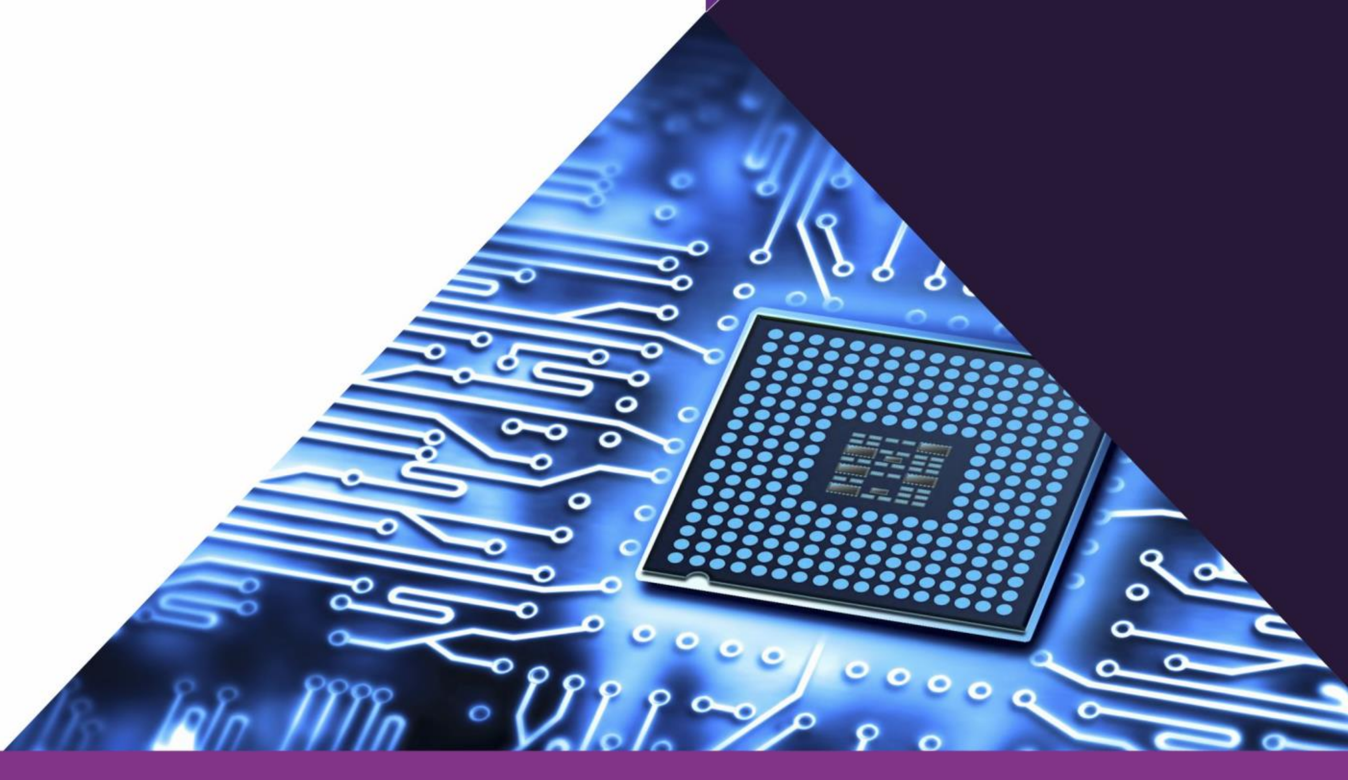
BYJU'S
EXAM PREP

# Electronics Engineering

## Communication Systems

**SHORT NOTES**



# IMPORTANT FORMULAS TO REMEMBER

## CHAPTER-1: BASICS OF COMMUNICATION SYSTEM

1. **Introduction**

**Communication** is the process of establishing a connection or link between two points for information exchange.

OR

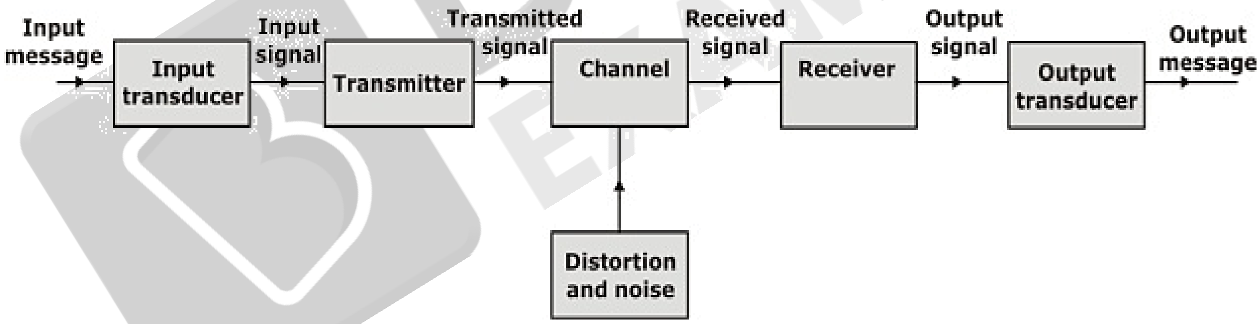
Communication is simply the basic process of exchanging information.

**Communication system**.

Typical examples of a communication system are line telephony and line telegraphy, radio telephony and radio telegraphy, radio broadcasting, point to point communication and mobile communication, computer communication, radar communication, satellite communication television broadcasting, radio telemetry, radio aids to navigation, radio aids to aircraft landing etc.

2. **The Communication Process: Elements Of A Communication System**

The whole idea of presenting the communication model is to analyse the key concepts used in communication in isolated parts and combine them to form the complete picture.




3. **Concept Of Bandwidth & Frequency Spectrum**

**3.1. Bandwidth:**

Different types of passband signals such as voice signal, music signal, TV signal, etc. Each of these signals will have its frequency range. This frequency range of a signal is known as its bandwidth.

Thus, we write $\boxed{BW = f_2 - f_1}$

The bandwidth of different signals has been listed in table 1.

**TABLE 1**

| S. No. | Type of the signal | Range of frequency in Hz | Bandwidth in Hz |
|---|---|---|---|
| 1. | Voice signal (speech) for telephony | 300 – 3400 | 3,100 |
| 2. | Music signal | 20 – 15000 | 14,980 |
| 3. | TV signals (picture) | 0 – 5 MHz | 5 MHz |
| 4. | Digital data | 300 – 3400 (If it is using the telephone line for its transmission) | 3,100 |

### 3.2. Frequency Spectrum

The frequency spectrum may be defined as the presentation of a signal in the frequency domain. It can be obtained by using either the Fourier series or the Fourier transform. It consists of the amplitude and phase spectrums of the signal.

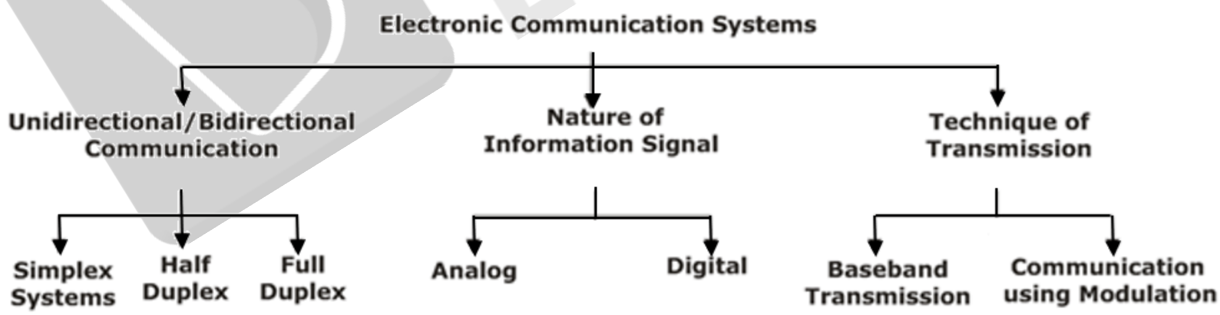
## 4. Classification Of Communication System

It shows that the electronic communication system may be categorized into three groups based on:

(i) Whether the system is unidirectional or bidirectional.

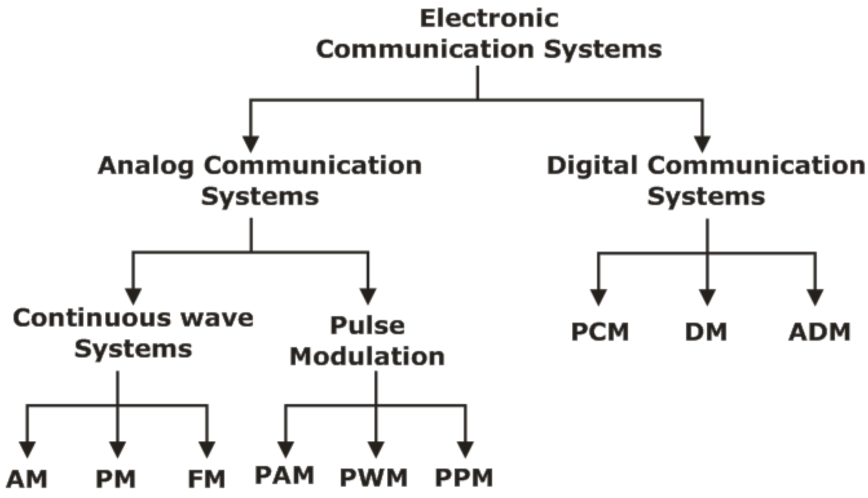
(ii) Whether it uses an analogue or digital information signal.

(iii) Whether the system uses baseband transmission or uses some modulation.




**Classification of Electronic Communication Systems**

## 5. Classification Based On The Nature Of Information SignaL




**Classification based on analogue or digital communication**

## 6. Analog Communication

The modulation system or techniques in which one of the characteristics of the carrier is varied in proportion with the instantaneous value of modulating signal is called an analogue modulation system.

### 6.1. Advantages Of Analogue Communication

Some of the advantages of analogue communication are as under:

(i) Transmitters and receivers are simple.

(ii) Low bandwidth requirement

(iii) All natural signals are analogue, so they don't have to be converted into digital signals for modulation, so the output is free from quantization errors.

### 6.2. Drawbacks Of Analogue Communication

Some of the drawbacks are as under:

(i) Noise affects the signal quality

(ii) It is not possible to separate noise and signal.

.

## 7. Digital Communication:

The modulation system or technique in which the transmitted signal is in the form of discreet pulses of constant amplitude, constant frequency and phase is called a digital modulation system.

### 7.1. Advantages Of Digital Communication

Some of the advantages of digital communication are as under:

(i) Due to the digital nature of the transmitted signal, the interference of additive noise does not introduce many errors. Hence, digital communication has a better noise immunity.

(ii) Due to the channel coding techniques used in digital communication, it is possible to detect and correct the errors introduced during the data transmission.

(iii) Repeaters can be used between transmitter and receiver to regenerate the digital signal. This improves the noise immunity further and increases the range at which the signal can be transmitted.

(iv) Multiplexing is easier in digital communications

### 7.2. Drawbacks Of Digital Communication

Some of the important drawbacks of digital communication are as under:

(i) The bit rates of digital systems are high. Therefore, they require a larger channel bandwidth as compared to analogue systems.

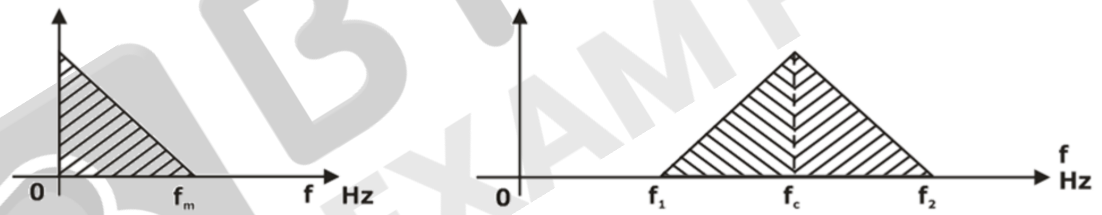
(ii) Digital modulation needs synchronization between transmitter and receiver in case of synchronous modulation.

(iii) System has an increased complexity.

## 8. Baseband And Bandpass Signals

### 8.1. Baseband Signal

The information or the input signal to a communication system can be analogue, i.e. sound, picture, or digital, e.g., computer data. The electrical equivalent of this original information signal is known as the baseband signal. Baseband is a signal that has a near-zero frequency range (or a narrow frequency "bandwidth") from close to zero hertz up to a higher cut-off frequency



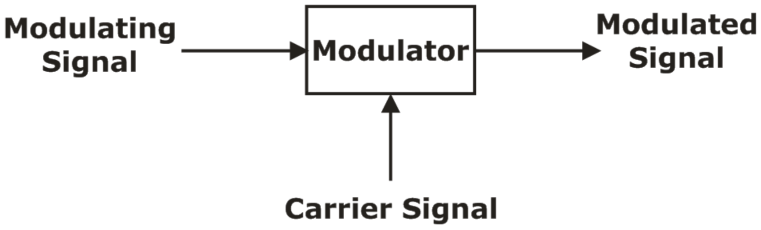

(a) Spectrum of a baseband signal (b) spectrum of a bandpass signal

### 8.2. Bandpass Signal

It may be defined as a signal which has a non-zero lowest frequency in its spectrum. This means that the frequency spectrum of a bandpass signal extends from $f_1$ to $f_2$ Hz. The modulated signal is called the bandpass signal.

## 9. Modulation

In the modulation process, two signals are used, namely the **modulating signal** and the **carrier signal**. The modulating signal is only the baseband or information signal, while the carrier is a high-frequency sinusoidal signal.




### 9.1 Need Of Modulation

**i.** To avoid interference between signals

**ii.** To decrease the length of transmitting and receiving antenna

**iii.** To allow the multiplexing of signals

## 10. Comparison Between Analog & Digital Communication

| S. No. | Analogue modulation | Digital modulation |
|---|---|---|
| (i) | The transmitted modulated signal is analogue. | The transmitted signal is digital, i.e. train of digital pulses. |
| (ii) | Amplitude, frequency or phase variations in the transmitted signal represent the information or message. | The amplitude, width or position of the transmitted pulses is constant. The message is transmitted in the form of code words. |
| (iii) | Noise immunity is poor for AM but improved for FM and PM. | Noise immunity is excellent. |
| (iv) | It is not possible to separate noise and signal. Therefore, repeaters cannot be used. | It is possible to separate signal from noise. Therefore, repeaters can be used. |
| (v) | Coding is not possible. | Coding techniques can be used to detect and correct errors. |
| (vi) | The bandwidth required is lower than that for the digital modulation methods. | Due to higher bit rates, a higher channel bandwidth is required. |
| (vii) | FDM is used for multiplexing. | TDM is used for multiplexing. |
| (viii) | Less secure. | More secure |
| (ix) | Analogue modulation systems are AM, FM, PM, PAM, PWM etc. | Digital modulation systems are PCM, DM, ADM, DPCM, etc. |

# CHAPTER-2: AMPLITUDE MODULATION

## 1. Amplitude Modulation

Amplitude modulation is defined as a process in which the amplitude of the carrier wave c(t) is varied, with the message signal m(t) keeping other parameters constant.

### 1.1 Time-Domain Description

The standard form of an amplitude-modulated (AM) wave is defined by

$x(t) = A_C [1 + k_a m(t)] \cos(2\pi f_c t)$

Where $k_a$ is a constant called the **amplitude sensitivity** of the modulator, the modulated wave so defined is said to be a "standard" AM wave because its frequency content is fully representative of amplitude modulation.

- The amplitude of the time function multiplying $\cos(2\pi f_c t)$ is called the envelope of the AM wave s(t). Using a(t) to denote this envelope, we may thus write

$a(t) = A_c |1 + k_a m(t)|$

- Two cases arise, depending on the magnitude of $k_a m(t)$, compared to unity.

**case 1:**

$|k_a m(t)| \leq 1$, for all t

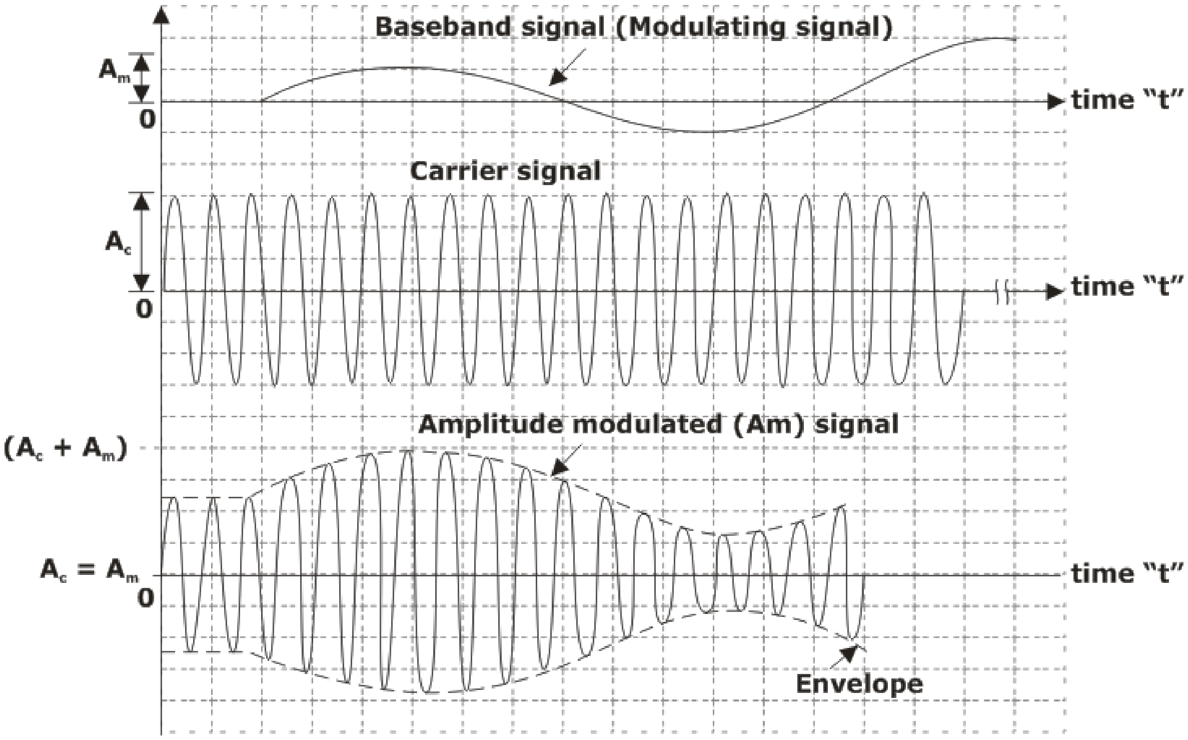
Under this condition, the term $1 + k_a m(t)$ is always non-negative. We may therefore simplify the expression for the envelope of the AM wave by writing

$a(t) = A_c(1 + k_a m(t))$, for all t

**case 2**:

$|k_a m(t)| > 1$, for all t

The maximum absolute value of $k_a m(t)$ multiplied by 100 is the **percentage modulation**. Accordingly, case 1 corresponds to a percentage modulation less than or equal to 100%, whereas case 2 corresponds to a percentage modulation of more than 100%.

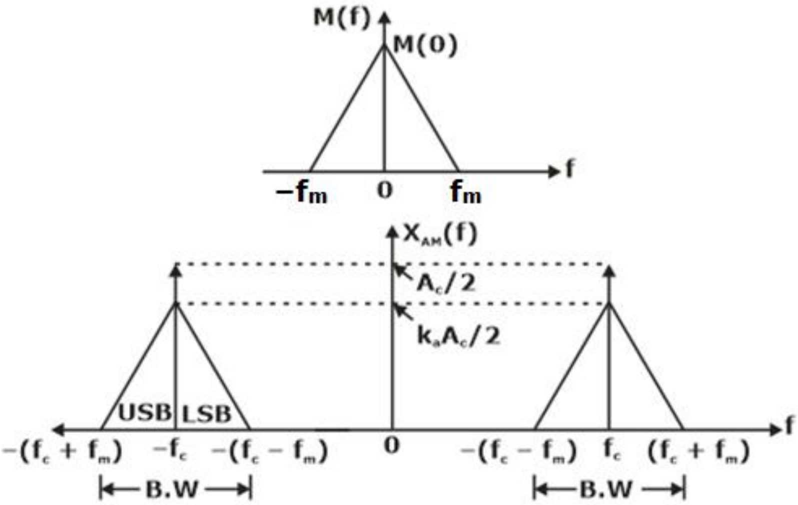



**AM waveform for sinusoidal modulating signal**

### 1.2 Frequency Domain Description

To develop the frequency description of the AM wave, we take the Fourier transform of both sides. Let S(f) denote the Fourier transform of s(t), and M(f) denote the Fourier transform of the message signal m(t); we refer to M(f) as the message spectrum. Accordingly, using the Fourier transform of the cosine function $A_C \cos(2\pi t_c t)$ and the frequency-shifting property of the Fourier transform. we may write

$$S(t) = \frac{A_c}{2}[\delta(f-f_c)+\delta(f+f_c)] + \frac{k_a A_c}{2}[M(f-f_c)+M(f+f_c)]$$




$B.W = (f_c + f_m) - (f_c - f_m)$

$B.W = 2f_m$ Hz or kHz

$B.W = 2\omega_m$ rad/s

## 3. Single Tone Amplitude Modulation

Let carrier signal,

$x(t) = A_C \cos \omega_c t$

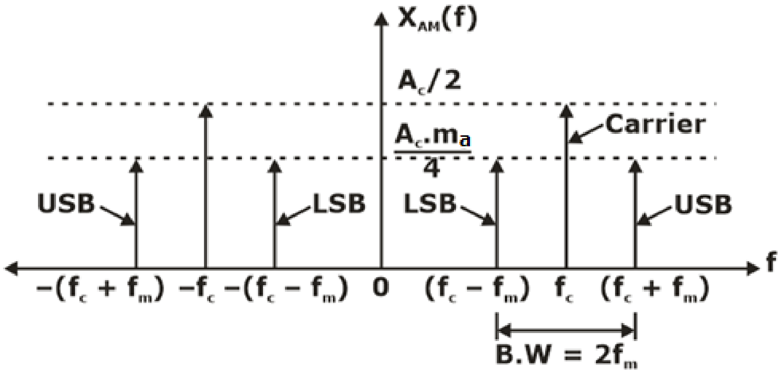
And the message signal,

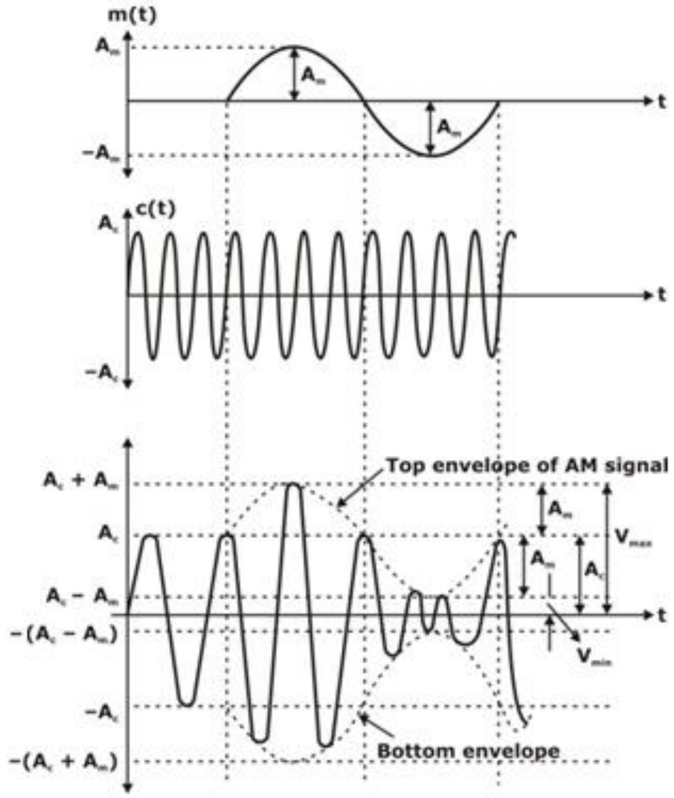
$m(t) = A_m \cos \omega_m t$

then after modulation, we get

$$X_{AM}(t) = \underbrace{A_C \cos\omega_c t}_{\text{Full carrier}} + \frac{1}{2}m_a A_c \underbrace{\cos(\omega_c+\omega_m)t}_{\text{Upper Side Band}} + \frac{1}{2}m_a A_C \underbrace{\cos(\omega_c-\omega_m)t}_{\text{Lower Side Band}}$$

### 3.1 The Spectrum Of Sinusoidal AM Signal




$2A_m = V_{max} - V_{min}$

$$\Rightarrow A_m = \frac{V_{max} - V_{min}}{2}$$

$$A_C = \frac{V_{max} + V_{min}}{2}$$

$$V_{max} = A_c(1 + m_a)$$

$$V_{min} = A_c(1 - m_a)$$

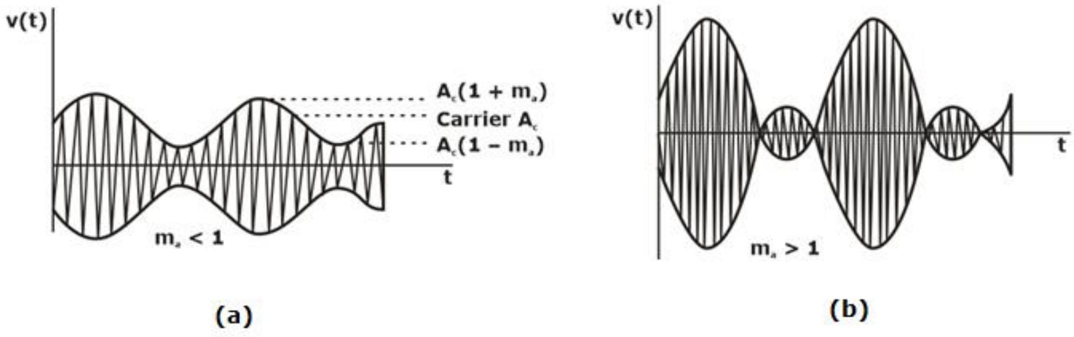
Finally, we get,

$$\boxed{m_a = \frac{A_m}{A_C} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}} \to \text{modulation index}$$

% modulation = $m_a \times 100$

4. **Overmodulation**

When $m_a > 1$, i.e. $A_m > A_C$, overmodulation occurs, and the signal gets distorted. Because the negative part of the waveform gets cut from the waveform leaving behind a "square wave-like" signal, which generates an infinite number of harmonics, this type of distortion is known as "**Non-linear distortion**" or "**Envelope distortion.**"




**(a) Under modulated AM wave (b) Over modulated AM wave**

## 6. Power Relations In AM

### 6.1. The Total Power In AM Full Carrier Dual Side-Band System

The total power in an AM wave is given by,

$P_t$ = [Carrier Power] + [Power in USB] + [Power in LSB]

$$\therefore\ P_t = \frac{E^2}{R} + \frac{E^2_{USB}}{R} + \frac{E^2_{LSB}}{R}$$

E, $E_{USB}$ and $E_{LSB}$ are the RMS values of the carrier and side-band amplitudes, and R is the characteristic resistance of the antenna in which the total power is dissipated.

### 6.2. Carrier Power ($P_c$)

The carrier power is given by

$$P_c = \frac{E^2}{R} = \frac{[E_c/\sqrt{2}]^2}{R} = \frac{E_C^2}{2R}$$

### 6.3. Power In The Side-Bands

The power in the two side-bands is given as

$$P_{USB} = P_{LSB} = \frac{E_{SB}^2}{R}$$

As we know, the peak amplitude of each side-band is $\frac{m_a E_c}{2}$

$$P_{USB} = P_{LSB} = \frac{[m_a E_c/2\sqrt{2}]^2}{R} = \frac{m_a^2 E_c^2}{8R}$$

$$P_{USB} = P_{LSB} = \frac{m_a^2}{4} \times \frac{E_c^2}{2R}$$

$$P_{USB} = P_{LSB} = \frac{m_a^2}{4} P_c$$

### 6.4 Total Power

The total power is given by

$P_t = P_c + P_{USB} + P_{LSB}$

$$= P_c + \frac{m_a^2}{4}P_c + \frac{m_a^2}{4}P_c$$

$$\therefore P_t = \left[1 + \frac{m_a^2}{2}\right]P_c$$

Or, $$\frac{P_t}{P_C} = 1 + \frac{m_a^2}{2}$$

## 7. Transmission Efficiency

- The transmission efficiency of an AM wave is the ratio of the transmitted power which contains the information (i.e. the total side-band power) to the total transmitted power.

$$\therefore \eta = \frac{P_{LSB} + P_{USB}}{P_t} = \frac{\left[\frac{m_a^2}{4} + \frac{m_a^2}{4}\right]P_c}{\left[1 + \frac{m_a^2}{2}\right]P_C} = \frac{m_a^2/2}{1 + \frac{m_a^2}{2}} = \frac{m_a^2}{2 + m_a^2}$$

- The percentage transmission efficiency is given by

$$\eta\% = \frac{m_a^2}{2 + m_a^2} \times 100\%$$

## 8. AM power in Terms of Current

Assume $I_C$ to be the RMS current corresponding to the unmodulated carrier and the RMS current AM wave.

$P_c = I_c^2 R$ and $P_t = I_t^2 R$

$$\therefore \frac{P_t}{P_c} = \frac{I_t^2}{I_c^2} \times \frac{R}{R} = \left[\frac{I_t}{I_c}\right]^2$$

$$\frac{P_t}{P_c} = \left[1 + \frac{m_a^2}{2}\right]$$

$$\left[\frac{I_t}{I_c}\right]^2 = \left[1 + \frac{m_a^2}{2}\right]$$

$$I_t = I_c\sqrt{1 + \frac{m_a^2}{2}}$$

## 9. Multiple Single-Tone Amplitude Modulation

Let us assume that there are two modulating signals.

$x_1(t) = E_{m1} \cos\omega_{m1}t$

and $x_2(t) = E_{m2} \cos\omega_{m2}t$

$$e_{AM} = E_c\left[1 + \frac{E_{m1}}{E_c}\cos\omega_{m1}t + \frac{E_{m2}}{E_c}\cos\omega_{m2}t\right]\cos\omega_c t$$

Where, $\frac{E_{m1}}{E_c} = m_1$

and $\frac{E_{m2}}{E_c} = m_2$

Use the following identity to simplify the equation

$$\cos A\cos B = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$$

$$e_{AM} = E_c\cos\omega_c t + \frac{m_1E_c}{2}\cos(\omega_c + \omega_{m1})t + \frac{m_1E_c}{2}\cos(\omega_c - \omega_{m1})t$$

$$+\frac{m_2E_c}{2}\cos(\omega_c + \omega_{m2})t + \frac{m_2E_c}{2}\cos(\omega_c - \omega_{m2})t$$

### 9.1. Total Power in AM Wave

The total power is given as,

$P_t = P_c + P_{USB1} + P_{LSB1} + P_{USB2} + P_{LSB2}$

Extending the concept to the AM wave with n number of modulating signals with modulating indices $m_1, m_2 \ldots m_n$ the total power is given by,

$$P_t = P_c\left[1 + \frac{m_1^2}{2} + \frac{m_2^2}{2} + \ldots + \frac{m_n^2}{2}\right]$$

### 9.2. Effective Modulation Index ($m_t$)

We know that $P_t = P_c\left[1 + \frac{m_t^2}{2}\right]$

$$m_t = \left[m_1^2 + m_2^2 + \ldots m_n^2\right]^{1/2}$$

## 10. GENERATION OF AM WAVES USING NON-LINEAR PROPERTY

The circuit that generates the AM waves is called an amplitude modulator

They are of the following type,

**i.** Square law modulator

**ii.** Switching modulator

## 11. Disadvantages of AM (DSBFC)

The AM signal is also called as "**Double Side-band Full Carrier (DSBFC) signal**. The main disadvantage of this technique is:

- Power wastage occurs as the carrier does not contain any information, so it is needlessly transmitted.
- AM needs larger bandwidth.
- AM wave gets affected due to noise as the amount of noise is directly proportional to the bandwidth.

## 12. Detection Of AM Waves

### 12.1. Square-Law Detector

A square-law detector is essentially obtained by using a square-law modulator for detection. Consider the characteristic transfer equation of a non-linear device, which is reproduced here for convenience

$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$

where $v_1(t)$ and $v_2(t)$ are the input and output voltages, respectively and $a_1$ and $a_2$ are constants.

### 12.2. Envelope detector

Charging time constant = $RC >> \frac{1}{f_c}$

Discharging time constant = $RC << \frac{1}{f_m}$

As the varying voltage across R follows the envelope.

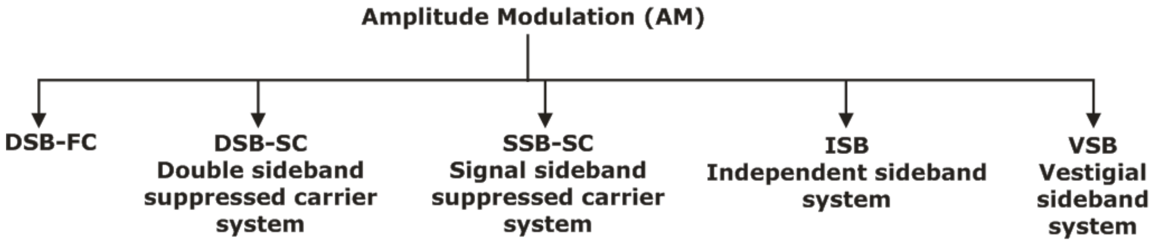
So that, $\frac{1}{f_c} << RC << \frac{1}{f_m}$

If RC is very small or RC is very large, then we can't get the envelope of the message signal waveform in both cases. If RC is very large, then diagonal clipping occurs.

For getting an envelope of m(t), the exact value of RC is given as,

$$RC \le \frac{1}{\omega_m} \cdot \frac{\sqrt{1 - m_a^2}}{m_a}$$

## 13. Types Of Am




## 14. Double-Sideband Suppressed-Carrier Modulation

### 14.1 Time-Domain Description

To describe a double-sideband suppressed-carrier (DSBSC) modulated wave as a function of time, we write

$s(t) = c(t)m(t)$

$\quad = A_c \cos(2\pi f_c t)\ m(t)$

### 14.2. Frequency-Domain Description

The suppression of the carrier from the modulated wave is well-appreciated by examining its spectrum. Specifically, by taking the Fourier transform

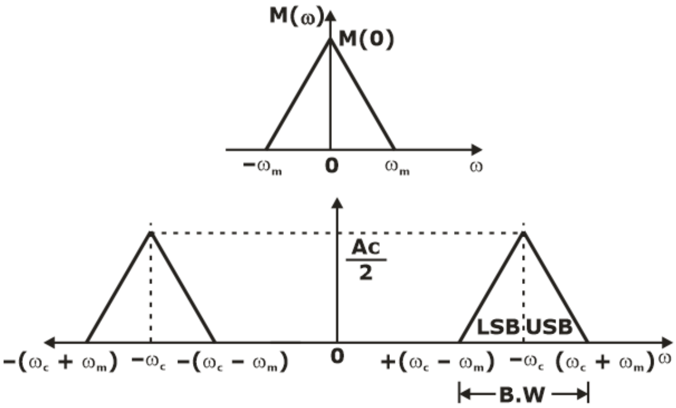
$$S(f) = \frac{1}{2} A_c [M(f - f_c) + M(f + f_c)]$$

Where, as before, S(f) is the Fourier transform of the modulated wave s(t), and M(f) is the Fourier transform of the message signal m(t).

### 14.3. Generation of DSBSC Waves

A double-sideband suppressed-carrier modulated wave consists simply of the product of the message signal and the carrier wave. A device for achieving this requirement is called a product modulator.

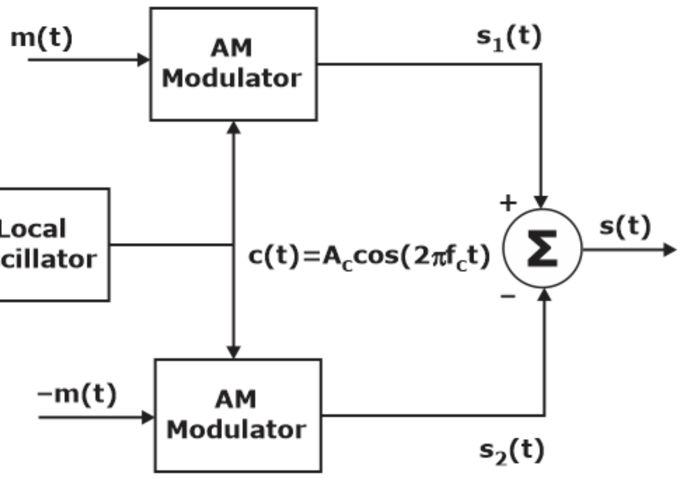
### 14.4. The Spectrum Of DSB-SC Signal



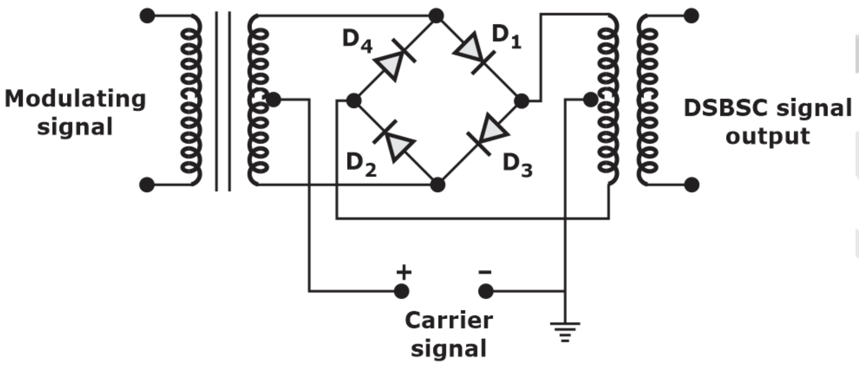

**Modulated DSBSC signal**

Transmission B.W = $2\omega_m$

### 14.5. Balanced Modulator



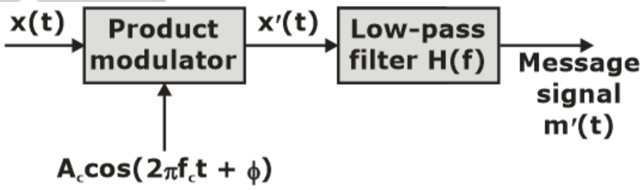

### 14.6. Ring Modulator




### 14.7. Coherent (Synchronous) Detection of DSB-SC Waves

Let x(t) be the DSB-SC signal at the input of the product modulator and the local oscillator having frequency $A_c \cos(2\pi f_c t + \phi)$. The signal x(t) can be represented as

$x(t) = m(t) \times A_c \cos(2\pi f_c t)$




Hence the output of the product modulator is given by

$x'(t) = m(t).\ A_c \cos(2\pi f_c t) \cos(2\pi f_c t + \phi)$

$x'(t) = m(t).\ A_c \cos(2\pi f_c t + \phi) \cos(2\pi f_c t)$

But $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

Therefore, $x'(t) = \frac{1}{2} m(t) A_c [\cos(4\pi f_c t + \phi) + \cos\phi]$

$x'(t) = \frac{1}{2} A_c (\cos\phi)(m(t)) + \frac{1}{2} m(t) A_c \cos(4\pi f_c t + \phi)$

Signal x′(t) is passed through a low pass filter. Which allows only the first term to pass through and will reject the second term. Hence the filter output is given by,

$$m'(t) = \frac{1}{2} A_c \cos\phi\, m(t)$$

If Φ=90° the cosΦ=0, so the output becomes 0 this is called the quadrature null effect

**15. Hilbert Transform**

Hilbert transform of x(t) is represented with $\hat{x}(t)$ , and it is given by

$$H(\omega) = \begin{cases} i = e^{\frac{+i\pi}{2}}, \text{ for } \omega < 0 \\ 0, \text{ for } \omega = 0 \\ -i = e^{-\frac{i\pi}{2}}, \text{ for } \omega > 0 \end{cases}$$

$$\hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(k)}{t-k} dk$$

$$F(H(u))(\omega) = -i \operatorname{sgn}(\omega) F(u)(\omega)$$

Where sgn() is the signum function

**16. Single Side-Band Supressed Carrier**

Let m(t) is modulating signal and $\hat{m}(t)$ is Hilbert transform of m(t) then,

$$\boxed{X_{SSB\text{-}SC}(t) = m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t} \Rightarrow \text{LSB}$$
$$\boxed{X_{SSB\text{-}SC}(t) = m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t} \Rightarrow \text{USB}$$

Also,

B.W = $\omega_c + \omega_m - \omega_c$

**B.W = $\omega_m$**

**16.1. Power Saving**

**DSB-SC:**

The power saved in DSBSC $= \frac{P_c}{P_t} \times 100$

$$P_{save} = \frac{2}{2 + m_a^2} \times 100\%$$

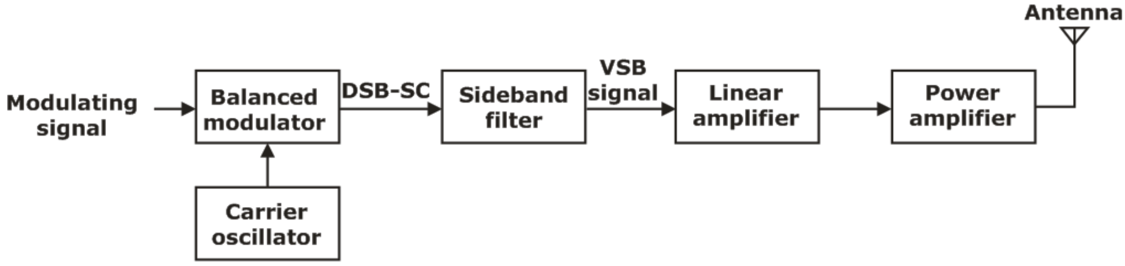
**SSB-SC:**

Power saved in SSB $= \frac{P_c + P_{USB} \text{ or } P_{LSB}}{P_t} \times 100$

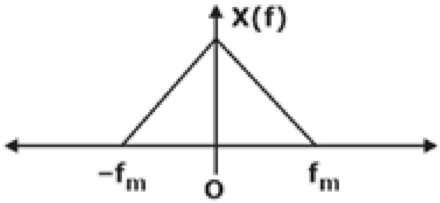
$$P_{save} = \frac{4 + m_a^2}{4 + 2m_a^2} \times 100\%$$

## 17. Vestigial Side-Band Modulation (VSB)

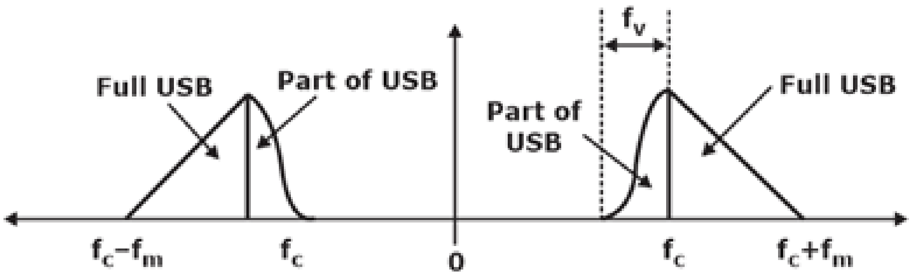
VSB transmission is similar to single-sideband (SSB) transmission, in which one of the side-bands is completely removed. HOWEVER, in VSB transmission, the second side-band is not completely removed but is filtered to remove all but the desired range of frequencies.




**VSB Transmitter**




Spectrum of message signal




Spectrum of VSB signal

### 17.1. Transmission bandwidth

The transmission bandwidth of the VSB modulated wave is given by,

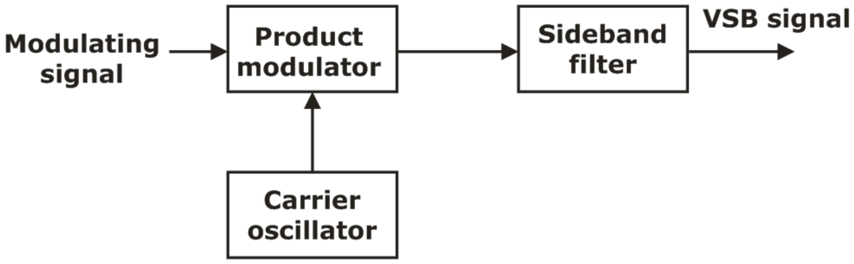
$B = (f_m + f_v)$ Hz

where $f_m$ = Message bandwidth

And $f_v$ = Width of the vestigial sideband.

### 17.2. Generation of VSB Modulated Wave

The modulating signal x(t) is applied to a product modulator. The output of the carrier oscillator is also applied to the other input of the product modulator. The output of the product modulator is given by




**Generation of VSB signal**

m(t) = x(t).c(t)

= x(t).$V_c$ cos(2πf$_c$t)

The spectrum of the VSB modulated signal is given by,

$$S(f) = \frac{V_c}{2}[X(f - f_c) + X(f + f_c)]H(f)$$

**18. Noise in amplitude modulation**

$$N_i(noise\ \ power) = \eta f_m$$

Where η=white noise power density

$$Figure\ of\ \ merit(FOM) = \frac{\frac{S_o}{N_o}}{\frac{S_i}{N_i}}$$

$$FOM_{DSBSC} = 1$$

$$FOM_{SSBSC} = 1$$

$$FOM_{DSBFC} = \frac{m_a^2}{2 + m_a^2}$$

# CHAPTER-3: FREQUENCY AND PHASE MODULATION

## 1. Angle Modulation

An angle modulated wave can be expressed mathematically as
$s(t) = A_c \cos[\omega_c t + \theta(t)]$

Where $A_c$ is the peak carrier amplitude,

$\omega_c$ is the carrier frequency and

$\theta(t)$ is the instantaneous phase deviation.

In angle modulation, $\theta(t)$ is a function of modulating signal.

## 2. Phase Modulation (PM)

### 2.1. Mathematical Expression for PM

A phase-modulated can be mathematically expressed in the time domain as follows:

$$s(t) = \underbrace{A_c}_{\text{Constant amplitude}} \cos[2\pi f_c t + \underbrace{k_p m(t)}_{\text{Varying phase angle}}]$$

$K_p$ is the phase sensitivity in radians per volt, and m(t) is the message waveform.

### 2.2. Mathematical Expression for FM

We can use $\theta(t)$ to write the expression for FM in the time domain as under:

$s(t) = A_c \cos \theta(t)$

FM wave:

$$s(t) = \underbrace{A_c}_{\text{Constant amplitude}} \cos\left[\underbrace{2\pi f_c t}_{\text{Unmodulated carrier}} + 2\pi K_f \int_0^t m(t)\, dt\right]$$

$K_f$ is the frequency sensitivity in rad/V, and m(t) is the message signal.

## 3. Single-Tone Frequency Modulation

Changing the frequency of the carrier according to the-message signal is called Frequency Modulation.

$f_i(t) = f_c + K_f m(t)$ Hz

$K_f$ = Frequency sensitivity (Hz/V)

$f_i(t) = f_c + K_f A_m \cos 2\pi f_m t$

$f_{i,\,max} = f_c + K_fA_m$ Hz

$f_{i,\,min} = f_c - K_fA_m$ Hz

$\Delta f = K_fA_m$ Hz = frequency deviation

### 3.1. Maximum Frequency of FM Wave:

The maximum frequency of FM wave is given by

$f_{max}=f_c\pm \Delta f$ Hz

### 3.2. For a single tone modulation:

$m(t) = A_m \cos 2\pi f_m t$

$$s(t) = A_c\left(\cos\left(2\pi f_c t + \frac{2\pi K_f A_m}{2\pi f_m}\sin 2\pi f_m t\right)\right)$$

$$= A_c\left(\cos\left(2\pi f_c t + \frac{K_f A_m}{f_m}\sin 2\pi f_m t\right)\right)$$

### 3.3. Modulation Index:

The modulation index of FM wave is defined as under:

$$\beta_f = \frac{\text{Frequency deviation}}{\text{Modulating frequency}}$$

$$\beta = \frac{K_f A_m}{f_m} = \frac{\Delta f}{f_m}$$

### 3.4. Deviation Ratio:

The modulation index corresponding to the maximum deviation and maximum modulating frequency is called the deviation ratio.

$$\text{Deviation ratio} = \frac{\text{Maximum deviation in frequency}}{\text{Maximum modulating frequency}}$$

### 3.5. Percentage Modulating of FM Wave:

$$\%\ \text{Modulation} = \frac{\text{Actual frequncy deviation}}{\text{Maximum allowable deviation}}$$

### 3.6. TYPES OF FM

The FM systems are classified into the following two types:

**i.** Narrowband FM ($\beta << 1$)

**ii.** Wideband FM ($\beta >> 1$)

BW of NBFM = $2f_m$

The spectrum of AM and NBFM are identical except that the spectral component at $f_c - f_m$ is 180° out of phase.

### 3.7. Narrow-band FM

Narrowband FM is very similar to AM; therefore, it is rarely used

Figure 6 shows the generation of narrowband FM using a balanced modulator.

$BW_{NB}=2f_m$

$$s_{NBFM}(t) = A\cos(\omega_c t) - AK_f m(t)\sin(\omega_c t)$$

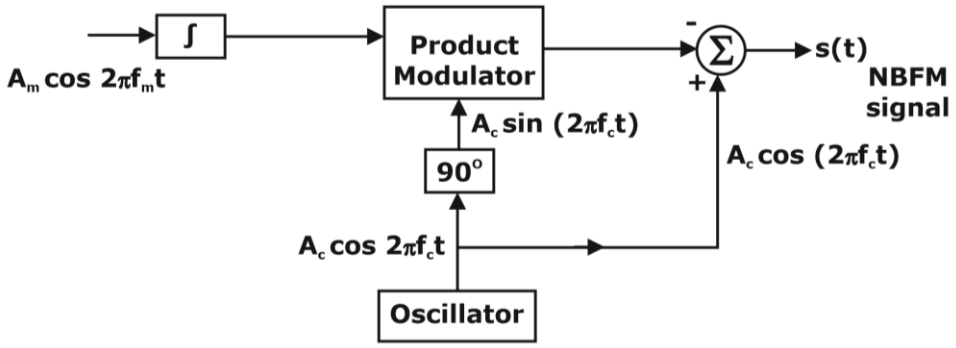



**Figure 6: Generation of Narrowband FM**

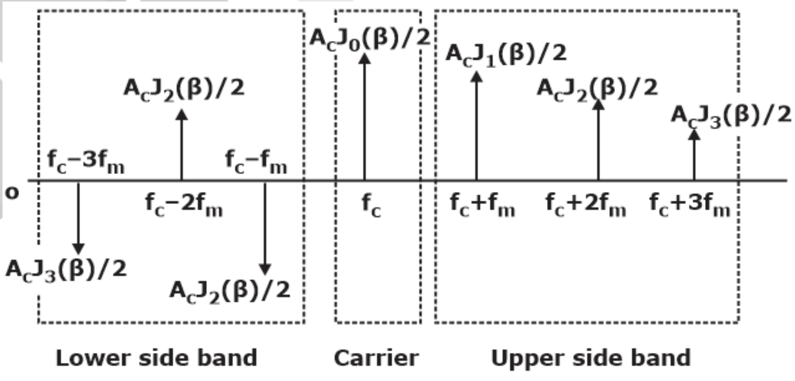
### 3.8. Wideband FM

Bessel function of order 'n' is given by

$J_n(x) = (1/2\pi)\int e^{j(x\sin\theta - n\theta)}\, d\theta$

s(t) is wideband FM

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos\left(2\pi\left(f_c + nf_m\right)t\right)$$




**Wideband FM spectrum**

### Generation Of WBFM Signals

WBFM signal can be generated by two methods

i. Direct Method or parameter variation method

ii. Indirect Method or Armstrong Method

**Direct Method**

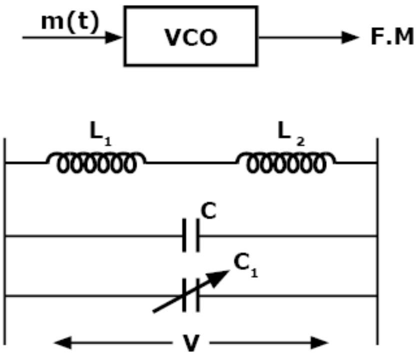
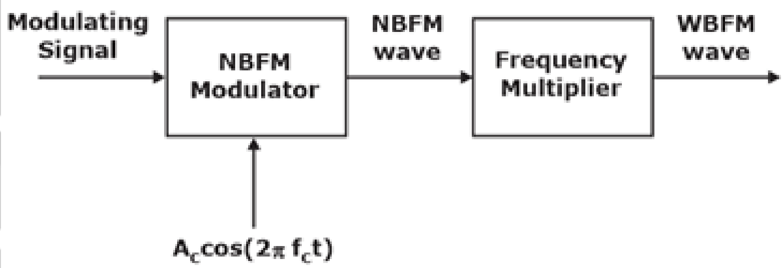



**Figure 14(a): Voltage control oscillator**

Frequency of oscillation, $f = \dfrac{1}{2\pi\sqrt{(L_1 + L_2)(C + C_1)}}$

Difficult to obtain higher-order frequency stability in carrier frequency because the carrier generation is directly affected by the message signal, so highly stable sources like crystal oscillators cannot be used. So it is rarely used.

**Indirect Method (Armstrong's method)**




In WBFM, after going through a frequency multiplier with a multiplication factor of n

$\Delta f = n\Delta f$

$\beta = n\beta$

$f_m = f_m$

$f_c = nf_c$

### 3.9. Power Calculation

Total Power

$$P_t = \frac{A_C^{\,2}}{2R}\sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

According to the property of Bessel function

$$P_t = \frac{A_C^2}{2R}.1$$

$$\boxed{\text{Total power} = \frac{A_C^2}{2R}}$$

Same as unmodulated carrier power

i.e. $\boxed{P_t = P_C}$

The total power is independent of the modulation index. AM takes more power compared to FM for the same message and carrier.

### 3.10. Calculation Of Practical B.W Of WBFM Using Carson's Rule

Carson has proved that β + 1 number of side-bands having significant amplitudes contain 99% of the total power. This is called Carson's rule

B.W. = $2(\beta + 1)f_m$

$$= 2\left(\frac{\Delta f}{f_m} + 1\right) f_m$$
$$= 2\Delta f + 2f_m$$

### 3.11. FM Demodulation Using Frequency Discriminators And Envelop Detectors

The FM signal is demodulated by a two-step process. Here first, the FM signal is converted into an AM signal using frequency discriminators and then that AM signal is demodulated using an envelope detector to get our message signal.

FM discriminators can be divided into two types

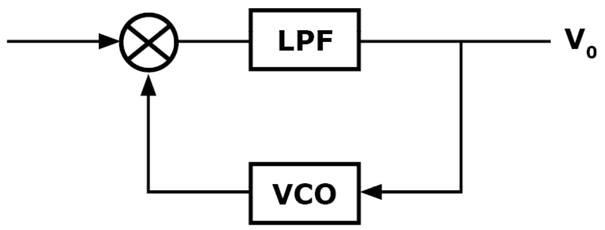
**Slope Detectors**

- **Single tuned or simple slope detector**
  Not used because it has harmonic distortions.
  It is also sensitive to amplitude variations
- **Stagger tuned or balanced slope detector**
  Limited to small frequency deviation signals

**Phase difference discriminators**

- **Foster Seeley discriminator**
  Widely used
  Sensitive to amplitude variations caused due to noise
- **Slope detector**
  Insensitive to amplitude variations due to noise
  Requires and AGC signal

### 3.12. FM Demodulation Using Pll




**First-order PLL**

When the input to the PLL is of the form cos[2πft + ϕ], the output voltage of VCO is

$\boxed{V_0 \propto \frac{d}{dt}[\phi]}$.

When the input to the PLL is an FM signal, $A_c \cos[2\pi f_c t + 2\pi K_f \int m(t)\,dt]$, the output voltage is

$$V_0 \propto \frac{d}{dt} 2\pi K_f \int m(t)\,dt$$

$$V_0 \propto 2\pi K_f m(t)$$

$$V_0 = \frac{1}{2\pi K_V}\left[2\pi K_f m(t)\right]$$

Where, $\frac{1}{2\pi K_V}$ =proportionality constant

$\boxed{V_0 = \frac{K_f}{K_v} m(t)}$ $K_f$ – frequency sensitivity of VCO at transmitter
$K_v$ – frequency sensitivity of VCO at Receiver

For perfect sync, $f_{VCO}=f_c$, which is called the lock mode.

And $\Phi_{VCO}= \Phi_c$, which is called the capture mode

$f_{VCO}=f_c$ is achieved very quickly due to the negative feedback due to the VCO

LPF is responsible for the capture mode (phase sync).

Lock range > Capture range

PLL operation is a differentiation operation

## 4. PHASE MODULATION

In phase modulation, the phase of the carrier is varied according to the message signal. Time-domain equation of PM modulated signal can be written as,

$\boxed{\begin{array}{l} S(t) = A_c \cos[2\pi f_c t + K_p m(t)] \\ \phi = K_p m(t) \end{array}}$ multitone modulation

Where, $K_p$ =phase sensitivity (units =rad/V)

$\boxed{s(t) = A_c \cos\left[2\pi f_c t + K_p A_m \cos 2\pi f_m t\right]}$ single tone modulation

where $\boxed{K_p A_m = \Delta\phi}$ is called phase deviation

$$\boxed{s(t) = A_c \cos[2\pi f_c t + \beta \cos 2\pi f_m t]}$$

$\boxed{\Delta\phi = \beta}$ = modulation index

**5. Comparison Between AM and FM**

| | **Parameter** | **Amplitude Modulation** | **Frequency Modulation (Angle modulation)** |
|---|---|---|---|
| 1 | Noise performance | Bad | Better, but FM has a more severe threshold effect |
| 2 | Common channel interference | More than FM | Less due to the capture effect |
| 3 | Externally generated noise effect | More affected but does not require exact tuning | Less affected but requires more accurate tuning |
| 4 | Channel bandwidth | Low | High |
| 5 | operating carrier frequency | Low operates in MF and HF bands | High >30MHz |
| 6 | Transmission efficiency | Less | More |

**6. Capture effect**

The capture effect is defined as the complete suppression of the weaker signal at, or near, the same frequency or channel at the receiver's limiter (if present), where the weaker signal is greatly attenuated.
It is only present in FM, not in AM(amplitude modulation)

**7. Threshold effect**

When the carrier-to-noise ratio decreases below a certain point, below this critical point, the signal-to-noise ratio decreases significantly. This is known as the FM threshold effect.
It is more severe in FM than AM(amplitude modulation)

**8. Pre-emphasis and de-emphasis**

Pre-emphasis is boosting the amplitudes of higher frequencies at the transmitter.
De-emphasis is the attenuation of the higher frequencies by the same amount at the receiver.

**9. Relation between PM and FM**

$$\Delta\phi_{PM} = \frac{K_f A_m}{f_m}$$

$$\Delta f_{FM} = K_p A_m f_m$$

**10. Figure of merit**

$$FOM_{FM} = \frac{3}{2}\beta^2$$

$$FOM_{PM} = \frac{1}{2}\beta^2$$

## CHAPTER-4: RECEIVERS

1. **Features of a receiver**

- **Selectivity**

  It is the ability of the receiver to accurately distinguish between two carrier frequencies and select the correct frequency. Selectivity depends on the sharpness of the resonance curve of the tuned circuits in the circuit.

- **Sensitivity**

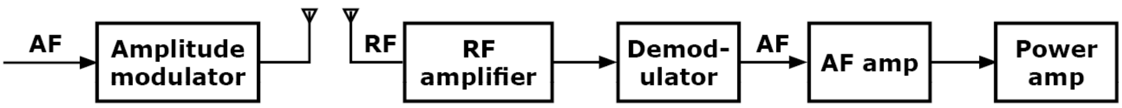
  It is the ability of the receiver to detect the weakest possible signal. It depends on the gain of the amplifying stages.

- **Fidelity**

  It is the ability to reproduce all frequencies faithfully in the message signal at the output it depends on the bandwidth of the amplifier, which amplifies the baseband signal

2. **TRF Receiver**

Block diagram for TRF Receiver is as follows




**Tuned radio frequency receiver**

Carrier frequencies allotted from FM = (88– 108) MHz

Carrier frequencies allotted from AM =(550 – 1650) kHz

BW allotted to each AM broadcasting station = 10kHz

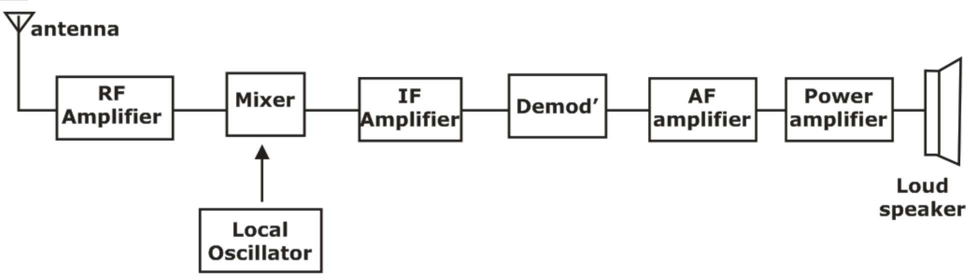
Don't suffer from problems of image frequency rejection and tracking and alignment.

A very high value of Q is required for higher frequencies

Selectivity declines by a lot on higher frequencies.

3. **SUPERHETERODYNE RECEIVER**

Block diagram for TRF Receiver is as follows




**Superheterodyne Receiver**

The mixer will change the carrier frequency from $f_s$ to $f_{IF}$.

The intermediate frequency for MW is 455 kHz.

### 3.1. Choice Of Intermediate Frequency

For adjacent channel selectivity and easy tracking, the $f_{IF}$ should be low

For good image signal rejection, $f_{IF}$ should be high

### 3.2. Image Frequency:

$F_{si} = f_s + 2\,IF$

Where IF is the image frequency

The resonant frequency of the IF tuned amplifier is constant, i.e., IF.

$f_l - f_s = IF$

Where $f_l$ is the local oscillator frequency

In SHR, the local oscillator frequency is always kept higher than the signal frequency

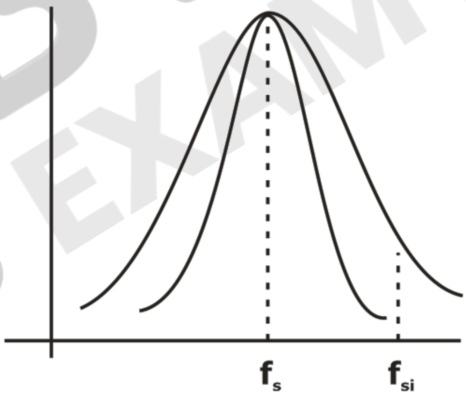
### 3.3. Capacitance Ratio

$$\frac{C_{MAX}}{C_{MIN}} = \left(\frac{f_H + f_{IF}}{f_L + f_{IF}}\right)$$

### 3.4. Image (Frequency) Rejection Ratio:

$$IRR = \frac{\text{Gain at } f_s}{\text{Gain at } f_{si}}$$

Gain at $f_{si} \ll 1$




By increasing the Intermediate frequency, IRR can be increased. By increasing the bandwidth, the gain at $f_{si}$ can be decreased so that IRR increases.

$$IRR \propto \frac{1}{B.W}$$

$$IRR \propto Q$$

$$IRR = \sqrt{1 + Q^2\rho^2}$$

Where, $\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$

Since the frequency of the local oscillator is given by

$$f_{LO} = \frac{1}{2\pi\sqrt{LC}}$$

Where L is inductance and C is capacitance. Now, for a fixed value of L, we have $1/f_{LO}$

$$\sqrt{C} = \frac{1}{2\pi\sqrt{L}}$$

$$\text{or} \quad C = \frac{1}{2\pi L}\frac{1}{f_{LO}^2}$$

So, the maximum value of capacitance exists for the minimum value of $f_{LO}$, i.e

$$C_{max} = \frac{1}{2\pi L}\frac{1}{f_{LO,min}^2}$$

Similarly, we get

$$C_{min} = \frac{1}{2\pi L}\frac{1}{f_{LO,\,max}^2}$$

# CHAPTER-5: NOISE IN ANALOG COMMUNICATION

## 1. Classification Of Noise

The fundamental noise sources produce different types of noise. They may be listed as under:

(i) Shot noise

(ii) Thermal noise

(iii) Partition noise

(iv) Low frequency or flicker noise

(v) High frequency or transit time noise

### 1.1. Shot Noise

The mean square shot noise current for a diode is given as

$I_n^2 = 2(I + 2I_0)q\ B\ A^2$

Where I = direct current across the junction (in the amp.)

$I_0$ = reverse saturation current (in amp.)

q = electronic charge = $1.6 \times 10^{-19}$C

B = effective noise bandwidth in Hz.

Schottky's formula

$$i_o^2 = 2qI_o(\Delta f)$$

Where $i_o$=shot noise current

### 1.2. Thermal Noise or Johnson Noise

The average thermal noise power is given by,

$P_n$ = kTB Watts

Where k = Boltzmann's constant = $1.38 \times 10^{-23}$ J/K

B = Bandwidth of the noise spectrum (Hz)

T = Temperature of the conductor in Kelvin

Also $P_n$=2kTBR

Noise voltage

$$v_n = \sqrt{2kTBR}$$

### 1.3 Partition Noise

When a circuit is divided between two or more paths, the noise generated is Partition noise. The reason for the generation is random fluctuation during division.

### 1.4 Flicker Noise

This type of noise is generally observed at a frequency range below a few kHz. The power spectral density of this noise increases with the decrease in frequency. That is why the name is given as Low- Frequency Noise.

### 1.5 Transit Time Noise

This noise is also known as TRANSIT- TIME Noise. This is observed in semiconductor devices when the transit time of a charge carrier while crossing a junction is comparable with the time period of that signal.

### 2. White Gaussian Noise

White noise is the noise whose power spectral density is uniform over the entire frequency range of interest, as shown in figure 2.

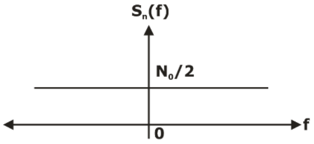
3. 


**Figure: Power Spectral Density of White Noise**

The white noise contains all the frequency components in equal proportion. This is analogous to white light, which is a superposition of all visible spectral components.

The white noise has a gaussian distribution. This means that the PDF of white noise has the shape of a Gaussian PDF Hence, it is called gaussian noise.

As shown in figure 2, the power spectral density (PSD) of white noise is given by,

$$S_n(f) = \frac{N_0}{2}$$

This equation shows that the power spectral density of white noise is independent of frequency. As $N_0$ is constant, the PSD is uniform over the entire frequency range, including the positive and the negative frequencies. $N_0$ is defined as under:

$N_0 = kT_e$

where K = Boltzmann's constant and

$T_e$ = Equivalent noise temperature or the system

An example of white noise is the thermal or Johnson noise.

### 3. Signal To Noise Ratio

It is defined as the ratio of signal power to the noise power at the same point.

Therefore, $\frac{S}{N} = \frac{P_s}{P_n}$

where $P_s$ = Signal power

$P_n$ = Noise power at the same point.

$S/N\ (dB) = 10 \log_{10} (P_s/P_n)$

### 4. SINAD

This is another variation of signal to noise ratio. SINAD stands for signal noise and distortion, and it is defined as,

$$\text{SINAD} = \frac{S+N+D}{N+D}$$

Where, S = Signal, N = Noise and D = Distortion

SINAD is generally used in the specifications of the FM receiver.

**5. Noise Factor**

It is defined as,

$$F = \frac{\text{S/N ratioat the input}}{\text{S/N ratio at the output}}$$

$$F = \frac{P_{si}}{P_{ni}} \times \frac{P_{n0}}{P_{s0}}$$

Where $P_{si}$ and $P_{ni}$ = Signal and noise power at the input

and $P_{so}$ and $P_{no}$ = Signal and noise power at the output

**6. Noise Figure**

Sometimes, the noise factor is expressed in decibels. When noise factor is expressed in decibels, it is known as noise figure.

Noise figure $F_{dB} = 10 \log_{10} F$

Substituting the expression for the noise factor, we get

$$\text{Noise figure} = 10 \log_{10} \left[\frac{\text{S/N at the input}}{\text{S/N at the output}}\right] = 10 \log_{10} (S/N)_i - 10 \log_{10} (S/N)_0$$

Hence, Noise figure $F_{dB} = (S/N)_i \text{ dB} - (S/N)_0 \text{ dB}$

The ideal value of noise figure is 0 dB.

**7. Noise Temperature**

The equivalent noise temperature of a system is defined as the temperature at which the noise resistor has to be maintained so that by connecting this resistor to the input of a noiseless version of the system, it will produce the same amount of noise power at the system output as that produced by the actual system.

The equivalent noise temperature of the amplifier is given by,

$T_{eq} = (F - 1)\, T_0$

Where F is noise factor and $T_0$ is temperature

**8. The Figure of Merit:**

$(S/N)_i = (S_i/N_i) = \dfrac{\text{Powerof the modulated signal}}{\text{Power of noise in message bandwidth}}$

$(S/N)_0 = (S_0/N_0) = \dfrac{\text{Powerof the demodulated signal}}{\text{Power of noise in message bandwidth}}$

Figure of Merit $= \dfrac{(S/N)_0}{(S/N)_i} = \dfrac{1}{\text{Noise Figure}}$

Noise Figure $= \dfrac{(S/N)_i}{(S/N)_0}$

$(S/N)_0$ depends mainly on the modulation scheme and receiver characteristics.

**9. Equivalent Noise Temperature In Cascaded States**

$$T_e = T_{e1} + \frac{T_{e2}}{G_{a1}} + \frac{T_{e2}}{G_{a1}G_{a2}} + \bullet\bullet\bullet\bullet + \frac{T_{en}}{G_{a1}G_{a2}\bullet\bullet\bullet\bullet G_{a(n-1)}}$$

Where $G_{an}$=gain of the $n^{th}$ stage

$T_{en}$=noise temperature of the $n^{th}$ stage

**10. Equivalent Noise Resistance In Cascaded States**

$$R_{eq} = R_1 + \frac{R_2}{A_1^2} + \frac{R_2}{A_2^2 A_1^2} + \bullet\bullet\bullet\bullet + \frac{R_N}{A_1^2 A_2^2 A_3^2 \bullet\bullet\bullet\bullet A_{(n-1)}^2}$$

**11. Noise Figure In Cascaded States**

$$F = F_1 + \frac{F_2 - 1}{G_{a1}} + \frac{F_3 - 1}{G_{a1}G_{a2}} + \bullet\bullet\bullet\bullet + \frac{F_N}{G_{a1}G_{a2}G_{a3}\bullet\bullet\bullet\bullet G_{a(n-1)}}$$

Where $G_{an}$=gain of the $n^{th}$ stage

$F_n$= noise figure of the $n^{th}$ stage

$$F = 1 + \frac{R_{eq}}{R_a}$$

# CHAPTER-6: RANDOM VARIABLES

## 1. Probability

### 1.1. Probability Axioms

1. (Nonnegativity) $P(A) \geq 0$, for every event A.

2. (Additivity) If A and B are two disjoint events, then the probability of their union satisfies $P(A \cup B) = P(A) + P(B)$.

Furthermore, if the sample space has an infinite number of elements and $A_1$, $A_2$, ... is a sequence of disjoint events, then the probability of their union satisfies

$P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots P(A_n)$

3. (Normalization) The probability of the entire sample space $=\Omega$ is equal to 1, that is, $P(\Omega) = 1$.

4.For an event, A $0 \leq P(A) \leq 1$

5.For an absolute certain event A $P(A)=1$

6.For an impossible event, A $P(A)=0$

### 1.2. Properties of Probability Laws

Consider a probability law, and let A, B, and C be events.

(a) If $A \subset B$, then $P(A) \leq P(B)$.

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(c) $P(A \cup B) \leq P(A) + P(B)$.

(d) $P(A \cup B \cup C) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C)$.

### 1.3. Conditional probability for event A and B:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

## 2. Total Probability Theorem And Bayes' Rule

### 2.1. Total Probability Theorem

Let $A_1,\ldots, A_n$ be disjoint events that form a partition of the sample space (each possible outcome is included in one and only one of the events $A_1,\ldots, A_n$) and assume that $P(A_i) > 0$, for all $i = 1,\ldots, n$. Then, for any event B, we have

$P(B) = P(A_1 \cap B) + \ldots + P(A_n \cap B)$

$= P(A_1)P(B \mid A_1) + \ldots + P(A_n)P(B \mid A_n)$.

### 2.2. Bayes' Rule

Let $A_1$, $A_2,\ldots, A_n$ be disjoint events that form a partition of the sample space, and assume that $P(A_i) > 0$, for all i. Then, for any event B such that $P(B) > 0$, we have

$$P(A_i / B) = \frac{P(A_i) P(B / A_i)}{\sum_{i=1}^{n} P(A_i) P(B / A_i)}$$

**3. Independent Events:**

We say that A is independent of B

P(A | B) = P(A).

P(A | B) =P(A ∩ B)/P(B), this is equivalent to

P(A ∩ B) = P(A)P(B).

**4. Introduction To Random Variables**

A random variable is a real value function and defined over a sample space of a random experiment.

It is also known as stochastic function, stochastic variable and random function

The random variables can be distinguished as

1. Discrete Random Variable
2. Continuous Random Variable

3 Cumulative Distribution Function

4 Mean, Variance and Standard Deviation of a Random Variable

5 Different types of Random Variable

**5. Discrete Random Variable**

When the random variable takes only a discrete set of values, it is called a discrete random variable. For example, we flip a coin, the possible outcomes are head (H) and tail (T), so S contains two points labelled H and T. Suppose we define a function X(S) such that

$$X(S) = \begin{cases} 1 \text{ for} & S = H \\ -1 \text{ for} & S = T \end{cases}$$

Thus, we have mapped the two outcomes into the two points on the real line. So, this type of random variable is called a discrete random variable.

**5.1. Probability Density Function of Discrete Random Variable**

Let a discrete random variable X having the possible outcomes, X = $\{X_1, X_2, \ldots X_n\}$

So, the probability density function (PDF) of the discrete random variable is defined as

$$P(X = x_j) = f(x_j), j = 1, 2, 3, \ldots\ldots.$$

$$f(x) \geq 0$$

$$\sum_x f(x) = 1$$

### 5.2. Probability Mass Function of Discrete Random Variable

Suppose that the jumps in $F_x(x)$ of a discrete random variable X occur at the points $x_1$, $x_2$, ..., where the sequence may be finite or countably infinite, and we assume $x_i < x_j$ if $i < j$.

Then $F_x(x_i) - F_x(x_{i-1}) = P(X \le x_i) - P(X \le x_{i-1}) = P(X = x_i)$

Let $P_x(x) = P(X = x)$

The function $p_x(x)$ is called the probability mass function (pmf) of the discrete random variable X.

**Properties of $p_x(x)$:**

I. $0 \le p_x(x_k) \le 1 \quad k = 1, 2, \ldots$

2. $p_x(x) = 0$ if $x \# x_k$ $(k = 1, 2, \ldots)$

3. $\sum_k p_x(x_k) = 1$

The CDF $F_x(x)$ of a discrete random variable X can be obtained by

$F_x(x) = P(X \le x) = \sum_{x_k \le x} p_x(x_k)$

## 6. Continuous Random Variable

A random variable that takes on an infinite number of values is known as a continuous random variable. As there are infinite possible values of X probability that it takes a single value is $\frac{1}{\infty} = 0$

### 6.1 CONTINUOUS RANDOM VARIABLES AND PDFs

For every subset B of the real line. In particular, the probability that the value of X falls within an interval is

$P(X \in B) = \int_B f_x(x)dx,$

Note that to qualify as a PDF, a function $f_x$ must be non-negative, i.e., $f_x(x) \ge 0$ for every x, and must also satisfy the normalization equation

.

### 6.2 Properties of PDF of Continuous Random Variable:

1. $f_x(x) \ge 0$

2. $\int_{-\infty}^{\infty} f_x(x)dx = 1$

3. $P(X \le x) = F_x(x) = \int_{-\infty}^{x} f_x(\lambda)d\lambda$

4. $P(a < x \le b) = \int_a^b f_x(x)dx$

5. Excluding the endpoints of an interval has no effect on its probability:
$P(a \le X \le b) = P(a < X < b) = P(a \le X < b) = P(a < X \le b)$.

## 7. CUMULATIVE DISTRIBUTION FUNCTIONS

The CDF of a random variable X is denoted by $F_x$ and provides the probability $P(X \le x)$. In particular, for every x we have

$$F_x(x) = P(X \le x) = \begin{cases} \sum_{k \le x} p_x(k) & X : \text{discrete}, \\ \int_{-\infty}^{x} f_x(t)dt & X : \text{continuous} \end{cases}$$

### 7.1. Properties of a CDF

The CDF $F_x$ of a random variable X is defined by $F_x(x) = P(X \le x)$, for all x,

and has the following properties.

$F_x$ is nondecreasing

if $x \le y$, then $F_x(x) \le F_x(Y)$.

$F_x(x)$ tends to 0 as $x \to -\infty$ and 1 as $x \to \infty$.

If X is discrete, then $F_x$ has a piecewise constant and has a staircase-like graph.

If X is continuous, then $F_x$ has a continuously varying graph.

If X is discrete, the PDF and the CDF can be obtained from each other by summing or differencing:

$$F_x(k) = \sum_{i=-\infty}^{k} P_x(i),$$

$p_x(k) = P(X \le k) - P(X \le k-1) = F_x(k) - F_x(k-1)$,

for all integers k.

- If X is continuous, the PDF and the CDF can be obtained from each other by integration or differentiation:

$$F_x(x) = \int_{-\infty}^{x} f_x(t)dt,$$

## 8. The Statistical Average Of Random Variable:

### 8.1 Mean or Expected Value:

Let a random variable X characterized by its PDF $f_x(x)$. The mean or expected value of X is defined as

$$E(X) = \overline{X} = \int_{-\infty}^{\infty} x f_x(x)dx$$

Similarly, we obtain the expected value of a function g(X) as $E[g(X)] = \overline{g(X)} = \int_{-\infty}^{\infty} g(x) f_x(x)dx$

If X is a discretely distributed random variable, then the expected value of X is given by

$$E[X] = \overline{X} = \mu_x = \sum_{i=1}^{n} x_i f_x(x_i)$$

**Properties of expected values**

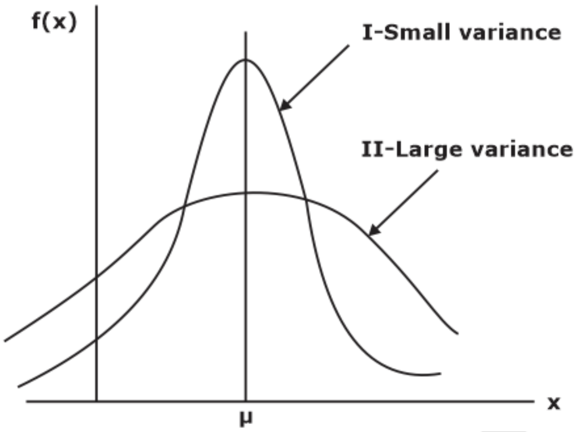
$E(cX)=cE(X)$

$E(X+Y)=E(X)+E(Y)$

E(XY)=E(X)E(Y)

### 8.2 Variance

The variance $\sigma_x^2$ of a random variable X is the second moment taken about its mean. i.e.




Var [X] = $\sigma_x^2$ = E [(X − $\mu_x$)$^2$]

$$= \int_{-\infty}^{\infty} (x-\mu_x)^2 f_x(x)dx$$

**Properties of variance**

$\sigma^2=E((X-\mu)^2)=E(X^2)-\mu^2$

$Var(cX)=c^2Var(X)$

$Var(X+Y)=Var(X)+Var(Y)$

$Var(X-Y)=Var(X)+Var(Y)$

### 8.3. Standard Deviation

The standard deviation $\sigma_x$ of a random variable is the square root of its variance, i.e.,

$$\sigma_x = \sqrt{\text{var}[x]} = \sqrt{E\left[(X-\mu)^2\right]}$$

## 9. Tchebycheff's Inequality

$$P\left[|X-c| \geq \varepsilon\right] \leq \frac{E\left[(X-c)^2\right]}{\varepsilon^2}$$

## 10. Normalized Random Variable

$$X^* = \frac{X-\mu}{\sigma}$$

$$E(X^*) = 0$$

$$Var(X^*) = 1$$

**11. Central Limit Theorem**

The probability density of a sum of N independent random variables tends to approach the probability density of a Normal distribution as N increases. It becomes equal to that of the normal distribution function as N tends to infinity.

**12. Covariance**

The covariance of the random variables X and Y is defined as:

$\mathrm{cov}(X,Y) = \sigma_{xy} = E\,[(X - \mu_x)\,(Y - \mu_y)] = \overline{(X-\mu_x)(Y-\mu_y)}$

where $\mu_x$ and $\mu_y$ are the mean of random variables X and Y, respectively. We may expand the above result as

$\mathrm{cov}(X,Y) = \sigma_{xy} = E[XY] - \mu_x\mu_y = \overline{XY} - \mu_x\mu_y$

it can also be written as

cov(X,Y)=E[XY]-E[X]E[y]

**Properties of Covariance**

If X and Y are real valued random variables and c and d are real valued constants

cov(X,X)=var[X]

cov(X,c)=0

cov(X,Y)=cov(Y,X)

cov(cX,dY)=cd cov(X,Y)

cov(X+c,Y+d)=cov(X,Y)

**13. Correlation Coefficient**

The correlation coefficient of random variables X and Y can be defined as

$$\rho_{xy} = \frac{\mathrm{cov}[xy]}{\sigma_x\sigma_y}$$

Where cov [X, Y] is the covariance of X and Y, and $\sigma_x$, $\sigma_y$ are the standard deviations of random variables.

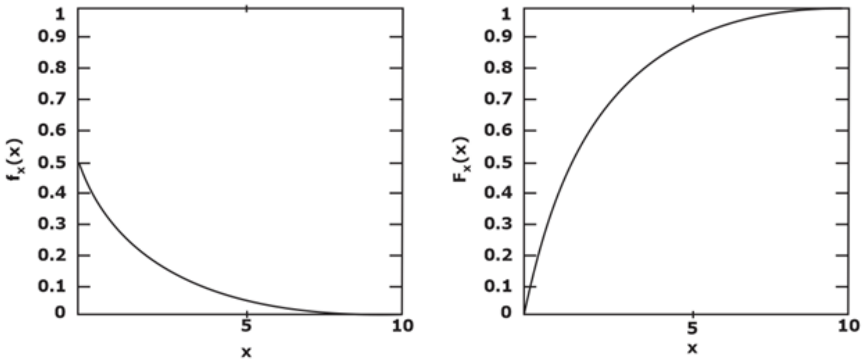
**NOTE:**

1. The random variables X and Y are uncorrelated if and only if their covariance is zero, i.e. cov[XY] = 0

2. The random variables X and Y are orthogonal if and only if their correlation is zero, i.e. E[X, Y] = 0

## 14. Different types of Random Variable distribution:

### 14.1. Exponential Distribution




**PDF and CDF**

**PDF**

$$f_x(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

**CDF**

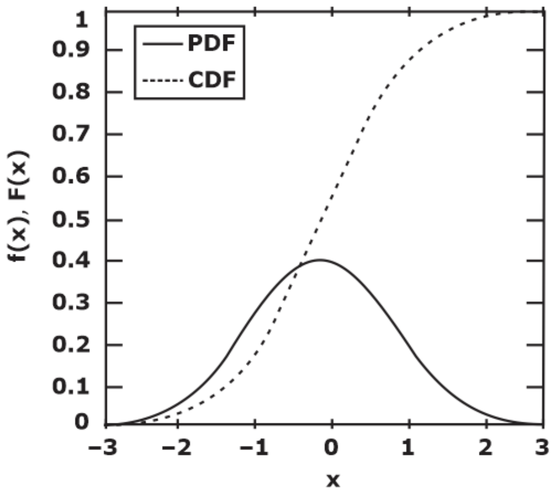
$$F_x(x) = \begin{cases} 1 - e^{-\lambda x} & \text{if } x \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

Where λ is a positive parameter

Mean=E[X]= $\frac{1}{\lambda}$

$\text{var}(X) = \frac{1}{\lambda^2}$ .

### 14.2. Normal Or Gaussian Distribution




**PDF and CDF**

**PDF**

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

**CDF**

$$F_x(x) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

Where μ and σ are two scalar parameters characterizing the PDF

The mean and the variance are

E[X] = μ,

var(X) = $\sigma^2$.

**14.3. Bernoulli Random Variable**

$$0 \le p \le 1$$
$$q = p - 1$$

**PDF**

$$f_x(x) = \begin{cases} q = 1 - p & x = 0 \\ p & x = 1 \end{cases}$$

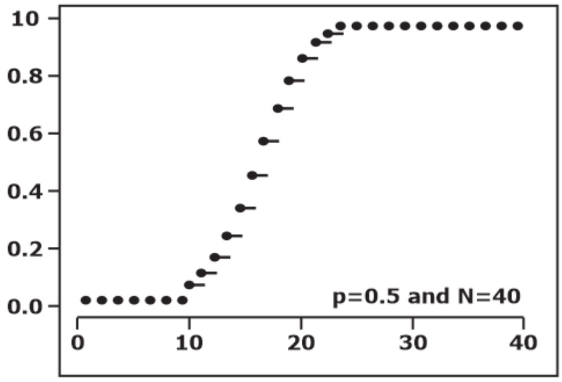
**CDF**

$$F_x(x) = \begin{cases} 0, \ if \ x < 0 \\ 1 - p, \ if 1 \le x < 1 \\ 1, \ if \ x \ge 1 \end{cases}$$

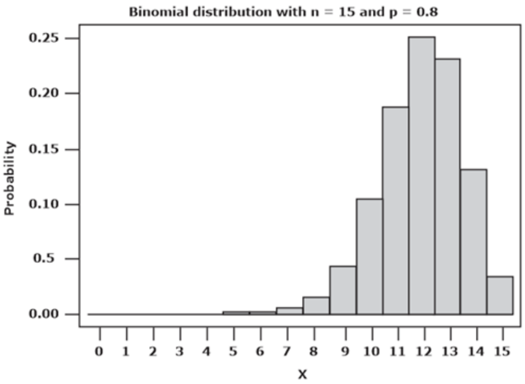
$$\mu_x = E(X) = p$$

$$\sigma_x^2 = Var(X) = pq$$

### 14.4. Binomial Distribution




CDF




PDF

**PDF**

$$f_x(x) = {}^nC_x p^x q^{n-x}$$

**CDF**

$$F_x(x) = \sum_{i=0}^{\lfloor x \rfloor} {}^nC_i p^i q^{n-i}$$

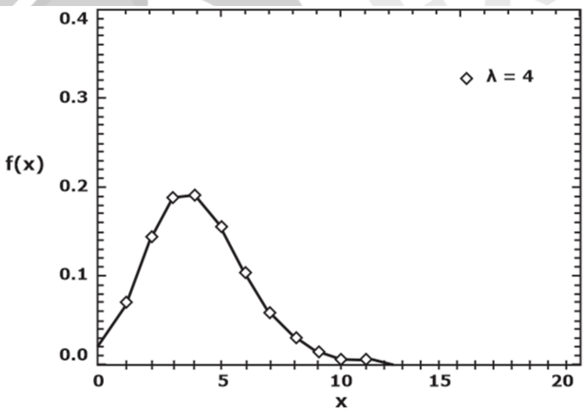
Where p=1-q

$\mu_x = E(X) = np$

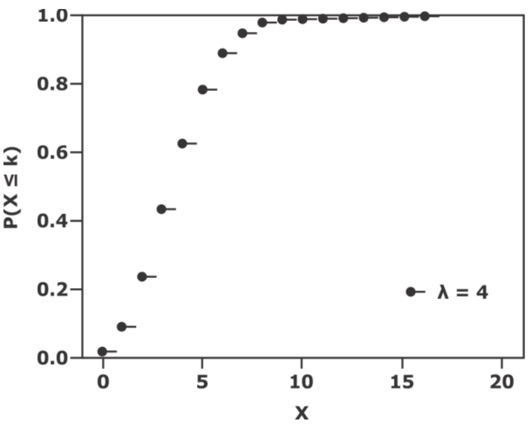
$\sigma_x^2 = Var(X) = npq$

In a binomial distribution, if the n is large and p is small or close to zero, then it could be approximated to a Poisson distribution with mean=np

### 14.5. Poisson Distribution:




PDF




CDF

A random variable X is called a Poisson random variable with parameter $\lambda(>0)$

**PDF**

$$f_x(x) = e^{-\lambda} \frac{\lambda^x}{x!} \qquad x = 0, 1, 2....$$

**CDF**

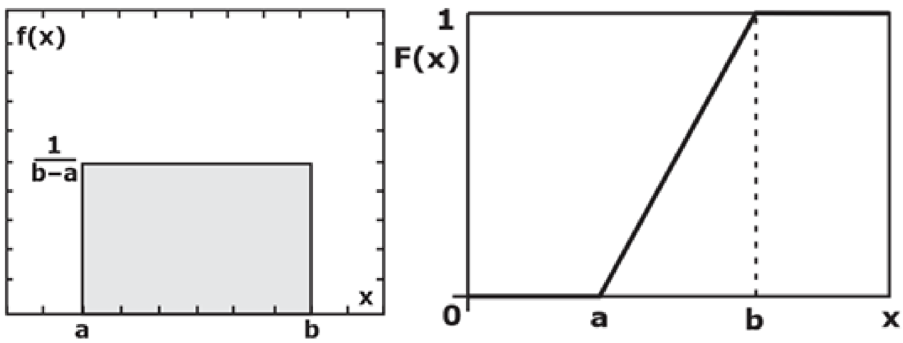
$$F_x(x) = e^{-\lambda}\sum_{i=0}^{\lfloor x \rfloor}\frac{\lambda^i}{i!}$$

$$\mu_x = E(X) = \lambda$$

$$\sigma_x^2 = Var(X) = \lambda$$

If $\lambda \to \infty$ then Poisson distribution approaches the normal distribution

### 14.6. Uniform Distribution:




**PDF** **CDF**

A random variable X is called a uniform random variable over (a, b) if its PDF is given by

$$f_x(x) = \begin{cases} \dfrac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

The corresponding CDF of X is

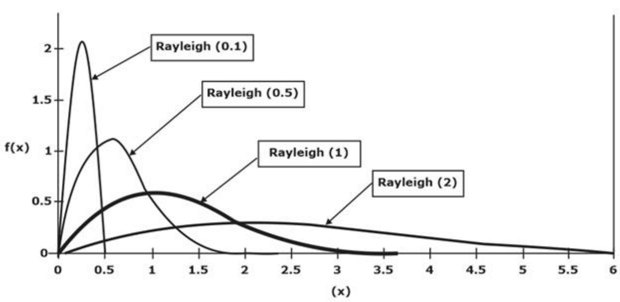
$$F_x(x) = \begin{cases} 0 & x \le a \\ \dfrac{x-a}{b-a} & a < x < b \\ 1 & x \ge b \end{cases}$$

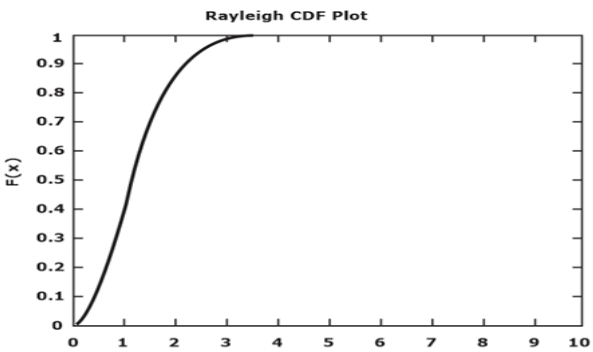
The mean and variance are:

$$\mu_x = E(x) = \frac{a+b}{2}$$

$$\sigma_x^{\,2} = Var(X) = \frac{(b-a)^2}{12}$$

### 14.7. Rayleigh Distribution



**PDF** **CDF**

**PDF**

$$f_x(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$$

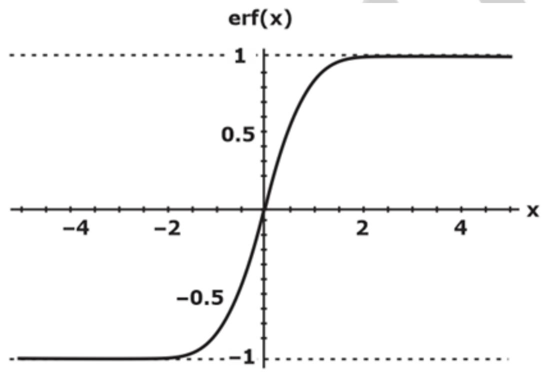
**CDF**

$$F_x(x) = 1 - e^{-\frac{x^2}{2\sigma^2}}$$

$$\mu_x = E(x) = \sigma\sqrt{\frac{\pi}{2}}$$

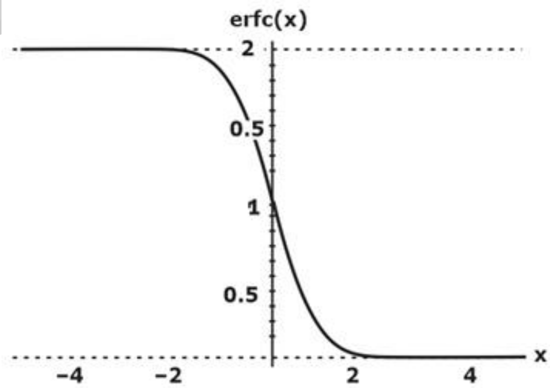
$$\sigma_x^2 = Var(X) = \frac{4-\pi}{2}\sigma^2$$

## 15. Error Function



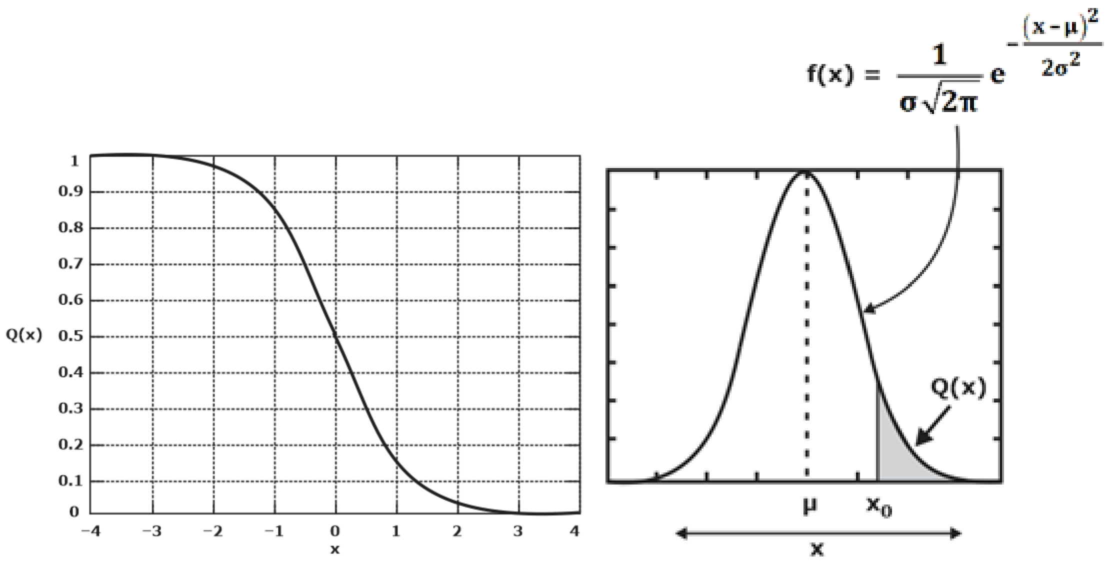

$$erf(x) = \frac{2}{\pi}\int_0^x e^{-t^2} dt$$

## 16. Complementary error function




erfc(x)=1-erf(x)

**17. Q function**




$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{u^2}{2}} du$$

$$Q(x) = 1 - Q(-x)$$

$$Q(x) = \frac{1}{2} - \frac{1}{2} erf\left(\frac{x}{\sqrt{2}}\right)$$

$$Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right)$$

$$Q(0) = 0.5$$

**18. Multiple Random Variables**

**18.1. Expectation**

If Y and X are continuous random variables, and g is some function, then Z = g(X, Y) is also a random variable. For now, let us note that the expected value rule is still applicable and

$$E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f_{x,y}(x,y)\,dxdy$$

As an important special case, fur any scalars a, is, we have
E(aX + by] = aE[X] + bE[Y].

**18.2. Independence of Continuous Random Variables**

Suppose that X and Y are independent, that is,
$f_{x,y}(x,Y) = f_x(x) f_y(y)$, for all x,y.

**18.3. Independent Random Variables:**

If X and Y are independent random variable's, then

$p_{XY}(x_i, y_j) = p_X(x_i)\, p_Y(y_j)$

## 19. Joint Probability Density Functions:

Let (X, Y) be a continuous bivariate random variable with CDF $F_{XY}(x, y)$ and let

$$f_{XY}(x,y) = \frac{\partial^2 F_{xy}(x,y)}{\partial x\, \partial y}$$

The function $f_{XY}(x, y)$ is called the joint probability density (joint pdf) of (X, Y).

### 19.1. Properties of $f_{XY}(x,)$:

1. $f_{XY}(x, y) \geq 0$
2. $\iint_{R_A} f_{XY}(x, y)\, dx\, dy = 1$
3. $f_{XY}(x, y)$ is continuous for all values of x or concept, possible a finite set.
4. $P[(X, Y)] \in A] = \iint_{R_A} f_{XY}(x, y)\, dx\, dy$
5. $P(a < X \leq b, c \leq d) = \int_c^d \int_a^b f_{XY}(x, y)\, dx\, dy$

   Since P[(X = a)] = 0 = P(Y = c)
   it follows that
   $P(a < X \leq b, c < Y \leq d) = P(a \leq X \leq b, c \leq Y \leq d) = P(a \leq X < b, c \leq Y < d)$
   $= P(a < X \leq b, c < Y < d) = \int_c^d \int_a^b f_{XY}(x, y)\, dx\, dy$

### 19.2. Properties of $F_{XY}(x, y)$:

The joint CDF of two random variable's has many properties analogous to those of the CDF of a single random variable

1. $0 \leq F_{XY}(x, y) \leq 1$
2. If $x_1 \leq x_2$, and $y_1 \leq y_2$, the
$F_{XY}(x_1, y_1) \leq F_{XY}(x_2, y_1) \leq F_{XY}(x_2, y_2)$
$F_{XT}(x_1, y_1) \leq F_{XY}(y_1, y_2 \leq F_{XY}(x_2, y_2)$
3. $\lim_{\substack{x\to\infty \\ y\to\infty}} F_{XY}(x,y) = F_{XY}(\infty, \infty) = 1$
4. $\lim_{x\to-\infty} F_{XY}(x,y) = F_{XY}(-\infty, y) = 0$
5. $\lim_{x\to a^+} F_{XY}(x,y) = F_{XY}(-a^+, y) = F_{XY}(a,y)$
6. $P(X_1 < X \leq x_2, Y \leq y) = F_{XY}(x_2, y) - F_{XY}(x_1, y)$
7. $P(X \leq x, y_1 < Y \leq y_2) = F_{XY}(x, y_2) - F_{XY}(x, y_1)$
8. If $x_1 \leq x_2$ and $y_1 \leq y_2$, then
$F_{XY}(x_2, y_2) - F_{XY}(x_1, y_2) - F_{XY}(x_2, y_1) + F_{XY}(x_1, y_1) \geq 0$

Properties are almost same for the discreet case just use summation instead if integration.

**20. Marginal Probability Density Functions:**

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$f_Y(x) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

$$F_Y(x) = \lim_{y\to\infty} F(x,y)$$

$$F_Y(y) = \lim_{x\to\infty} F(x,y)$$

$$f_{y|x}(y\,|\,x) = \frac{f_{X,Y}(x,y)}{f_x(x)}$$

$$f_{x|y}(x\,|\,y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

# CHAPTER-7: RANDOM PROCESS

## 1. Classification of Random Process

Random processes may be classified as continuous or discrete.

**1.1. Continuous Random Process:** A continuous random process consists of a random process with associated continuously distributed random variables $X(t, s_i)$. The Gaussian random process is an example of the continuous random process.

**1.2. Discrete Random Process**: A discrete random process consists of random variables with discrete distributions. For example, the output of an ideal (hard) limiter in a binary (discrete with two levels) random process.

## 2. Probability Density Function of Random Process

A complete description of a random process $\{X(t, s)\}$ is given by the N-fold joint pdf that probabilistically describes the possible values assumed by a typical sample function at time $t_N > t_{N-1} > \ldots > t_1$, when N is arbitrary.

For N = 1, we can interpret this joint pdf as

$$f_{X_1}(x_1, t_1)\,dx_1 = P(x_1 - dx_1 < X_1 \le x_1 \text{ at time } t_1)$$

Where $X_1 = X(t_1, s)$. Similarly, for N = 2, we can interpret the joint pdf as $f_{X_1X_2}(x_1, t_1 : x_2, t_2)$ $dx_1dx_2 = P(x_1 - dx_1 < X_1 \le x_1 \text{ and } x_2 - dx_2 < X_2 \le x_2)$ where $X_2 = X(t_2, s)$. In general, we denote the N-dimensional PDF of a random process as

$$f_{X(t)}(x) = f_{X(t)}(x(t_1), x(t_2), \ldots, x(t_N))$$

## 3. Stationary Random Process

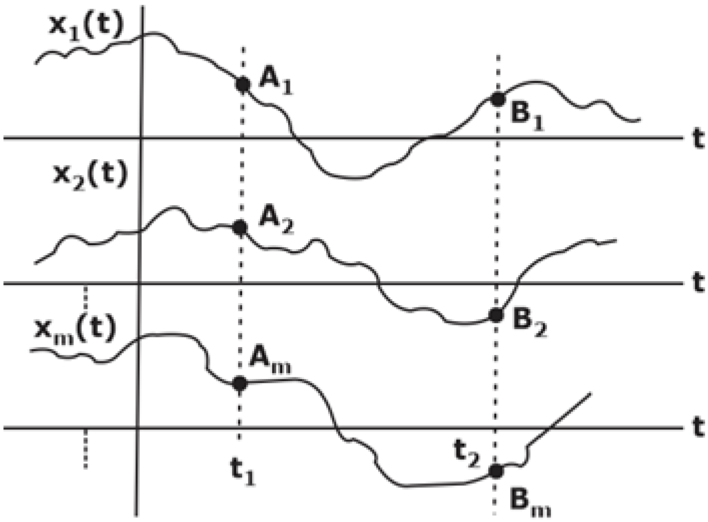
A random process X(t) is said to be stationary to the order N if, for any $t_1, t_2 \ldots, t_N$

$$f_{X(t)}(x(t_1), x(t_2), \ldots x(t_N)) = f_{X(t)}(x(t_1 + t_0), x(t_2 + t_0), \ldots x(t_N + t_0))$$

Where $t_0$ is an arbitrary real constant, the process is strictly stationary if it is stationary to the order $N \to \infty$.

## 4. Averages of Random Process:

We may define the time average and ensemble average of a random process in the following ways:




A random process with its random variables

### 4.1. Ensemble of a Random Process

If we fix t to some value, let's say $t_1$, then the result is a random variable $X(t_1, S)=[A_1A_2.....A_N]$ then the mean of $X(t_1)$, $E[X(t_1)]$ can be calculated, which is known as an ensemble average.

Ensemble average is a function of time. There is an ensemble average with respect to each time, and thus ensemble average with respect to each time can be found.

### 4.2. Time Average of a Random Process

We can consider a sample function, let's say $x_1(t)$ over an entire time scale the mean value of $x_1(t)$ is defined as

$$\langle x_1(t)\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-\infty}^{\infty} x_1(t)\,dt$$

Similarly, we can find the value for other functions in the variable

The expected value $E[X(t)]$ is known as the time average where $X(t)=\left(\langle x_1(t)\rangle,\langle x_2(t)\rangle,\langle x_3(t)\rangle.......\langle x_N(t)\rangle\right)$

**Property of Ensemble average:**

The mean (ensemble average) of a stationary process is constant, i.e.

$\mu_X(t) = \mu_X$ for all t

## 5. Autocorrelation function

The autocorrelation function of a random process X(t) is defined as the expectation of the product of two random variables $X(t_1)$ and $X(t_2)$, i.e.

$$R_X(t_1, t_2) = E[X(t_1)\,X(t_2)] = \overline{X(t_1)\,X(t_2)}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1x_2\, f_{x(t)}(x_1, x_2)\,dx_1dx_2$$

where $x_1 = x(t_1)$ and $x_2 = x(t_2)$. Some important properties of the autocorrelation function of a stationary random process are given below.

**5.1. Properties of Auto Correlation Function:**

- The autocorrelation function of a strictly stationary random process is a function only of the time difference $\tau = t_2 - t_1$, i.e.

  $R_X(\tau) = E[X(t)\,X(t + \tau)] = \overline{X(t)\,X(t+\tau)}$

- The mean square value of the process may be obtained from $R_x(\tau)$ simply by putting $\tau = 0$ in the above equation, i.e.

  $R_X(0) = E[X^2(t)] = \overline{X^2(t)}$

- If the autocorrelation function of a random process has no periodic component then, we may define

  $\overline{X(t)}^2 = R_X(\pm\infty)$ or $\overline{X(t)} = \sqrt{R_X(\pm\infty)}$

- The autocorrelation function is the even function of $\tau$, i.e.

  $R_X(\tau) = R_X(-\tau)$

- The autocorrelation has its maximum magnitude at $\tau = 0$, i.e.

  $R_X(0) \geq R_X(\tau)$

## 6. Cross-Correlation Function

The cross-correlation function for two random processes, X(t) and Y(t), is defined as:

$R_{XY}(t_1, t_2) = E[X(t_1)\,Y(t_2)] = \overline{X(t_1)\,Y(t_2)}$

Where $t_1$ and $t_2$ denote the two values of time at which the processes are observed, the following are some important properties of the cross-correlation function of two jointly stationary random processes, X(t) and Y(t):

**6.1. Properties of Cross-Correlation Function:**

- The cross-correlation function of jointly random processes X(t) and Y(t) is a function only of the time difference $\tau = t_2 - t_1$, i.e.

  $R_{XY}(t_1, t_2) = R_{XY}(\tau)$
- The cross-correlation function is the even function of $\tau$, i.e.

  $R_{XY}(-\tau) = R_{YX}(\tau)$
- The random processes X(t) and Y(t) are said to be uncorrelated if, for all $t_1$ and $t_2$,

  $R_{XY}(t_1, t_2) = \overline{X(t_1)} \times \overline{Y(t_2)}$

- $|R_{XY}(\tau)| \leq \sqrt{R_X(0)\,R_Y(0)}$

- $|R_{XY}(\tau)| \leq \frac{1}{2}[R_X(0) + R_Y(0)]$

## 7. ERGODIC PROCESS

A random process is ergodic if the all-time averages of any sample function are equal to the corresponding ensemble averages (expectations). As the ergodic process has its ensemble average equal to its time average, we may deduce the following properties for the ergodic process:

### 7.1 Properties of Ergodic Process:

- The dc value of an ergodic process can be defined in terms of ensemble average as

  $X_{dc} = \langle X(t) \rangle = E[X(t)] = \overline{X(t)}$
- The dc power of an ergodic process can be defined as

  $P_{dc} = \langle X(t) \rangle^2 = \{E[X(t)]\}^2 = \{\overline{X(t)}\}^2$
- The power in the ac (time-varying) component is given by

  $P_{ac} = \sigma_x^2 = \langle X^2(t) \rangle = \langle X(t) \rangle^2 = \overline{X^2(t)} - \overline{X(t)}^2$
- The RMS power (total power) of an ergodic process is defined as

  $P_{total} = \langle X^2(t) \rangle = E[X^2(t)] = \overline{X^2(t)} = \sigma^2 x + \overline{X(t)}^2$
- The RMS value of an ergodic process can be defined as

  $X_{rms} = \sqrt{\langle X^2(t) \rangle} = \sqrt{E[X^2(t)]} = \sqrt{\overline{X^2(t)}}$

  $= \sqrt{\sigma_x^2 + \overline{X(t)}^2}$

## 8. Wide Sense Stationary Process:

A random process is said to be wide-sense stationery (WSS) if its mean is independent of time and autocorrelation function depends on the time difference, i.e.

$E[X(t)] = \mu_x$ where $\mu_x$ is a constant

and $R_X(t_1, t_2) = R_X(\tau)$, where $\tau = t_2 - t_1$

## 9. Power Spectral Density

The power spectral density of a given signal describes the distribution of power into frequency components making up that signal

Following are some important properties of the power spectral density of a stationary process.

### 9.1. Properties of Power Spectral Density:

- The power spectral density $S_X(f)$ is always real and non-negative, i.e.

  $S_X(f) \geq 0$
- The power spectral density of a real-valued random process is an even function of frequency, i.e.

  $S_X(-f) = S_X(f)$

- The total normalized power of a random process is defined in terms of power spectral density as

  $$P_{total} = \int_{-\infty}^{\infty} S_X(f)\,df$$

  or $$E[X^2(t)] = \int_{-\infty}^{\infty} S_X(f)\,df$$

  or $$R_X(0) = \int_{-\infty}^{\infty} S_X(f)\,df$$

- The zero-frequency value of the power spectral density of a stationary process equals the total area under the graph of the autocorrelation function, i.e.

  $$S_X(0) = \int_{-\infty}^{\infty} R_X(\tau)\,d\tau$$

**10. Cross Spectral Density**

Let X(t) and Y(t) be two jointly stationary processes with their cross-correlation functions denoted by $R_{XY}(\tau)$ and $R_{YX}(\tau)$. We then define the cross-spectral densities for the random processes as

$$S_{XY}(f) = \int_{-\infty}^{\infty} R_{XY}(\tau)\,e^{-j2\pi fr}\,d\tau$$

and $$S_{YX}(f) = \int_{-\infty}^{\infty} R_{YX}(\tau)\,e^{-j2\pi fr}\,d\tau$$

Accordingly, using the formula for inverse Fourier transformation, we may also write

$$R_{XY}(\tau) = \int_{-\infty}^{\infty} S_{XY}(f)\,e^{j2\pi fr}\,df$$

and $$R_{YX}(\tau) = \int_{-\infty}^{\infty} S_{YX}(f)\,e^{j2\pi fr}\,df$$

**10.1. Properties of Cross Spectral Density:**

The cross-spectral densities $S_{XY}(f)$ and $S_{YX}(f)$ are not necessarily the real function or f. However, using the property of the autocorrelation function, we may deduce that

$S_{XY}(f) = S_{YX}(-f) = S_{YX}^{*}(f)$

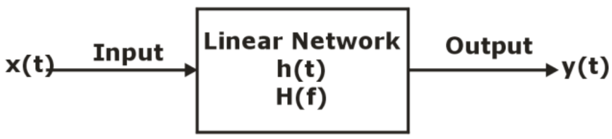
**11. Linear System:**

Consider a linens system shown in Figure 10. The input-output relationship for the system is

$y(t) = h(t) * x(t)$

The corresponding Fourier transform relationship is

$Y(f) = H(f)\,X(f)$




| | |
|---|---|
| $X(f)$ | $Y(f)$ |
| $R_X(\tau)$ | $R_Y(\tau)$ |
| $S_X(f)$ | $S_Y(f)$ |

**11.1. Input-Output Relationship of a Linear System**

If x(t) is a wide-sense stationary random process, then the output autocorrelation is defined as

$R_Y(\tau) = h(-\tau) * h(\tau) * R_X(\tau)$

Correspondingly, the output power spectral density is given by

$S_Y(f) = |H(f)|^2 S_X(f)$

Thus, the power transfer function of the network is

$$G(f) = \frac{S_Y(f)}{S_X(f)} = |H(f)|^2$$

# CHAPTER-8: BASEBAND MODULATION

## 1. Sampling Process

Sampling is the process of measuring the instantaneous values of the continuous-time signal in a discrete form. The sample is a piece of data taken from the whole data, which is continuous in the time domain.

The time interval between two consecutive samples is called the sampling period.

**Sampling frequency:** The reciprocal of the sampling period is referred to as sampling frequency, i.e.

$\mathbf{f_S = 1/T_S}$

## 2. Sampling Theorem

A band-limited signal having no frequency components higher than $f_m$ Hz is completely described by its sample values at uniform intervals less than or equal to $1/2f_m$ Hz

A band-limited signal having no frequency components higher than $f_m$ Hz may be completely recovered from the knowledge of its samples taken at the rate of at least $2f_m$ samples per second.

## 3. Nyquist Rate

Nyquist rate is defined as the minimum sampling frequency allowed to reconstruct a band-limited waveform without error, i.e.

$f_N = \min\{f_S\} = 2W$

Where W is the message signal bandwidth, and $f_S$ is the sampling frequency.

## 4. Nyquist Interval

The reciprocal of Nyquist rate is called the Nyquist interval (measured in seconds), i.e.

$$T_N = \frac{1}{f_N} = \frac{1}{2W}$$

Where $f_N$ is the Nyquist rate, and W is the message signal bandwidth.

## 5. Sampling of Bandpass Signals

The bandpass signal x(t), whose maximum bandwidth is $2f_m$, can be completely represented and recovered from its samples if sampled at the minimum rate of twice the bandwidth.

Hence if the bandwidth is $2f_m$, the minimum sampling rate for bandpass signal must be $4f_m$ samples per second.

This bandpass signal is first represented in terms of its in-phase and quadrature components

Let $x_I(t)$ = Inphase component of x(t)

And $x_Q(t)$ = Quadrature component of x(t)

Thus, the signal x(t) in terms of inphase and quadrature components will be expressed as

$x(t) = x_I(t)\cos(2\pi f_c t) - X_Q(t)\sin(2\pi f_c t)$

Thus, if $4f_m$ samples per second are taken, the bandpass signal of bandwidth $2f_m$ can be completely recovered from its samples. Hence, for bandpass signals of bandwidth $2f_m$.

Minimum sampling rate = Twice of bandwidth

= $4f_m$ samples per second.

### 6. Sampling Technique

There are three types of sampling techniques as under:

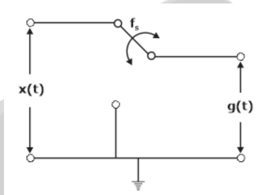
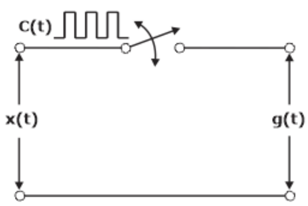
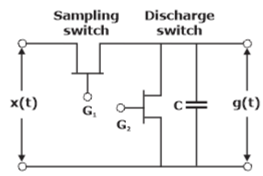
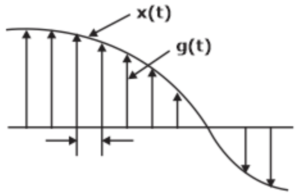
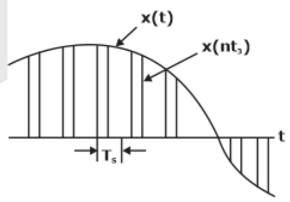
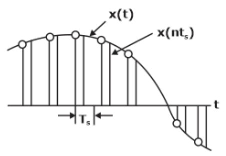
**i.** Instantaneous sampling

**ii.** Natural sampling

**iii.** Flat top sampling

Out of these three, instantaneous sampling is ideal sampling, whereas natural sampling and flat-top sampling are **practical sampling** methods.

**Table 1: Performance Comparison of three Sampling Technique**

| S. No | Parameter of comparison | Ideal or instantaneous sampling | Natural sampling | Flat top sampling |
|---|---|---|---|---|
| 1. | Sampling principle | It uses multiplication | It uses the chopping principle | It uses sample and hold circuit |
| 2. | Generation circuit |  |  |  |
| 3. | Waveforms involved |  |  |  |
| 4. | Feasibility | This is not a practically possible method | This method is used practically | This method is also used practically |
| 5. | Sampling rate | The sampling rate tends to infinity | The sampling rate satisfies Nyquist criteria | The sampling rate satisfies Nyquist criteria |
| 6. | Noise | Noise is maximum | Noise is minimum noise | noise is maximum |
| 7. | Time-domain representation | $g(t)=\sum_{n=-\infty}^{\infty} x(nT_s)\delta(t-nT_s)$ | $g(t)=\frac{\tau A}{T_s}\sum_{n=-\infty}^{\infty} x(t)\,\mathrm{sinc}(nf_s\tau)e^{j2\pi nf_s t}$ | $g(t)=\sum_{n=-\infty}^{\infty} x(nT_s)h(t-nT_s)$ |
| 8. | Frequency domain representation | $G(f)=f_s\sum_{n=-\infty}^{\infty} X(f-nf_s)$ | $G(f)=\frac{\tau A}{T_s}\sum_{n=-\infty}^{\infty} \mathrm{sinc}(nf_s\tau)X(f-nf_s)$ | $G(f)=f_s\sum_{n=-\infty}^{\infty} X(f-nf_s)H(f)$ |

## 7. Inter symbol Interference

Intersymbol interference (ISI) is a phenomenon in which one symbol interferes with subsequent symbols. This is unwanted as the previous symbols have a similar effect as noise, thus making the communication less reliable. The spreading of the pulse beyond its allotted time interval causes it to interfere with neighbouring pulses. ISI is usually caused by multipath propagation or the inherent linear or non-linear frequency response of a communication channel causing successive symbols to blur together.

## 8. Aliasing

Aliasing is a phenomenon that causes different signals to become indistinguishable when sampled. It also often refers to the distortion that results when a signal is reconstructed from the samples different from the original continuous signal.

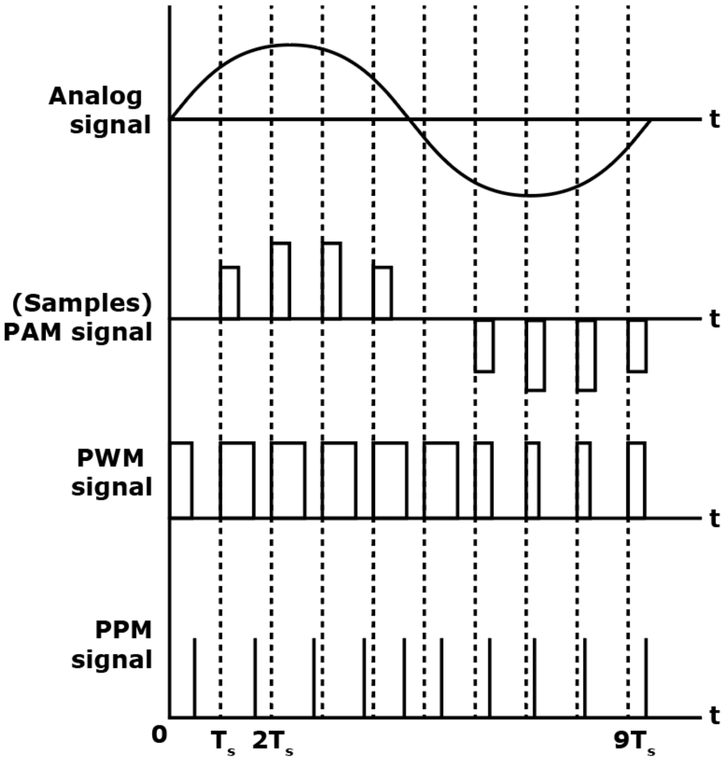
## 9. Pulse Modulation

Pulse modulation is the process of changing a binary pulse signal according to the information to be transmitted. Pulse modulation can be either analogue or digital.

### 9.1. Analog Pulse Modulation

Analog pulse modulation results when some pulse attribute varies continuously in one-to-one correspondence with a sample value. In analogue pulse modulation systems, the amplitude, width, or position of a pulse can vary over a continuous range in accordance with the message amplitude at the sampling instant.

**i.** Pulse Amplitude Modulation (PAM)
**ii.** Pulse Width Modulation (PWM)
**iii.** Pulse Position Modulation (PPM)




Representation of Various Analog Pulse Modulation

**9.2. Digital Pulse Modulation**

**i.** Delta Modulation (DM)

**ii.** Pulse Code Modulation (PCM)

**iii.** Differential Pulse Code Modulation (DPCM)

**iv.** Adaptive Delta Modulation (ADM)

## 10. Pulse Code Modulation

The PCM is done by using the following steps

**i.** Sampling

**ii.** Quantizing

**iii.** Encoding


**Block Diagram Representation of PCM System**

**10.1. Quantization**

Quantization is the process of mapping input values from a large set or a continuous signal to output values in a (countable) smaller set, with a finite number of elements

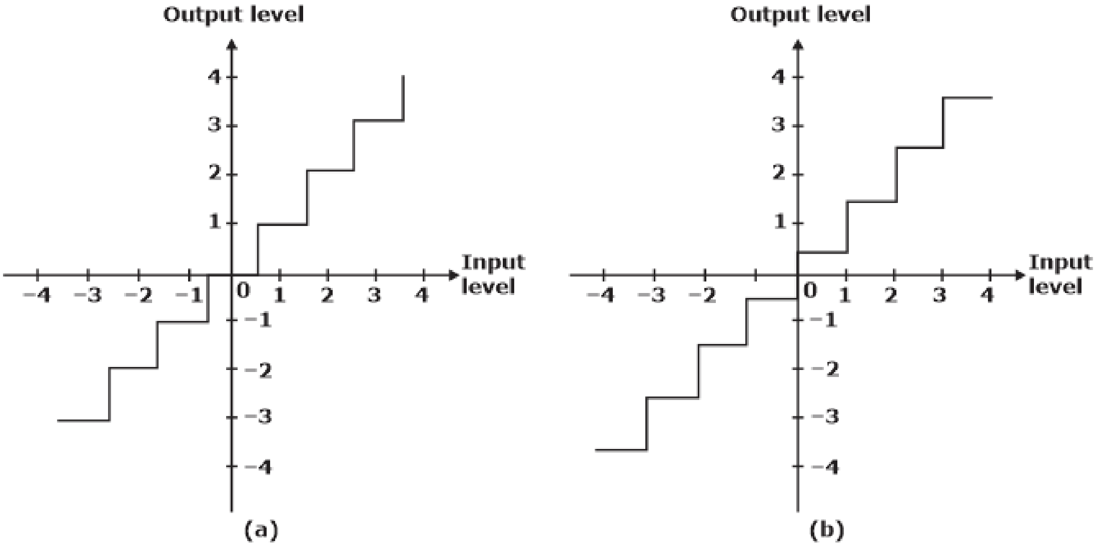
### 10.1.1 Uniform Quantizer

A quantizer is called a uniform quantizer if the step size remains constant throughout the input range.

There are two types of quantizers:

**i.** midtread type

**ii.** midrise type




**Two types of Uniform Quantization (a) Midtread, and (b) Midrise**

### 10.1.2. Nonuniform Quantizer

Nonuniform quantization is a quantizer whose step size is not fixed but varies based on different techniques like μ law given below

$$|m_q| = \frac{1n(1+\mu|m|)}{1n(1+\mu)}$$

Where m and $m_q$ are the normalized input and output voltages, and μ is a positive constant.

## 10.2. Encoding

An encoder translates the quantized samples into digital code words. The encoder works with M-ary digits and produces a codeword of n digits parallel for each sample. Since there are $M^n$ possible M-ary codewords with n digits per word, unique encoding of the q different levels requires that

$M^n \geq q$

The parameters M, n, and q should be chosen so that

$q = M^n$ or $n = \log_M q$

### 10.2.1. Encoding in Binary PCM

For binary PCM, each digit may be either of two distinct values, 0 or 1, i.e.

M = 2

If the code word of binary PCM consists of n digits, then the number of quantization levels is defined as

$q = 2^n$

or $n = \log_2 q$

### 10.3. Characteristics of PCM System

- A sampled waveform is quantized into q quantization levels, where q is an integer.
- If the message signal is defined in the range $(-m_p, m_p)$, then the step size of the quantizer is

$$\delta = \frac{2m_p}{q}$$

- For a binary PCM system with n digit codes, the number of quantization levels is defined as

$q = 2^n$

- If the message signal is sampled at the sampling rate $f_S$ and encoded to n number of bits per sample; then the bit rate in (bits/s) of the PCM is defined as

$R_b = nf_S$

### 10.4. Transmission Bandwidth In A PCM System

$$B_{PCM} \geq \frac{1}{2}R_b = \frac{1}{2}nf_s$$

$R_b$ is the bit rate, n is the number of bits in PCM word, and $f_S$ is the sampling rate. Since the required sampling rate for no aliasing is

$f_S \geq 2W$

Where W is the bandwidth of the message signal (that is to be converted to the PCM signal). Thus, the bandwidth of the PCM signal has a lower bound given by

$B_{PCM} \geq nW$

When k synchronising bits are included in the signal

$BW=2(nN+k)f_s$ Hz

### 10.5. NOISE IN PCM

In PCM (pulse code modulation), there are two sources of error:

**i.** Quantization noise

**ii.** Channel noise

**Quantization Noise**

For a PCM system, the kth sample of quantized message signal is represented by

$M_q(kT_S) = m(kT_S) + \varepsilon(kT_S)$

Where $m(kT_S)$ is the sampled waveform, and $\varepsilon(kT_S)$ is the quantization error. Let the quantization levels having a uniform step size δ. Then, we have

$$-\frac{\delta}{2} \leq \varepsilon \leq \frac{\delta}{2}$$

So, the mean-square error due to quantization is

$$\overline{\varepsilon^2} = \frac{1}{\delta}\int_{-\delta/2}^{\delta/2} \varepsilon^2 d\varepsilon = \frac{\delta^2}{12} \text{...........(i)}$$

### 10.6. Signal to Quantization Noise Ratio

For the PCM system, we have the message signal m(t) and quantization error of ε. So, we define the signal to quantization noise ratio as

$$(SNR)_Q = \frac{\overline{m^2(t)}}{\overline{\varepsilon^2}} = \frac{\overline{m^2(t)}}{\delta^2/12} \text{ ..............(ii)}$$

Where δ is the step size of the quantized signal defined as

$$\delta = \frac{2m_p}{q} \text{ ....................(iii)}$$

Substituting equation (iii) in equation (ii), we get the expression for the signal to quantization noise ratio as

$$(SNR)_Q = 12\frac{\overline{m^2(t)}}{(2m_p/q)^2}$$

$$(SNR)_Q = 3q^2\frac{\overline{m^2(t)}}{m_p^2} \text{ ...............(iv)}$$

Where $m_p$ is the peak amplitude of message signal m(t), and q is the number of quantization levels.

### 10.7. Channel Noise

If a PCM signal is composed of the data that are transmitted over the channel having bit error rate $P_e$, then peak signal to average quantization noise ratio is defined as

$$(SNR)_{peak} = \frac{3q^2}{1+4(q^2-1)P_e}$$

Similarly, for the channel with bit error probability $P_e$, the average signal to average quantization noise ratio is defined as

$$(SNR)_{avg} = \frac{q^2}{1+4(q^2-1)P_e}$$

### 10.8. Companding

Companding is a non-linear technique used in PCM which compresses the data at the transmitter and expands the same data at the receiver.
It is used to mitigate the effects of noise and crosstalk.

μ law companding

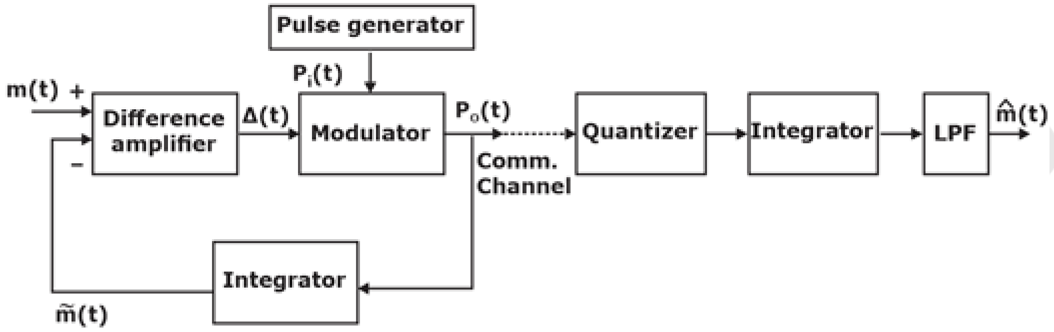
$$y = \frac{1}{\ln(1+\mu)}\ln\left(1+\frac{\mu m}{m_p}\right)'0 \le \frac{m}{m_p} \le 1$$

A-law companding

$$y=\begin{cases}\dfrac{A}{1+\ln A}\left(\dfrac{m}{m_p}\right), & 0\le\dfrac{m}{m_p}\le\dfrac{1}{A}\\[2ex] \dfrac{1}{1+\ln A}\left(1+\ln\dfrac{Am}{m_p}\right), & \dfrac{1}{A}\le\dfrac{m}{m_p}\le 1\end{cases}$$

**11. Delta Modulation**

Delta Modulation is the simplest form of differential pulse-code modulation (DPCM), where the difference between successive samples is encoded into n-bit data streams. In delta modulation, the transmitted data are reduced to a 1-bit data stream




**Delta modulation block diagram**

**11.1 NOISE IN DELTA MODULATION**

The noise in delta modulation can be classified into two types:

**i.** Slope Overload Noise

**ii.** Granular Noise

**11.1.1. Slope Overload Noise**

This distortion arises because of a large dynamic range of the input signal

The maximum slope that the accumulator output can generate is

$$\frac{\delta}{T_s}=\delta f_s$$

condition to avoid slope overload is,

$$\max\left|\frac{dm(t)}{dt}\right|\le\delta f_s$$

Where m(t) is the message signal, δ is the step size of the quantized signal, and $f_s$ is the sampling rate.

**11.1.2. Granular Noise**

Granular or Idle noise occurs when the step size is too large compared to a small variation in the input signal

Form equation (i), we have the total quantizing noise for the PCM system,

$$(\overline{\varepsilon^2})_{PCM} = \frac{1}{\delta}\int_{-\delta/2}^{\delta/2} \varepsilon^2 d\varepsilon = \frac{\delta^2}{12} = \frac{(\delta/2)^2}{3}$$

Replacing δ/2 of PCM by δ for DM, we obtain the total granular quantizing noise as

$$(\overline{\varepsilon^2})_{DM} = \frac{\delta^2}{3}$$

Thus, the power spectral density for granular noise in delta modulation system is obtained as

$$S_N(f) = \frac{\delta^2/3}{2f_s} = \frac{\delta^2}{6f_s}$$

Where δ is the step size, and $f_S$ is the sampling frequency.

### 11.2. Finding Minimum Step Size In Delta Modulation

$$\delta f_s \geq \max\left|\frac{dm(t)}{dt}\right| = 2\pi f_m A_m$$

## 12. Multilevel Signaling

### 12.1. Baud Rate

$$D = \frac{1}{T_s}$$

Where D is the symbol rate which is called baud.

### 12.2. Bits per Symbol

we define the bits per symbol as

$K = \log_2 M$

### 12.3. Relation Between Baud and Bit Rate

For a multilevel signalling scheme, the bit rate and baud (symbols per second) are related as

$R_b = kD = D\log_2 M$ ……………..(v)

$R_b$ is the bit rate, $k = \log_2 M$ is the bits per symbol, and D is the baud (symbols per second).

### 12.4. Relation Between Bit Duration and Symbol Duration

For a multilevel signalling scheme, the bit duration is given by

$$T_b = \frac{1}{R_b}$$

Where $R_b$ is the bit rate. Also, we have the symbol duration

$$T_s = \frac{1}{D}$$

Where D is the symbol rate. Thus, by substituting this expression in equation (v), we get the relation

$T_S = kT_b = T_b \log_2 M$

Where $k = \log_2 M$ is the bits per symbol.

### 13. Transmission Bandwidth

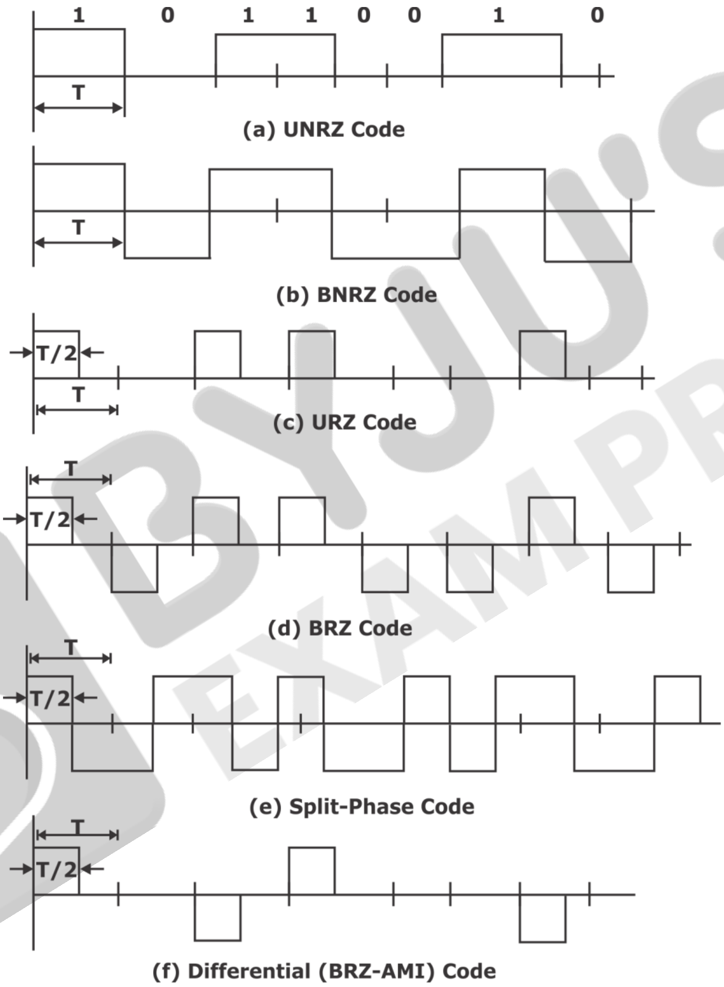
The null to null transmission bandwidth of the rectangular pulse multilevel waveform is defined as

$B_T$ = D symbols/sec

The absolute transmission bandwidth for $\frac{\sin x}{z}$ pulse multilevel waveform is defined as

$B_T = \frac{D}{2}$ symbols / sec

### 14. Line Codes




| Line Code | Minimum Bandwidth | Average DC | Clock recovery | Error Detection |
|---|---|---|---|---|
| UNRZ | 1/2T | V/2 | Poor | No |
| BNRZ | 1/2T | 0 | Poor | No |
| URZ | 1/T | V/2 | Good | No |
| BRZ | 1/T | 0 | Very good | No |
| Manchester | 1/T | 0 | Best | No |
| BRZ-AMI | 1/2T | 0 | Good | Yes |

## CHAPTER-9: BANDPASS MODULATION

### 1. Digital Bandpass Modulation

There are three basic modulation schemes:

- Amplitude shift keying (ASK)
- Frequency shift keying (FSK)
- Phase shift keying (PSK)

#### 1.1. Amplitude-Shift Keying (ASK):

In ASK, the modulated signal is expressed as

$$x_c(t) = \begin{cases} A\cos\omega_c t & \text{symbol 1} \\ 0 & \text{symbol 2} \end{cases}$$

Note that the modulated signal is still an on-off signal. Thus, ASK is also known as on-off keying (00K).

#### 1.2. Frequency-Shift Keying (FSK):

In FSK, the modulated signal can be expressed as

$$x_c(t) = \begin{cases} A\cos\omega_1 t & \text{symbol 1} \\ A\cos\omega_2 t & \text{symbol 0} \end{cases}$$

#### 1.3. Phase shift keying (PSK):

In PSK, the modulated signal can be expressed as

$$x_c(t) = \begin{cases} A\cos\omega_c t & \text{symbol 1} \\ A\cos(\omega_c t + \pi) & \text{symbol 0} \end{cases}$$

### 2. Amplitude Shift Keying

ASK is often referred to as on-off keying (OOK). The ASK signal is represented by

$s(t) = A_c\, m(t) \cos\omega_c t$

where m(t) is a unipolar baseband data signal.

$$s(t) = A_c \cos\omega_c t = \begin{cases} A_c\cos 2\pi_c t & \to 1 \\ 0 & \to 0 \end{cases}$$

$$P_S = \frac{A_C^{\,2}}{2}$$

$$s(t) = \begin{matrix} \sqrt{2p_s}\cos 2\pi f_c t \to 1 & 0 \le t \le T_2 \\ 0 \qquad\qquad\quad \to 0 & \text{elsewhere} \end{matrix}$$

Because of this, it is called **"ON-OFF KEYING."**

Ask is one dimensional

### 2.1. Transmission Bandwidth of ASK Signal

For ASK signal, the transmission bandwidth is given by

$B_T = 2R_b$

If raised cosine-roll off is used (to conserve bandwidth), the absolute transmission bandwidth (for rectangular pulse waveform) of the ASK signal is obtained as

$B_T = (1+\alpha)R_b$

Where α is the roll-off factor of the filter.

## 3. Binary Phase Shift Keying

**3.1.** Binary Phase-shift keying (BPSK) is a digital modulation scheme that conveys data by changing two different phases of a carrier wave. The constellation points chosen are usually positioned with uniform angular spacing around a circle.

### 3.2. The bandwidth of BPSK Signal

The null-to-null transmission bandwidth for the BPSK system is the same as that found for amplitude shift keying (ASK). The null-to-null transmission bandwidth for the BPSK system is given by

$B_T = 2R_b$

Where $R_b$ is the bit rate of the digital signal.

## 4. FSK

BFSK uses a pair of discrete frequencies to transmit binary (0s and 1s) information. With this scheme, the 1 is called the mark frequency, and the 0 is the space frequency.

### 4.1. Transmission Bandwidth of Coherent Binary FSK Signal

The transmission bandwidth for the FSK signal may be expressed as

$B_T = 2(\Delta f + R_b)$

Where $R_b$ is the bit rate of the modulating signal and Δf is the peak frequency deviation.

The above expression can be more generalized for the following cases:

**Case I: Narrowband FSK**

For narrowband FSK signal, $\Delta f \ll R_b$. So, the transmission bandwidth of narrowband FSK is given by

$B_T = 2R_b$

**Case II: Wideband FSK**

For wideband GSK signal, $\Delta f \gg R_b$. So, the transmission bandwidth of wideband FSK given by

$B_T = 2\Delta f$

**Case III: FSK with Raised Cosine Roll-off Factor**

If a raised cosine roll-off factor α is used, equation (ii) becomes

$B_T = 2\Delta f + (1 + \alpha) R_b$

5. **Noncoherent Binary Systems**

The most common noncoherent bandpass modulation techniques are:

**i.** Differential phase-shift keying (DPSK)

**ii.** Noncoherent frequency-shift keying

6. **Differential phase-shift keying (DPSK)**

Following are the steps involved in the differential encoding of a message sequence:

**Step 1:** An arbitrary reference binary digit is assumed for the initial digit of the encoded sequence. In the example shown in Table 1, a 1 has been chosen.

**Step 2:** For each digit of the encoded sequence, the present digit is used to reference the following digit in the sequence.

**Step 3:** A 0 in the message sequence is encoded as a transition from the state of the reference digit to the opposite state in the encoded message sequence; a 1 is encoded as no change of state. In the example shown, the first digit in the message sequence is a 1, so no change in state is made in the encoded sequence, and a 1 appears as the next digit.

**Step 4:** This serves as the reference for the next digit to be encoded. Since the next digit appearing in the message sequence is a 0, the next encoded digit is the opposite of the reference digit, or a 0.

**Step 5:** The encoded message sequence then phase-shift keys a carrier with the phases 0 and π as shown in the table.

**Differential Encoding Example**

| | **Reference Digit** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Message Sequence** | | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| **Encoded Sequence** | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| **Transmitted Phase** | 0 | 0 | π | 0 | 0 | 0 | 0 | Π | 0 | π |

**7. Continuous Phase Frequency Shift Keying (CPFSK)**

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos \omega_i t$$

$$\omega_i t = \omega_c t + \theta(t)$$

Here $\theta(t) = \theta(0) \pm \frac{n\pi ht}{T_b}$ where h = Deviation Ratio

$$\omega_1 t = \omega_c t + \theta(0) + \frac{n\pi ht}{T_b}$$

$$\omega_2 t = \omega_c t + \theta(0) - \frac{n\pi ht}{T_b}$$

Since $\theta(t)$ is changing with time, hence it is known as CPFSK.

$$(\omega_1 - \omega_2)\, t = \frac{2n\pi ht}{T_b}$$

$$\boxed{(f_1 - f_2) = \frac{nh}{T_b}}$$

$s_1(t) = \sqrt{\frac{2\,E_b}{T_b}}\cos(2\pi f_1 t) \rightarrow +1$

$s_2(t) = \sqrt{\frac{2\,E_b}{T_b}}\cos(2\pi f_2 t) \rightarrow -1/0$

n → integer taken as '1' for smallest value.

**8. MSK (Minimum Shift Keying)**

Here, $h = \frac{1}{2}$ (Deviation ratio)

Type of CPFSK in which $\left(h = \frac{1}{2}\right)$

In the case of MSK, both $S_1$ and $S_2$ will be orthogonal to each other. This type of FSK is also known as fast FSK.

**For MSK:**

$(f_1 - f_2) = \frac{h}{T_b} = \frac{1}{2T_b}$ (for n =1) $= \frac{\text{Bit rate}}{2}$

**9. Relation Between Bit Rate and Symbol Rate**

Since $k = \log_2 M$ bits per symbol are transmitted, so symbol rate for the MPSK system can be defined in terms of bit rate $R_b$ as

$$R_s = \frac{R_b}{k} = \frac{R_b}{\log_2 M}$$

**10. Relation Between Bit Energy and Symbol Energy**

For a multilevel Signalling scheme, assume that the signal energy per bit is $E_b$, and signal energy per symbol is $E_s$. We express the relationship between these two quantities as

$E_s = E_b(\log_2 M)$

**11. M-ary Phase Shift Keying (MPSK)**

**11.1. Transmission Bandwidth**

For an M-ary PSK signal, we define the transmission bandwidth as

$B_T = 2R_s$

where $R_s$ is the symbol rate. Substituting equation (iii) in the above expression, We get the transmission bandwidth of the MPSK system as

$$B_T = \frac{2R_b}{\log_2 M} \quad \text{..................(vii)}$$

Where $R_b$ is the bit rate for the system. Also, we have overall absolute transmission bandwidth with raised cosine filtered pulses as

$$B_T = \frac{(1+\alpha)R_s}{\log_2 M}$$

Where α is the roll-off factor.

## 12. Quadrature Phase Shift Keying (QPSK)

### 12.1. Transmission Bandwidth

Substituting M = 4 in equation (vii), we get the transmission bandwidth for the QPSK system as

$$B_T = \frac{2R_b}{\log_2 4} = R_b$$

## 13. M-ary Frequency Shift Keying (MFSK)

### 13.1. Transmission Bandwidth

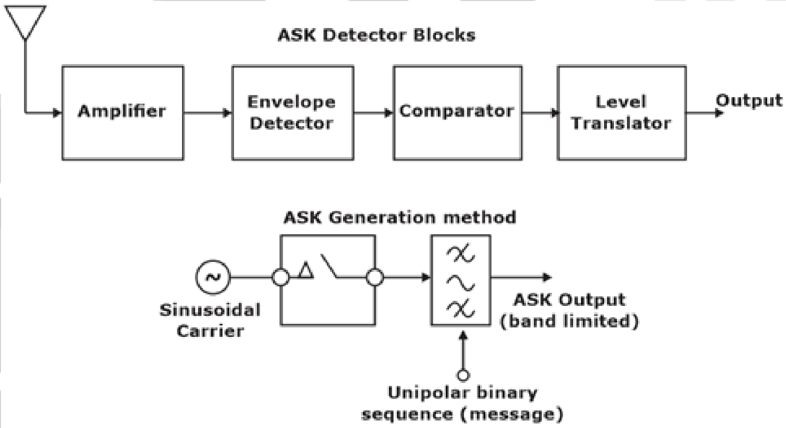
The transmission bandwidth for an M-ary FSK system is defined as

$$B_T = \frac{R_b M}{2\log_2 M}$$

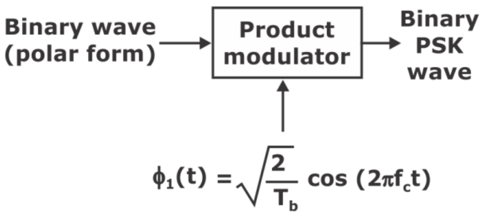
Where $R_b$ is the bit rate, and $M = 2^k$ is the size of the symbol.

## 14. Keying Techniques Block Diagrams

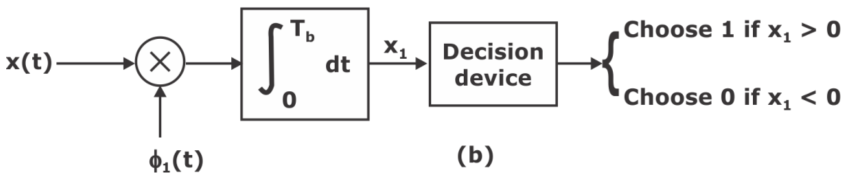
### 14.1. ASK transmitter and receiver




### 14.2. PSK transmitter and receiver

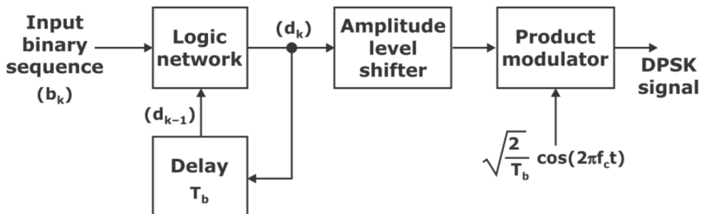



(a)

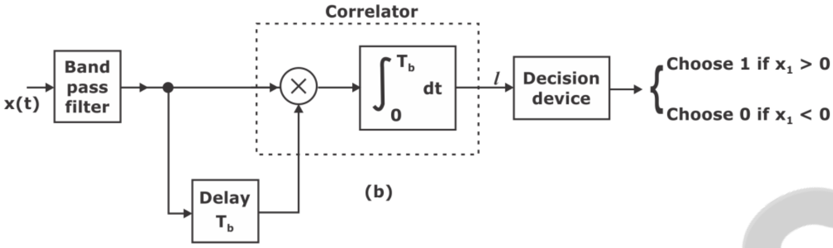



(b)

### 14.3. DPSK transmitter and receiver

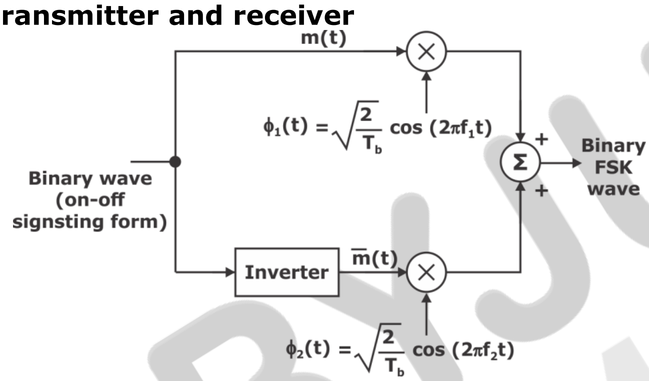



(a)

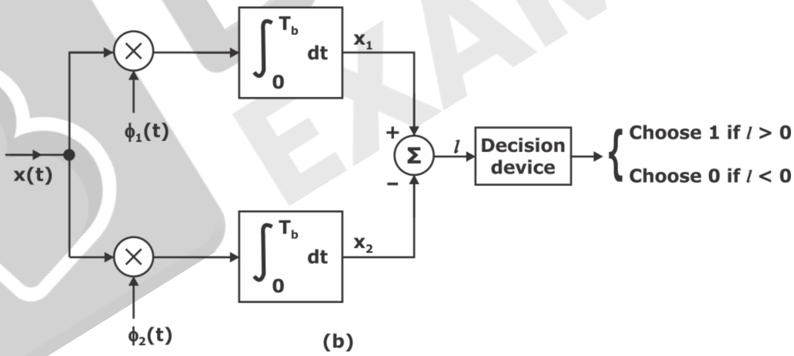



(b)

### 14.4. FSK transmitter and receiver

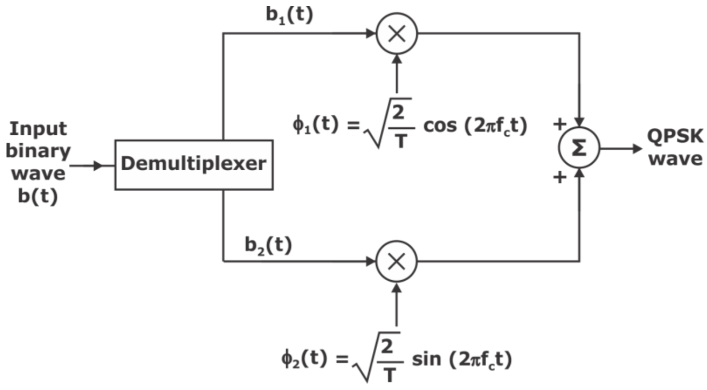



(a)

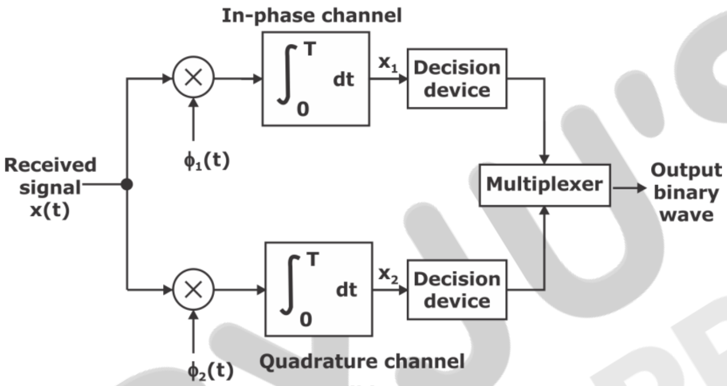



(b)

### 14.5. QPSK transmitter and receiver

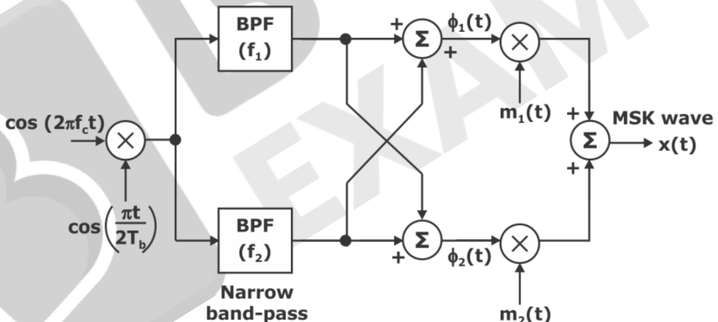



(a)

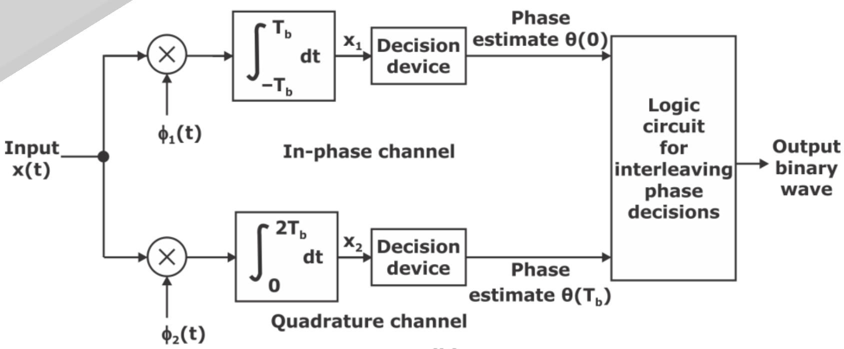



(b)

### 14.6. MSK transmitter and receiver

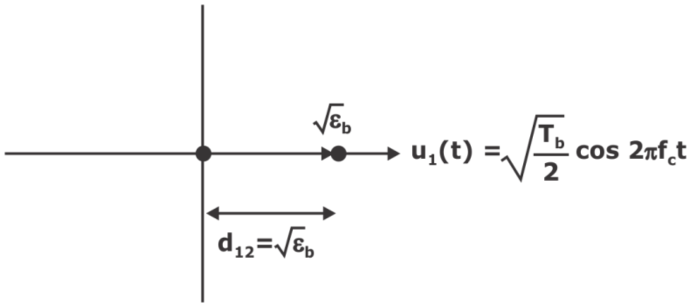



(a)

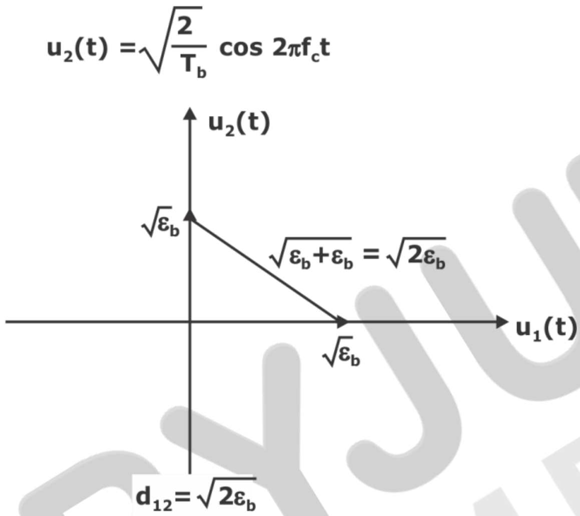



(b)

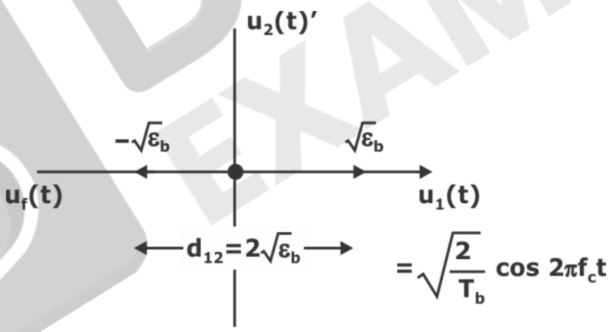
**15. Constellation diagrams**



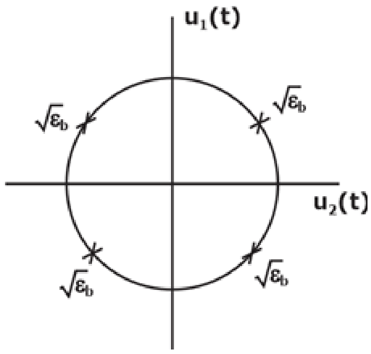

**ASK**




**FSK**




**PSK**




**MSK**

## 16. Overall comparison

| S. No | Parameter | ASK | BPSK | QPSK | QAM | M-ary | BFSK | M-ary FSK | MSK |
|---|---|---|---|---|---|---|---|---|---|
| 1. | Information is transmitted by change in | Amplitude | Phase | Phase | Amplitude and phase | Phase | Frequency | Frequency | Frequency |
| 2. | Expression for transmitted signal | $A_c \cos\omega_c t$ for symbol 1<br>0 for symbol 0 | $A_c\, b(t) \cos\omega_c t$<br>$b(t) = 1$ for 1<br>And $b(t) = -1$ for 0 | $A_c \cos\left[\omega_c t + (2i-1)\frac{\pi}{4}\right]$<br>Where, m = 0, 1, 2, 2 | $k_1\sqrt{0.2P_s}$<br>$\cos \omega_c t$<br>Where, m= 0, 1,2, 3 | | | | |
| 3. | Number of bits per symbol | N = 1 | N = 1 | N =2 | N | N | N = 1 | N | N = 2 |
| 4. | Number of possible symbol $M = 2^N$ | Two | Two | Four | $M = 2^N$ | $M = 2^N$ | Two | $M = 2^N$ | Four |
| 5. | Detection method | Coherent | Coherent | Coherent | Coherent | Coherent | Non- coherent | Non- coherent | coherent |
| 6. | Minimum Euclidean distance | $\sqrt{E_b}$ | $2\sqrt{E_b}$ | $2\sqrt{E_b}$ | $\sqrt{0.2E_s}$<br>for M = 16 | $2\sqrt{E_s}$ | $2\sqrt{E_b}$ | $2\sqrt{2N\, E_b}$ | $2\sqrt{E_b}$ |
| 7. | Minimum bandwidth | $2f_b$ | $2f_b$ | $f_b$ | $2\frac{f_b}{N}$ | $2\frac{f_b}{N}$ | $4f_b$ | $\frac{2^{N+1} f_b}{N}$ | $1.5f_b$ |
| 8. | Symbol duration $T_s$ | $T_b$ | $T_b$ | $2T_b$ | $NT_b$ | $NT_b$ | $T_b$ | $NT_b$ | $2T_b$ |

## 17. Matched Filter

The matched filter is the optimal linear filter for maximizing the signal-to-noise ratio (SNR) in the presence of additive stochastic noise

Impulse response of matched filter, h(t) is

- $h(t) = s_i\,(T - t)$ $\quad s_i(t) \rightarrow$ Real
- $h(t) = s_i\,(T - t)$ $\quad s_i(t) \rightarrow$ Imaginary
- $h(t) = s_i{}^*(T - t)$ $\quad s_i(t) \rightarrow$ Complex

# CHAPTER-10: PROBABILITY OF ERROR AND DECISION THEORY

## 1. Bit Error Probability of ASK Signal

The probability of bit error for a coherent ASK system is given by

$$P_e = Q\sqrt{(E_b / N_0)} = Q\left(\sqrt{\gamma_b}\right)$$

$E_b$ is the bit energy, $N_0$ is the noise power density, and $\gamma_b$ is the bit energy to noise density ratio.

## 2. Calculation of probability of error of Binary Phase Shift Keying

$$P_e(\min) = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{r_{max}{}^2}{8}}$$

$$\gamma^2{}_{max} = \frac{2}{N_0}\int_0^{T_b}[s_1(t) - s_2(t)]^2 dt$$

**NOTE: Probability of error in terms of distance (d)**

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\frac{d}{2\sqrt{N_0}}\right) = Q\left(\frac{d}{\sqrt{2}\sqrt{N_o}}\right) = Q\left(\frac{d}{\sqrt{2N_o}}\right)$$

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\frac{2\sqrt{E_b}}{\sqrt{2N_b}}\right) = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

If the distance between two manage points is decreased, then the probability of error will increase.

## 3. Probability Of error in case of Non-Synchronized BPSK:

If $\phi$ is the error between local oscillation and modulated signal, then $P_e$ will become

$$P_e = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b\cos^2\phi}{N_0}} = Q\left(\frac{2E_b\cos^2\phi}{N_0}\right)^{1/2}$$

$$\boxed{SNR = \frac{E_b\cos^2\phi}{N_0}}$$

## 4. Bit Error Probability of BPSK Signal

If we consider phase error $\phi$ in demodulation, then the bit error probability is expressed as

$$P_e = Q\left(\sqrt{2\gamma_b\cos^2\phi}\right)$$

## 5. Bit Error Probability of Coherent Binary FSK Signal

For coherent binary FSK signal, we define the bit error probability as

$$P_e = Q\left(\sqrt{E_b / N_0}\right) = Q\left(\sqrt{\gamma_b}\right)$$

$E_b$ is the bit energy, $N_0$ is the noise power density, $\gamma_b$ is the bit energy to noise density ratio.

**Note:**

For a larger value of z, the Q(z) function can be approximated as

$$Q(z) \approx \frac{1}{\sqrt{2\pi}z} e^{-z^2/2}, \quad z >> 1$$

Q(z) function can be expressed in terms of complementary error function as

$$Q(z) = \frac{1}{2}\text{erfc}\left(\frac{z}{\sqrt{2}}\right)$$

**5. Probability of bit error for binary DPSK (Non-coherent PSK)**

$$P_{e\ (\text{bit-error})} = \frac{1}{2}\exp\left(\frac{-E_b}{N_0}\right)$$

The probability of bit error for binary DPSK is higher than binary BPSK.

$P_e$ (DPSK) > $P_e$ (BPSK)

**6. Bit Error Probability for Noncoherent Frequency Shift Keying**

The bit error probability for noncoherent frequency shift keying is defined

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right) = \frac{1}{2}\exp\left(-\frac{\gamma_b}{2}\right)$$

$E_b$ is the bit energy, $N_a$ is the noise power density, and $\gamma_b$ is the bit energy to noise density ratio.

**7. Relation Between Probability Of Bit Error And Probability Of Symbol Error For Orthogonal Signals**

Let $P_E$ be the average probability of symbol error, and $P_e$ be the average probability of bit error (bit error rate) for an M-ary orthogonal system (such as MFSK).

$$\frac{P_e}{P_E} = \frac{2^{k-1}}{2^k - 1} = \frac{M/2}{M-1} \text{......(v)}$$

In the limit as k increases, we get

$$\lim_{x\to\infty}\frac{P_e}{P_E} = \frac{1}{2}$$

**8. Relation Between Probability Of Bit Error And Probability Of Symbol Error For Multiple Phase Signals**

For a multiple phase system (such as MPSK), the probability of bit error ($P_e$) can be expressed in terms of probability error ($P_E$) as

$$P_e = \frac{P_E}{\log_2 M}$$

**9. Probability of Symbol Error for MPSK**

The probability of symbol error for the MPSK system is defined as

$$P_E \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right)$$

Where M = $2^k$ is the size of the symbol set, and $E_s$ is the energy per symbol. Since the symbol energy, $E_s$, is given by

$E_s = E_b(\log_2 M) = kE_b$

Where k = $\log_2 M$ is the number of bits transmitted per symbol. So, we can express the probability of symbol error in terms of $E_b/N_0$ as

$$P_E = 2Q\left(\sqrt{\frac{2kE_b}{N_0}}\sin\frac{\pi}{M}\right)$$

**10. Probability of Bit Error for M-ary PSK**

The bit error probability in terms of symbol error probability for an M-ary PSK system as

$$P_e = \frac{P_E}{\log_2 M} = \frac{P_E}{k}$$

The probability of bit error for M-ary PSK system as

$$P_e = \frac{2}{k}Q\left(\sqrt{\frac{2kE_b}{N_0}}\sin\frac{\pi}{M}\right)$$

$$= \frac{2}{k}Q\left(\sqrt{2k\gamma_b \sin^2\frac{\pi}{M}}\right)$$

**11. Probability of Symbol Error for QPSK**

For QPSK M = 4, we get the probability of symbol error for the QPSK system as

$$P_E \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{4}\right)$$

Or $P_E = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right)$

Since, the symbol energy $E_s$ is given by

$E_s = E_b(\log_2 M) = E_b(\log_2 4) = 2E_b$

So, we can express the probability of symbol error in terms of $E_b/N_0$ as

$$P_E = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

**12. Probability of Symbol Error for M-ary QAM**

The probability of symbol error for an M-ary QAM system is given by

$$P_E \approx \left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3k}{M-1}\frac{E_b}{N_0}}\right)$$

$$P_E \approx 4\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3k}{M-1}\gamma_b}\right)$$

Where k = $\log_2 M$ is the number of bits transmitted per symbol, $E_b$ is the bit energy, $N_0$ is the noise power density, and $\gamma_b$ is the bit energy to noise density ratio.

**13. Probability of Bit Error for M-ary QAM**

The bit error probability for an M-ary QAM system as

$$P_e = \frac{P_E}{\log_2 M} = \frac{P_E}{k}$$

$$= \frac{4}{k}\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3k}{M-1}\gamma_b}\right)$$

**14. Probability of Symbol Error for M- ary FSK**

The probability of symbol error for an M-ary FSK system is given by

$$P_E \le (M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

$$= (M-1)Q\left(\sqrt{\frac{E_b \log_2 M}{N_0}}\right)$$

or $$P_E \le (M-1)Q\left(\sqrt{\gamma_b \log_2 M}\right)$$

**15. Probability of Bit Error for M-ary FSK**

The bit error probability for an M-ary FSK system as

$$P_e = \frac{\frac{M}{2}}{M-1}P_E$$

or $$P_e \le \frac{M}{2}Q\left(\sqrt{\gamma_b \log_2 M}\right)$$

**16. OVERALL CONCLUSION OF FORMULAE**

**16.1. Probability of error of ASK, FSK, PSK and QPSK using constellation diagram**

$$P_e = Q\left(\frac{d}{2\sigma}\right) = Q\left(\frac{d}{\sqrt{2N_0}}\right) \qquad (d = d_{min})$$

For ASK : $d_{min} = \sqrt{E_b}$ $\left(E_b = \text{Bit energy} = \frac{A_c^2 T_b}{2}\right)$

$$P_e = Q\left(\sqrt{\frac{E_b}{2N_0}}\right) = Q\left(\sqrt{\frac{A_c^2 T_b}{4N_0}}\right)$$

For PSK : $d_{min} = 2\sqrt{E_b}$

$$P_e = Q\left(2\sqrt{\frac{E_b}{2N_0}}\right) = Q\left(\sqrt{\frac{A_c^2 T_b}{N_0}}\right)$$

For FSK : $d_{min} = \sqrt{2E_b}$

$$P_e = Q\left(\sqrt{\frac{2E_b}{2N_0}}\right) = Q\left(\sqrt{\frac{A_c^2 T_b}{2N_0}}\right)$$

For QPSK : $d_{min} = \sqrt{2E_s} = \sqrt{4E_b}$ $\quad (E_s = 2E_b)$

$$P_e = Q\left(\sqrt{\frac{4E_b}{2N_0}}\right) = Q\left(\sqrt{\frac{A_c^2 T_b}{N_0}}\right)$$ ($P_e$= Bitt error probability)

**16.2. Probability of error for various signalling schemes:**

QPSK : $P_{e(symbol)} \simeq 2Q\left(\sqrt{\frac{E}{N_0}}\right)$; $P_{e(bit)} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left(E_b = \frac{A_c^2 T_b}{2}\right)$

DPSK : $P_e = \frac{1}{2} e^{-Es/N_0}$ $\quad \left(E_s = \frac{A_c^2 T}{2}\right)$

16-QAM : $P_e = 3Q\left(\sqrt{\frac{E_s}{N_0}}\right) - 2.25\left[Q\left(\sqrt{\frac{E_s}{5N_0}}\right)\right]^2$

MSK : $P_e = Q\left[\sqrt{\frac{d^2}{2\eta}}\right] = Q\left[\sqrt{\frac{2E_b}{\eta}}\right]$

**CHAPTER-11: INFORMATION THEORY**

## 1. Information

The amount of the information associated with $x_i$ is defined as

$$I(x_i) = \log_a \frac{1}{P(x_i)}$$

or $$I_i = \log_a \frac{1}{p_i}$$

Units of f=information based on the value of a

If a=2 unit is bits

If a=e unit is nat

If a=10 unit is decit

However, bits is the most commonly used unit

Entropy is also known as the measure of uncertainty.

### 1.1. Properties of Information:

- If we are sure of the outcome of an event, even before it occurs, there is no information gained, i.e.
  $I_i = 0$ for $p_i = 1$
- The occurrence of an event either provides some or no information but never brings about a loss of information, i.e.
  $I_i \geq 0$ for $0 \leq p_i \leq 1$
- The less probable an event is, the more information we gain when it occurs.
  $I_j > I_i$ for $p_j < p_i$
- If two events $x_i$ and $x_j$ are statistically independent, then
  $I(x_i x_j) = I(x_i) + I(x_j)$

## 2. Entropy:

If each symbol $x_i$ occurs with probability $p_i$ and conveys the information $I_i$, then the average information per symbol is obtained as entropy, and it is given by

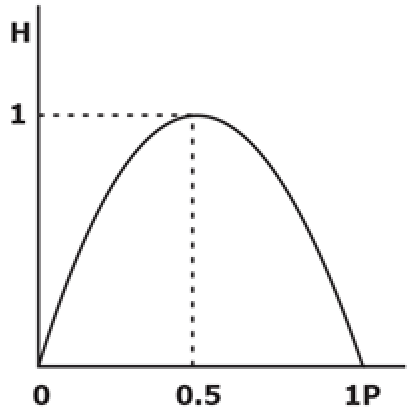
$$H(X) = E\left[I(x_i)\right] = \sum_{i=1}^{n} p_i I_i$$

$$H(X) = \sum_{i=1}^{n} p_i \log_2 \frac{1}{p_i}$$

### 2.1. Properties of Entropy:

These are some important properties of entropy.

- In a set of symbol X, if the probability $p_i = 1$ for some i and the remaining probabilities in the set are all zero; then the entropy of the source is zero, i.e
  $H(X) = 0$
- If all the n symbols emitted from a source are equiprobable, then the entropy of the source is
  $H(X) = \log_2 n$
- From the above two results, we can easily conclude that the source entropy is bounded as
  $0 \leq H(X) < \log_2 n$




## 3. Information Rate:

Information rate for a source with entropy H is given by

$$R = \frac{H}{T} \text{bits / sec}$$

Where T is the time required to send a message.

If the message source generates messages at the rate of r messages per second, then we have

$$T = \frac{1}{r}$$

The information rate of the source as

**R = rH bits / sec**

### 3.1. Methodology to evaluate source Information Rate:

For a given set of source symbols, we evaluate the information rate in the following steps:

**Step 1:** Obtain the probability $p_i$ of each symbol emitted by a source.

**Step 2:** Deduce the amount of information conveyed in each symbol using the expression,

$$I_i = \log_2 \frac{1}{p_i} \text{bits}$$

**Step 3:** Obtain the source entropy by substituting the above results in the expression

$$H = \sum_{i=1}^{n} p_i I_i = \sum_{i=1}^{n} p_i \log_2\left(\frac{1}{p_i}\right)$$

**Step 4:** Obtain the average message transmission rate using the expression

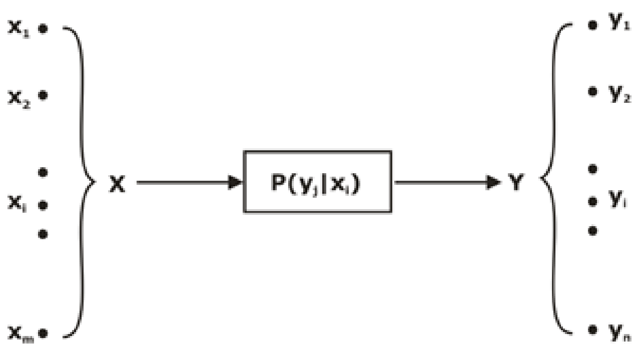
$$r = \frac{1}{T}$$

where T is the time required to send a message

**Step 5:** Evaluate the information rate of the source by substituting the above results in the expression

R = rH bits / s

### 4. Discrete Memoryless Channels




Discrete memoryless channel

The matrix of transition probabilities [P(Y|X)], given by:

$$[P(Y|X)] = \begin{bmatrix} P(y_1|x_1) & P(y_2|x_1) & \cdots & P(y_n|x_1) \\ P(y_1|x_2) & P(y_2|x_2) & \cdots & P(y_n|x_2) \\ \cdots & \cdots & \cdots & \cdots \\ P(y_1|x_m) & P(y_2|x_m) & \cdots & P(y_n|x_m) \end{bmatrix}$$

if the input probabilities P(X) are represented by the row matrix

$[P(X)] = [P(x_1) \quad P(x_2) \quad \ldots \quad P(X_m)]$

and the output probabilities P(Y) arc represented by the row matrix

$[P(X)] = [P(y_1) \quad P(y_2) \quad \ldots \quad P(y_m)]$

then [P(Y)] = [P(X)][P(Y|X)]

If P(X) is represented as a diagonal matrix

$$[P(X)]_d = \begin{bmatrix} P(x_1) & 0 & \cdots & 0 \\ 0 & P(x_2) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & P(x_m) \end{bmatrix}$$

then $[P(X, Y)] = [P(X)_d[P(Y|X)]$

**5. Entropy functions for Discrete Memoryless Channel**

If the channel has n inputs and m outputs, then we can define several entropy functions for input and output as

$$H(X) = -\sum_{i=1}^{n} P(x_1)\log_2 P(x_i)$$
$$H(Y) = -\sum_{j=1}^{m} P(y_j)\log_2 P(y_j)$$

**6. Joint Entropy**

The joint entropy is given as

$$H(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2 P(x_i, x_j)$$

**7. Conditional Entropy**

The several conditional entropy functions for the discrete memoryless channel are defined as

$$H(Y \mid x_i) = -\sum_{j=1}^{m} p(y_j \mid x_i)\log_2 P(y_j \mid x_i)$$
$$H(X \mid y_j) = -\sum_{i=1}^{n} P(x_i \mid y_j)\log_2 (x_i \mid y_j)$$
$$H(Y \mid X) = -\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2 P(y_j \mid x_i)$$
$$H(X \mid Y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2 P(x_i \mid y_j)$$

**8. Mutual Information**

The mutual information I(X; Y) of a channel is defined by

**I(X; Y) = H(X) – H(X|Y) b/symbol**

Also, we can define the mutual information as

I (X; Y) = H(Y) – H(Y | X)

I (X; Y) = H(X) – H(X | Y)

For a noise-free channel

I (X; Y) = H(X,Y)

**9. Channel Capacity**

The channel capacity is defined as the maximum mutual information, i.e.

**C = max {I(X;Y)}**

Substituting above equation, we get the channel capacity as

**C = max {H(X) – H(X |Y)}**

This result can be more generalized for the Gaussian channel. The information capacity of a continuous channel of bandwidth B hertz is defined as

**C = Blog$_2$ (1 + S/N)**

where S/N is the signal to noise ratio. This relationship is known as the **Hartley – Shannon law** that sets an upper limit on the performance of a communication system.

**10. Channel Efficiency**

The channel efficiency is defined as the ratio of actual transformation to the maximum transformation, i.e.

$$\eta = \frac{I(X;Y)}{\max\{I(X;Y)\}}$$

$$\eta = \frac{I(X;Y)}{C}$$

Redundancy in the channel is defined as R=1-η

**11. Capacities of Special Channels:**

**11.1. Lossless Channel:**

For a lossless channel, H(X|Y) = 0 and

I(X; Y)= H(X)

Thus, the mutual information (information transfer) is equal to the input (source) entropy, and no source information is lost in transmission. Consequently, the channel capacity per symbol is

$$C_s = \max_{\{P(x_i)\}} H(X) = \log_2 m$$

where m is the number of symbols in X.

**11.2. Deterministic Channel:**

For a deterministic channel, H(Y|X) = 0 for all input distributions $P(x_i)$, and

I(X; Y) = H(Y)

Thus, the information transfer is equal to the output entropy. The channel capacity per symbol is

$$C_s = \max_{\{P(x_i)\}} H(Y) = \log_2 n$$

where n is the number of symbols in Y.

**11.3. Noiseless Channel:**

Since a noiseless channel is both lossless and deterministic, we have

I(X; Y) = H(X) = H(Y)

and the channel capacity per symbol is

$C_s = \log_2 m = \log_2 n$

**12. Binary Symmetric Channel:**

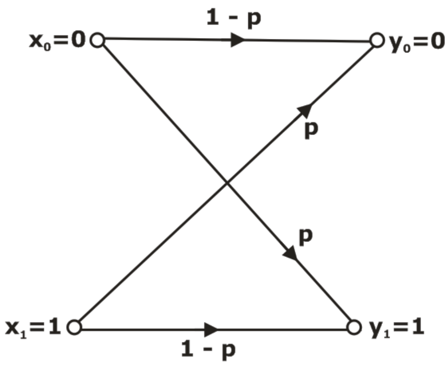
For the BSC of Figure below, the mutual information is:

$I(X; Y) = H(Y) + p\log_2 p + (1 - p)\log_2(1 - p)$

Where p is the probability of error

and the channel capacity per symbol is

$C_s = 1 + p\log_2 p + (1 - p)\log_2(1 - p)$




**Binary Symmetric Channel**

**13. Differential Entropy:**

The average amount of information per sample value of x(t) is measured by

$$H(X) = -\int_{-\infty}^{\infty} f_X(x)\log_2 f_X(x)\,dx \text{ b/sample}$$

The entropy H(X) defined above is known as the differential entropy of X.

$$H(Y) = -\int_{-\infty}^{\infty} f_Y(y)\log_2 f_Y(y)\,dy$$

$$H(X|Y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\log_2 f_X(x\,|\,y)\,dxdy$$

$$H(X|Y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\log_2 f_Y(y\,|\,x)\,dxdy$$

**14. Shannon's theorem**

It says that for any given degree of noise contamination of a communication channel, it is possible to communicate discrete data (digital information) nearly error-free up to a computable maximum rate through the channel

**15. Bandwidth and S/N tradeoff**

A noiseless channel can have an infinite capacity

However, if the bandwidth approaches infinity, the channel capacity does not become infinite because the noise power increases with the increase in bandwidth. Thus for fixed signal power and white Gaussian noise, the channel approaches an upper limit for capacity known as Shannon's limit

$$\lim_{\omega\to\infty} C = \frac{S}{\eta}\log e = 1.44\frac{S}{\eta} = \mathbf{R_{MAX}}$$

# CHAPTER-12: CODING TECHNIQUE

**1. Average codeword length:**

Average codeword length is given by

$$\bar{L} = \sum_{i=1}^{N} n_i p(n_i) \text{ letters/message}$$

Where $n_i$=number of symbols in the ith message

$p(n_i)$=probability of the ith message

**2. Source Coding theorem**

Source encoding theorem states that the minimum average codeword length for any distortion less source encoding scheme is defined as

$$\bar{L}_{min} = \frac{H(X)}{\log_2 k}$$

Where H(X) is the source's entropy, and k is the number of symbols in the encoding alphabet. Thus, for the binary alphabet (k = 2), we get the minimum average codeword length as

$$\bar{L}_{min} = H(X)$$

**3. Coding Efficiency**

$$\eta = \frac{\bar{L}_{min}}{\bar{L}}$$

$$\eta = \frac{H(X)}{\bar{L}\log_2 k}$$

**4. Shannon-Fano Coding:**

**Methodology: Shannon – Fano encoding algorithm:**

**Step 1**: The source symbols are first arranged in order of decreasing probability.

**Step 2:** The set is then divided into two sets that are as equiprobable as possible

**Step 3**: 0's are assigned to the upper set, and 1's to the lower set.

**Step 4:** The above process continues, each time partitioning the sets with as nearly equal probabilities as possible until further partitioning is impossible.

**5. Huffman coding:**

**Methodology: Huffman encoding algorithm:**

**Step 1:** The source symbols are arranged in order of decreasing probability.

**Step 2**: The two source symbols of lowest probability are assigned a 0 and a 1.

**Step 3:** These two source symbols are then combined into a new source symbol with probability equal to the sum of the two original probabilities. (the list source symbols, and therefore source statistics, is thereby reduced in size by one.)

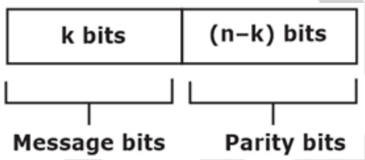
**Step 4**: The probability of the new symbol is placed in the list per its value.

**Step 5:** The above procedure is repeated till we have a final list of source statistics (symbols) of only two for which a 0 and a 1 are assigned.

**Step 6:** The code for each (original) source symbol is found by working backwards and tracing the sequence of 0s and 1s assigned to that symbol as well as its successors.

## 6. Linear block codes

**Linear block code** is an error-correcting code in which the actual information bits are linearly combined with the parity check bits to generate a linear codeword transmitted through the channel.




### 6.1. Error Detection and Correction Capabilities:

A linear code C with minimum distance $d_{min}$ can detect up to t errors if

**$d_{min} \geq t + 1$.”**

A linear code C of minimum distance $d_{min}$ can correct up to t errors if

**$d_{min} \geq 2t + 1$."**

### 6.2. Parity-Check Matrix:

A parity-check matrix in a linear block code is a matrix that describes the linear relationships that the components of a codeword must satisfy. It is used to decide whether a particular vector is a codeword or not and is also used in decoding algorithms.

H is an m x n matrix defined by

$H = [P \quad I_m]$

where m = n – k and $I_m$ is the mth-order identity matrix. Then

$$H^T = \begin{bmatrix} P^T \\ I_m \end{bmatrix}$$

Matrix **H is called the parity-check matrix** of C. Note that the rank of H is m = n – k, and the rows of H are linearly independent. The minimum distance $d_{min}$ of a linear block code C is closely related to the structure of the parity-check matrix H of C.

**7. SYNDROME DECODING**

With syndrome decoding, an (n, k) linear block code can correct up to t errors per codeword if n and k satisfy the following Hamming bound.

$$2^{n-k} \geq \sum_{i=0}^{t} \binom{n}{i}$$

Where $\binom{n}{i} = \frac{n!}{(n-1)!i!}$

A block code for which equality holds is known as the perfect code. Single error-correcting perfect codes are called Hamming codes.

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