration, I is the Moment of Inertia, and Ais the cross-section area.

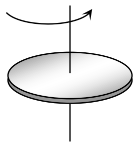
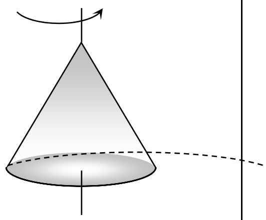
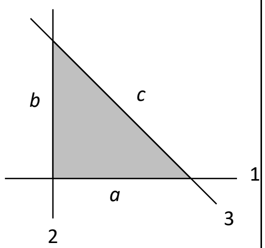
Body	Axis of Rotation	Figure	Moment of inertia	***k***	***k²/R²***
Ring	About an axis passing through C.G. and perpendicular to its plane		MR^2	R	1
Ring	About its diameter		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Ring	About a tangential axis in its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Ring	About a tangential axis perpendicular to its own plane		$2MR^2$	$\sqrt{2}R$	2
Disc	About an axis passing through C.G. and perpendicular to its plane		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Disc	About its Diameter		$\frac{1}{4}MR^2$	$\frac{R}{2}$	$\frac{1}{4}$
Disc	About a tangential axis in its own plane		$\frac{5}{4}MR^2$	$\frac{\sqrt{5}}{2}R$	$\frac{5}{4}$
Disc	About a tangential axis perpendicular to its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Annular disc inner radius = R_1 and outer radius = R_2	Passing through the center and perpendicular to the plane		$\frac{M}{2}[R_1^2 + R_2^2]$	–	–
Annular disc	Diameter		$\frac{M}{4}[R_1^2 + R_2^2]$	–	–
Annular disc	Tangential and Parallel to the diameter		$\frac{M}{4}[5R_1^2 + R_2^2]$	–	–
Annular disc	Tangential and perpendicular to the plane		$\frac{M}{2}[3R_1^2 + R_2^2]$	–	–
Solid cylinder	About its own axis		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Solid cylinder	Tangential (Generator)		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Solid cylinder	About an axis passing through its C.G. and perpendicular to its own axis		$M\left[\frac{L^2}{12} + \frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{12} + \frac{R^2}{4}}$	-

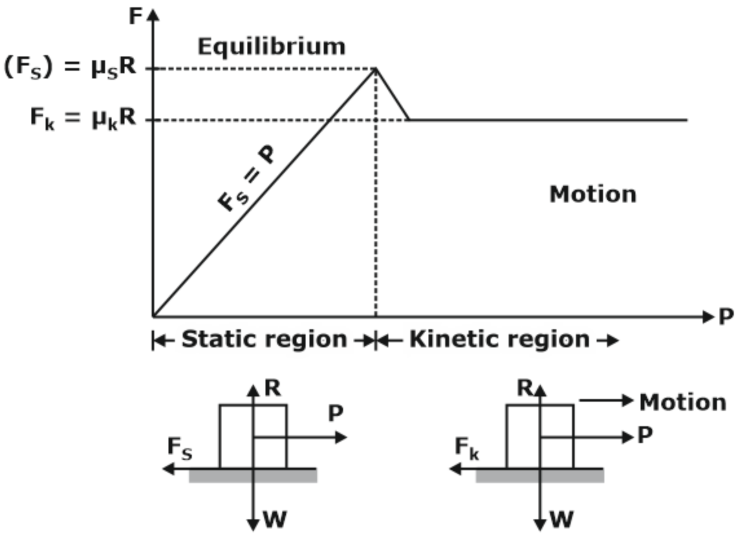
Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Solid cylinder	About the diameter of one of faces of the cylinder		$M\left[\frac{L^2}{3}+\frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{3}+\frac{R^2}{4}}$	-
Cylindrical shell	About its own axis		MR^2	R	1
Cylindrical shell	Tangential (Generator)		$2MR^2$	$\sqrt{2}R$	2
Cylindrical shell	About an axis passing through its C.G. and perpendicular to its own axis		$M\left[\frac{L^2}{12}+\frac{R^2}{2}\right]$	$\sqrt{\frac{L^2}{12}+\frac{R^2}{2}}$	-
Cylindrical shell	About the diameter of one of faces of the cylinder		$M\left[\frac{L^2}{3}+\frac{R^2}{2}\right]$	$\sqrt{\frac{L^2}{3}+\frac{R^2}{2}}$	
Hollow cylinder with inner radius = R_1 and outer radius = R_2	Axis of cylinder		$\frac{M}{2}(R_1^2+R_2^2)$	-	-

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Hollow cylinder with inner radius = R_1 and outer radius = R_2	Tangential		$\frac{M}{2}(R_1^2+3R_2^2)$	-	-
Solid Sphere	About its diametric axis		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$	$\frac{2}{5}$
Solid sphere	About a tangential axis		$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$	$\frac{7}{5}$
Spherical shell	About its diametric axis		$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$	$\frac{2}{3}$
Spherical shell	About a tangential axis		$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$	$\frac{5}{3}$
Hollow sphere of inner radius R_1 and outer radius R_2	About its diametric axis		$\frac{2}{5}M\left[\frac{R_2^5-R_1^5}{R_2^3-R_1^3}\right]$	-	-
Hollow sphere	Tangential		$\frac{2M[R_2^5-R_1^5]}{5(R_2^3-R_1^3)}+MR_2^2$	-	-

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Long thin rod	About on axis passing through its center of mass and perpendicular to the rod.		$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$	-
Long thin rod	About an axis passing through its edge and perpendicular to the rod		$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$	-
Rectangular lamina of length *l* and breadth *b*	Passing through the center of mass and perpendicular to the plane		$\frac{M}{12}[l^2 + b^2]$	-	-
Rectangular lamina	Tangential perpendicular to the plane and at the mid-point of breadth		$\frac{M}{12}[4l^2 + b^2]$	-	-
Rectangular lamina	Tangential perpendicular to the plane and at the mid-point of length		$\frac{M}{12}[l^2 + 4b^2]$	-	-
Rectangular parallelepiped length *l*, breadth *b*, thickness *t*	Passing through center of mass and parallel to (i) Length (x) (ii) breadth (z) (iii) thickness (y)		(i) $\frac{M[b^2 + t^2]}{12}$ (ii) $\frac{M[l^2 + t^2]}{12}$ (iii) $\frac{M[b^2 + l^2]}{12}$	-	-
Rectangular parallelepiped length *l*, breath *b*, thickness *t*	Tangential and parallel to (i) length (x) (ii) breadth (y) (iii) thickness(z)		(i) $\frac{M}{12}[3l^2 + b^2 + t^2]$ (ii) $\frac{M}{12}[l^2 + 3b^2 + t^2]$ (iii) $\frac{M}{12}[l^2 + b^2 + 3t^2]$	-	-

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Elliptical disc of semimajor axis = a and semi minor axis = b	Passing through CM and perpendicular to the plane		$\frac{M}{4}[a^2+b^2]$	-	-
Solid cone of radius R and height h	Axis joining the vertex and center of the base		$\frac{3}{10}MR^2$	$\sqrt{\frac{3}{10}}R$	$\frac{3}{10}$
Equilateral triangular lamina with side a	Passing through CM and perpendicular to the plane		$\frac{Ma^2}{6}$	$\frac{a}{\sqrt{6}}$	-
Right angled triangular lamina of sides a, b, c	Along the edges		(1) $\frac{Mb^2}{6}$ (2) $\frac{Ma^2}{6}$ (3) $\frac{M}{6}\left[\frac{a^2b^2}{a^2+b^2}\right]$	-	-

CHAPTER-5- FRICTION & ITS APPLICATIONS



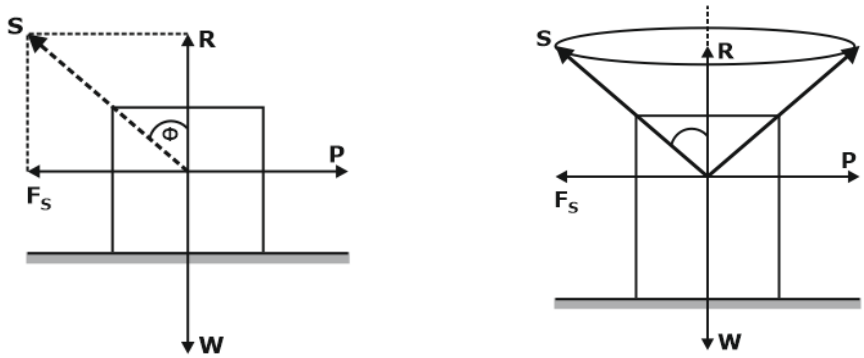
- The graph between the applied load P and the friction force F is linear until limiting friction is reached.

- The force of static friction between any two surfaces in contact is opposite the applied force and can have values given by $F_s \le \mu_s R$, where μ_s is called the coefficient of static friction.
- When the block is on the verge of slipping, the force of static friction is given by $F_s = (F_s)_{max} = \mu_s R$
- The force of kinetic friction acting on an object is opposite to the object's direction of motion and is given by $F_k = \mu_k R$, where μ_k is a constantly called coefficient of kinetic friction.
- The values of coefficients of friction μ_s and μ_k do not depend upon the area of the surfaces in contact.
- However, both the coefficients depend strongly on the nature of the surface in contact,i.e. roughness/smoothness.
- μ_k is generally less than μ_s

ANGLE OF FRICTION

- Angle θ, which resultant S subtends with the normal plane when the body just starts sliding over the horizontal plane.
- This is also known as the limiting angle of reaction or friction.
- It is sometimes found convenient to replace the normal force R and the friction force F_s by their resultant S. From the geometry of Fig, and we note that

$$\tan\phi = \frac{F_s}{R} = \frac{\mu R}{R} = \mu$$

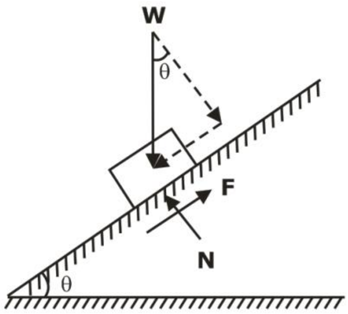


- limiting angle of friction $\phi = \tan^{-1}\mu$
- This indicates that the friction coefficient is tangent tothe angle of friction.
- We know F_s vary from 0 to limiting value, So if F_s form a circle, then resultant S will describe a right circular cone of apex angle 2ϕ about the line of action Ras its axis, then this cone is called a **Cone of friction**.
- The body will be stationary if the resultant is within the cone of friction.

ANGLE OF REPOSE

- The maximum inclination of the plane on which a body, free from external forces, experiences repose (sleep) is called the Angle of Repose.

- Applying the Equilibrium equation to the block shown above, we get $\mu = \tan\theta$
- In terms of angle of friction φ, $\theta = \phi$



The direction of friction on DRIVING AND DRIVEN WHEELS OF AN AUTOMOBILE

- On a smooth surface driving wheels will rotate about the axis of the axle, but the vehicle will not move.
- Frictional torque on driving wheels is overcome by engine torque,i.e. frictional torque direction opposite to engine torque. It provides driving force or tractive force to the vehicle.

$$0 \le (F = T / r) \le F_s$$

Where F_s is limiting friction, T is engine torque, and r is the wheel's radius.

- Friction on driven wheels provides motion to driven wheels,i.e. friction direction is along with the wheel's motion.

FRICTION IN WHEELS

WHEELS BEING PULLED BY A FORCE

- Say a wheel of radius R is being pulled by a force Power a rough surface, as shown in Fig. If the coefficient of friction between wheel and surface is μ, the frictional force $F = \mu N$,

N=normal reaction

Turning moment, $T = FR = \mu WR$

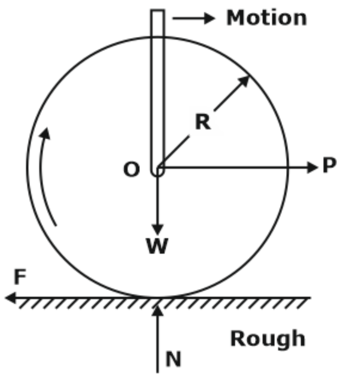


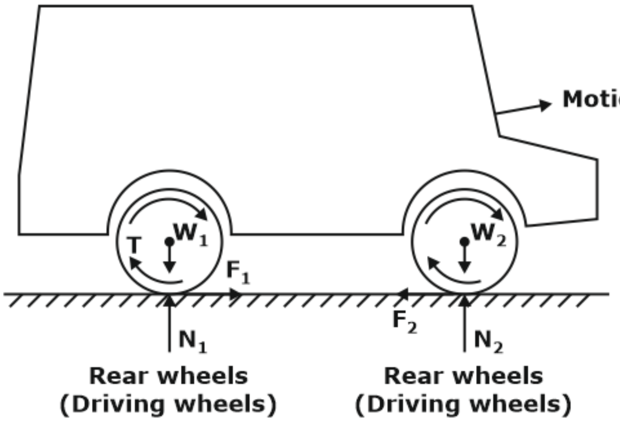
Fig:13

- If the surface is smooth $\mu = 0, T = 0$, the wheel will not rotate.
- The wheel moves in the clockwise direction, and the wheel moves towards the right.
- The magnitude of frictional force F will be maximum equal to μN.

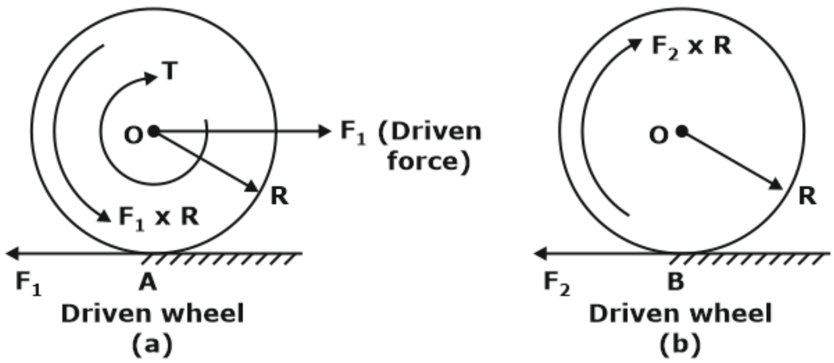
- Force P applied at axis O cannot provide a turning moment about axis O. Frictional force provides a clockwise turning moment.

DRIVING AND DRIVEN WHEELS OF AN AUTOMOBILE

- Consider a rear-wheel-drive vehicle as shown in Fig travelling towards the right as shown.
- Driving torque on rear wheels is T. The engine provides this torque through the propeller shaft.



However, the road is generally rough, and a frictional force



- F_1 acts on the outer surface of the wheel, if R is the radius of the wheel, then frictional torque $T_f = F_1 \times R$ (where F_1 becomes the propelling force on the vehicle)
- In the direction of motion. Friction force F_1 at contact point A anticlockwise couple replace a case $F_1 \times R$ and a force F_1 at axle axis O as shown. So this is the frictional force on the driving wheel responsible for producing motion in the vehicle.
- $F_2 < F_1$ The following points must be remembered in an engine-powered vehicle:

  (a) On a smooth surface, driving wheels will rotate about the axis of the axle, but the vehicle will not move.

  (b) Frictional torque on driving wheels is overcome by engine torque.

  (c) Friction on driven wheels provides motion to driven wheels.

  (d) Friction on driving wheels provides driving force or tractive force to the vehicle.

CHAPTER-6- FREE VIBRATIONS OF UNDAMPED SDOF SYSTEM

1.1. Fundamental of vibrations

a) **Free vibration –** Vibrations due to inherent forces of the system is called free vibration.

b) **Force vibration-** Vibrations caused due to external forces is called forced vibrations.

c) **Degree of freedom-** The number of independent coordinates required to solve the vibrating system is the degree of freedom.

1.2. Damping

- Damping is the ability of a building to disintegrate the energy of the earthquake ground shaking.

2. FUNDAMENTAL PERIOD OF VIBRATIONS

The following formula is used for vibrations

(a)$T = 0.075\ H^{0.75}$

H= Height of building above ground level

T= Fundamental period of vibration

(b)If the shear wall is provided

$$T = \frac{0.09H}{\sqrt{D}}$$

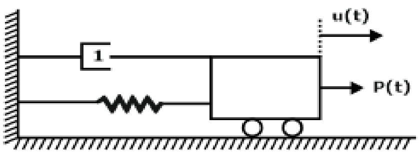
D= depth of the building

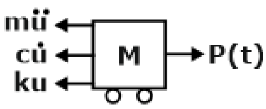
Note: The following relationship can be used to find the approximate fundamental period of vibrations

$$T = 0.1 \times N$$

N= number of storeys above ground level

3. EQUATION OF MOTION





$m\ddot{u}$: fictitious inertia force

$c\dot{u}$: damping force

Ku: spring force

u: displacement at any time t

$\dot{u}$: velocity at any time t

$\ddot{u}$: acceleration at any time t

P(t) : External force

The general equation of motion is,

P(t) = $m\ddot{u} + c\dot{u} + ku$

For undamped (c= 0) free vibration (P = 0) the above equation reduces to

$m\ddot{u} + ku = 0$

$$\Rightarrow \quad \ddot{u} + \frac{k}{m}u = 0$$

$$\text{or} \quad \ddot{u} + w_n^2\, u = 0$$

Where, $w_n^2 = k/m$

$w_n = \sqrt{k/m}$: Natural frequency of vibration

The solution of the above equation of motion is

$$u(t) = u(0)\cos w_n t + \frac{\vec{u}(0)}{w_n}\sin w_n t$$

where u(0) : initial displacement

$\dot{u}$ (0) : initial velocity

This motion repeats itself after every $\frac{2\pi}{w_n}$ seconds

Motion executed is simple harmonic as the restoring force on the particle is directly proportional to the displacement.

a) **Undamped natural circular frequency (ω_n)**

$$\omega_n = \sqrt{\frac{k}{m}}$$

k= spring constant

m= mass

b) Time period (T)

$$T = \frac{2\pi}{\omega_n}$$

c) Frequency

$$f = 1/T$$

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