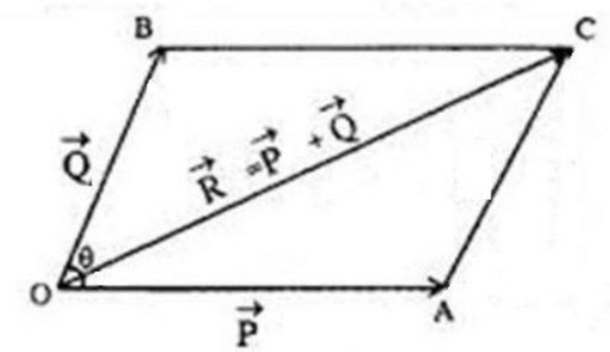
BYJU'S EXAM PREP

Parallelogram Law of Vectors

Suppose the magnitude and direction of two vectors acting simultaneously on a particle are represented by the two adjacent sides of a parallelogram drawn from a point. In that case, the resultant is wholly represented in magnitude and direction by the diagonal of that parallelogram formed from that point.



Let two vectors, P and Q operate on a particle O simultaneously. They are represented by the neighboring sides OA and OB of a parallelogram OACB formed from a point O in magnitude and direction. The diagonal OC running across O will thus indicate the size and direction of the consequent R.

If a body of unknown weight (say S) is suspended from the middle hanger of Gravesand's apparatus, and balancing weights P and Q are strung from the other two hangers, then

$$\vec{R} = \vec{P} + \vec{Q}$$

or

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\Theta}$$

Formula of the Parallelogram Law of Vectors

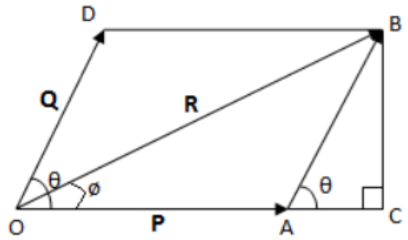
Consider two vectors P and Q separated by an angle θ. The vector R, the resultant sum vector of the parallelogram law of vector addition, gives the sum of vectors P and Q. The formulas for the magnitude and direction of the resulting vector R, if it forms an angle ϕ with the vector P, are:

- $|R| = \sqrt{(P^2 + Q^2 + 2PQ\cos\theta)}$
- $\beta = \tan^{-1}[(Q\sin\theta)/(P + Q\cos\theta)]$

Derivation of Parallelogram Law of Vectors

Let P and Q be two vectors acting at the same time at a point and represented in magnitude and direction by two adjacent sides OA and OD of the parallelogram OABD seen in the figure.

Let θ the angle formed by P and Q, and R be the resulting vector. The resultant of P and Q is therefore represented by diagonal OB, according to the parallelogram law of vector addition.



The magnitude of resultant

So, we have

R = P + Q

Now, expand A to C and draw BC perpendicular to OC.

From triangle OCB,

$$OB^2 = OC^2 + BC^2$$
$$or, OB^2 = (OA + AC)^2 + BC^2 \quad ......(i)$$

In triangle ABC,

$$\cos\theta = \frac{AC}{AB}$$
$$or, AC = AB\cos\theta$$
$$or, AC = OD\cos\theta = Q\cos\theta \quad [\because AB = OD = Q]$$

Also,

$$\cos\theta = \frac{BC}{AB}$$
$$or, BC = AB\sin\theta$$
$$or, BC = OD\sin\theta = Q\sin\theta \quad [\because AB = OD = Q]$$

Direction of Resultant

Let ø be the angle formed by the resultant R with P. Then, From the OBC triangle given in figure shown above.

Substituting value of AC and BC in (i), we get

$$\begin{aligned}
R^2 &= (P + Q\cos\theta)^2 + (Q\sin\theta)^2 \\
or, R^2 &= P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta \\
or, R^2 &= P^2 + 2PQ\cos\theta + Q^2 \\
\therefore R &= \sqrt{P^2 + 2PQ\cos\theta + Q^2}
\end{aligned}$$

which is the magnitude of resultant.