Parallelogram Law of Forces

Suppose two vectors acting on a particle simultaneously are represented in magnitude and direction by the two adjacent sides of a parallelogram formed from a point. In that case, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point, according to the law of

parallelogram of forces.

Assume that two vectors, A and B, both inclined at angle θ, are operating on a particle at the same time. Let them be represented by two neighbouring sides, OA and OB, of the parallelogram OACB, drawn from a point O, in magnitude and direction. This law is widely used in solving problems in the GATE exam. According to the parallelogram law of vectors, the resultant vector will be represented by the parallelogram's diagonal.

- It should be observed that the size of the resultant of two vectors is greatest when they act in the same direction and least when they act in opposing directions.
- It should be noted that when using the parallelogram law of vector addition to obtain the resulting vector of two vectors, the two vectors A and B should either act towards or away from the point.

Parallelogram Law of Forces Formula

Suppose the magnitude and direction of two forces acting at a point are represented by the two adjacent sides of a parallelogram. In that case, the resultant is represented by the diagonal of the parallelogram formed from the same point. Following is the formula for resultant force by using the Parallelogram Law of Forces.

$$\left|\vec{R}\right| = \sqrt{\left|\vec{A}\right|^2 + \left|\vec{B}\right|^2 + 2\left|\vec{A}\right| \bullet \left|\vec{B}\right| \cos\theta}$$

Special Case of Parallelogram Law of Forces

Let us see a few important and special cases of the parallelogram law of forces that will help solve the problems in the GATE question paper.

- Case 1: When two vectors act in the same direction, then θ=0°, cosθ = 1.

$$\left|\vec{R}\right| = \sqrt{A^2 + B^2 + 2AB}$$
$$\Rightarrow \left|\vec{R}\right| = \sqrt{(A+B)^2}$$
$$\Rightarrow \left|\vec{R}\right| = A + B$$

- Case 2: When two vectors act in opposite directions, then θ =180° and cosθ = −1.

$$\left|\vec{R}\right| = \sqrt{A^2 + B^2 - 2AB}$$
$$\Rightarrow \left|\vec{R}\right| = \sqrt{(A-B)^2}$$
$$\therefore \left|\vec{R}\right| = A - B$$

Thus, when two vectors act in different directions, the magnitude of the resulting vector equals the difference between the magnitudes of the two vectors and acts in the direction of the larger vector.

Newton's Proof of the Parallelogram Law of Forces

Assume two forces act on a particle at the figure's origin below. Let the lengths of the vectors F_1 and F_2 reflect the velocities that the two forces may induce in the particle if they were to act for a given duration, and the direction of each represents the direction in which they act. Each force acts separately and will produce its own velocity whether or not the other force acts. The particle has both velocities at the end of the allotted time. According to the previous proof, they are identical to a single velocity, Fnet. According to Newton's second law, this vector is also a measure of the force that would cause that velocity; hence the two forces are equivalent.