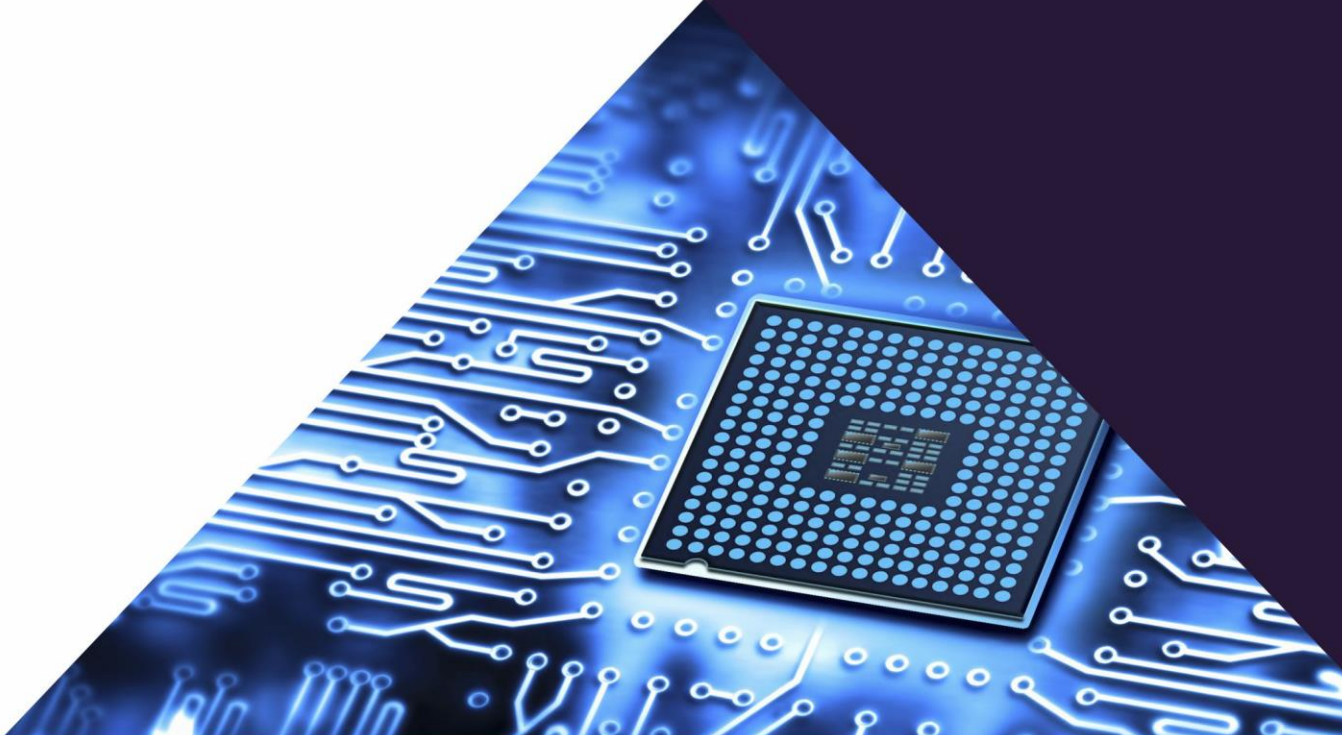


Electronics Engineering

Electromagnetics

SHORT NOTES



IMPORTANT FORMULAS TO REMEMBER

CHAPTER 1: VECTOR CALCULUS

1. Vector Quantity

A physical quantity which has both magnitude and definite direction is called a vector quantity. The various examples of vector quantity are force, velocity, displacement, electric field intensity, magnetic field intensity, acceleration etc.

1.1. Representation of a Vector

To distinguish between a scalar and a vector it is customary to represent a vector by a letter with an arrow on top of it, such as \vec{a} and \vec{b} , or by a letter in boldface type such as \bar{A} and \bar{B} .

1.2. Unit Vector

A unit vector consists both magnitude and direction. Its direction is same as that of the main vector however, its magnitude is unity. It can be written in various as i_A , \hat{a}_A or u_A . A unit vector is defined as the ratio of the main vector itself to its magnitude. For example, the unit

vector of A is given as $\alpha_A = \frac{\vec{A}}{|A|}$

Where $|A|$ is the magnitude of the vector and α_A is the unit vector of A.

2. Basic Vector Operations

2.1. Scaling of a Vector

When a vector is multiplied by a scalar it results in a vector quantity.

Consider a vector \vec{A} and a scalar k. The product \vec{R} of the two quantities is given as

$$\vec{R} = k\vec{A}$$

Following are some important properties of scaling operation:

Properties of scaling operation

1. Consider the scaling operation $\vec{R} = k\vec{A}$. The direction of \vec{R} is same as that of \vec{A} if k is positive, and opposite to that of A if k is negative.

2. In rectangular coordinates, assume that the scaling operation is given by

$$R_x\vec{a}_x + R_y\vec{a}_y + R_z\vec{a}_z = k(A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z)$$

The above equality is satisfied if each component of the LHS is equal to the corresponding component of RHS, i.e.

$$R_x = kA_x, R_y = kA_y, R_z = kA_z$$

The magnitude of R is

$$|R| = k\sqrt{A_x^2 + A_y^2 + A_z^2} = kA$$

3. Let k_1, k_2 be the scalars, and A, B be the vectors then,

$$(k_1+k_2)\vec{A} = k_1\vec{A} + k_2\vec{A}$$

2.2. Addition of Vectors

Consider the two vectors,

$$\vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z \text{ and}$$

$$\vec{B} = B_x \vec{a}_x + B_y \vec{a}_y + B_z \vec{a}_z$$

The addition of these two vectors is given by

$$\vec{A} + \vec{B} = (A_x + B_x) \vec{a}_x + (A_y + B_y) \vec{a}_y + (A_z + B_z) \vec{a}_z$$

Properties of Vector Addition

1. Vector addition follows the commutative law, i.e.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

2. Vector addition follows the associative law, i.e.

$$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$$

3. Similar to the vector addition, the subtraction of the vectors is defined as

$$\vec{A} - \vec{B} = (A_x - B_x) \vec{a}_x + (A_y - B_y) \vec{a}_y + (A_z - B_z) \vec{a}_z$$

$$k_1(\vec{A} + \vec{B}) = k_1 \vec{A} + k_1 \vec{B}$$

2.3. Multiplication of vectors

When two vectors A and B are multiplied, the result is either a scalar or a vector depending on how they are multiplied. There are two types of vector multiplication:

1. Scalar (or dot) product: $\vec{A} \cdot \vec{B}$

2. Vector (or cross) product: $\vec{A} \times \vec{B}$

2.3.1 Scalar Product

The dot product of the vectors A and B is defined as

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

Following are some important properties of dot product of two vectors.

Properties of Dot product

1. The dot product of two orthogonal vectors is always zero, i.e.

$$\vec{A} \cdot \vec{B} = 0, \text{ if } \theta = 90^\circ$$

2. The dot product of two parallel vectors is equal to the product of their magnitudes, i.e.

$$\vec{A} \cdot \vec{B} = AB, \text{ if } \theta = 0^\circ$$

3. In rectangular coordinate systems, the dot products of the unit vectors are given as

$$\vec{a}_x \cdot \vec{a}_y = \vec{a}_y \cdot \vec{a}_z = \vec{a}_z \cdot \vec{a}_x = 0$$

$$\vec{a}_x \cdot \vec{a}_x = \vec{a}_y \cdot \vec{a}_y = \vec{a}_z \cdot \vec{a}_z = 1$$

4. If the two vectors are defined in rectangular coordinates as

$$\vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z$$

$$\vec{B} = B_x \vec{a}_x + B_y \vec{a}_y + B_z \vec{a}_z$$

Then, their dot product is evaluated as

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

5. The dot product follows the commutative law, i.e.

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

6. The dot product also follows the distributive law, i.e.

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

2.3.2 Vector of Cross Product

The cross product of two vectors A and B is Defined as

$$\vec{A} \times \vec{B} = (AB \sin \theta) a_n$$

Where a_n is the unit vector normal to the plane containing A and B, θ is the angle between the vector A and B as shown in Figure 1.4 As there are the normal unit vector a_n we use the right- hand rule.

Right hand rule

Let your fingers point in the direction of the first vector and curl around (via the smaller angle) toward second; then your thumb indicates the direction of a_n . the cross-product $A \times B$ points upward.

Properties of cross product

1. The cross product of two orthogonal vectors is equal to the product of their magnitudes with the direction perpendicular to the plane, i.e.

$$\vec{A} \times \vec{B} = AB a_n, \quad \text{if, } \theta = 90^\circ$$

2. The cross product of two parallel vectors is always zero, i.e.

$$\vec{A} \times \vec{B} = 0, \quad \text{if } \theta = 0^\circ$$

3. In rectangular coordinate system, the cross product of the unit vectors are given as

$$a_x \times a_x = a_y \times a_y = a_z \times a_z = 0$$

$$a_x \times a_y = a_z$$

$$a_y \times a_z = a_x$$

$$a_z \times a_x = a_y$$

4. If the two vectors are defined in rectangular coordinates as

$$\vec{A} = A_x a_x + A_y a_y + A_z a_z$$

$$\vec{B} = B_x a_x + B_y a_y + B_z a_z$$

Then, their cross product is evaluated as

$$\vec{A} \times \vec{B} = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

5. The cross product is anti-commutative, i.e.

$$\vec{A} \times \vec{B} = - \vec{B} \times \vec{A}$$

6. The cross product follows the distributive law, i.e.

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

3. COORDINATE SYSTEMS

The following three most useful coordinate systems:

1. Cartesian or rectangular coordinates,
2. Circular or cylindrical coordinates, and
3. Spherical coordinates.

3.1. Rectangular or Cartesian Coordinate System

The three coordinate axes are designated as x, y and z which are mutually perpendicular to each other. The variables x, y and z can have any values in the range

$$-\infty < x < \infty, -\infty < y < \infty, -\infty < z < \infty$$

Vector Representation in Rectangular Coordinate System

A vector A in rectangular coordinate system is represented as

$$A = A_x a_x + A_y a_y + A_z a_z$$

Where a_x, a_y, a_z are the unit vectors along the x, y and z directions.

The magnitude of A is given by

$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

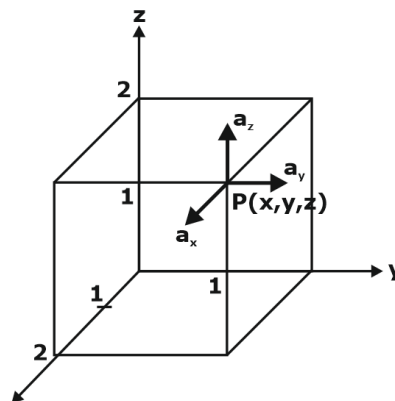


Figure: Representation of cartesian coordinates

3.2. Cylindrical Coordinates

The cylindrical coordinate system is very convenient whenever we are dealing with problems having cylindrical symmetry.

A Point P in cylindrical coordinates is represented as (ρ, ϕ, z) and is as shown in figure below.

The ranges of the variables are:

$$0 \leq \rho \leq \infty$$

$$0 \leq \phi \leq 2\pi$$

$$-\infty \leq z \leq +\infty$$

A vector \vec{A} in cylindrical coordinates can be written as

$$(A_\rho, A_\phi, A_z) \text{ or } A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

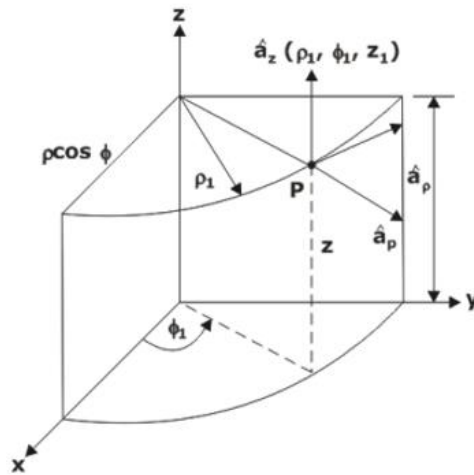


Figure: Representation of cylindrical coordinates

Notice that the unit vectors $\hat{a}_\rho, \hat{a}_\phi$ and \hat{a}_z are mutually perpendicular because our coordinates system is orthogonal.

$$\hat{a}_\rho \times \hat{a}_\phi = \hat{a}_\phi \times \hat{a}_z = \hat{a}_z \times \hat{a}_\rho = 0$$

$$\hat{a}_\rho \times \hat{a}_\rho = \hat{a}_\phi \times \hat{a}_\phi = \hat{a}_z \times \hat{a}_z = 0$$

$$\hat{a}_\rho \times \hat{a}_\phi = \hat{a}_z$$

$$\hat{a}_\phi \times \hat{a}_z = \hat{a}_\rho$$

$$\hat{a}_z \times \hat{a}_\rho = \hat{a}_\phi$$

Conversion of cartesian coordinate to cylindrical coordinate and vice-versa

Point transformation,

$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1} \frac{y}{x}$$

$$z = z$$

or

$$x = \rho \cos \phi$$

$$y = \rho \sin \phi$$

$$z = z$$

The relationship between $\hat{a}_x, \hat{a}_y, \hat{a}_z$ and $\hat{a}_\rho, \hat{a}_\phi, \hat{a}_z$ are vector transformation,

$$\hat{a}_x = \cos \phi \hat{a}_\rho - \sin \phi \hat{a}_\phi$$

$$\hat{a}_y = \sin \phi \hat{a}_\rho + \cos \phi \hat{a}_\phi$$

$$\hat{a}_z = \hat{a}_z$$

$$\text{or } \hat{a}_\rho = \cos \phi \hat{a}_x + \sin \phi \hat{a}_y$$

$$\hat{a}_\phi = -\sin \phi \hat{a}_x + \cos \phi \hat{a}_y$$

$$\hat{a}_z = \hat{a}_z$$

Finally, the relationship between (A_x, A_y, A_z) and (A_ρ, A_ϕ, A_z) are

$$\begin{vmatrix} A_\rho \\ A_\phi \\ A_z \end{vmatrix} = \begin{vmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix}$$

$$\begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix} = \begin{vmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} A_\rho \\ A_\phi \\ A_z \end{vmatrix}$$

3.3. Spherical Coordinates

A point P can be represented as $((r, \theta, \phi))$ and is illustrated in figure below. From figure, we notice that r is defined as the distance from the origin to point P or the radius of a sphere centered at the origin and passing through P; θ (called the colatitudes) is the angle between the z-axis and the position vector of P; and ϕ is measured from the x-axis (the same azimuthal angle in cylindrical coordinates). According to these definitions, the ranges of the variables are

$$0 \leq r \leq \infty$$

$$0 \leq \theta \leq \pi$$

$$0 \leq \phi \leq 2\pi$$

Note: the unit vectors $\hat{a}_r, \hat{a}_\theta$ and \hat{a}_ϕ are mutually perpendicular because our coordinate system is orthogonal.

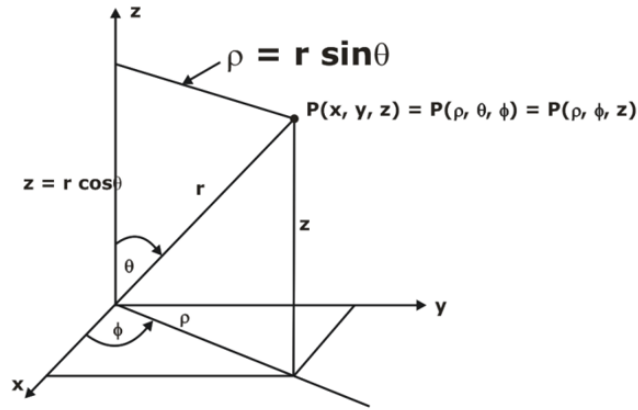
$$\hat{a}_r \times \hat{a}_\theta = \hat{a}_\theta \times \hat{a}_\phi = \hat{a}_\phi \times \hat{a}_r = 0$$

$$\hat{a}_r \times \hat{a}_r = \hat{a}_\theta \times \hat{a}_\theta = \hat{a}_\phi \times \hat{a}_\phi = 0$$

$$\hat{a}_r \times \hat{a}_\theta = \hat{a}_\phi$$

$$\hat{a}_\theta \times \hat{a}_\phi = \hat{a}_r$$

$$\hat{a}_\phi \times \hat{a}_r = \hat{a}_\theta$$



Conversion of cartesian coordinate to spherical coordinate and vice-versa

Point transformation,

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z}$$

$$\phi = \tan^{-1} \frac{y}{x}$$

Or

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

The relationship between $\hat{a}_x, \hat{a}_y, \hat{a}_z$ and $\hat{a}_r, \hat{a}_\theta, \hat{a}_\phi$ are

$$\hat{a}_x = \sin \theta \cos \phi \hat{a}_r + \cos \theta \cos \phi \hat{a}_\theta - \sin \phi \hat{a}_\phi$$

$$\hat{a}_y = \sin \theta \sin \phi \hat{a}_r + \cos \theta \sin \phi \hat{a}_\theta + \cos \phi \hat{a}_\phi$$

$$\hat{a}_z = \cos \theta \hat{a}_r - \sin \theta \hat{a}_\theta$$

$$\hat{a}_r = \sin \theta \cos \phi \hat{a}_x + \sin \theta \sin \phi \hat{a}_y + \cos \theta \hat{a}_z$$

$$\hat{a}_\theta = \cos \theta \cos \phi \hat{a}_x + \cos \theta \sin \phi \hat{a}_y - \sin \theta \hat{a}_z$$

$$\hat{a}_\phi = -\sin \phi \hat{a}_x + \cos \phi \hat{a}_y$$

Finally, the relationship between (A_x, A_y, A_z) and (A_r, A_θ, A_ϕ) are

Vector transformation,

$$\begin{vmatrix} A_r \\ A_\theta \\ A_\phi \end{vmatrix} = \begin{vmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{vmatrix} \begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix}$$

$$\begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix} = \begin{vmatrix} \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\ \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\ \cos \phi & -\sin \phi & 0 \end{vmatrix} \begin{vmatrix} A_r \\ A_\theta \\ A_\phi \end{vmatrix}$$

4. DIFFERENTIAL ELEMENTS IN COORDINATE SYSTEMS

4.1. Differential Elements in Rectangular Coordinate System

The differential elements in rectangular coordinate system are defined as follows:

1. Differential length in rectangular coordinate system:

$$dL = dx\alpha_x + dy\alpha_y + dz\alpha_z$$

2. Differential area in rectangular coordinate system:

$$dS = dydz\alpha_x = dx dz\alpha_y = dx dy\alpha_z$$

3. Differential volume in rectangular coordinate system:

$$dV = dx dy dz$$

4.2. Differential Elements in Cylindrical Coordinate System

The differential elements in cylindrical coordinate system are defined as follows:

1. Differential length in cylindrical coordinate system:

$$dL = d\rho\alpha_\rho + \rho d\phi\alpha_\phi + dz\alpha_z$$

2. Differential area in cylindrical coordinate system:

$$dS = \rho d\phi dz\alpha_\rho = \rho dz d\phi\alpha_\phi = \rho d\rho dz\alpha_z$$

3. Differential volume in cylindrical coordinate system:

$$dV = \rho d\rho d\phi dz$$

4.3. Differential Elements in spherical Coordinate System

The differential elements in spherical coordinate system are defined as follows:

1. Differential length in spherical coordinate system

$$dL = dr\alpha_r + r d\theta\alpha_\theta + r \sin\theta d\phi\alpha_\phi$$

2. Differential area in spherical coordinate system

$$dS = r^2 \sin\theta d\theta d\phi\alpha_r = r \sin\theta dr d\phi\alpha_\theta = r dr d\phi\alpha_\phi$$

3. Differential volume in spherical coordinate system

$$dV = r^2 \sin\theta dr d\theta d\phi$$

5. DIFFERENTIAL CALCULUS

The Del operator (∇), in the different coordinate system, is defined as

$$\nabla = \frac{\partial}{\partial x}\alpha_x + \frac{\partial}{\partial y}\alpha_y + \frac{\partial}{\partial z}\alpha_z \quad (\text{Rectangular coordinates})$$

$$\nabla = \frac{\partial}{\partial \rho}\alpha_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\alpha_\phi + \frac{\partial}{\partial z}\alpha_z \quad (\text{Cylindrical Coordinates})$$

$$\nabla = \frac{\partial}{\partial r}\alpha_r + \frac{1}{r}\frac{\partial}{\partial \theta}\alpha_\theta + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}\alpha_\phi \quad (\text{Spherical coordinates})$$

6. Gradient of a Scalar

The gradient (or grad) is defined by the operation of the Del operator on a scalar field. For a scalar Field V , we define the gradient in the different coordinates as

$$\nabla V = \frac{\partial V}{\partial x}\alpha_x + \frac{\partial V}{\partial y}\alpha_y + \frac{\partial V}{\partial z}\alpha_z \quad (\text{Rectangular coordinates})$$

$$\nabla V = \frac{\partial V}{\partial \rho} \alpha_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \alpha_\phi + \frac{\partial V}{\partial z} \alpha_z \quad \text{(Cylindrical coordinates)}$$

$$\nabla V = \frac{\partial V}{\partial r} \alpha_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \alpha_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \alpha_\phi \quad \text{(Spherical coordinates)}$$

7. Divergence of a Vector

Divergence of a vector function is a scalar and defined as the net outward flux per unit volume over the elementary closed surface. For a vector function A, we define the divergence in the different coordinates as

$$\nabla \cdot A = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) \quad \text{(Rectangular)}$$

$$\nabla \cdot A = \frac{1}{\rho} \frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \quad \text{(Cylindrical)}$$

$$\nabla \cdot A = \frac{1}{r^2} \frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\sin \theta A_\theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi} \quad \text{(Spherical)}$$

8. Curl of a Vector

The curl of a vector plays a very important role in electromagnetic theory. We define the curl of vector A in different coordinate systems as

$$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \quad \text{(Rectangular coordinates)}$$

$$\nabla \times A = \left(\frac{1}{\rho} \right) \begin{vmatrix} a_x & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} \quad \text{(Cylindrical coordinates)}$$

$$\nabla \times A = \left(\frac{1}{r^2 \sin \theta} \right) \begin{vmatrix} a_\rho & r a_\theta & r \sin \theta a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix} \quad \text{(Spherical coordinates)}$$

9. Laplacian Operator

The Laplacian Operator is the square of the Del operator and written as (∇^2). It can operate both on scalar as well as vector field. The Laplacian of a scalar field is a scalar field whereas the Laplacian of a vector is a vector field.

9.1. Laplacian of a Scalar

The Laplacian of a scalar field V in different coordinate systems is defined as

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \quad (\text{Rectangular coordinates})$$

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \quad (\text{Cylindrical coordinates})$$

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2} \quad (\text{Spherical coordinates})$$

9.2. Laplacian of a Vector

The Laplacian of a vector is defined as the gradient of divergence of the vector minus the curl of the curl of vector, i.e.

$$\nabla^2 \mathbf{A} = \nabla (\nabla \cdot \mathbf{A}) - \nabla \times \nabla \times \mathbf{A}$$

10. DIVERGENCE THEOREM

According to divergence theorem, the surface integral of a vector field over a closed surface is equal to the volume integral of the divergence of the vector field over the volume. Mathematically, the divergence theorem is written as

$$\oint_s \mathbf{A} \cdot d\mathbf{S} = \int_v (\nabla \cdot \mathbf{A}) dV$$

Where A is a vector field and V is the volume bounded by the closed surface S.

11. STOKE'S THEOREM

According to Stoke's theorem, the line integral of a vector field around a closed path is equal to the surface integral of the curl of vector field over the open surface bounded by the closed path. Mathematically, the Stoke's theorem is written as

$$\oint_L \mathbf{A} \cdot d\mathbf{L} = \int_s (\nabla \times \mathbf{A}) \cdot d\mathbf{S}$$

Where A is a vector field and S is the open surface bounded by the closed path L.

CHAPTER 2: ELECTROSTATICS

1. Electric charge

Electric Charge is a fundamental conserved property of some subatomic particles, which determines their electromagnetic interaction.

1.1 Point Charge

Point charges are very small charges assumed to be of infinitesimally small volume, although they have finite volume considered as a single charge.

1.2. Line Charge

The charge per unit length along the line charge is called line charge density. It is denoted by ρ_L and defined as

$$\rho_L = \lim_{\Delta L \rightarrow 0} \frac{\Delta Q}{\Delta L} = \frac{dQ}{dL}$$

where ΔQ is small charge, and ΔL is small length.

1.3. Surface Charge

The charge per unit area over the surface is called the surface charge density. It is denoted by ρ_s and defined as

$$\rho_s = \lim_{\Delta S \rightarrow 0} \frac{\Delta Q}{\Delta S} = \frac{dQ}{dS}$$

where ΔQ is small charge, and ΔS is small area.

1.4. Volume Charge

The charge per unit volume in the region is called volume charge density. It is denoted by ρ_v and defined as

$$\rho_v = \lim_{\Delta v \rightarrow 0} \frac{\Delta Q}{\Delta v} = \frac{dQ}{dv}$$

where ΔQ is small charge, and Δv is small volume.

2. Electric flux Density

The electric flux density vector \vec{D} in a medium is defined as the product of the permittivity and the electric field vector

$$\vec{D} = \epsilon \vec{E}$$

The permittivity of the medium is defined in terms of the free space permittivity and the relative permittivity (ϵ_r) as

$$\epsilon = \epsilon_r \epsilon_0$$

Electric flux density is independent of the medium properties

For point charge $\vec{E} = \frac{Q}{4\pi \epsilon R^2} \hat{a}_R, \vec{D} = \frac{Q}{4\pi R^2} \hat{a}_R$

For line charge $\vec{E} = \frac{\rho_L}{2\pi \epsilon_p} \hat{a}_p, \vec{D} = \frac{\rho_L}{2\pi_p} \hat{a}_p$

The unit of electric flux density are

$$\frac{F}{m} \times \frac{F}{m} = \frac{C}{m^2}$$

NOTE: The units of \vec{D} are equivalent to surface density i.e. C/m²

3. Gauss's Law – Maxwell Equations

The total outward electric flux Ψ through any closed surface is equal to the total charge enclosed by the surface.

In equation form, Gauss's law is written as

$$\psi = \oint_S \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}$$

Where $d\vec{S} = \hat{a}_n dS$ and \hat{a}_n is the outward pointing unit normal to closed surface S.

$$\psi = \oint_S \vec{D} \cdot d\vec{S} = \text{total charge enclosed}$$

$$Q = \oint_V \rho_v dv$$

$$\text{or } Q = \oint_S \vec{D} \cdot d\vec{S} = \int_V \rho_v dv$$

By applying divergence theorem to the middle term, we have

$$\oint_S \vec{D} \cdot d\vec{S} = \int_V \nabla \cdot \vec{D} dv$$

Comparing the two volume integrals

$$\nabla \cdot \vec{D} = \rho_v$$

It states that the volume charge density is the same as the divergence of the electric flux density.

4. Electric field due to a point charge

$$\vec{D} = \frac{Q}{4\pi r^2} \hat{a}_r$$

$$\vec{E} = \frac{Q}{4\pi \epsilon_0 r^2} \hat{a}_r$$

Where, Q is the point charge and r is the distance between point where electric field is calculated and point charge.

5. Electric field due to an Infinite line charge

$$\Rightarrow \vec{D} = \frac{P_L}{2\pi \rho} \hat{a}_\rho$$

$$\text{and } \vec{E} = \frac{P_L}{2\pi \epsilon_0 \rho} \hat{a}_\rho$$

Where, P_L is linear charge density, ρ is distance of the point P (P is the point where electric field is calculated) from line charge and \hat{a}_ρ is position vector of point P.

6. Electric Field due to an infinite sheet of charge

$$\vec{D} = \frac{\rho_s}{2} \hat{a}_z$$

$$\text{Or, } \vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{\rho_s}{2\epsilon_0} \hat{a}_z$$

Where, ρ_s is surface charge density and \hat{a}_z is the unit normal vector from sheet to the point where electric field is calculated.

7. Field due to a uniformly charged sphere

$$\vec{D} = \begin{cases} \frac{r}{3} \rho_v \hat{a}_r; & 0 < r \leq a \\ \frac{a^3}{3r^2} \rho_v \hat{a}_r; & r \geq a \end{cases} \quad \text{Where, } \rho_v \text{ is volume charge density.}$$

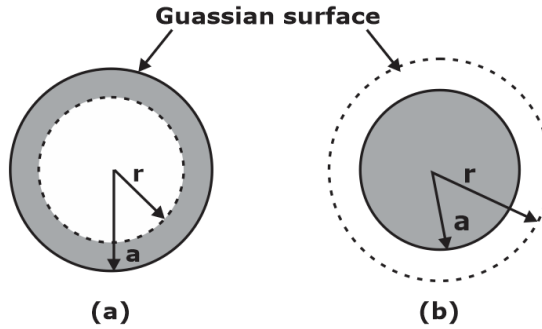


Figure: Gaussian surface for a uniformly charged when (a) $r \geq a$ and (b) $r \leq a$

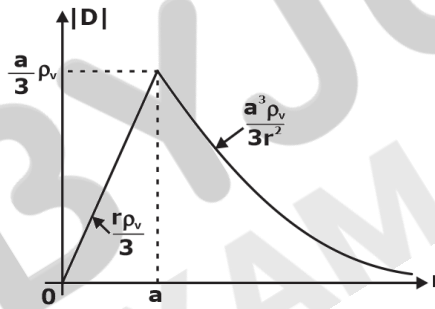


Figure: Sketch of $|D|$ against r for a uniformly charged sphere.

8. Electric field due to multiple point charger

The electric field due to multiple points chargers can be determined using the principle of superposition. for N point charges Q_1, Q_2, \dots, Q_N located at $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_N$ the electric fields intensity at point \vec{r} is obtained by equations.

$$\vec{E} = \frac{Q_1(\vec{r} - \vec{r}_1)}{4\pi \epsilon_0 |\vec{r} - \vec{r}_1|^3} + \frac{Q_2(\vec{r} - \vec{r}_2)}{4\pi \epsilon_0 |\vec{r} - \vec{r}_2|^3} \dots + \frac{Q_N(\vec{r} - \vec{r}_N)}{4\pi \epsilon_0 |\vec{r} - \vec{r}_N|^3}$$

$$\vec{E} = \frac{1}{4\pi \epsilon_0} \sum_{k=1}^N \frac{Q_k(\vec{r} - \vec{r}_k)}{|\vec{r}_1 - \vec{r}_k|^3}$$

9. Electric field due to charge distributions

$$dQ = \rho_L dl \Rightarrow Q = \int_L \rho_L dl \text{ (line charge)}$$

$$dQ = \rho_S dS \Rightarrow Q = \int_S \rho_S dS \text{ (surface charge)}$$

$$dQ = \rho_V dV \Rightarrow Q = \int_V \rho_V dV \text{ (volume charge)}$$

10. Electric field on the axis of a charged ring

Consider a circular ring of radius a with uniform line charge density ρ_L (C/m) and a point P on the axis of ring as shown in figure

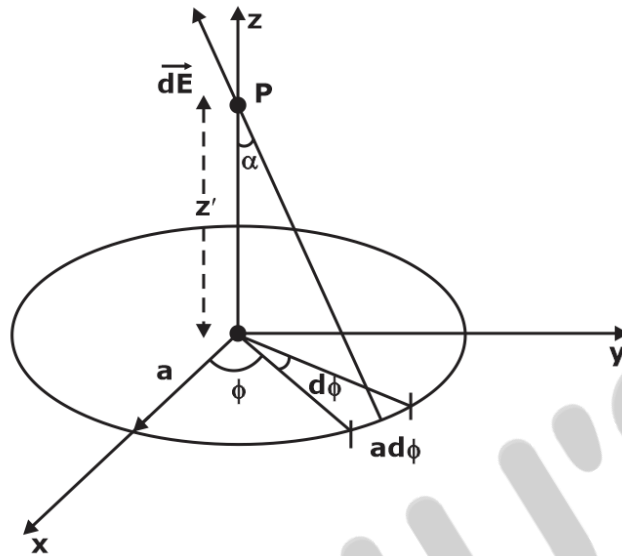


Figure: electric field to circular ring

The total electric field is therefore

$$\vec{E} = \frac{Qz}{4\pi\epsilon_0(z^2 + a^2)^{3/2}} \hat{a}_z$$

Note: As $z \rightarrow \infty$, \vec{E} tends to $\frac{Q}{4\pi\epsilon_0 z^2}$

11. Electric field of a Charged Circular Disk

The electric field due to a uniformly charged circular disk at a point on its axis can be calculated using the result for a ring. Consider a disk of radius a , surface charge density ρ_s (C/m²) and point P as shown in the figure

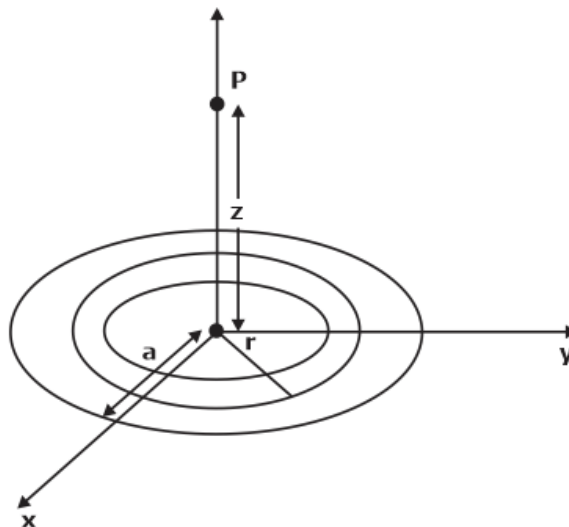


Figure: Electric field due to charged circular

$$\vec{E} = \frac{\rho_s}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + a^2}} \right) \hat{a}_z$$

Note: If $z \gg a$ then $\vec{E} = \frac{Q}{4\pi \epsilon_0 z^2} \hat{a}_z$

12. Potential difference in the electric field of a point charge

The potential difference between two points A and B in the electric fields of the point charge is

$$V_{AB} = -\int_A^B \vec{E} \cdot d\vec{l} = \int_A^B dV = V_B - V_A$$

13. Energy Density in Electrostatic Field

$$W_E = \frac{1}{2} \int_{vol.} (D \cdot E) dv = \frac{1}{2} \int_{vol.} \epsilon_0 E^2 dv \text{ (J)}$$

Where, D is electric flux density and E is electric field intensity.

We defined energy density in (J/m³)

14. Boundary Conditions

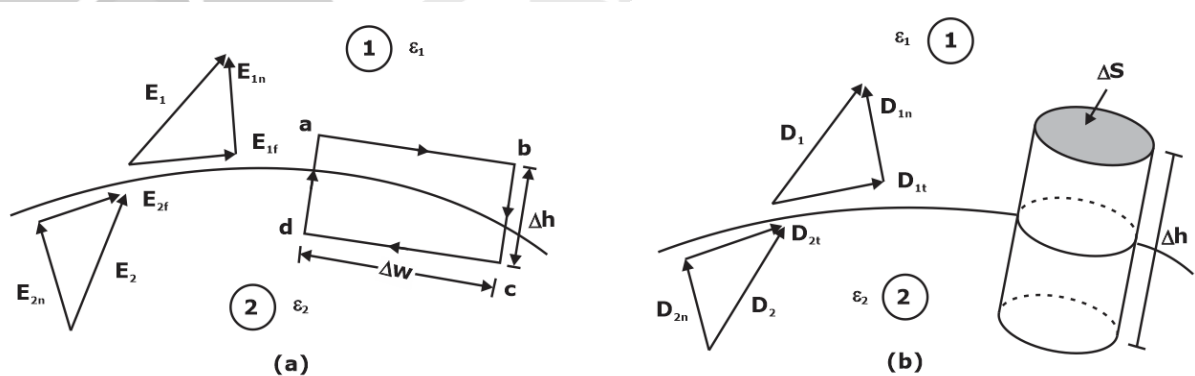
Electric field intensity \vec{E} into two orthogonal components

$$\vec{E} = \vec{E}_t + \vec{E}_n$$

where \vec{E}_t and \vec{E}_n are tangential and normal components of \vec{E} respectively.

14.1. Dielectric-Dielectric Boundary Conditions

Consider the \vec{E} field existing in a region that consist of two different dielectrics characterized by $\epsilon_1 = \epsilon_0 \epsilon_n$ and $\epsilon_2 = \epsilon_0 \epsilon_a$ as shown in figure.



**Figure: Dielectric-dielectric boundary: (a) determining $E_{1t} = E_{2t}$
(b) determining $D_{1\sigma} = D_{2\sigma}$.**

The fields and \vec{E}_1 and \vec{E}_2 can be decomposed as

$$\vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n}$$

$$\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n}$$

then, $E_{1t} = E_{2t}$

14.2. Conductor-Dielectric Boundary Conditions

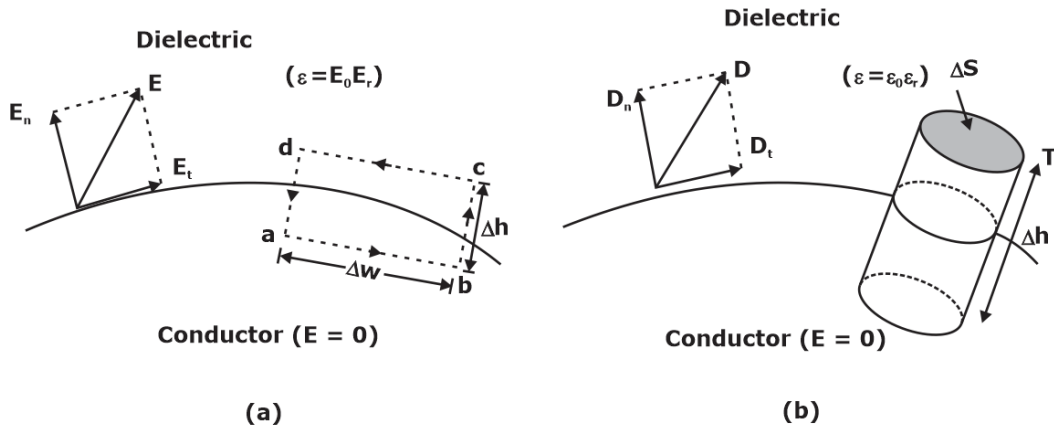


Figure: Conductor-dielectric boundary.

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_s$$

or, $D_n = \rho_s$

14.3. Conductor-Free Space Boundary Conditions

The boundary conditions at the interface of conductor and free space can be obtained from conductor-dielectric boundary conditions with $\epsilon_r = 1$.

Thus the boundary conditions are

$$E_t = 0, D_t = \epsilon_0 E_t = 0$$

$$D_n = \frac{\rho_s}{\epsilon_0}$$

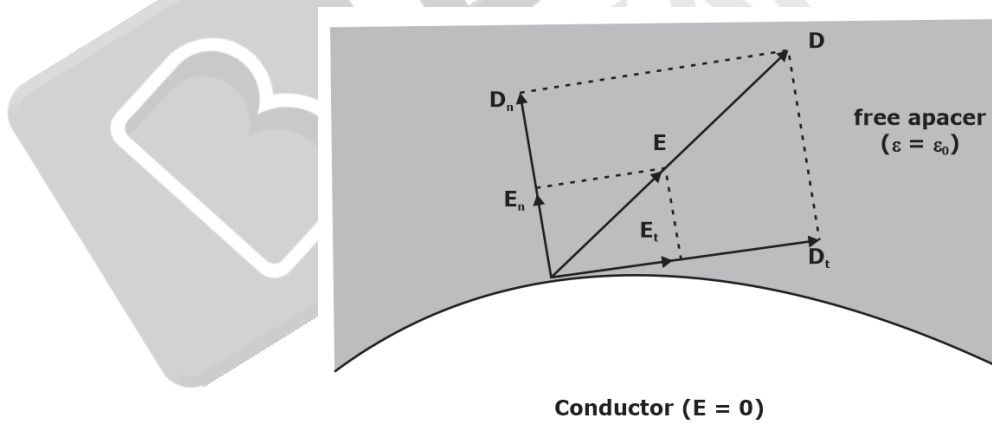


Figure: Conductor-free space boundary

15. Poisson's and Laplace's Equations

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}, \text{ Where } V \text{ is electrostatic potential and } \rho_v \text{ is volume charge density.}$$

This is known as Poisson's equation.

As special case of this equation occurs when $\rho_v = 0$ (i.e., for a charge free region)

$$\nabla^2 V = 0$$

Which is known as Laplace's equation.

16. Coaxial Capacitor

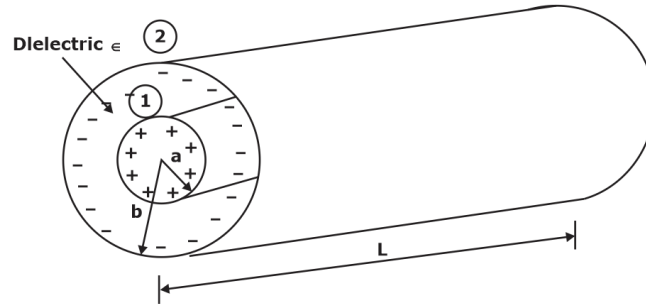


Figure: A coaxial capacitor

The capacitance of a coaxial cylinder is given by

$$C = \frac{Q}{V} = \frac{2\pi \epsilon}{\ln \frac{b}{a}}$$

17. Spherical Capacitor

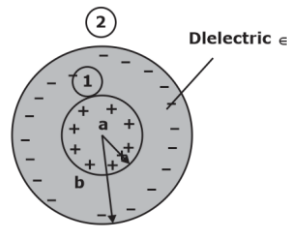


Figure: A spherical capacitor

The capacitance of the spherical capacitor is

$$C = \frac{Q}{V} = \frac{4\pi \epsilon}{\frac{1}{a} - \frac{1}{b}}$$

CHAPTER 3: MAGNETOSTATICS

1. MAGNETIC FLUX DENSITY

Magnetic flux density is the amount of magnetic flux per unit area of a section, perpendicular to the direction of magnetic flux.

It is denoted by B. Mathematically,

$$B = \frac{d\Phi}{dS} a_n$$

Where dΦ is a small amount of magnetic flux through small area dS of the section perpendicular to magnetic flux a_n is the unit vector normal to the surface area.

also expressed as

$$\Phi = \int_S B \cdot dS$$

2. Relation between Magnetic field Intensity (H) and Magnetic Flux Density (B):

The magnetic field intensity is related to the magnetic flux density as

$$B = \mu H = \mu_0 \mu_r H$$

Where, μ is the permeability of the medium, μ₀ = 4 π × 10⁻⁷ H/m is the permeability of free space, and μ_r is the relative permeability of the medium.

3. Biot-Savart's Law

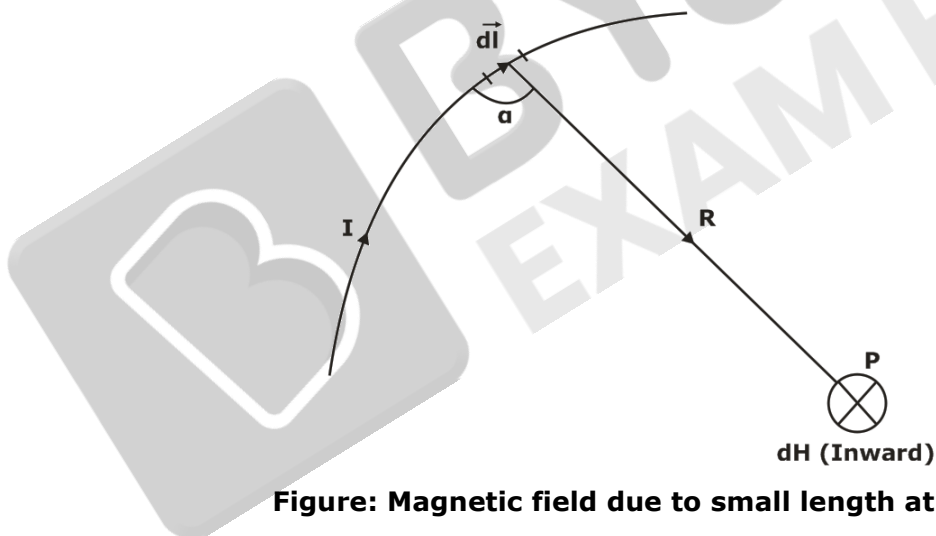


Figure: Magnetic field due to small length at P

Line current $\vec{H} = \int_L \frac{I d\vec{l} \times \vec{a}_R}{4\pi R^2}$

Surface current $\vec{H} = \int_L \frac{\vec{K} dS \times \vec{a}_R}{4\pi R^2}$

Volume current $\vec{H} = \int_V \frac{\vec{J} dv \times \vec{a}_R}{4\pi R^2}$

(H direction = I direction × Radial vector)

4. Ampere’s Circuital Law

According to Ampere’s circuital law the line integral of magnetic field intensity H around the closed loop L is equal to I, i.e.

$$\oint_L H \cdot dL = I$$

Differential Form of Ampere’s Circuital Law

In differential form Ampere’s circuital law is defined as

$$\nabla \times H = J$$

i.e. the curl of the magnetic field intensity (H) is equal to the current density (J) at the point in space.

5. H-field for finite length of current I carrying wire:

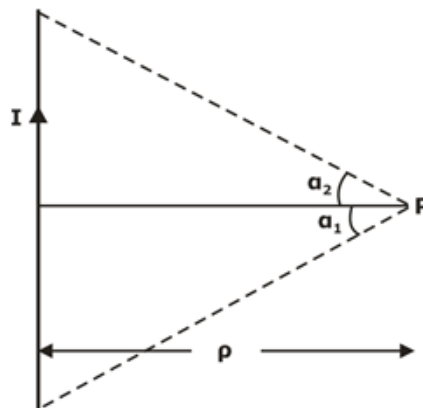


Figure: Field at P due to line conductor

$$\vec{H} = \frac{I}{4\pi\rho} (\sin \alpha_1 + \sin \alpha_2) \hat{a}_\phi$$

Note: Notice from the above equation that \vec{H} is always along the unit vector \hat{a}_ϕ (i.e., along concentric circular paths) irrespective of the length of the wire or the point of interest P.

6. H-field for infinite length of current I carrying wire:

$$\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\phi$$

The unit vector \hat{a}_ϕ must be found carefully. A simple approach is to determine \hat{a}_ϕ from

$$\hat{a}_\phi = \hat{a}_l \times \hat{a}_p$$

Where \hat{a}_l is a unit vector along the line current and \hat{a}_p is a unit vector along the perpendicular line from the line current to the filed point.

	Electric force ($F_e = QE$)	Magnetic Force ($F_m = Qu \times B$)
1.	It is in the same direction as the field E.	It is perpendicular to both u and B.
2.	It can perform work.	It cannot perform work.

3.	It is independent of the velocity of charge.	It depends upon the velocity of charge.
4.	It can produce change in kinetic energy.	It cannot produce change in kinetic energy.

Table: Comparison between Electric Force and Magnetic Force

7. Force on a Differential Current Element in Magnetic Field

The differential magnetic force experienced by various differential current elements are given below:

$$F_m = \int_L IdL \times B \text{ (Line current)}$$

$$F_m = \int_S K \times BdS \text{ (Surface current)}$$

$$F_m = \int_V J \times Bdv \text{ (Volume current)}$$

Where IdL is the line current element, KdS is surface current element, Jdv is volume current element, and F_m is the magnetic force exerted on the respective elements in presence of magnetic field B

8. Magnetic Force Between Two Current Elements

Consider the two differential current elements I_1dL_1 and I_2dL_2 separated by a distance r . The magnetic force between the two current elements is given by

$$F = \frac{\mu I_1 I_2}{4\pi} \int_{L_1} \int_{L_2} \frac{dL_2 \times (dL_1 \times a_r)}{r^2}$$

This equation is also called Ampere’s force law.

9. Magnetic Susceptibility

In a linear material, magnetization is directly proportional to field intensity. i.e.

$$M \propto H$$

or $M = \chi_m H$

where χ_m is the magnetic susceptibility of the medium. The magnetic susceptibility of a magnetic material is a measure of the degree of magnetization of a material in response to an applied magnetic field.

10. Relation between Magnetic Field Intensity and Magnetic Flux Density

In a magnetic material, magnetic flux density is expressed in terms of magnetic field intensity as

$$B = \mu_0(H + M) = \mu_0(1 + \chi_m)H$$

$$= \mu_0\mu_r H = \mu H$$

where

$\mu = \mu_0\mu_r$ is called permeability of the medium, expressed in Henry per metre (H/m),

$\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of free space, known as absolute permeability,

$\mu_r = (1 + \chi_m) = \frac{\mu}{\mu_0}$ is the relative permeability of the medium, it is dimensionless.

11. Energy Density in a Magnetic Field

In a magnetic field with flux density B, the stored magnetic energy density is given by

$$w_m = \frac{1}{2}(B \cdot H)$$

where H is the magnetic field intensity in the region. The total magnetic energy stored in a region is obtained by taking the volume integral of the energy density, i.e.

$$W_m = \int_v w_m dv = \int_v \frac{1}{2}(B \cdot H) dv$$

12. Boundary Conditions for Magnetostatic Fields:

$B_{1n} = B_{2n}$ States that Normal component of B is continuous across an interface. $\mu_1 H_{1n} = \mu_2 H_{2n}$

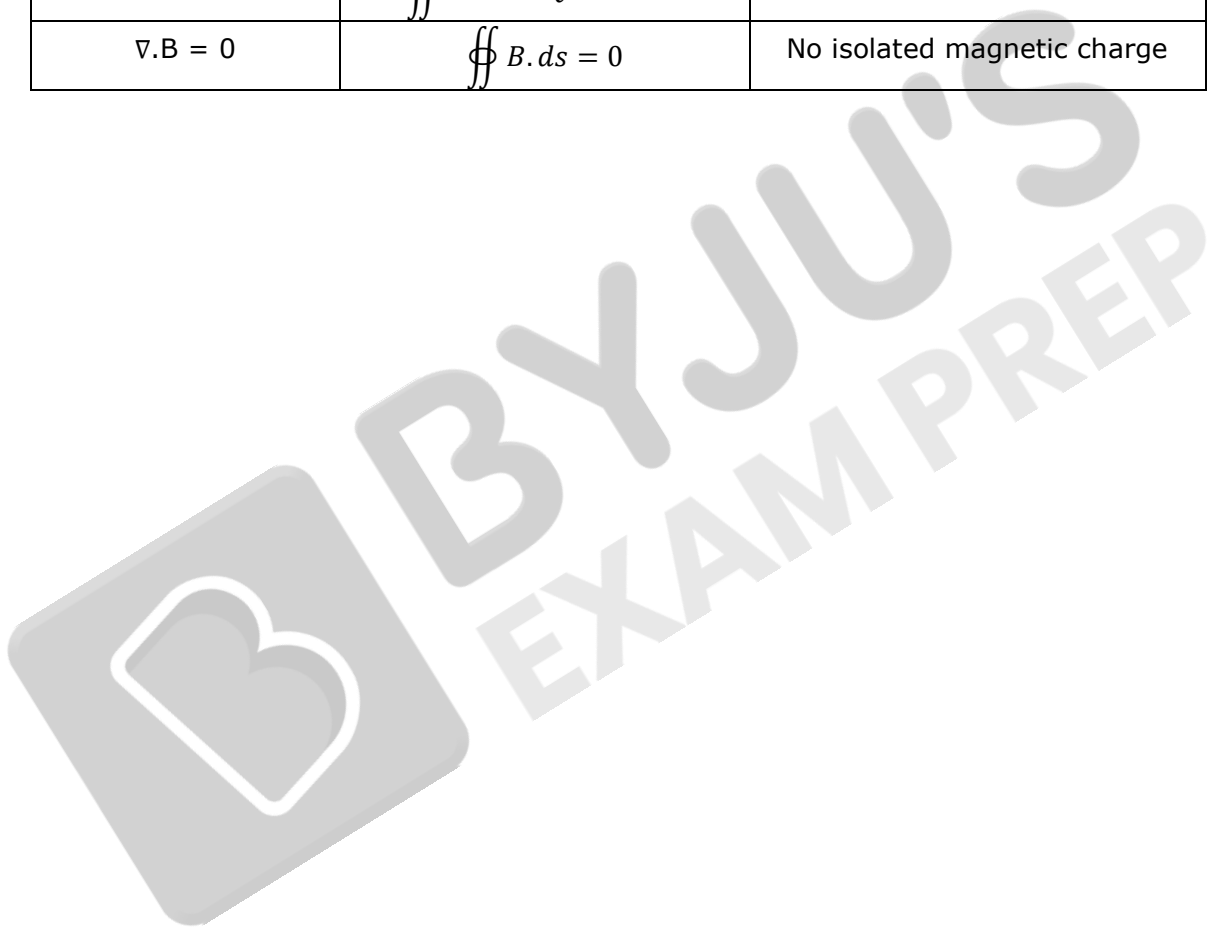
$H_{1t} - H_{2t} = J_{sn}$ States that the Tangential component of H field is discontinuous across an interface where free surface current exist-amount the amount of discontinuity being equal to the surface current density.

When conductivities of both media are finite, current are defined by volume current densities and free surface currents don't exist on interface hence j equal to zero, and the Tangential component of H field is continuous across the boundary of almost all physical media; it is discontinuous only when an interface with an ideal conductor or a super conductor is assumed.



CHAPTER 4: MAXWELL'S EQUATION
Maxwell Equations

Differential form	integral form	Significance
$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint E \cdot dl = -\iint \frac{\partial B}{\partial t} \cdot ds$	Faraday's Law
$\nabla \times H$ $= J + \frac{\partial D}{\partial t}$	$\oint H \cdot dl = \iint (J + \frac{\partial D}{\partial t}) \cdot ds$	Ampere's Circuital Law
$\nabla \cdot D = \rho_v$	$\oiint D \cdot ds = Q_{enclosed}$	Gauss Law
$\nabla \cdot B = 0$	$\oiint B \cdot ds = 0$	No isolated magnetic charge



CHAPTER 5: EM WAVE PROPAGATION

1. General wave equation for electromagnetic waves

The three-dimensional vector wave equation or Helmholtz equation in an absorbing medium or lossy dielectric medium is defined as

$$\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \epsilon\mu \frac{\partial^2 H}{\partial t^2} \quad \text{and} \quad \nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \epsilon\mu \frac{\partial^2 E}{\partial t^2}$$

2. Wave equation for perfect dielectric medium

In a perfect dielectric medium, the conductivity is zero, i.e. $\sigma = 0$.

$$\nabla^2 E = \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

$$\text{and} \quad \nabla^2 H = \mu\epsilon \frac{\partial^2 H}{\partial t^2}$$

These are the wave equations for perfect dielectric medium.

3. Wave equation for Time-Harmonic Fields

The standard form of wave equations for time harmonic fields (in phasor form) are defined as

$$\nabla^2 E_s - \gamma^2 E_s = 0$$

$$\text{and} \quad \nabla^2 H_s - \gamma^2 H_s = 0$$

Where γ is a complex constant called the propagation constant.

4. Propagation Constant

For a medium with permittivity ϵ , permeability μ , and conductivity σ , the propagation constant is given by

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

Propagation constant is expressed in per meter (m^{-1}). It can be also defined as

$$\gamma = \alpha + j\beta$$

where α is the attenuation constant, and β is the phase constant.

5.1. Propagation Constant in lossy dielectrics

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\text{and} \quad \gamma = \alpha + j\beta$$

where α is the attenuation constant, and β is the phase constant.

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} - 1 \right]}$$

$$\text{and} \quad \beta = \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} + 1 \right]}$$

5.2. Intrinsic Impedance

$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

or
$$\eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\epsilon)}}$$

Thus, the intrinsic impedance is complex quantity.

5.3. Loss Tangent

This ratio is defined as the loss tangent or loss angle of the medium i.e.

$$\frac{|\epsilon''|}{|\epsilon'|} = \frac{\sigma}{\omega\epsilon} = \frac{|\sigma E_s|}{|\omega\epsilon E_s|} = \frac{|J_{\text{conduction}}|}{|J_{\text{displacement}}|} = \tan \theta$$

6. WAVE PROPAGATION IN LOSSLESS DIELECTRICS

6.1. Attenuation Constant

$$\alpha = 0$$

6.2. Phase Constant

$$\beta = \omega\sqrt{\mu\epsilon}$$

6.3. Propagation Constant

Propagation constant in a medium is defined by

$$\gamma = \alpha + j\beta$$

The propagation constant in the lossless dielectric medium as

$$\gamma = j\omega\sqrt{\mu\epsilon}$$

6.4. Velocity of Wave Propagation

The velocity of wave propagation in a medium is given by

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{3 \times 10^8}{\sqrt{\mu_r \epsilon_r}}$$

6.5. Intrinsic Impedance

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = 120\pi \sqrt{\frac{\mu_r}{\epsilon_r}}$$

7. WAVE PROPAGATION IN PERFECT CONDUCTORS

7.1. Attenuation Constant

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

7.2. Phase Constant

$$\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

7.3. Propagation Constant

The propagation constant in the good conductors as

$$\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$$

7.4. Velocity of wave propagation

$$v_p = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}} = \sqrt{\frac{2\omega}{\mu\sigma}}$$

7.5. Intrinsic Impedance

The intrinsic impedance

$$\eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\epsilon)}} = \sqrt{\frac{j\omega\mu}{\sigma\left(1 + \frac{j\omega\epsilon}{\sigma}\right)}}$$

Since, $\sigma \gg \omega\epsilon$, i.e. $\frac{\omega\epsilon}{\sigma} \ll 1$. So, we can write

$$1 + \frac{j\omega\epsilon}{\sigma} \approx 1$$

Hence, the intrinsic impedance in a good conductor reduce is obtained as

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} e^{j\frac{\pi}{4}}$$

7.6. Skin Depth

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

8. WAVE PROPAGATION IN FREE SPACE

8.1. Attenuation Constant

$$\alpha = 0$$

8.2. Phase Constant

$$\beta = \omega\sqrt{\mu_0\epsilon_0}$$

8.3. Propagation Constant

$$\gamma = j\omega\sqrt{\mu_0\epsilon_0}$$

8.4. Velocity of Wave Propagation

$$v_p = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3 \times 10^8 \text{ m / s}$$

8.5. Intrinsic Impedance

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \approx 377\Omega$$

9. Average Power Flow in Uniform Plane Waves

$$P_{av} = \frac{1}{2} \text{Re}[E_s \times H_s^*]$$

P_{av} is time-average power density vector in a uniform plane wave which is expressed in Watt per squared meter (W/m^2).

CHAPTER 6: REFLECTION & REFRACTION OF WAVES

1. Reflection and Transmission Coefficients

The ratio E_{r0}/E_{i0} is called reflection coefficient denoted by Γ and given by

$$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

The ratio E_{t0}/E_{i0} is called transmission coefficient denoted by τ and given by

$$\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

2. Standing Wave Ratio

The ratio of the maximum amplitude to the minimum of the total electric field $|E_t|$ is called standing wave ratio, i.e.

$$S = \frac{|E_t|_{max}}{|E_t|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Important Point

- Both Γ and τ are dimensionless and may be complex.

$$1 + \Gamma = \tau$$

$$0 \leq |\Gamma| \leq 1$$

- Since $|\Gamma| \leq 1$, standing wave ratio (S) is always positive and greater than or equal to unity, i.e. $S \geq 1$

3. Reflection and Transmission Coefficients for Parallel Polarization

The reflection and transmission coefficients for the parallel polarised wave are defined as

$$\Gamma_p = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} \quad \text{and}$$

$$\tau_p = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2 \cos \theta_t}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}$$

4. Brewster Angle for Parallel Polarized Wave

The incident angle at which there is no reflection (i.e. $E_{r0} = 0$) is called Brewster angle. For the parallel polarized wave propagating through lossless mediums, the Brewster angle $\theta_{B||}$ is expressed as

$$\sin^2 \theta_{B||} = \frac{1 - \mu_2 \epsilon_1 / \mu_1 \epsilon_2}{1 - (\epsilon_1 / \epsilon_2)^2}$$

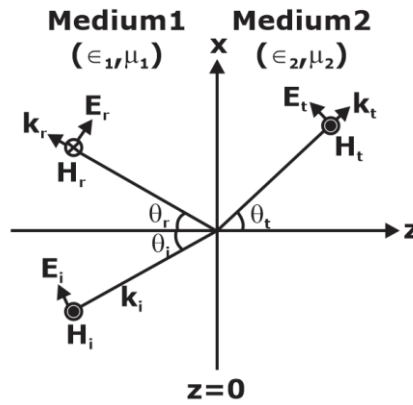


Figure: Oblique Incidence of Parallel Polarized Wave at the Interface between Two Lossless Dielectrics

5. Reflection and Transmission Coefficients for Perpendicular Polarization

The reflection and transmission coefficients for perpendicular polarized wave are given by

$$\Gamma_{\perp} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$$

and
$$\tau_{\perp} = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$$

6. Brewster Angle for Perpendicular Polarized Wave

For the perpendicular polarized wave propagating through lossless mediums, the Brewster angle, $\theta_{B\perp}$ is expressed as

$$\sin^2 \theta_{B\perp} = \frac{1 - \mu_1 \epsilon_2 / \mu_2 \epsilon_1}{1 - (\mu_1 / \mu_2)^2}$$

CHAPTER 7: TRANSMISSION LINES

1. Transmission Line:

For a transmission line with primary constants R , L , C and G , the propagation constant is given by

$$\begin{aligned}\gamma &= \sqrt{\text{series impedance} \times \text{shunt admittance}} \\ &= \sqrt{(R + j\omega L)(G + j\omega C)}\end{aligned}$$

Or $\gamma = \alpha + j\beta$

2. For a transmission line with primary constants R , L , C and G , the characteristic impedance is defined as

$$Z_0 = \sqrt{\frac{\text{series impedance}}{\text{shunt admittance}}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

3. The input impedance can be expressed as:

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right]$$

Points to Remember

1. For a short circuited transmission line, $Z_L = 0$, the input impedance is

$$(Z_{in})_{sc} = Z_0 \tanh \gamma l$$

2. For open circuited line, $Z_L = \infty$, the input impedance is given by

$$(Z_{in})_{oc} = Z_0 \coth \gamma l$$

3. From the above two results, we have

$$(Z_{in})_{sc} (Z_{in})_{oc} = Z_0^2 \text{ and } Z_0 = \sqrt{(Z_{in})_{sc} (Z_{in})_{oc}}$$

4. For matched line, $Z_L = Z_0$, the input impedance is

$$Z_{in} = Z_0$$

4. Reflection Coefficient:

$$0 \leq \Gamma_L \leq 1 \text{ and } \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

5. The propagation constant of a lossless transmission line is obtained as

$$\begin{aligned}\gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \sqrt{(0 + j\omega L)(0 + j\omega C)} \\ &= j\omega\sqrt{LC}\end{aligned}$$

$$\text{or } \gamma = j\beta = j\omega\sqrt{LC}$$

thus, the attenuation and phase constants of transmission line is given by

$$\alpha = 0, \beta = \omega\sqrt{LC}$$

6. The characteristic impedance of a lossless transmission line is obtained as

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{0 + j\omega L}{0 + j\omega C}} = \sqrt{\frac{L}{C}}$$

7. The velocity of propagation in a lossless transmission line is given by

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

8. The input impedance for a lossless line is obtained as

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right] = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

9. The propagation constant of a distortion less transmission line is obtained as

$$\begin{aligned} \gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \sqrt{RG \left(1 + \frac{j\omega L}{R}\right) \left(1 + \frac{j\omega C}{G}\right)} \\ &= \sqrt{RG} \sqrt{\left(1 + \frac{j\omega C}{G}\right) \left(1 + \frac{j\omega L}{R}\right)} = \alpha + j\beta \end{aligned}$$

Thus, we obtain the attenuation and phase constants as

$$\alpha = \sqrt{RG}$$

$$\beta = \omega\sqrt{LC}$$

10. The characteristic impedance of a distortion less line is obtained as

$$\begin{aligned} Z_0 &= \sqrt{\frac{R + j\omega L}{G + j\omega C}} \\ &= \sqrt{\frac{R(1 + j\omega L / R)}{G(1 + j\omega C / G)}} \\ &= \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} \end{aligned}$$

11. Standing wave ratio is defined as the ratio of the maximum voltage (or current) to the minimum voltage (or current) of a line having standing waves.

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

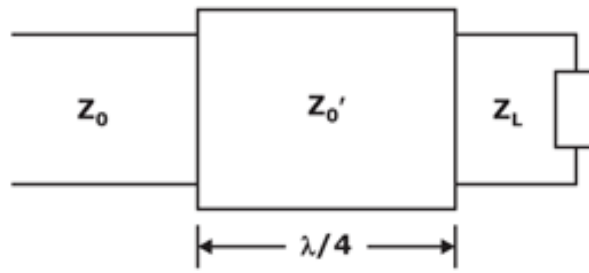
11.1. Impedance Matching Techniques

Most load impedances are typically large and line impedances are small so we have to use additional elements for matching

- Series -λ/4- Quarter wave transformer

The Z_0 line has a termination = Z_{in} of the λ/4 line of Z_L load = $\frac{Z_0^2}{Z_L}$

Therefore, $Z'_0 = \sqrt{Z_0 \cdot Z_L}$



- With the use of the quarter wave transformer the line is completely matched but the load is not completely matched but the miss match is reduced as compare without the $\lambda/4$ transformer.

12.2. Shunt-Stub Matching

A stub is a short circuit line of l_t length placed at l_s distance from the load such that the function impedance is Z_0 and the line is matched, from that point towards the source.

Design of a stub

- Identity a position on the line from the load where $Z(x) = Z_0 \pm jx$ then x value is ' s ' is position of stub

$$l_s = \frac{\lambda}{2\pi} \tan^{-1} \left(\sqrt{\frac{Z_L}{Z_0}} \right)$$

- At this stub position an equal and opposite reactance is placed in shunt that cancels existing reactance so that the junction impedance.
- This junction impedance is Z_0 as the cancelling reactance is designed form a short-circuit line of ' l_t ' length

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \left(\sqrt{\frac{Z_L Z_0}{Z_L - Z_0}} \right)$$

- For the miss-matched region to be small ' l_s ' should be as close to the load as possible

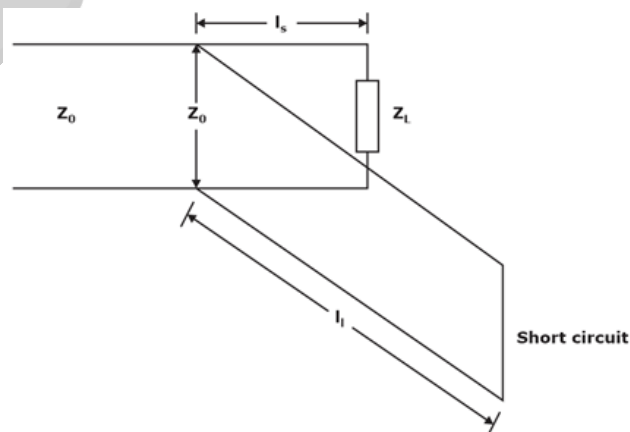


Figure: Matching with a single stub tuner

CHAPTER 8: RECTANGULAR WAVEGUIDE

1. Transverse Electromagnetic (TEM) Modes

In TEM mode, the electric and magnetic fields are transverse to the direction of wave propagation with no longitudinal components, i.e.

$$E_z = H_z = 0$$

2. Transverse Electric (TE) Modes

In TE mode, the electric field is transverse to the direction of propagation (no longitudinal electric field component), while the magnetic field has both transverse and longitudinal components, i.e.

$$E_z = 0, H_z \neq 0$$

3. Transverse Magnetic (TM) Modes

In TM mode, the magnetic field is transverse to the direction of propagation (no longitudinal magnetic field component), while the electric field has both transverse and longitudinal components, i.e. $H_z = 0, E_z \neq 0$

4. For TM_{mn} mode in a rectangular waveguide, the propagation constant is defined as

$$\gamma = \sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2} - k^2$$

Where $k = \omega\sqrt{\mu\epsilon}$. Following are the three-special case of different values of k , m , and n .

Case 1: cut-off

$$\text{If } k^2 = \omega^2\mu\epsilon = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$$

$$\text{Then, } \gamma = 0 \text{ or } \alpha = 0 = \beta$$

The value of ω that causes this called the cut off angular frequency ω_c ; that is,

No propagation takes place at this frequency.

Case 2: Evanescent

$$\text{If } k^2 = \omega^2\mu\epsilon < \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$$

$$\text{Then, } \gamma = \alpha \quad \beta = 0$$

In this case, we have no wave propagation at all. These non propagating modes are said to be evanescent.

Case 3: Propagation

$$\text{If } k^2 = \omega^2\mu\epsilon > \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$$

$$\text{Then, } \gamma = j\beta, \alpha = 0$$

So, the phase constant β becomes

$$\beta = \sqrt{k^2 - \left[\frac{m\pi}{a}\right]^2 - \left[\frac{n\pi}{b}\right]^2}$$

This is the only case in which propagation takes place because all field components will have the factor $e^{-\gamma z} = e^{-j\beta z}$.

5. The cut-off frequency for TE_{mn} and TM_{mn} mode in rectangular waveguide is given by:

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2}$$

$$= \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left[\frac{m}{a}\right]^2 + \left[\frac{n}{b}\right]^2}$$

6. The cut-off wavelength of TE_{mn} and TM_{mn} mode in rectangular waveguide is given by

$$\lambda_c = \frac{v_p}{f_c} = \frac{\frac{1}{\sqrt{\mu\epsilon}}}{\frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} = \frac{2}{\sqrt{\left[\frac{m}{a}\right]^2 + \left[\frac{n}{b}\right]^2}}$$

7. The phase constant β for TE_{mn} and TM_{mn} mode is also given by

$$\beta = \omega\sqrt{\mu\epsilon} \sqrt{1 - \left[\frac{f_c}{f}\right]^2}$$

$$\beta = \beta' \sqrt{1 - \left[\frac{f_c}{f}\right]^2}$$

8. The intrinsic wave impedance for TE_{mn} mode is given by

$$\eta_{TE} = \frac{\eta'}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

The intrinsic wave impedance of TM_{mn} mode is obtained as

$$\eta_{TM} = \eta' \sqrt{1 - \left[\frac{f_c}{f}\right]^2}$$

9. **Guided Wavelength:**

$$\lambda_g = \frac{\lambda'}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

10. **Phase velocity:**

$$\lambda_p = \frac{v'}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

11. Group velocity:

$$v_g = \frac{1}{\frac{\partial \beta}{\partial \omega}}$$
$$= v' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

12. $v_p v_g = v'^2$

13. The resonant frequency for TM_{mnp} mode in a waveguide resonator is given by

$$f_r = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}$$
$$= \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

14. The resonant frequency for TE_{mnp} mode in a waveguide resonator is given by

$$f = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

15. The resonant wavelength for TE_{mnp} mode in a waveguide resonator is given by

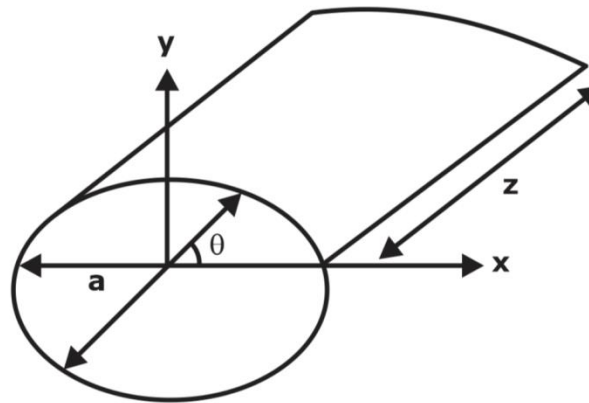
$$\lambda_r = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}}$$

CHAPTER 9: CIRCULAR WAVEGUIDE

1. Circular Waveguide: A circular waveguide is a tubular circular conductor. Figure shows circular waveguide of radius a and length z , placed in cylindrical coordinate systems.

- A plane wave propagating through a circular waveguide results in TE and TM modes.
- The vector Helmholtz wave equation for a TE and TM wave travelling in a z -direction in a circular waveguide is given as,

$$\nabla^2 H_z = 0 \text{ and } \nabla^2 E_z = 0$$



Circular Waveguide

2. TE Modes in Circular Waveguide: Helmholtz equation of H_z in circular guide is given as

$$\nabla^2 H_z = \gamma^2 \cdot H_z$$

TE_{mn} modes in circular waveguide

$$E_r = E_{0r} J_n \left(\frac{P'_{nm} r}{a} \right) \sin(n\phi) \cdot e^{-j\beta z}$$

$$E_\phi = E_{0\phi} J'_n \left(\frac{P'_{nm} r}{a} \right) \cos(n\phi) \cdot e^{-j\beta z}$$

$$E_z = 0$$

$$H_r = -\frac{E_{0\phi}}{Z_g} J'_n \left(\frac{P'_{nm} r}{a} \right) \cos(n\phi) \cdot e^{-j\beta z}$$

$$H_\phi = \frac{E_{0r}}{Z_g} J_n \left(\frac{P'_{nm} r}{a} \right) \sin(n\phi) \cdot e^{-j\beta z}$$

$$H_z = H_{0z} J_n \left(\frac{P'_{nm} r}{a} \right) \cos(n\phi) \cdot e^{-j\beta z}$$

$$Z_g = \frac{E_r}{H_\phi} = -\frac{E_\phi}{H_r} = \text{represent characteristic wave impedance in the guide,}$$

when $n = 0, 1, 2, 3$ and $m = 1, 2, 3, 4, \dots$

The first subscript n represents, number of full cycles of field variation in one revolution through 2π radian of ϕ , while second subscript m indicates the number of zeros of E_ϕ i.e.,

$J'_n(P'_{nm}r/a)$ along the radius of a guide.

$$\beta = \sqrt{\omega^2 \mu \epsilon - \left(\frac{P'_{nm}}{a}\right)^2}$$

$$h = \frac{P'_{nm}}{a} = \omega \sqrt{\mu \epsilon}, f_c = \frac{P'_{nm}}{2\pi a \sqrt{\mu \epsilon}}$$

The phase velocity, group velocity and guide wavelength remains same as that of rectangular waveguide.

3. TM Modes in Circular Waveguide: The TM_{nm} modes in a circular guide are defined as $H_z = 0$. But $E_z \neq 0$, in order to transmit energy in the guide.

Helmholtz equation in terms of E_z in circular guide is

$$\nabla^2 E_z = \gamma^2 E_z$$

The field equation for TM_{nm} modes are given as

$$E_r = E_{or} \left(\frac{P_{nm}r}{a}\right) \cos(n\phi) \cdot e^{-j\beta z}$$

$$E_\phi = E_{o\phi} J_n \left(\frac{P_{nm}r}{a}\right) \sin(n\phi) \cdot e^{-j\beta z}$$

$$E_z = E_{oz} J_n \left(\frac{P_{nm}r}{a}\right) \cos(n\phi) \cdot e^{-j\beta z}$$

$$H_r = \frac{E_{o\phi}}{Z_g} \cdot J_n \left(\frac{P_{nm}r}{a}\right) \sin(n\phi) \cdot e^{-j\beta z}$$

$$H_\phi = \frac{E_{or}}{Z_g} \cdot J'_n \left(\frac{P_{nm}r}{a}\right) \cos(n\phi) \cdot e^{-j\beta z}$$

$$H_z = 0$$

$$h = \frac{P_{nm}}{a}, n = 0, 1, 2, 3 \text{ and } m = 1, 2, 3, 4$$

Key Points

- For TE wave $h = \frac{P'_{nm}}{a}$ and $h = \frac{P_{nm}}{a}$ for TM waves
- $\lambda_c = \frac{2\pi}{h} = \frac{2\pi a}{P_{nm}}$
- TE_{11} is the dominant mode in circular waveguide for TE_{11} , $P'_{nm} = 1.841$ So λ_c for $TE_{11} = \frac{2\pi a}{1.841}$ also for TM wave $\lambda_c = \frac{2\pi}{h} = \frac{2\pi a}{P_{nm}}$

Note: TEM mode cannot exist in circular waveguide.

4. Power Handling Capacity: For rectangular waveguide:(in watt)

$$P_{max} = 27 \left(\frac{E_d}{f_{max}}\right) \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

where, E_d = Dielectric strength of material, f_c = Cut off frequency for TE₁₀ mode, f = Operating frequency, and f_{max} = Maximum frequency

- For circular waveguide:

$$P_{max} TE_{11} = 1790a^2 \sqrt{\left[1 - \left(\frac{f_{c11}}{f}\right)^2\right]} kW$$

$$P_{max} TE_{01} = 1805a^2 \sqrt{\left[1 - \left(\frac{f_{01}}{f}\right)^2\right]} kW$$

5. Power Transmission in Circular Waveguide or Coaxial Lines

For a loss less dielectric:

$$P_0 = \frac{1}{2Z_g} \int_0^{2\pi} \int_0^a \left[|E_r|^2 + |E_\phi|^2 \right] r dr d\phi$$

$$P_0 = \frac{Z_g}{2} \int_0^{2\pi} \int_0^a \left[|H_r|^2 + |H_\phi|^2 \right] r dr d\phi$$

where, $Z_g = \frac{E_r}{H_\phi} = -\frac{E_\phi}{H_r}$ = Wave impedance in guide, a Radius of the circular guide,

- The average power transmitted through a circular waveguide for TE_{np} modes is given by

$$P_0 = \frac{\sqrt{1 - (f_c / f)^2}}{2\eta} \int_0^{2\pi} \int_0^a \left[|E_r|^2 + |E_\phi|^2 \right] r dr d\phi$$

- For a TM_{np} modes

$$P_0 = \frac{1}{2\eta\sqrt{1 - (f_c / f)^2}} \int_0^{2\pi} \int_0^a \left[|E_r|^2 + |E_\phi|^2 \right] r dr d\phi$$

CHAPTER 10: ANTENNA

1. Basic Antenna Elements:

Following are some basic antenna elements:

1.1. Alternating Current Element or Hertzian Dipole:

It is a very short linear antenna in which the current along its length is approximately constant.

1.2. Short Dipole:

It is a linear antenna whose length is less than $\lambda / 10$ and the approximate current distribution is triangular.

1.3. Short Monopole:

It is a linear antenna whose length is less than $\lambda / 20$ and the approximate current distribution is triangular.

1.4. Half Wave Dipole:

It is a linear antenna whose length is $\lambda / 2$ and the current distribution is sinusoidal.

1.5. Quarter Wave Monopole:

It is a linear antenna whose length is $\lambda / 4$ and the current distribution is sinusoidal.

2. Basic Antenna Elements:

2.1. Effective Length or Transmitting Antenna:

$$(L_{\text{eff}})_{T_x} = \frac{1}{I_0} \int_{-H}^H I(z) dz$$

2.2. Effective Length of Receiving Antenna:

$$(L_{\text{eff}})_{R_x} = \frac{V_{\text{oc}}}{E}$$

Effective length of an antenna is always less than the actual length.

2.3. Radiation Intensity: It is defined as the power radiated in a given direction per unit solid angle, i.e.

$$U(\theta, \phi) = r^2 P_{\text{avg}} = \frac{r^2 E^2}{2\eta_0}$$

Where

η_0 = intrinsic impedance of the medium, (Ω)

r = radius of the sphere, (m)

P_{avg} = Average power

E = electric field strength, (V/m)

2.4. Directive Gain: Directive gain is defined as the ratio of radiation intensity in that direction to the average radiation intensity, that is,

$$G_d(\theta, \phi) = \frac{U(\theta, \phi)}{U_{av}} = \frac{U(\theta, \phi)}{P_{rad} / 4\pi} = \frac{4\pi}{P_{rad}} U(\theta, \phi)$$

2.5. Directivity: It is defined as the ratio of the maximum radiation intensity to the average radiation intensity, i.e.

$$D = \frac{\max\{U(\theta, \phi)\}}{U_{av}} = G_{d,max}$$

2.6. Power Gain: The power gain of an antenna is defined as

$$G_p = \frac{4\pi}{P_t} U(\theta, \phi)$$

where P_t is the total input power given by

$$P_t = P_{rad} + P_l$$

Where P_{rad} is the radiated power, and P_l is ohmic losses in the antenna

2.7. Antenna Efficiency: It is defined as that ratio of radiated power to the total input power, i.e.

$$\eta = \frac{P_{rad}}{P_t} = \frac{P_{rad}}{P_{rad} + P_l} = \frac{G_p}{G_d}$$

2.8. Effective Area: The effective area of a receiving antenna is the ratio of the time-average power received P_r (or delivered to the load) to the time average power density \mathcal{P}_{ave} of the incident wave of antenna, i.e.

$$A = \frac{P_r}{\mathcal{P}_{ave}}$$

This expression can be further generalised as

$$A_e = \frac{\lambda^2}{4\pi} G_d(\theta, \phi)$$

3. Power Flow from Hertzian Dipole

The time-average radiated power from a Hertzian dipole is defines as

$$P_{rad} = \frac{\eta I_0^2 \pi}{3} \left(\frac{dl}{\lambda}\right)^2$$

NOTE: The radiated power in free space is given by

$$P_{rad} = 40\pi^2 \left(\frac{dl}{\lambda}\right)^2 I_0^2$$

4. Radiation Resistance of Hertzian Dipole:

The radiated power in a Hertzian dipole is equivalent to the power dissipated in an imaginary resistance R_{rad} by current $I = I_0 \cos \omega t$. i.e.

$$P_{\text{rad}} = I_{\text{rms}}^2 R_{\text{rad}}$$

$$= \frac{1}{2} I_0^2 R_{\text{rad}}$$

In free space, intrinsic impedance is $\eta_0 = 120\pi$, so the radiation resistance in free space is given by

$$R_{\text{rad}} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

5. RADIATION FROM SHORT DIPOLE ($d < (\lambda / 10)$):

The radiation resistance of the short dipole antenna is given by:

$$(R_{\text{rad}})_{\text{short dipole}} = \frac{1}{4} \times (R_{\text{rad}})_{\text{current element}}$$

$$= 20\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

$$\approx 20\pi^2 \left(\frac{d}{\lambda}\right)^2$$

6. RADIATION FROM SHORT MONOPOLE ($d < (\lambda / 20)$)

The radiation resistance of the short monopole antenna is given by

$$(R_{\text{rad}})_{\text{short dipole}} = \frac{1}{2} (R_{\text{rad}})_{\text{short dipole}}$$

$$= 10\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

7. Radiation Resistance of Half Wave Dipole Antenna

The Radiation resistance for the half wave dipole antenna can be given directly as

$$R_{\text{rad}} = \frac{2P_{\text{rad}}}{I_0^2}$$

$$= \frac{2 \times (36.54I_0^2)}{I_0^2} \approx 73\Omega$$

8. Radiation Resistance of Quarter Wave Monopole Antenna

The radiation resistance for the quarter wave monopole antenna can be given directly as

$$R_{\text{rad}} = \frac{2P_{\text{rad}}}{I_0^2}$$

$$= \frac{2 \times (18.28I_0^2)}{I_0^2} \approx 36.54\Omega$$

9. Array Factor

The array factor of the two-element array is defined as

$$AF = 2 \cos \left[\frac{1}{2} (\beta d \cos \theta + \delta) \right] e^{j\delta/2}$$

So, the normalized array factor of the two-element array is given by

$$|AF| = \left| \cos \left(\frac{\beta d \cos \theta + \delta}{2} \right) \right| = \left| \cos \left(\frac{\Psi}{2} \right) \right|$$

In the H-plane, $\theta = \pi / 2$, and the normalized array factor becomes

$$|AF| = \left| \cos \left(\frac{\beta d \cos \phi + \delta}{2} \right) \right| = \left| \cos \left(\frac{\Psi}{2} \right) \right|$$

9.1. Array Factor

Array factor of an N-element array is defined as

$$|AF| = \left| \frac{\sin \frac{N\Psi}{2}}{\sin \frac{\Psi}{2}} \right|$$

where $\Psi = \beta d \cos \phi + \alpha$

Following are some important points about an N-element array

Important Points

Since $|AF|$ has the maximum value of N, the normalized $|AF|$ is obtained by dividing $|AF|$ by N, i.e.

$$|AF| = \frac{1}{N} \left| \frac{\sin \frac{N\Psi}{2}}{\sin \frac{\Psi}{2}} \right|$$

The principal maximum occurs when $\psi = 0$, i. e

$$0 = \beta d \cos \phi + \alpha$$

$$\text{or } \cos \phi = -\frac{\alpha}{\beta d}$$

When $|AF| = 0$, $|AF|$ has nulls (or zeros); i.e.

$$\frac{N\Psi}{2} = \pm k\pi, \quad k = 1, 2, 3, \dots$$

Where k is not a multiple of N.

A broadside array has its maximum radiation directed normal to the axis of the array, i.e. $\psi = 0, \phi = 90^\circ$ so that $\alpha = 0$.

And end-fire array has its maximum radiation directed along the axis of the array, i.e.

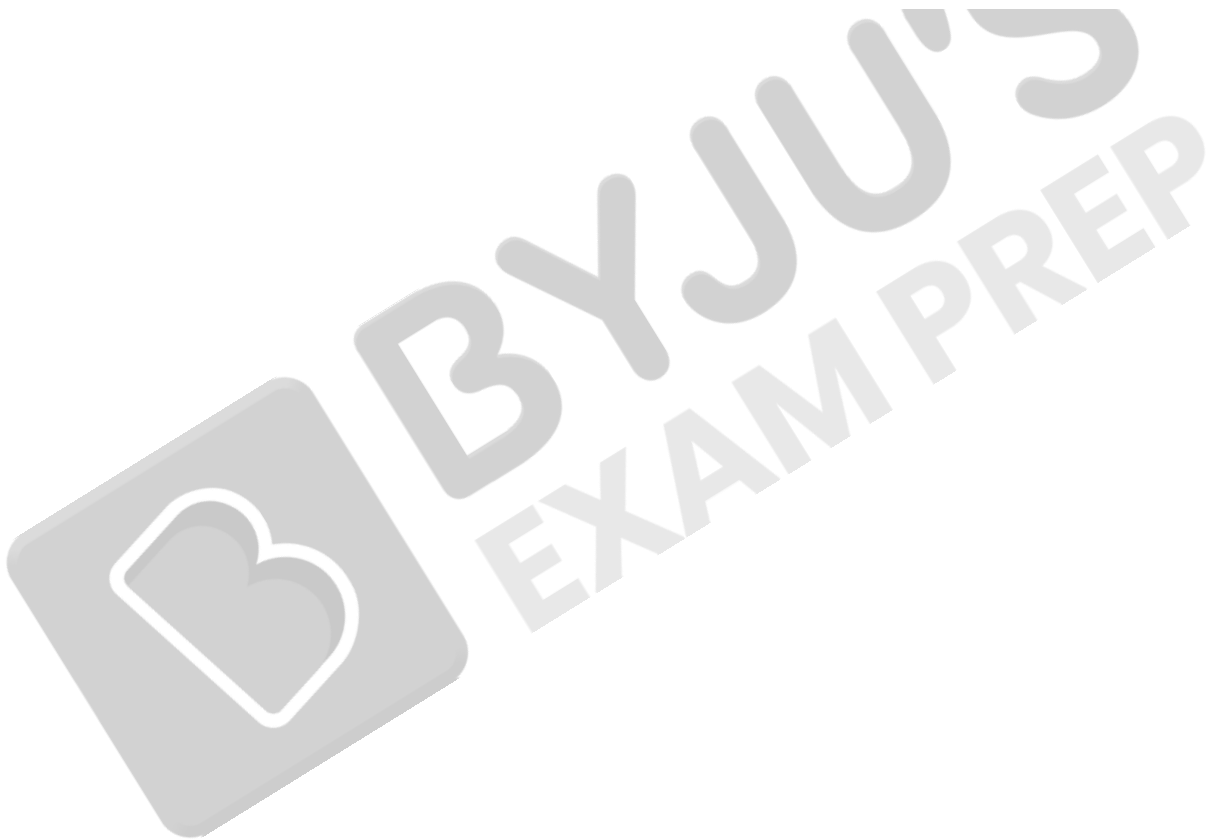
$$\psi = 0, \phi = \begin{bmatrix} 0 \\ \pi \end{bmatrix} \text{ so that } \alpha = \begin{bmatrix} -\beta d \\ \beta d \end{bmatrix}$$

10. FRIIS EQUATION

The Friis Transmission equation relates the power received to the power transmitted between two antennas separated by $R > \frac{2D^2}{\lambda}$, D is the largest dimension of either antenna.

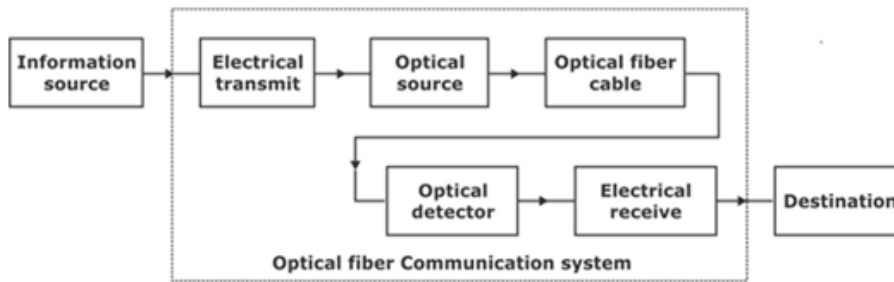
$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{ot} G_{or} = \frac{G_{ot} G_{or}}{\left(\frac{4\pi R}{\lambda}\right)^2}$$

Above equation is known as the Friis Transmission Equation, and it relates the power P_r served to the receiver load) to the input power of the transmitting antenna P_t . The term $\left(\frac{4\pi R}{\lambda}\right)^2$ is called the space loss factor, and it takes into account the losses due to the spreading of the energy by the antenna.



CHAPTER 11: OPTICAL FIBRE

1. Block diagram of Optical Communication system:

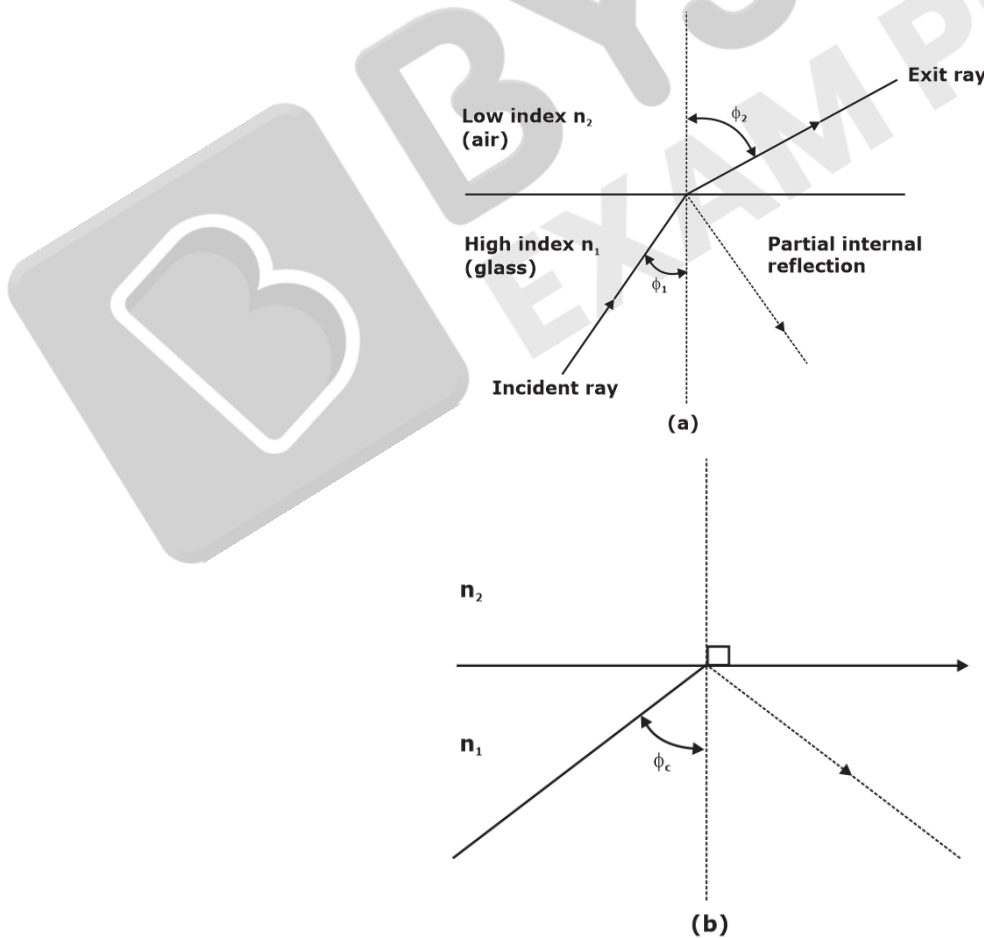


The optical fibre communication system

2. Total internal reflection

The angles of incidence ϕ_1 and refraction ϕ_2 are related to each other and to the refractive indices of the dielectrics by Snell's law of refraction which states that:

$$n_1 \sin \phi_1 = n_2 \sin \phi_2$$



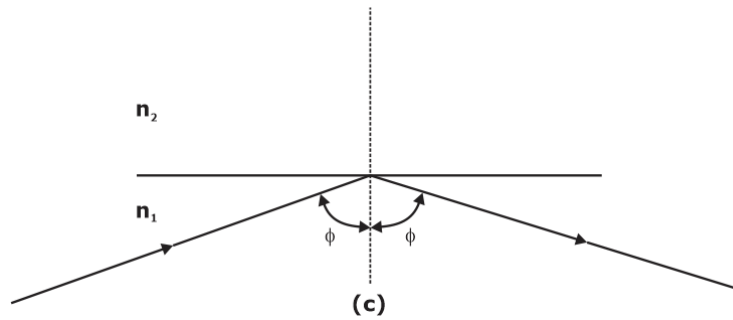


Figure: Light rays incident on a high to low refractive index interface (e.g. glass-air): (a) refraction; (b) the limiting case of refraction showing the critical ray at an angle ϕ_{ci} (c) total internal reflection where $\phi > \phi_c$

Or:

$$\frac{\sin \phi_1}{\sin \phi_2} = \frac{n_2}{n_1}$$

From above equation, the value of the critical angle is given by:

$$\sin \phi_c = \frac{n_2}{n_1}$$

When the angle of refraction is 90° and the refracted ray emerges parallel to the interface between the dielectrics, the angle of incidence must be less than 90° . This is the limiting case of refraction and the angle of incidence is now known as the critical angle ϕ_c .

3. Acceptance angle

For rays to be transmitted by total internal reflection within the fiber core they must be incident on the fiber core within an acceptance cone defined by the conical half angle θ_a . Hence θ_a is the maximum angle to the axis at which light may enter the fiber in order to be propagated, and is often referred to as the acceptance angle for the fiber.

4. Numerical aperture

the NA is defined as;

$$NA = n_0 \sin \theta_a = (n_1^2 - n_2^2)^{\frac{1}{2}}$$

Where n_1 is core refractive index and n_2 is cladding refractive index, θ_a is the acceptance angle.

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

$$\approx \frac{n_1 - n_2}{n_1} \quad \text{For } \Delta \ll 1$$

Hence combining above equations, we can write:

$$NA = n_1 (2\Delta)^{\frac{1}{2}}$$

5. Step index fibres

The refractive index profile may be defined as:

$$n(r) = \begin{cases} n_1 & r < a \text{ (core)} \\ n_2 & r \geq a \text{ (cladding)} \end{cases}$$

The total number of guided modes or mode volume M_s for a step index fibre is related to the V value for the fibre by the approximate expression:

$$M_s \approx \frac{V^2}{2}$$

6. Single-mode fibers

Thus single-mode propagation of the LP_{01} mode in step index fibers is possible over the range:

$$0 \leq V < 2.405$$

Where V is normalized frequency.

7. Cut off wavelength

The cut off wavelength λ_c given by:

$$\lambda_c = \frac{\pi a n_1}{V_c} (2\Delta)^{\frac{1}{2}}$$

where V_c is the cut off normalized frequency. Hence λ_c is the wavelength above which a particular fiber becomes single-mode.

For step index fiber where $V_c = 2.405$, the cut off wavelength is given by

$$\lambda_c = \frac{V\lambda}{2.405}$$
