Inverse Trigonometric Function Notes PDF For NDA Exam

Inverse Trigonometric Functions is a scoring topic from NDA exam point of view as every year **many questions are asked from this topic**. Every candidate should take care of not letting go easy marks from this topic. To revise effectively read and revise from the Inverse Trigonometric Functions. You can also **download Inverse Trigonometric Functions notes PDF** at end of the post.

Introduction

If sin x = 1/2, we can write one value of x = π/6.

If sin x = 1/3, i.e. x is not a well known angle, then we can write x = $\sin^{-1}$ 1/3.

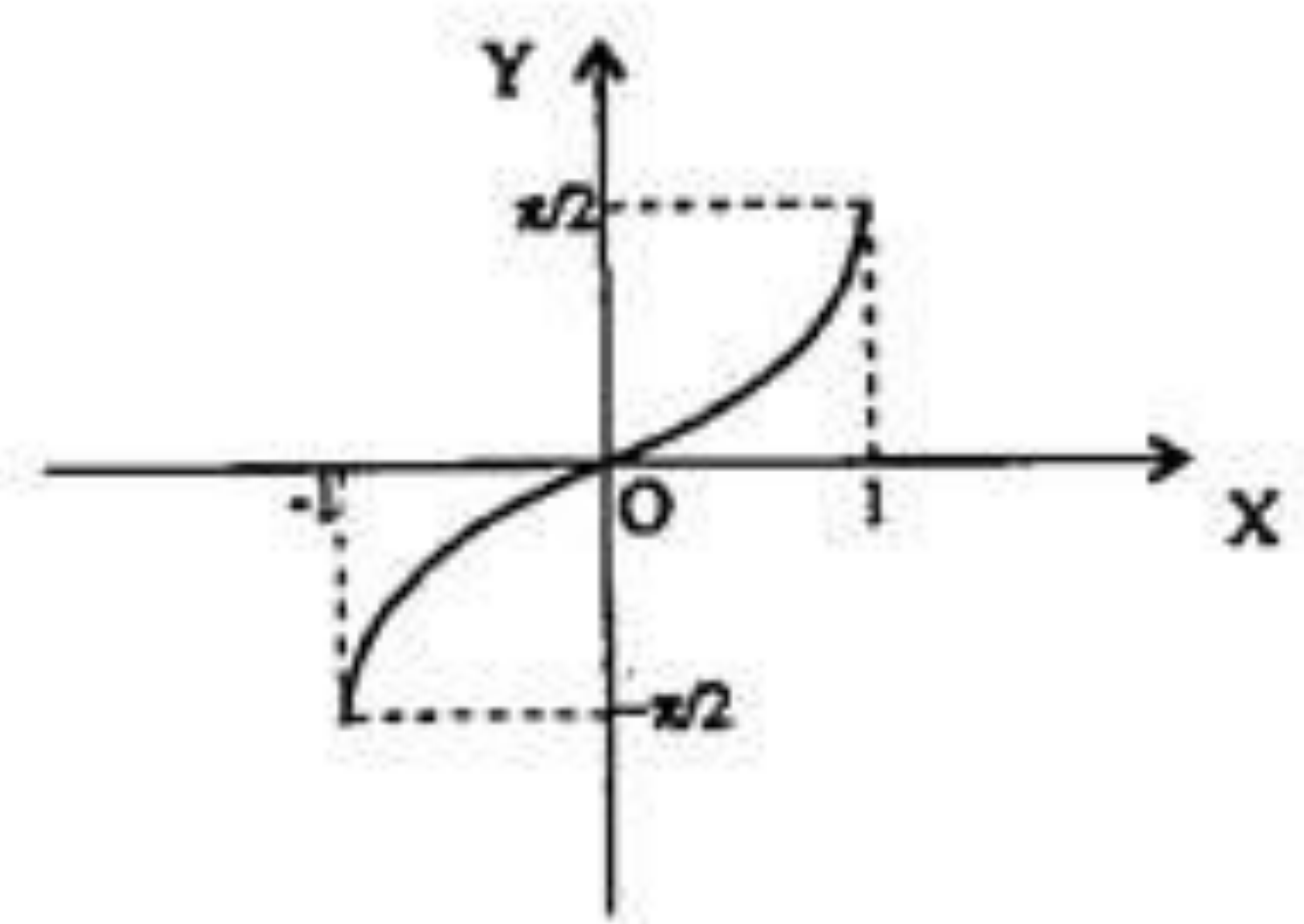
Similarly,

cos x = t ⇒ x = $\cos^{-1}$ t.

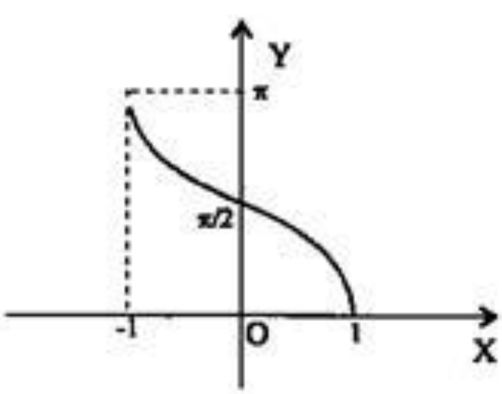
tan x = t ⇒ x = $\tan^{-1}$ t.

Rules for defined values of $\sin^{-1} x$, $\cos^{-1} x$

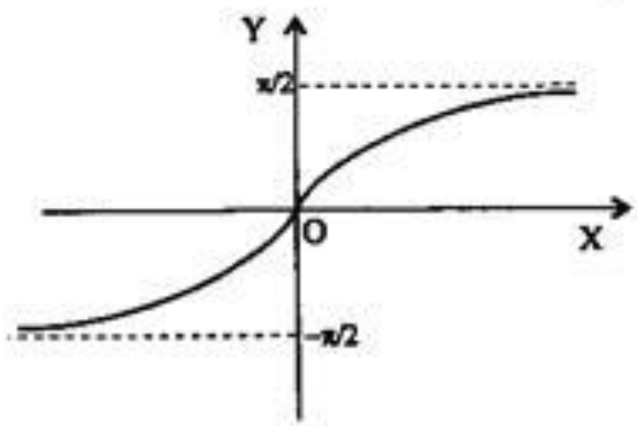
1. **y = $\sin^{-1}$ x:**
   Domain: x ∈ [-1, 1]
   Range (principal value branch of $\sin^{-1}$ x)
   y ∈ [-π/2, π/2]



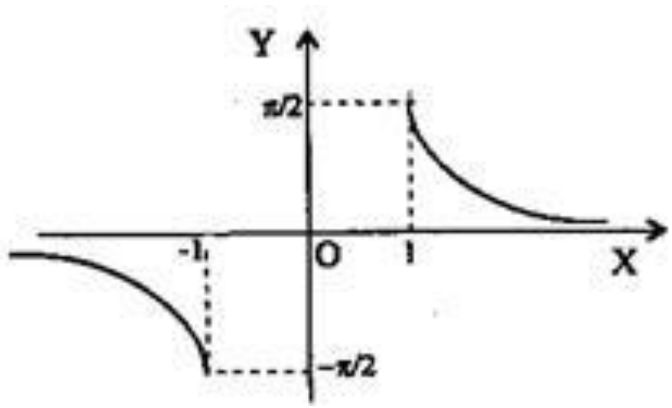
1. **$y = \cos^{-1} x$:**
   Domain: x ∈ [-1, 1]
   Range (principal value branch of $\cos^{-1} x$)
   y ∈ [0, π]



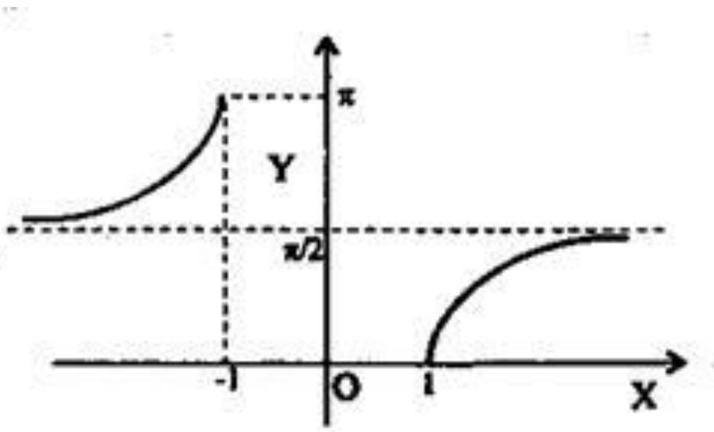
1. **$y = \tan^{-1} x$:**
   Domain: x ∈ R
   Range (principal value branch of $\tan^{-1} x$)
   y ∈ (-π/2, π/2)



1. **y = cosec⁻¹ x:**
   Domain: x ∈ (-∞, -1] ∪ [1, ∞)
   Range (principal value branch of cosec⁻¹ x)
   y ∈ [-π/2, 0) ∪ (0, π/2)



1. **y = sec⁻¹ x:**
   Domain: x ∈ (-∞, -1] ∪ [l, ∞)
   Range (principal value branch of sec⁻¹ x)
   y ∈ [0, π/2) ∪ (π/2, π]



1. **y = cot⁻¹ x:**
   Domain: X ∈ R

Range (principal value branch of $\cot^{-1} x$)
$y \in (0, \pi)$

Note the similarity in principal value branch of $\sin^{-1} x$, $\text{cosec}^{-1} x$, $\tan^{-1} x$.

Interval for allowed values of y is known as principal value branch of that inverse function.

Important Results

Important Results (I):

i. $\sin(\sin^{-1}x) = x$, $\cos(\cos^{-1}x) = x$, ...
ii. $\sin^{-1} \sin \theta = \theta$, $\cos^{-1} \cos \theta = \theta$
if θ allows the restrictions on y in the definition of corresponding inverse function.
e.g. $\sin^{-1} \sin 2\pi/3$ $2\pi/3$ because $2\pi/3$ does not lie in the principal value branch of $\sin^{-1} x$.
Hence $\sin^{-1} \sin 2\pi/3 = \sin^{-1} \sin(\pi - \pi/3) \sin^{-1} \sin \pi/3 = \pi/3$.
iii. $\sin^{-1}(-x) = -\sin^{-1}x$ $\cos^{-1}(-x) = \pi - \cos^{-1}x$
$\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x$ $\sec^{-1}(-x) = \pi - \sec^{-1}x$
$\tan^{-1}(-x) = -\tan^{-1}x$ $\cot^{-1}(-x) = \pi - \cot^{-1}x$

Important Results (II):

i. If x > 0, y > 0 then

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \ if \ xy < 1$$
$$= \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) \ if \ xy > 1$$

ii. If x > 0, y > 0 then

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

iii. If x > 0

$$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}, \ \cos^{-1}x = \sin^{-1}x\sqrt{1-x^2}$$
$$\sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}, \ \cos^{-1}x = \tan^{-1}\frac{\sqrt{1-x^2}}{x}$$
$$\cot^{-1}x = \tan^{-1}\frac{1}{x}, \ \sec^{-1}x = \cos^{-1}\frac{1}{x}, \ \text{cos}ec^{-1}x = \sin^{-1}x\frac{1}{x}$$

$$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$$

$$\operatorname{cosec}^{-1}x+\sec^{-1}x=\frac{\pi}{2}$$

iv. $$\tan^{-1}+\cot^{-1}x=\frac{\pi}{2}$$

Important Results (III):

$$2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2};\ -1<x<1$$

$$2\tan^{-1}x=\sin^{-1}\frac{2x}{1+x^2};\ -1\le x\le 1$$

$$2\tan^{-1}x=\cos^{-1}\frac{1-x^2}{1+x^2};\ x>0$$

$$\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{\pi}{4}-\tan^{-1}x;\ -1<x<\infty$$

$$\tan^{-1}\left(\frac{1+x}{1-x}\right)=\frac{\pi}{4}+\tan^{-1}x;\ -1\le x<1$$

Results (IV):

$\sin^{-1}x+\sin^{-1}y=\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]$, when $x\ge 0$, $y\ge 0$, $x^2+y^2\le 1$

$\sin^{-1}x+\sin^{-1}y=\pi-\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]$, when $x\ge 0$, $y\ge 0$, $x^2+y^2>1$

Results (V):

$\sin^{-1}x-\sin^{-1}y=\sin^{-1}\left[x\sqrt{1-y^2}-y\sqrt{1-x^2}\right]$, $0\le y\le x$

Results (VI):

$\cos^{-1}x+\cos^{-1}y=\cos^{-1}\left[xy-\sqrt{1-x^2}\sqrt{1-y^2}\right]$, where $x\ge 0$, $y\ge 0$

$\cos^{-1}x-\cos^{-1}y=\cos^{-1}\left[xy+\sqrt{1-x^2}\sqrt{1-y^2}\right]$, where $0\le x,\le y$