Volumetric Strain

Volumetric strain is defined as the ratio of change in the volume of a body to its original volume due to the application of some external deformation-causing forces. It is also known as Dilation and is important for the GATE exam. The general equation for volumetric strain is given as -

$$E_V = \Delta V/V$$

where

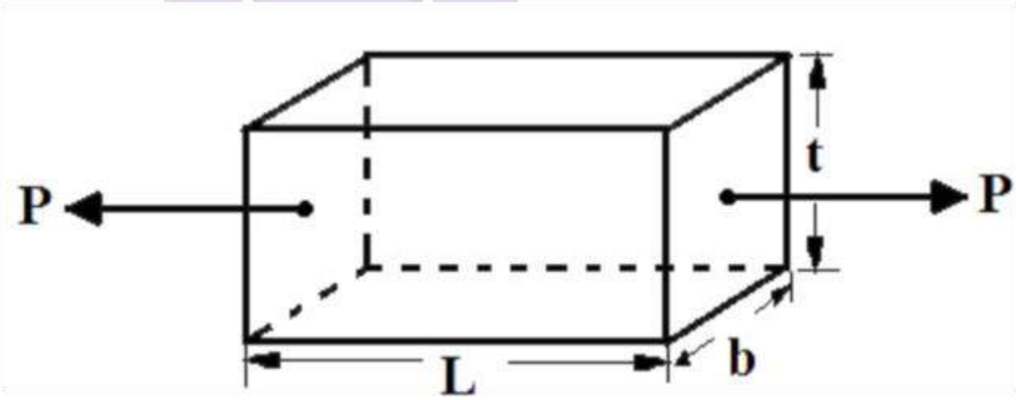
- ΔV = change in volume
- V = original volume

Bulk Modulus (K): When a body is subjected to stresses of equal intensity in 3 mutually perpendicular directions, then the ratio of this direct stress to the volumetric strain is called Bulk modulus. It is generally denoted by K.

$$K = \text{Direct Stress}/\text{Volumetric strain} = \sigma/E_V$$

Volumetric Strain for Rectangular Bar

This section will derive the volumetric strain formula for a rectangular bar. To define volumetric strain expression for a rectangular bar, let us assume a rectangular prismatic member of length L, width B, and depth D subjected to triaxial stresses, as shown in the figure below.



The initial volume of the rectangular bar,

$$V = L \times B \times D$$

The change in volume due to the applied stresses,

$$\Delta V = \delta L \times B \times D + L \times \delta B \times D + L \times B \times \delta D$$

We know that volumetric strain,

$$E_V = \Delta V / V$$

$$E_V = \delta L/L + \delta B/B + \delta D/D$$

We know that,

$$\delta L/L = E_x \text{ (strain in the x-direction)}$$

$$\delta B/B = E_y \text{ (strain in the y-direction)}$$

$$\text{and } \delta D/D = E_z \text{ (strain in the z-direction)}$$

So,

$$E_V = E_x + E_y + E_z \text{....(i)}$$

We also know that,

$$E_x = \sigma_x/E - \mu\sigma_y/E - \mu\sigma_z/E$$

$$E_y = \sigma_y/E - \mu\sigma_x/E - \mu\sigma_z/E \text{and}$$

$$E_z = \sigma_z/E - \mu\sigma_x/E - \mu\sigma_z/E$$

where

- μ = Poisson's ratio
- E = Young's modulus of elasticity

Putting the value of x, y and z in equation (i)

$$E_V = \sigma_x/E - \mu\sigma_y/E - \mu\sigma_z/E + \sigma_y/E - \mu\sigma_x/E - \mu\sigma_z/E + \sigma_z/E - \mu\sigma_x/E - \mu\sigma_z/E$$

$$E_V = (1-2\mu)(\sigma_x + \sigma_y + \sigma_z)/E$$

Volumetric Strain for Cylindrical Rod

In this section, we will derive the volumetric strain formula for a cylindrical rod. To define volumetric strain expression for a cylindrical rod, let us assume a cylindrical rod of length L and diameter d as shown in the figure below

The initial volume of the cylindrical rod,

$V=(\pi/4)d^2.L$

The change in volume due to applied stresses

$\Delta V=(\pi/4)[d^2.\delta L+L.2d\delta d]$

We know that volumetric strain,

$E_V=\Delta V/V$

$E_V=[\delta L/L+2.\ \delta d/d]$

We know that,

$\delta L/L=E_L$ (strain in the longitudinal direction)

$\delta d/d=E_d$ (strain in the radial direction)

So,

$$E_V=E_L+2E_d$$

Volumetric Strain for a Spherical Body

In this section, we will derive the volumetric strain formula for a spherical body. To define volumetric strain expression for a spherical body, let us assume a sphere of diameter d, as shown in the figure below.

The initial volume of the sphere,

$V=(\pi/6)d^3$

The change in volume due to applied stresses

$V=(\pi/6).3\delta d.d^2$

We know that volumetric strain,

$E_V=\Delta V/V$

$E_V=3\delta d/d$

$E_V=3E_d$