Varignon’s Theorem

Varignon's theorem is a method for calculating moments developed in 1687 by the French mathematician Pierre Varignon (1654–1722). This theorem is essential for the GATE ME question paper. The right-hand thumb rule can be used to determine the direction of moments. A moment can be completely defined by using the five characteristics of a moment (i.e., magnitude of force acting, direction of force, point of application of force, line of action of force, and the point of rotation).

Vaignon’s theorem is also known as the “principle of moments."

Varignon's theorem helps to figure out the location of the resultant force on an object acted upon by a non-concurrent coplanar system of forces and reduces the complications in calculating net moments on objects about the provided axis of rotation.

State and Explain Varignon's Theorem

According to Varignon's theorem, “The total of the moments of many coplanar forces around a point equals the moment of the resultant of those forces, or the moment of a force around a point equals the sum of its components.”

Varignon’s Theorem Equation

The formula gives Varignon’s Theorem equation:

$$M_o = r \times R$$

where M_o is the moment of resultant 'R' at point 'O'.

Varignon’s Theorem Proof

Varignon's theorem is a principle that is frequently utilized in conjunction with the Principle of Transmissibility to solve systems of forces acting on and/or within a structure. Varignon’s theorem can be easily understood with the help of a given an example.

Consider the force R acting in the plane of the body, as indicated in the above figure. Any two non-rectangular components of 'R' are represented by the forces 'P' and 'Q.' The moment of resultant 'R' at point 'O' is

$$M_o = r \times R \text{ --------(1)}$$

R is the resultant of P and Q; hence, R = P+ Q

Put the value in equation 1;

$$M_o = r \times (P+Q)$$

Use the distributive law of cross-product,

$$M_o = (r \times P)+(r \times Q) \text{ -------(2)}$$

From equations 1 and 2

$$r \times R = (r \times P)+(r \times Q)$$

It states that the moment of 'R' about 'O' is the total of the moments of 'P' and 'Q' about 'O'. This establishes the theorem.

Varignon's Theorem Problem

Let us understand Varignon's Theorem with the help of an example from the GATE exam.

Calculate the moment of the force F around point O illustrated in the image using Varignon's theorem if the magnitude of Force is 500 N.

Varignon's theorem makes it easier to compute the moment of the force F about the point O in the structure illustrated in the picture if the force is divided into rectangular components and the moment of each is calculated:

Varignon's Theorem Example

Force F can be decomposed in the following manner;

The horizontal component of force, $F_x = F.\ Cos\theta = 500 \times \cos 30 = 433N$

The vertical component of force, $F_y = F.\ Sin\theta = 500 \times Sin\ 30 = 250N$

Multiplying the force and the perpendicular distance yields the moment of each component of the force about O. Both forces will spin the frame in the same direction (clockwise) with an arbitrarily assigned positive sign.

As per Varignon's theorem;

Net moment about 'O' due to force F given as;

$$M_o = (F.\ Cos\theta) \times 2 + (F.\ Sin\theta) \times 1$$

$$M_o = (433) \times 2 + (250) \times 1$$

$$M_o = 1116 \text{ N-m, clockwise}$$