BYJU'S EXAM PREP

GATE/ESE

Civil Engineering

Design of Concrete Structures

Important Formula Notes

IMPORTANT FORMULAS ON DESIGN OF CONCRETE STRUCTURES

CHAPTER-1-INTRODUCTION

1. Important Codes considered in the design

IS 456:2000	RCC
IS 1893	Earthquake
IS 13920	Ductile Detailing
IS 1343	Prestress
IS 3370 (Part-I/II/III/IV)	Water tank
IS 800:2007	Steel
IS 1905	Masonry work

2. Permissible Limit for Impurities in the water as per IS 456:2000

Impurity	**Maximum permissible limit (mg/l)**
Organic	**200**
Inorganic	**300**
Sulphates (as SO_3)	**400**
Suspended matter	**2000**
Chloride as Cl	**2000- plain concrete** **500- Reinforce concrete work**

3. IMPORTANT TESTS ON CEMENT

i. Consistency- Vicat apparatus
ii. Initial and final setting time- Vicat apparatus
iii. Soundness test- Autoclave test
iv. Specific gravity- Le chatelier flask
v. Fineness by specific surface- Blaine air permeability test

Note: Water cement ratio $\propto \frac{1}{\text{Compressive strength}}$ (Abram's law)

4. IMPORTANT CRITERIA USED IN RCC

i. **Comparison of workability by various methods**

Degree of workability	Slump (mm)	Vee-Bee (sec)	Compacting
Very low	Nil	20-10	0.70 – 0.75
Low	0 – 25	10 – 5	0.75 – 0.80
Medium	25 – 75	5 - 3	0.80 – 0.85
High	75 – 150	3-0	0.85 – 0.92
Very high	> 150	-	> 0.92

ii. **Minimum cement contents and maximum w/c ratio for durability**

	Plain cement concrete PCC		Reinforcement cement concrete RCC		Minimum grade of concrete	
Exposure	**Minimum cement kg/m^3**	**Maximum Free w/c**	**Minimum cement kg/m^3**	**Maximum free w/c**	**PCC**	**RCC**
Mild	220	0.60	300	0.55	-	M20
Moderate	240	0.60	300	0.50	M15	M25
Severe	250	0.50	320	0.45	M20	M30
Very Severe	260	0.45	340	0.45	M20	M35
Extreme	280	0.40	360	0.40	M25	M40

iii. **Exposure conditions**

Environment	Exposure condition
Mild	Concrete surfaces protected against weather or aggressive conditions, except those situated in the coastal area
Moderate	Concrete surfaces sheltered from rain or freezing whilst wet Concrete exposed to condensation and rain Concrete continuously underwater Concrete in contact or buried under non-aggressive soil/groundwater Concrete surfaces sheltered from saturated salt air in the coastal area
Severe	Concrete surface exposed to severe rain, alternate wetting and drying or occasional freezing whilst wet or severe condensation. Concrete completely immersed in seawater. Concrete exposed to the coastal environment

Very severe	Concrete surfaces exposed to seawater spray. corrosive fumes or severe freezing conditions whilst wet concrete in contact or buried under aggressive subsoil groundwater
Extreme	The surface of members in the tidal zone. Members in direct contact with liquid/solid aggressive chemicals

5. NOMINAL COVER

It is minimum clear cover required for the outermost layer of steel reinforcement.

Minimum Nominal cover

Member	Mild (mm)	Moderate (mm)	Severe (mm)	Very severe (mm)	Extreme (mm)
Slab	20	30	45	50	75
Beam	25	30	45	50	75
Column	40	40	45	50	75
Foundation	40	50	50	50	75

6. CHARACTERISTIC STRENGTH OF CONCRETE (f_{ck})

i. The strength below which 5% of test results are expected to fail is called characteristic strength of concrete.

ii. f_{ck} value gives compressive strength at 28 days after casting.

iii. If f_m is targeted mean strength and f_{ck} is characteristics strength, then

$$f_m = f_{ck} + 1.65 \times \propto$$

$\propto$ = standard deviation

Value of standard deviation

Grade	standard deviation ($\propto$) N/mm²)
M10- M15	3.5
M 20-M25	4
M 30 & Higher	5

7. ACCEPTANCE CRITERIA FOR CONCRETE

As per IS code 456:2000

The avg. strength of four non-overlapping consecutive tests should not be

- $f_{av} \geq (f_{ck} + 0.825 \times \sigma)$ or $f_{avg} \geq (f_{ck} + 3)$ {whichever is more}
- For individual test results ITR $\geq (f_{CK} - 3)$
- Test results should be obtained after testing on atleast three cubes. The difference in each test block strength and average strength should not be more than 15%

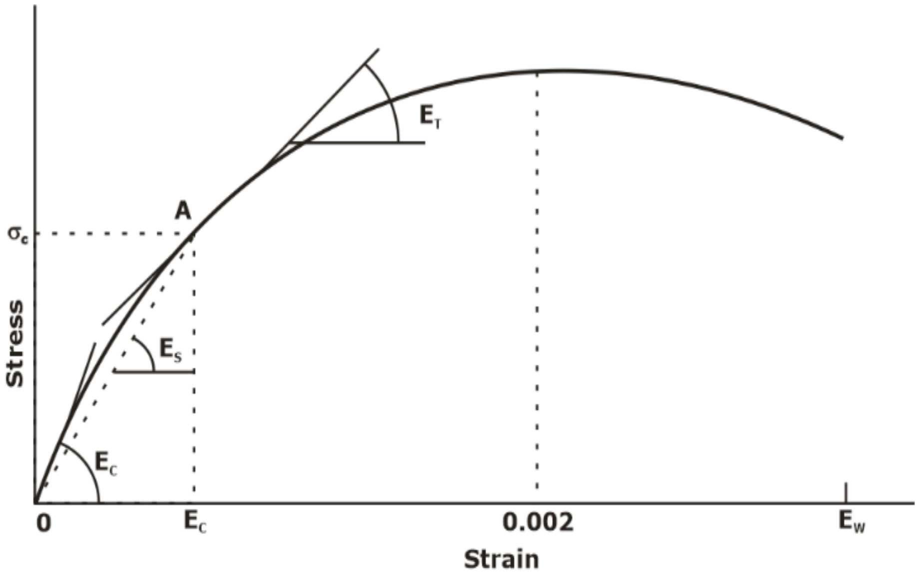
8. MODULUS OF ELASTICITY OF CONCRETE

i. **Initial Tangent modulus (E_T) -** Tangent's slope at any point on the curve is called initial tangent modulus. It gives the instant value of modulus of elasticity.

ii. **Secant modulus/ Static modulus (E_S) -** The slope of the line joining any point of the curve to origin is called the secant modulus of elasticity.

iii. **Initial tangent of elasticity/dynamic modulus of elasticity (E_C)-** It is the modulus of elasticity of concrete at the origin.

$$E_T = E_s = E_c = 5000\sqrt{f_{ck}}$$



The above formula holds only for the short term. For long term elastic coefficient (E_L)

$$E_L = \frac{5000\sqrt{f_{ck}}}{1+\theta}$$

θ = creep coefficient

Table for creep coefficient

Age of loading	Creep coefficient
7 days	2.2
28 days	1.6
1 year	1.1

9. STRIPPING TIME

Type of formwork	Minimum period before removing formwork
a. Vertical formwork to columns, walls, beams	16-24 hours
b. Soffit formwork to slabs (Props to be refixed immediately after removal of formwork)	3 days
c. Soffit formwork to beams (Props to be refixed immediately after removal of formwork)	7 days
d. Props to slabs i) Spanning upto 4.5 m ii) Spanning over 4.5 m	 7 days 14 days
e. Props to beams i) Spanning upto 6 m ii) Spanning over 6 m	 14 days 21 days

10. DESIGN METHODS

i) Working Stress Method or Elastic Theory

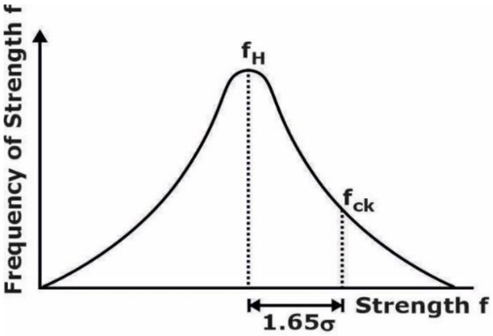
ii) Limit State Method

iii) Ultimate Load Method or Whitney's Theory

CHAPTER-2-LIMIT STATE METHOD OF DESIGN

1. Characteristic strength of materials

- The term 'characteristic strength 'means that the value of the strength of material below, which is not more than the minimum acceptable percentage of test results, are expected to fall.
- IS 456:2000 have accepted the minimum acceptable percentage as 5% for reinforced concrete structures.



f_H= Mean strength
f_{ck}= Characteristic strength
σ =Standard deviation
Frequency distribution curve for strength

- Characteristic strength = Mean strength – K x standard deviation or

$$f_k = f_m - K \times S_d$$

where, f_k = characteristic strength of the material

f_m = mean strength

K = constant = 1.65

S_d = standard deviation for a set of test results.

The value of standard deviation (s_d) is given by

$$S_d = \sqrt{\frac{\sum \delta^2}{n-1}}$$

Where δ = deviation of the individual test strength from the average or mean strength of n samples.

n = number of test results

IS 456:2000 has recommended minimum value of n = 30

2. Partial safety factor for loads

The partial safety for loads, as per IS 456:2000 are given in the table below

Load combination	Limit state of collapse			Limit state of Serviceability		
	DL	LL	WL/EL	DL	LL	WL/EL
DL+IL	1.5	1.5	-	1.0	1.0	-
DL+WL	1.5or 0.9*	-	1.5	1.0	-	1.0
DL+IL+WL	1.2	1.2	1.2	1.0	0.8	0.8

(* This value is to be considered when stability against overturning or stress reversal is critical)

LIMIT STATE OF COLLAPSE IN FLEXURE

- In bending, the maximum compressive strain in concrete (at the outermost fibre) ε_{cu} shall be taken as 0.0035.
- For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factorof 1.5 shall be applied in addition to this.

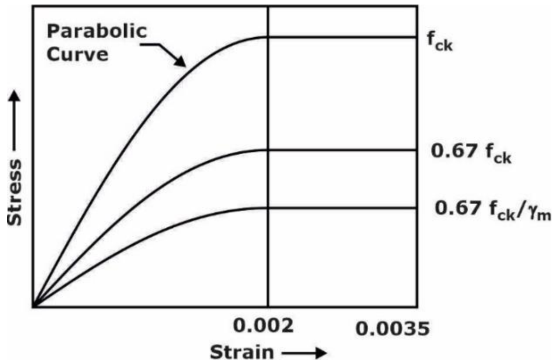
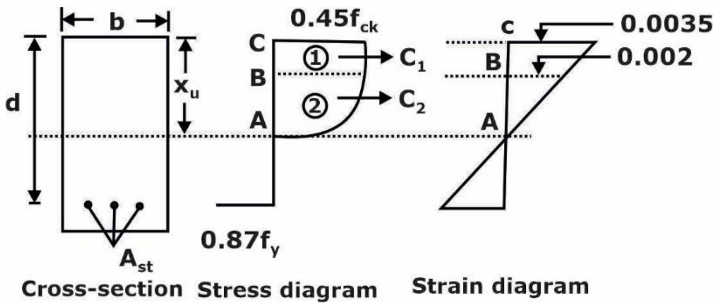


Figure: Stress-strain curve for concrete

- For the design purpose of reinforcement, the partial safety factor γ_m equal to 1.15 shall be applied.
- The maximum strain in the tension reinforcement in the section at failure shall not be less than :

$$\frac{f_y}{1.15E_s} + 0.002$$

3. Singly Reinforced Beam



$$\left(\frac{x_u}{d}\right)\text{lim} = \frac{0.0035}{\frac{0.87\,fy}{E_S}+0.0055}$$

Table: Limiting depth of neutral axis for different grades of steel

Steel Grade	Fe 250	Fe 415	Fe 500
Xu,lim/d	0.531	0.478	0.46

d= effective width

Note: Limiting depth of neutral axis depends only on the grade of steel and is independent of the grade of concrete.

i. Depth of neutral Axis

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

ii. Lever Arm= d-0.42x_u (d= effective width and x_u is depth of neutral axis)

iii. Ultimate Moment of resistance

M_u = 0.36 f_{ck}bx_u (d – 0.42x_u) ; for all x_u

Alternatively, in terms of the steel tensile stress,

M_u = 0.87 f_y× A_{st} (d – 0.42x_u) ; for all x_u

Case – 1: $\frac{x_u}{d}$ equal to the limiting value $\frac{x_{u,max}}{d}$: Balanced section

Case – 2: $\frac{x_u}{d}$ less than limiting value: under-reinforced section

Case – 3: $\frac{x_u}{d}$ more than limiting value: over-reinforced section.

iv. Computation of M_u

a. $x_u < x_{u,max}$

- In this case,steel yields before the crushing of concrete and the failure is ductile. In construction, under-reinforced sections are preferred as it gives warning before the collapse.
- M_u= 0.87 $f_y A_{st}$ (d – 0.42 x_u)(calculated from tension side)

b. $x_u = x_{u,max}$

- In this case,the yielding of steel and crushing of concrete takes place simultaneously.
- $$M_{u,lim} = 0.36 \frac{x_{u,max}}{d}\left(1 - 0.42\frac{x_{u,max}}{d}\right) f_{ck} bd^2$$

C. $x_u > x_{u,max}$

In this case, crushing of concrete occurs before yielding steel, and sudden failure occurs.

- On the other hand, when steel reaches $\frac{0.87 f_y}{E_s} + 0.002$, the strain of concrete far exceeds 0.0035. Hence, it is not possible. Therefore, such a design is avoided, and the section should be redesigned.
- The moment of resistance M_u for such an existing beam is calculated by restricting x_u to $x_{u,max}$ only, and the corresponding M_u will be as per the case when $x_u = x_{u,max}$.

Table: Limiting value of the moment of resistance for different grades of steel

Steel Grade	Fe 250	Fe 415	Fe 500
MOR_{lim}	0.148 f_{ck} bd^2	0.138 f_{ck} bd^2	0.133 f_{ck} bd^2

2. DOUBLY REINFORCED SECTION

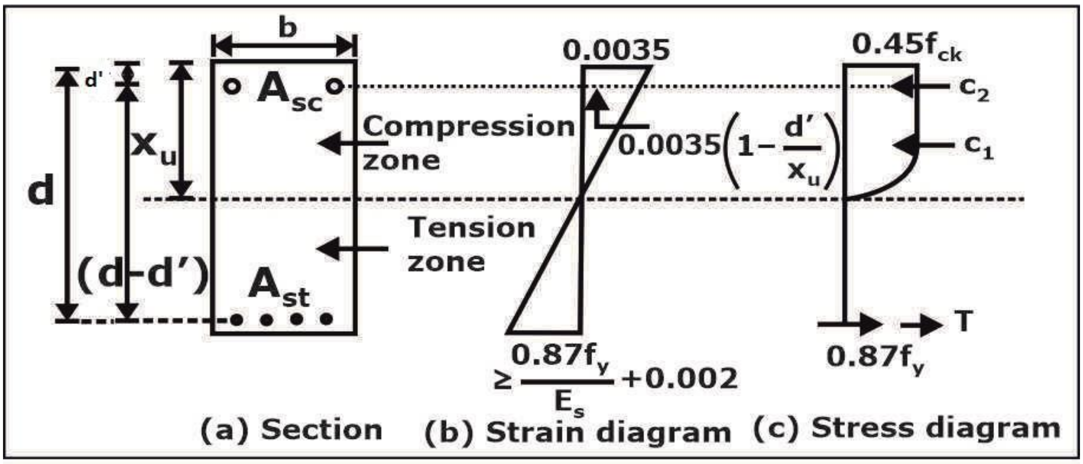
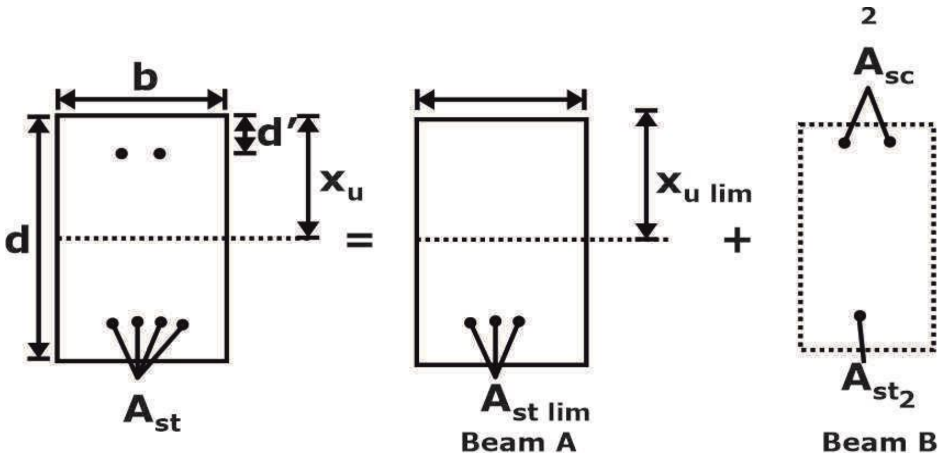


Fig. Doubly reinforced beam

Analysis of doubly reinforced beam



$M_u = M_1 + M_2$

$M_1 = MOR_{lim}$ (corresponding to limiting moment of resistance)

- $M_2 = M_u - M_1 = 0.87\ f_y\ A_{st2}\ (d-d') = A_{sc}\ (f_{sc} - f_{cc})\ (d - d')$

Where A_{st2} = Area of additional tensile reinforcement

A_{sc} = Area of compression reinforcement

f_{sc} = stress in compression reinforcement

f_{cc} = Compressive stress in concrete at the level of compression reinforcement

Since the addition of reinforcement is balanced by the additional compressive force.

$A_{sc}\ (f_{sc} - f_{cc}) = 0.87\ f_y A_{st2}$

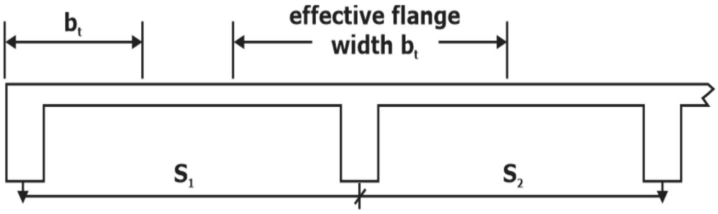
- The strain at the level of compression reinforcement is $0.0035\left(1-\frac{x_u}{x_{u,max}}\right)$
- The total area of reinforcement shall be obtained by

$A_{st} = A_{st1} + A_{st2}$

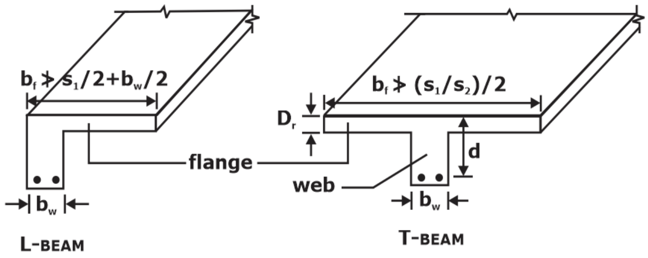
A_{st1} = Area of reinforcement for a singly reinforced section for $M_{u,lim}$

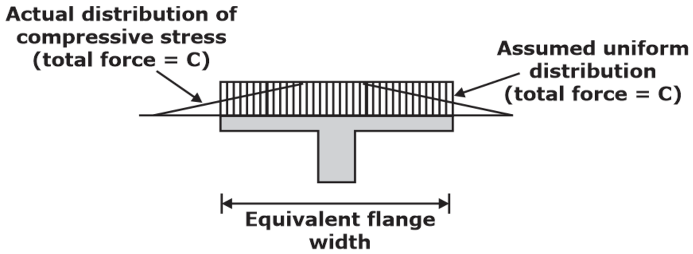
$$A_{st2} = \frac{A_{sc}\left(f_{sc} - f_{cc}\right)}{0.87f_y}$$

3. T BEAMS AND L BEAMS



Beam-supported floor slab system





i. Effective width of Flange

a. For beam casted monolithically with slab

effective width of flange b_f (Cl. 23.1.2 of Code) are given as follows:

$$b_f = \begin{cases} l_0/6 + b_w + 6D_f \text{ for T-Beam} \\ l_0/12 + b_w + 3D_f \text{ for L-Beam} \end{cases} \text{[Eq. 1]}$$

- b_w is the breadth of the web,
- D_f is the thickness of the Flange
- l_0 is the "distance between points of zero moments in the beam" (which may be assumed as 0.7 times the effective span in continuous beams and frames).

b. For Isolated Beams

$$b_f = \begin{cases} \dfrac{l_0}{l_0/b+4} + b_w \text{ for isolated T-Beam} \\ \dfrac{0.5l_0}{l_0/b+4} + b_w \text{ for isolated L-Beam} \end{cases}$$

ii. Analysis of Singly Reinforced Flanged Sections

Case A: If the neutral axis lies in the Flange area (i.e., $x_u < D_f$)

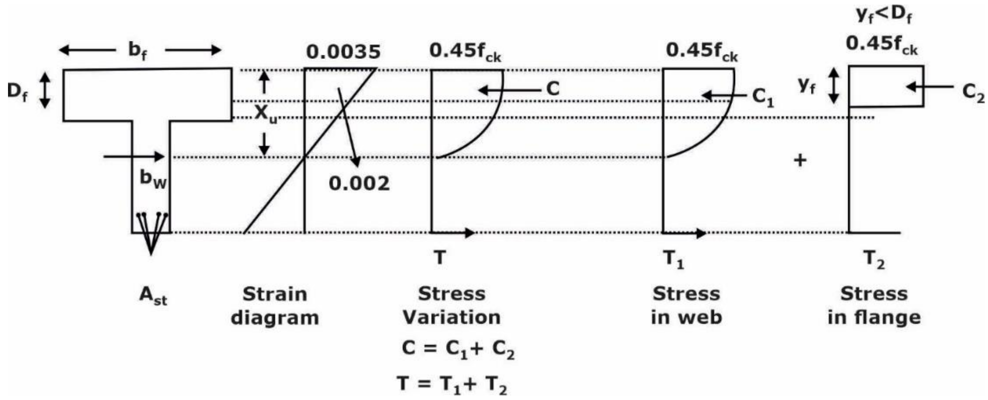
It will behave as rectangular section with width equal to that of flange.

$$x_u = \frac{0.87 \times f_y \times A_{st}}{0.36 \times f_{ck} \times b_f}$$

b_f = width of flange

CASE B: If theneutral axis lies in the web region (i.e., $x_u > D_f$)

I. When $x_u > D_f$ and $x_u < \frac{7}{3}D_f$



$$y_f = 0.15X_u + 0.65D_f$$

a. For calculation of NA

$$0.36 \times f_{ck} \times b_w \times X_u + 0.45 \times f_{ck} \times (b_f - b_w) \times y_f = 0.87 \times f_y \times A_{st}$$

b. For Moment of Resistance

- $M_u = 0.36 \times f_{ck} \times b_w \times X_u \times (d - 0.42 \times X_u) + 0.45 \times f_{ck} \times (b_f - b_w) \times y_f \left(d - \frac{y_f}{2}\right)$
- $M_u = 0.87 \times f_y \times A_{st1} \times (d - 0.42X_u) + 0.87 \times f_y \times A_{st2} \times \left(d - \frac{y_f}{2}\right)$
- $A_{st1} = \frac{0.36 \times f_{ck} \times b_w \times X_u}{0.87 \times f_y}$
- $A_{st2} = \frac{0.45 \times f_{ck} \times (b_f - b_w) \times y_f}{0.87 \times f_y}$

II. When $x_u > D_f$ and $x_u > \frac{7}{3}D_f$

Variation of stress and strain will be same as that of case I except that $y_f = D_f$

a. For calculation of NA

$$0.36 \times f_{ck} \times b_w \times X_u + 0.45 \times f_{ck} \times (b_f - b_w) \times D_f = 0.87 \times f_y \times A_{st}$$

b. Ultimate moment of resistance

- $M_u = 0.36 \times f_{ck} \times b_w \times X_u \times (d - 0.42 \times X_u) + 0.45 \times f_{ck} \times (b_f - b_w) \times D_f \left(d - \frac{D_f}{2}\right)$
- $M_u = 0.87 \times f_y \times A_{st1} \times (d - 0.42X_u) + 0.87 \times f_y \times A_{st2} \times \left(d - \frac{D_f}{2}\right)$
- $A_{st1} = \frac{0.36 \times f_{ck} \times b_w \times X_u}{0.87 \times f_y}$
- $A_{st2} = \frac{0.45 \times f_{ck} \times (b_f - b_w) \times D_f}{0.87 \times f_y}$

CHAPTER-3-WORKING STRESS METHOD OF DESIGN

1. WORKING STRESS METHOD/ MODULAR RATIO METHOD

1.1. Factor of safety

- In WSM, the structure's design is based on actual stresses developed in concrete and steel due to actual loads.
- These stresses are always kept within maximum permissible stress.

(ii) Assumptions

(a) At any section, a plain section before bending remains plain after bending.

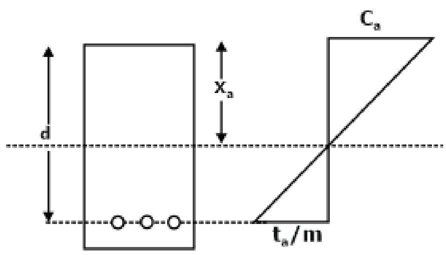
- The strain diagram of the section is linear.
- The stress diagram is also linear in WSM.
- Within the elastic limit, strain is directly proportional to stress.

(b) All tensile stresses are taken by steel only and none by concrete.

1.2. Value of modular ratio, (m)

- $m = \frac{280}{3\sigma_{cbc}}$
- $m = \frac{E_s}{E_c}$
- The value mentioned above of 'm' is a value with the only partial effect of creep.

2. ANALYSIS OF A RCC BEAM FOR FLEXURE



(i) Stress relationship

$$\frac{C_a}{x_a} = \frac{t_a / m}{d - x_a}$$

(ii) Actual depth by NA

$$\frac{Bx_a^2}{2} = m.A_{st}\left(d - x_a\right)$$

(iii) Total resultant compressive force

$$C = B \times x_a \times \left(\frac{C_a}{2}\right)$$

(iv) Total tensile force

$T = t_a \times A_{st}$

(v) Also, C = T

Therefore, $B \times x_a \times \frac{C_a}{2} = t_a \times Ast$

(vi) for Bending moment

$$BM = B \times x_a \times \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) \text{ (Compression approach)}$$

$$BM = t_a \times A_{st}\left(d - \frac{x_a}{3}\right) \text{ (tension approach)}$$

2.1. Stress behaviour

There are three types of sections.

(i) Under reinforced section

- The actual depth of neutral axis < critical depth
- At the maximum bending moment that can be allowed.

→ stress in concrete ⇒ $C_a < \sigma_{cbc}$

→ stress in steel ⇒ $t_a = \sigma_{st}$

- Steel reaches its maximum permissible stress first (prior to concrete)
- Under reinforced sections are always preferred to use.

(ii) Over reinforced section

- $X_a > x_c$
- At moment of resistance,

→ stress in concrete ⇒ $C_a = \sigma_{cbc}$

→ stress in steel ⇒ $t_a < \sigma_{st}$

- Concrete reaches maximum permissible stress prior to steel.

(iii) Balanced section

- $X_a = x_c$
- At the moment of resistance,

→ stress in concrete ⇒ $C_a = \sigma_{cbc}$

→ stress in steel ⇒ $t_a = \sigma_{st}$

- For balanced section.
- $x_c = k \times d = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right) \times d = \left(\frac{280}{3\sigma_{st} + 280}\right) \times d$ (Since, $m = \frac{280}{3\sigma_{cbc}}$)
- Value of x_c for different grades

x_c = 0.40 d → Fe 250

x_c = 0.289 d → Fe 145

x_c = 0.253 d → Fe 500

CHAPTER-4-DESIGN FOR SHEAR

1. NOMINAL SHEAR STRESS

- The average shear stress can be calculated using the following formula:

$$\tau_v = \frac{V_u}{bd}$$

Where,

V_u = ultimate shear stress at the section

b = width of the section

d = effective depth of the section

- For beams with varying depth

$$\tau_v = \left[\frac{V_u \pm (M_u/d)\tan\beta}{bd}\right]$$

Where,

β = inclination of flexural tensile force to the horizontal.

M_u = factored bending moment at the section.

+ sign is used when bending moment increases as the depth increases, and -ve sign is used when bending moment decreases as the depth increases.

2. DESIGN SHEAR STRENGTH

The design shear strength of concrete depends upon two factors:

(i) Grade of concrete

(ii) Percentage tensile reinforcement

3. MINIMUM SHEAR REINFORCEMENT

The minimum amount of shear reinforcement should always be provided in the RCC section to

- To prevent bursting of concrete cover.
- Avoid sudden shear failure.
- To hold the reinforcing bars together
- To prevent cracks in the concrete due to shrinkage, thermal stresses etc.
- IS 456 specifies the following formula for the calculation of minimum shear reinforcement.

$$\left(\frac{A_{sv}}{bS_v}\right) \geq \left(\frac{0.4}{0.87\ f_y}\right)$$

Where A_{sv} = total cross-sectional area of stirrup legs effective in shear.

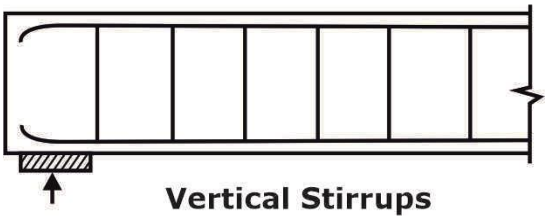
S_v = spacing of stirrups.

b = breadth of the beam or breadth of the web of flanged beams.

f_y = characteristic strength of stirrup reinforcement in N/mm^{2,} which shall not be taken greater than 415 N/mm^2.

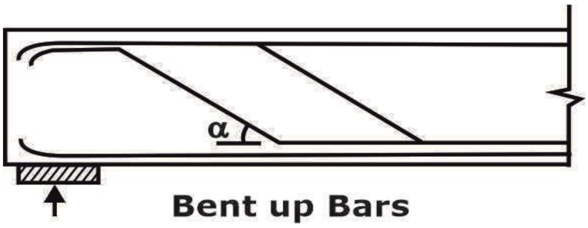
4. DESIGN OF SHEAR REINFORCEMENT

- When the nominal shear stress exceeds the design shear strength, extra shear reinforcement is provided in the form of
  - Vertical/Inclined stirrup



Vertical Stirrups

  - Bent up bars



Bent up Bars

- The following formula gives the design shear stress:

$$V_{us} = (V_u - V_c) = (\tau_v - \tau_c)\ bd$$

Where,

V_u = factorshear force.

V_c = shear resisted by concrete

V_{us} = shear resisted by reinforcements (Links or bent up bars)

τ_v = nominal shear stress.

τ_c = shear stress resisted by concrete

5. Vertical/Inclined Stirrup

- The spacing of the vertical stirrup can be calculated by using the following formula:

$$S_v = \left[\frac{0.87\ f_y\,.\,A_{sv}\,.\,d}{V_{us}}\right]$$

Where,

A_{sv} = total area of the legs of shear reinforcement.

S_v = spacing of the links.

d = effective depth of the section.

7. For inclined stirrup:

$$V_{us} = \frac{0.87\ f_y\ A_{st}\ d}{S_v}(\sin\alpha + \cos\alpha)$$

Where,

α = Angle of inclination of stirrup

8. The spacing between two stirrups shall be a minimum of the following values:

$$\left.\begin{array}{l} (i)\ S_v \ngtr (S_v)_{min\ shear\ rft} \\ (ii)\ S_v \ngtr 0.75d\ (for\ vertical\ stirrups) \\ \quad \ngtr d\ (for\ inclined\ stirrups) \\ (iii)\ S_v \ngtr 300\ mm \end{array}\right\}\ use\ minimum\ of\ three$$

9. Bent Up Bars

$V_{us} = 0.87 \times f_y \times A_{sb} \times \sin\alpha$ Where,

α = Angle of inclination of the bar with horizontal

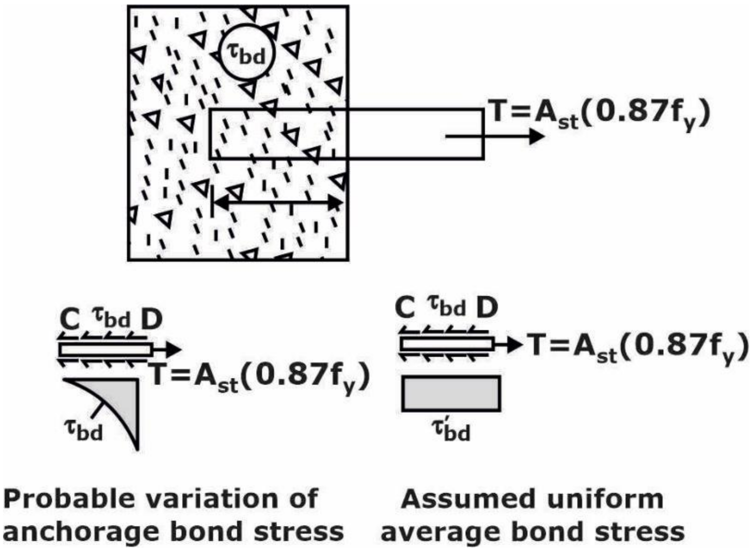
A_{sb} = area of bent up bar

CHAPTER-5-DESIGN FOR BOND, ANCHORAGE AND LAP LENGTH

1. BOND AND ANCHORAGE

➢ **Development Length**

- For LSM $L_d = \frac{0.87f_y}{4\tau_{bd}}\phi$
- For WSM $L_d = \frac{\phi \times \sigma_{st}}{4\tau_{bd}}$



➢ **Permissible bond stress in tension** τ_{bd}, (N/mm^2)

Grade of concrete	M15	M20	M25	M30	M35	M40 and above
Design bond stress (LSM)	–	1.2	1.4	1.5	1.7	1.9
Design bond stress (WSM)	0.6	0.8	0.9	1	1.1	1.2

- For deformed bars above value must be increased by 60%
- For a bar in compression, the above value must be increased by 25%

CHAPTER-6-DESIGN FOR TORSION

1. Equivalent Shear

The following formula calculates the equivalent shear:

$$V_e = V_u + 1.6\frac{T_u}{B}$$

Where,

V_e = Equivalent shear force

V_u = Shear force

T_u = Torsional moment

B = Width of the section

2. Longitudinal reinforcement

The longitudinal tension reinforcement should be designed to carry equivalent bending moment of

$$M_{e1} = M_u + M_t$$

Where M_u = Flexural moment

$$M_t = T_u\left(\frac{1+\frac{D}{b}}{1.7}\right)$$

T_u = Torsional moment

D = Overall depth of the section

3. Transverse Reinforcement

As per Is 456, transverse reinforcement is provided in the form of two-legged closed hoops. The following formula obtains the area of transverse reinforcement:

$$A_{sv} = \frac{T_u s_v}{b_1 d_1\left(0.87f_y\right)} + \frac{V_u s_v}{2.5d_1\left(0.87f_y\right)}$$

Subjected to a maximum value of $\frac{(\tau_{ve}-\tau_c)bs_v}{0.87f_y}$.

Where,

T_u = Torsional moment

V_u = Shear force

s_v = Spacing of shear reinforcement

b_1 = centre to centre distance between corner bar in the direction of width

d_1 = centre to centre distance between corner bar in the direction of depth

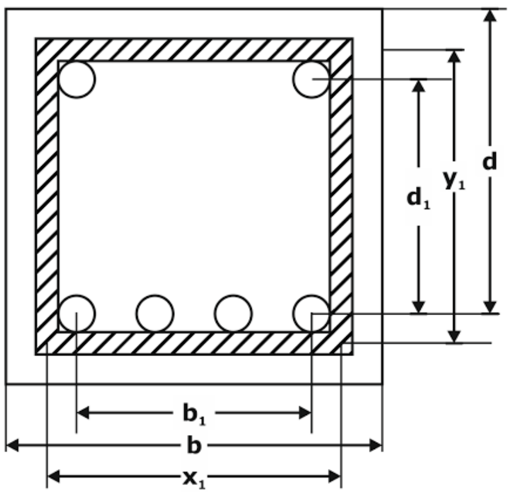
b = width of the member

f_y = Characteristics strength of stirrup reinforcement

τ_{ve} = equivalent nominal shear stress

τ_c = shear strength of concrete

Note: The distribution of transverse reinforcement should be such that the spacing should be a minimum value of x_1, $\frac{x_1+y_1}{4}$ or 300 mm where x_1 and y_1 are the short and long dimensions of the stirrup.



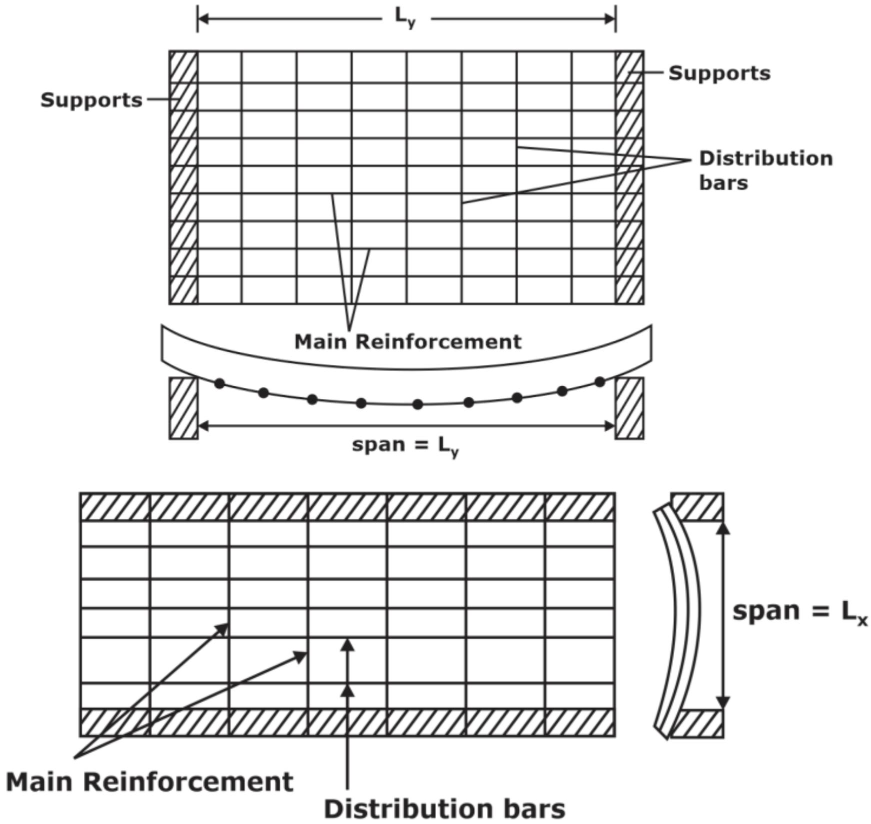
x_1 = b_1 + Diameter of longitudinal bar + Diameter of stirrup

y_1 = d_1 + Diameter of longitudinal bar + Diameter of stirrup

CHAPTER-7-DESIGN OF SLABS

1. ONE WAY SLAB

- If $\frac{L_y}{L_x}$ >, 2, slab is designed as a one-way slab.



Main Reinforcement distribution bars

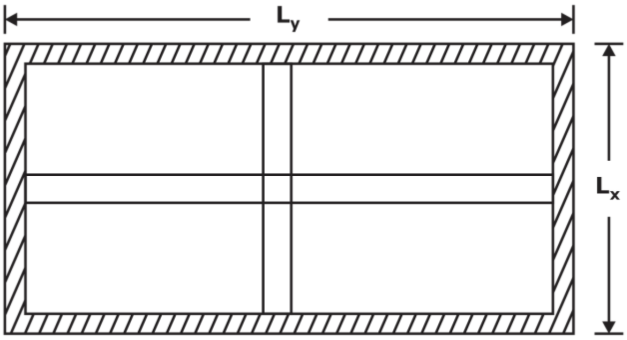
Ly = longer span

Lx = shorter span

Main reinforcement is always provided along with the supports

2. TWO WAY SLAB

- $\frac{Ly}{Lx} \leq 2$



3. GENERAL CONSIDERATIONS FOR DESIGN OF SLABS

L_C = clear span

w= width of support

d= depth of support

i. Effective length–

- For slabs that not built integrally with their supports

  L_{eff} = Minimum of {(L_C + d), (L_C + w)}

  L_C = clear span

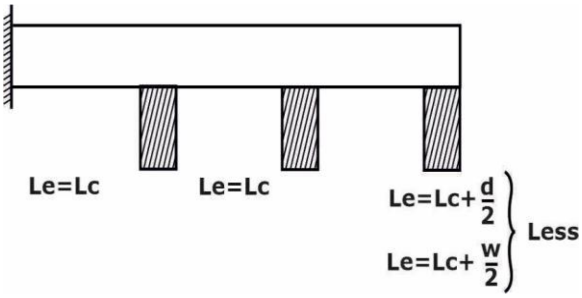
- For Continuous Slabs

  - If width of support $w \leq \frac{L_c}{12}$
    - L_{eff} = minimum {(L_C + d), (L_C + w)}

  - If the width of the support $w \geq \min\left(\frac{L_C}{12}, 600mm\right)$
    - One end is fixed other is continuous, or both ends continuous

      L_{eff} (L_e) =L_C
    - One end is continuous, and the other end is simply supported

$$L_{eff}\left(L_e\right) = \min\left(\left[L_C + \frac{W}{2}\right], \left[L_C + \frac{d}{2}\right]\right)$$



- For cantilevers

- $Le = Lc + \frac{d}{2}$ for fixed ends
- $Le = Lc + \frac{w}{2}$ for continuous supports

ii. Deflection

As per clause 23.2 of IS–456:2000,

- Final deflection due to all loads including the effect of temperature, creep and shrinkage should not exceed $\frac{span}{250}$
- Deflection including effect of creep, temperature and shrinkage occurring after creation of partition and application of finishes should not exceed $\frac{span}{350}$ or 20mm whichever is less.

iii. Span to depth ratio

- For span < 10m

Type of support	Span/depth
a. Cantilever	7
b. Simply supported	20
c. Continuous	26

- For span > 10m

$\frac{span}{depth}$ $A \times \frac{10}{\text{span in m}}$

iv. Concrete cover

- The cover at each end of the reinforcement bar should be not less than 25mm or twice the diameter of the bar

v. Reinforcement

The reinforcement for a slab spanning in one direction consists of main bars.

- The minimum reinforcement in either direction shall be 0.15% of the total cross-section area.
- This value is reduced to 0.12% when high strength deformed bars are used.
- Distribution Reinforcement
  - These are reinforcement provided running at right angles to the main steel to distribute the load and the temperature and shrinkage stresses.
- Diameter of bars
  - $\text{diameter of main bar} = \frac{\text{thickness of slab}}{8}$
  - Diameter of distribution bars = 8 mm

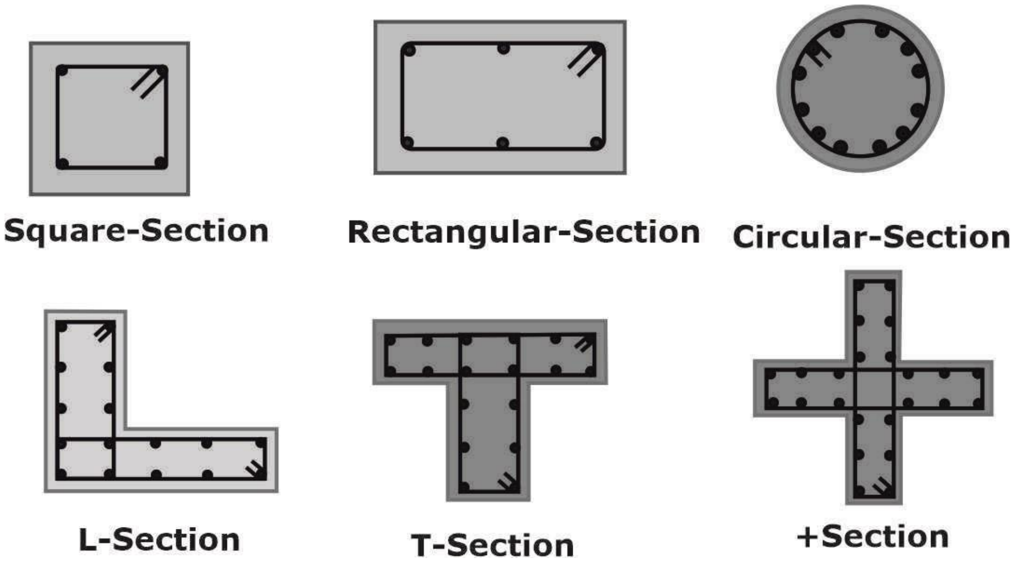
vi. Spacing between bars

The maximum spacing of main reinforcement should not exceed min(3d, 300 mm) and for distribution reinforcement it should not exceed min (5d, 300 mm).

CHAPTER-8-DESIGN OF COLUMNS

- If l_{eff}> 3Least Lateral Dimension, it is called a column
- If l_{eff}< 3 LLD, it is called a pedestal

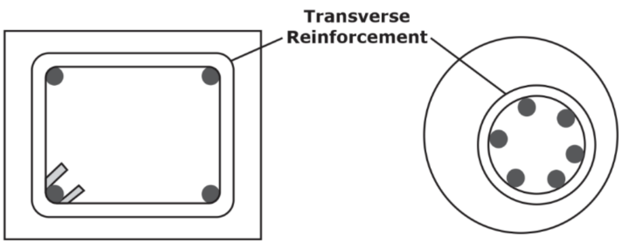
Note: LLD is the least lateral dimension



1. IS RECOMMENDATION REGARDING LONGITUDINAL REINFORCEMENT

- The minimum percentage of longitudinal reinforcement **should not be less than 0.8%** to prevent buckling of the column.
- The maximum percentage of longitudinal reinforcement **shall not be more than 6%** to avoid congestion of reinforcements, making it very difficult to place the concrete and consolidate it.
- The minimum number of longitudinal bars provided in a column shall be four in rectangular columns and sixfor circular columns.
- The bars shall not be less than **12 mm** in diameter
- Maximum spacing of longitudinal bars = **300 mm**
- Minimum cover to the column reinforcement equals **40 mm or diameter of the bar**, whichever is greater.

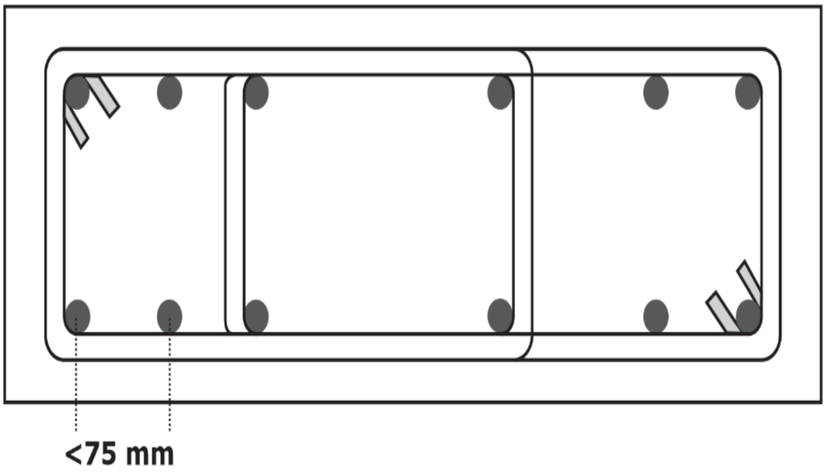
2. TRANSVERSE REINFORCEMENT



- Diameter shall not be less than the maximum of these values

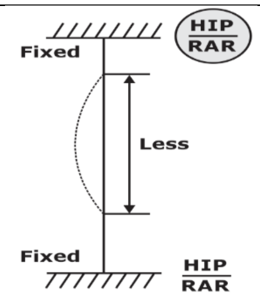
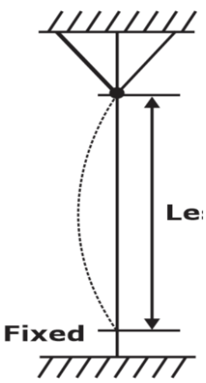
$$\begin{cases} \dfrac{\text{Diameter of longitudinal bar}}{4} \\ 6\text{mm} \end{cases}$$

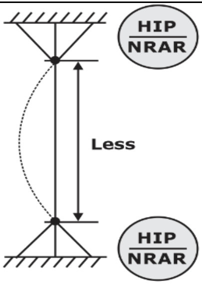
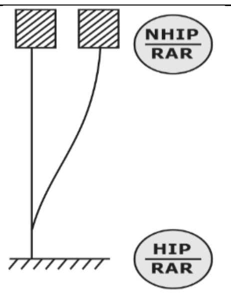
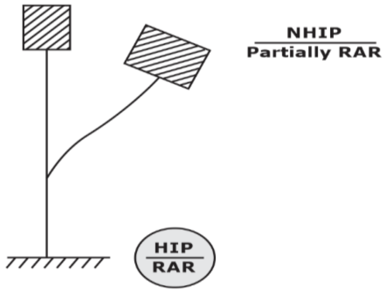
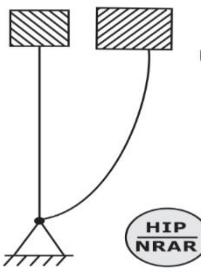
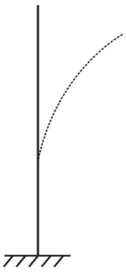
- Suppose the longitudinal bars are not spaced **more than 75 mm** on either side. In that case, transverse reinforcement needs only to go around the corner and alternate bars to provide effective lateral supports.



- The diameter of **transverse reinforcement need not exceed 20 mm.**
- Spacing of transverse reinforcement shall not exceed the least of the following
  - Least lateral dimension
  - Sixteen times the diameter of the smallest longitudinal reinforcing rod.
  - 48 times the diameter of transverse reinforcement

3. EFFECTIVE LENGTH

Case No.	End condition	Theoretical L_{eff}	Recommended L_{eff}
1		0.5 L_0	0.65 L_0
2	HIP NRAR  HIP RAR	0.7 L_0	0.80 L_0

3		1.0 L_0	1.0 L_0
4		1.0 L_0	1.20 L_0
5.		–	1.5 L_0
6.	**NHIP** **RAR** 	2.0 L_0	2.0 L_0
7.		2.0 L_0	2.0 L_0

HIP: Held in position

NHIP: Not held in position

RAR: Restrained against rotation

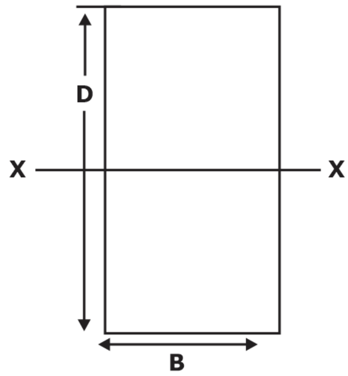
NRAR: Not restrained against rotation

4. SHORT COLUMNS AND LONG COLUMNS

- If $\frac{L_{eff}}{LLD} < 12$, short column
- If $\frac{L_{eff}}{LLD} > 12$, long column

5. SLENDERNESS RATIO (SR_{xx})

- It is the ratio of the length of a column and the least radius of gyration of its cross-section.
- $SR_{xx} = \frac{L_{ex}}{\text{Lateral dimension perpendicular to } x-x}$



$$Sr_{xx} = \frac{Le_x}{\text{lateral dimension}} = \frac{Le_x}{D}$$

- Maximum slenderness ratio for column = 60
- If one end is restrained, unsupported length

$$L > \frac{100\,B^2}{D}$$

- For a short column, $\left[\left(\frac{L_{ex}}{D}\right) \text{and} \left(\frac{L_{ey}}{B}\right)\right]_{both} < 12$
- For a long column, $\left[\left(\frac{L_{ex}}{D}\right) \text{and} \left(\frac{L_{ey}}{B}\right)\right]_{both} > 12$

6. MINIMUM ECCENTRICITY

$$e_{x,min} = \max\ \{\frac{l_{ex}}{500} + \frac{D_x}{30}, 20\text{ mm}\}$$

$$e_{y,min} = \max\ \{\frac{l_{ey}}{500} + \frac{D_y}{30}, 20\text{ mm}\}$$

7. DESIGN OF COLUMNS

All columns shall be designed for

- Axial load = P_0
- Moment about x – x = M_{ux} $M_{ux} > M_{uxmin}$
- Moment about y – y = M_{uy}

$$M_{uy} \not< M_{uy\,min}$$

(a) IS code method – WSM method

- The safe load on a short column is given by

$$W_c = \begin{bmatrix}\text{area of}\\ \text{concrete}\end{bmatrix} \times \begin{bmatrix}\text{safe stress}\\ \text{in concrete}\end{bmatrix} \times \begin{bmatrix}\text{Area of}\\ \text{steel}\end{bmatrix} \times \begin{bmatrix}\text{Safe stress}\\ \text{in steel}\end{bmatrix}$$

$W_C = \sigma_{cc} \times A_c + \sigma_{sc} \times A_s$

$A_c = (BD - A_{SC})$ = net area of concrete

- Safe stresses in concrete

	M20	M25	M30	M35	M40
σ_{cc}	5	6	8	9	10

- Safe stresses in concrete

	Fe 250	Fe350	Fe415	Fe500
σ_{sc}	130 N/mm^2	130 N/mm^2	190 N/mm^2	190 N/mm^2

- The load-carrying capacity of a long column

$P = C_r (\sigma_{cc} A_c + \sigma_{sc} A_{sc})$

C_r = reduction coefficient

- For Rectangular or square column

$$C_r = 1.25 - \frac{L_{eff}}{48B}$$

B= Least lateral dimension

- For irregular shape

$$C_r = 1.25 - \frac{L_{eff}}{160 \times i_{min}}$$

i_{min} = Minimum radius of gyration

$$i_{min} = \sqrt{\frac{I}{A}}$$

- Load carrying capacity of composite column

$P = C_r [\sigma_{cc} A_c + \sigma_{sc} A_{sc} + \sigma_{mc} A_{mc})$

Where A_{mc} = area of metal core (if provided) $\not>$ $0.2 \times BD$

σ_{mc} = stresses of metal core

σ_{mc} = 125 N/mm^2 for structural steel

= 70 N/mm^2 for cast Iron

(b) LSM METHOD

i. When column is subjected to only axial load

$P_U = 0.45 f_{ck} A_C + 0.75 f_y A_{SC}$

Note -The column section should be designed for combined axial load and bending moment due to the minimum specified eccentricity. If **minimum eccentricity (as per the above specification) is less than or equal to 0.05 D**, the column section can be designed as per the equation

$P_U = 0.4\, f_{Ck} \times A_C + 0.67\, f_y \times A_{SC}$

8. DESIGN OF CIRCULAR COLUMN

For a column with helical reinforcement

- If $e_{min} \leq 0.05$ D

Load-carrying capacity is increased by 5%

so$P_U = 1.05\, [0.4\, f_{Ck}\, A_C + 0.67\, f_y\, A_{SC}]$

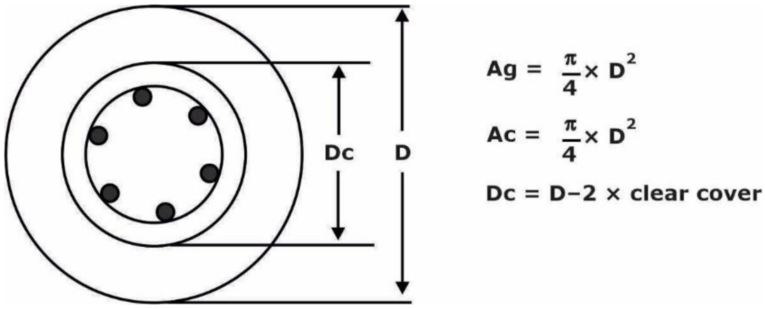
- For Helical reinforcement

$$\frac{0.36\, f_{ck}}{f_y}\left(\frac{A_g}{A_c} - 1\right) \leq \frac{V_h}{V_c}$$

Where,

A_g = Gross area of the section

A_C = Area of the core of the helically reinforced column



$$A_C = \frac{\pi}{4} \times D_C^2$$

$$A_g = \frac{\pi}{4} \times D^2$$

D_C = D – 2 × Clear cover

V_C = Volume of core portion in unit length of column = 1000 A_C mm^3

V_h= Volume of helical reinforcement in a unit length of the column

V_h= No. of turns × length of one turn × c/s are of helical reinforcement=$\frac{1000}{pitch} * (\Pi D_h) * \frac{\Pi}{4} \varphi_n{}^2$

Where, $D_n = D_c - \varphi_n$,φ_n is the diameter of helical reinforcement

- Pitch (p)

For helical reinforcement

$(i).p \not> 75mm$

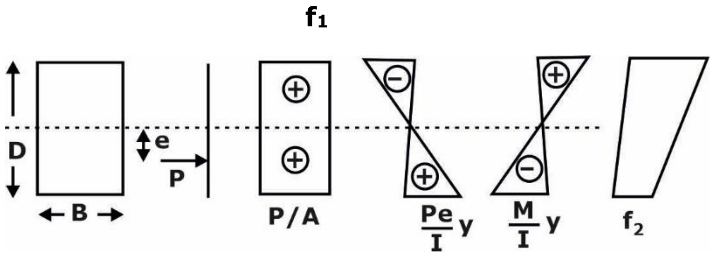
$(ii).p \not> \left(\frac{D_C}{6}\right)$

$(iii).p \not< 25mm$

$(iv).p \not< 3\phi_n$

CHAPTER-9-DESIGN OF PRESTRESSED CONCRETE

- Prestressed concrete is a block of concrete in which internal stress of suitable magnitude and distribution are introduced to counteract the stresses resulting from external load to the desired degree.
- Prestressed concrete is different from a conventional RCC structure due to the application of an initial load on the structure prior to its use.
- Important points:



Upto a desired degree

$$\left[f_1 / f_2 \Rightarrow \frac{p}{A} \pm \frac{Pe}{I} y \pm \frac{M}{I} y\right] \quad \underbrace{\left(\frac{P}{A} + \frac{Pe}{I} y\right)}_{\text{Internal stress Introduced}} \xleftrightarrow[\text{cach other}]{\text{Counter balance}} \underbrace{\left(\frac{M}{I} y\right)}_{\text{External loading stresses}}$$

#Pretesion [Hoyer method/Long line method]

- Tensioning → casting → distress → cutting of wires.
- Pre-stressing force is applied by bond action
- Straight wires or bent wire can also be used.
- Transmission length : length required to transmit full prestressing force to concrete.

[Example:- Railway sleepers, electric poles]

#Post-tension:- Prestress principal tensile stresses = 0.24 bulk uncrushed reaction.

- Casting with ducts – wire inserted & than tensioned from
  (1) one end jacking (2) Both end jacking.
- Straight, bend, parabolic /suitable for castin site
- Force is transformed by bearing action at ends, force transfer [r/f → wedge→ plate→ concrete]
- **Anchorage zone** : bursting and spalling forces occurs at end due to concentration of load **[compression forces],** after this zone only longitudinal stresses occurs

(a) Freyssinet system
- Conical steel wedges
- Multiple wires at a line.

(b) Magnel Blaton,
- Flat sandwich wedges
- 2 wires on either side of wedge.

- Normally total 8 wires

(c) LeMcall system

- High strength ruts are used on threaded r/f.

(b) Gifford Udall

- One wire with one set of half split cones

⇒ High strength steel is used as losses are high (10-20%) of initial prestress forces

⇒ Min. Grade of concrete (M-40) → pretension (M-30) → port tension.

⇒ Height [fck] concrete means less creep, more tensile strength **but** more brittle, more shrinkage ,less ductile as high cement content[prestressed concrete remains uncracked & reduction in steel corrosion]

⇒ Min. clear cover between cable or tendon

1. 40mm.
2. 5mm + largest size of aggregate.

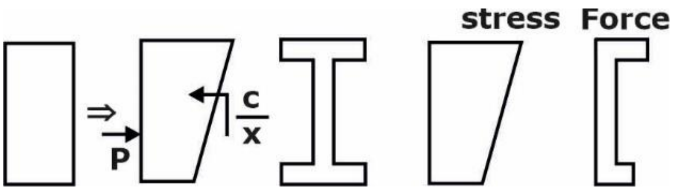
⇒Maximum stress in tendon just behind the anchorage zone is ⇒ 76% of ultimate strength of steel.

⇒ Types of section (as per cracking)

Class-I	Class-II	Class-III
• No tension • No cracking • Only compression • Section uncracked	• Tension allowed • No cracking • Tensile stress < $3N/m^2$ • Section uncracked	• Tension allowed • Cracking also allowed • Tensile stress <fcr. • Section uncracked

⇒ Some properties

- P(force) $= \frac{(f_1 + f_2)A}{2}$ (only for symmetrical section)
- C ⇒ (B × D) $\frac{(f_1 + b_2)A}{2}$



c-force ⇒ P-force at every section

BM Mx⇒ $P\bar{x}$ or $C\bar{x}$ ($\bar{x}$ = lever arm)

- Location of c-force is line of pressure & location of P-force is line of cable.

1. **Stress concept**

2. **Strength concept**→ check M_x (moment at section)'+' or '–' at section

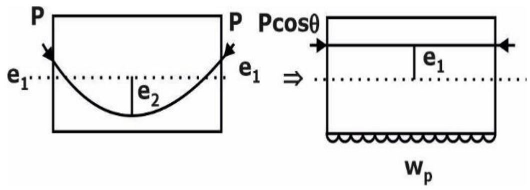
   + ⇒ C is above P

   - ⇒ C is below P

$\left(\bar{x} \Rightarrow \frac{M_p}{p}\right)$ now relate & find eccentricity (e_c)for c

$f_1 / f_2 \rightarrow \frac{C}{A} \pm \frac{Ce_c}{I} y$

3. Load balancing :-



Now find M_x net & find stress as with this condition for

- Bend wire W ⇒ 2P sin θ
- Parabolic = $= \frac{8P\cos\theta h\left(=(e_1+e_2)\right)}{L^2}$ $\qquad y = \frac{4h}{L^2}\left(Lx - x^2\right)$

⇒ Cracking moment : Moment at which tensile stresses → f_{cr}

All PSC beams are designed as uncracked section

$$FOS \Rightarrow \frac{M_{cr}}{BM_{warking}} \Rightarrow \frac{W_{cr}}{W_{warking}}$$

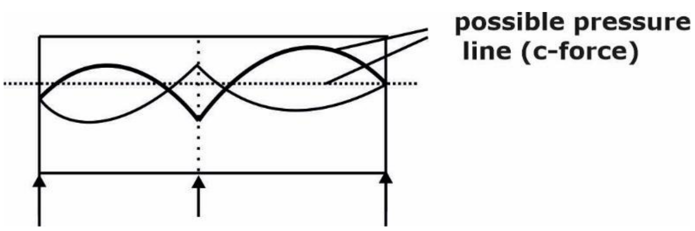
⇒ Load balancing profile of cable

$\left[e = \frac{M_x}{p}\right]$ (profile is mirror image (scale) of BMD)

⇒ Concordant profile of cable

To avoid (secondary moments ,secondary stress, change in support reaction)

[In continuous beam when simple P is applied on straight cable support reaction change so this concordant profile is used as mirror image of BMD]

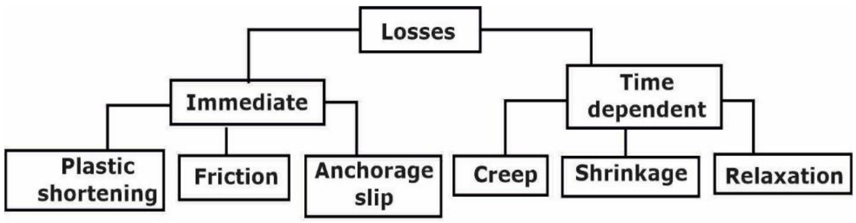


Kern Points & kern distance [No tension in section]

D/3 for  section, > D/3 for  section

A. LOSSES IN PRESTRESS

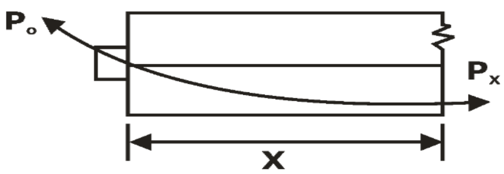
Pre-tensioning	Post-tensioning
1. Elastic deformation of concrete	1. No loss due to elastic shortening when all bars are simultaneously tensioned. If, however, wires are successively tensioned, there would be loss of prestress due to elastic deformation of concrete
2. Relaxation of stress in steel	2. Relaxation of stress in steel
3. Shrinkage of concrete	3. Shrinkage of Concrete
4. Creep of concrete	4. Creep of concrete
	5. Frictional losses
	6. Anchorage slip



Various losses in prestress

1. LOSS OF PRESTRESS DUE TO FRICTION

- The friction generated at the interface of concrete and steel during the stretching of a curved tendon in a post-tensioned member leads to a drop in the prestress along with the member from the stretching end.
- The loss due to friction does not occur in pre-tensioned members because there is no concrete during the stretching of the tendons.
- Force in the cable at a distance x from jacking end, after a frictional loss – P_x



$$P_x = P_0 e^{-(\mu\alpha + kx)}$$

Where P_x = Prestressing force at a distance x from jacking end.

P_0 = Prestressing force at jacking end.

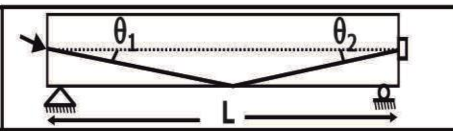
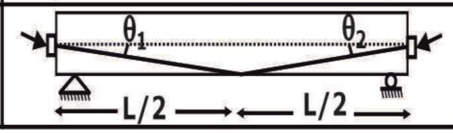
k = coefficient called wobble correction factor

μ = Coefficient for friction in the curve

α = Cumulative Angle in radian through which the tangent to the cable profile turned between any two points under consideration.

- For small (μ + kx) values, the Taylor series expansion can simplify the above expression.

$$P_x = P_o\,[1-(\mu\alpha + kx)]$$

		x	α
Jacking from one end		L	$\theta_1+\theta_2$
Jacking from both ends		L/2	$\max(\theta_1+\theta_2)$

For parabolic profile,

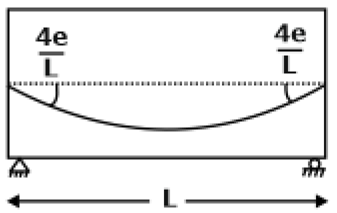
Jacking at one end, $\alpha = 2\theta = \dfrac{8e}{L}$

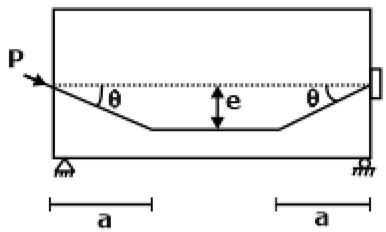
Jacking from both ends $\alpha = \theta = \dfrac{4e}{L}$

For trapezoidal profile,

Jacking at one end $\alpha = 2\theta = \dfrac{2e}{a}$

Jacking from both ends $\alpha = \theta = \dfrac{e}{a}$





2. LOSS OF PRESTRESS DUE TO ANCHORAGE SLIP

- In a post-tensioned member, when the prestress is transferred to the concrete, the wedges slip through a little distance before they get properly seated in the conical space.
- This loss due to anchorage slip = $\dfrac{E_s\Delta}{L}$

$$= \left(\frac{\Delta}{L}\right)E_s$$

E_s = Young modulus of steel in N/mm^2

Δ = Anchorage slip in mm

L = Length of cable in mm

Table:- Typical values of anchorage slip

Anchorage system	Anchorage slip (Δ)
Freyssinet	4 mm
12-5 mm φ strands	6 mm
12-8 mm φ strands	8 mm
Magnet	1 mm

3. LOSS OF PRESTRESS DUE TO CREEP OF CONCRETE

- Creep is the property of concrete by which it continues to deform with time under sustained loading.
- Creep coefficient is defined as

$$\phi = \frac{\text{creep strain}}{\text{elastic strain}} = \frac{\varepsilon_{cp}}{\varepsilon_c}$$

- Loss of stress = m φf_c
- Note that elastic shorting loss multiplied by creep co-efficient is equal to loss due to creep.

Age at loading	Creep co-efficient
7 days	2.2
28 days	1.6
1 year	1.1

4. LOSS DUE TO SHRINKAGE OF CONCRETE

- The loss of stress in steel due to the shrinkage of concrete is estimated as loss of stress $= \varepsilon_{cs} \times E_s$

Where E_s = modulus of elasticity of steel.

- ε_{cs} = total residual shrinkage strain having values of 3×10^{-4} for pre tensioning and $\varepsilon_{cs} = [(2 \times 10^{-4})/\log_{10}(t + 2)]$ for post-tensioning

  Where, t = age of concrete at transfer in days.

5. LOSS OF PRESTRESS DUE TO RELAXATION OF STEEL

Initial Stress (1)	Relaxation Loss N/mm^2
0.5 f_p	0
0.6 f_p	35
0.7 f_p	70
0.8 f_p	90

Note:

f_p is the characteristic strength of prestressing steel.

The conclusion of the above discussions:

Sr. No.	Type of loss	Equation
1	Wobble & curvature effect	$(\mu\alpha + kx)P_0$
2	Anchorage slip	$E_s\ \Delta/L$
3	Shrinkage loss	$\varepsilon_{sc}\ E_s$
4	Creep of concrete	$m\ \varphi\ f_c$
5	Elastic shortening of concrete	mf_c
6	Relaxation in steel	2 to 5% for initial stress in steel

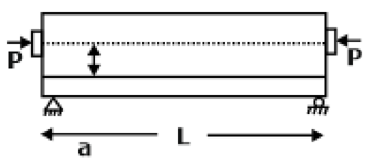
Type of loss	Pretensioned (%)	Post tensioned (%)
Elastic shorting of concrete	3	1
Shrinkage	7	6
Creep	6	5
Relaxation	2	3
Total Loss	18%	15%

Losses	Pretensioning	Post-tensioning
Length effect	No	Yes
Curvature effect	No	Yes
Anchorage slip	No	Yes
Shrinkage of concrete	Yes	Yes
Creep of concrete	Yes	Yes
Elastic deformation or shortening of concrete	Yes	No (If all wires are simultaneously tensioned) Yes (If wires are successively tensioned)

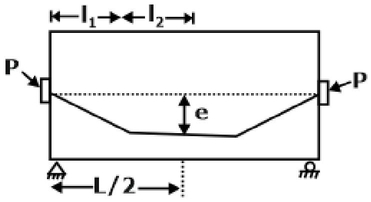
6. Deflection of Pre-stressed Beam

Short term deflection under uncracked condition can be computed using elastic theory by using area moment method (Mohr's method). Concrete beam deflects upwards on the application or transfer of prestress.

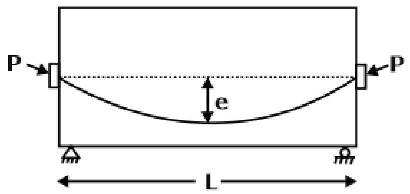
Bending moment at any section is the product of prestressing force and eccentricity at that section.



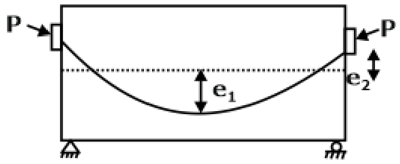
$$\Delta = \frac{PeL^2}{8EI}$$



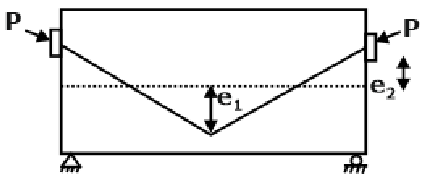
$$\Delta = \frac{-Pe}{6EI}\left(2l_1^2 + 6l_1l_2 + 3l_2^2\right)$$



$$\Delta = \frac{-5}{48}\frac{PeL^2}{EI}$$



$$\Delta = \left[\frac{-5}{48}\frac{PL^2}{EI}\left(e_1 + e_2\right)\right] + \left[\frac{Pe_2L^2}{8EI}\right]$$



$$\Delta = \left[\frac{-PL^2}{12EI}\left(e_1 + e_2\right)\right] + \left[\frac{Pe_2L^2}{8EI}\right]$$

Note:

Downward deflection due to self wt or imposed load is $\frac{5w}{384}\frac{L^4}{EI}$

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