

Matrix Method

The Matrix method of structural analysis is a method used for the analysis of indeterminate structures. A matrix is a tool for solving many civil engineering analysis problems. And elements of the matrix are based on either flexibility or stiffness coefficients. Based on the coefficient used as the matrix element, it can be classified as the flexibility and stiffness matrix methods. The matrix method is essential for the [GATE question paper](#). These coefficients are based on the compatibility conditions of the structure.

These methods analyse large structures, like multi-story buildings, railway bridges, and other large structures where analysis becomes complex. With the help of these methods, redundant forces can be determined in a structure, and with the help of that, forces and displacement analysis can be done.

Solution of an Equation by Matrix Method

Matrix methods are based on matrix algebra, and the matrix elements are flexibility and stiffness coefficients. These coefficients are based on the compatibility conditions of the indeterminate structure. Using various matrix operations, these unknown elements (redundant forces or displacements) can be found easily, and structural analysis can be done with the help of that. Matrix operation consists of addition, subtraction, etc. A system of linear equations in the matrix method of analysis can be solved with the help of these matrix operations.



Solving Equations by Matrix Method

$$\begin{aligned} a_1x + b_1y &= c_1 \dots\dots\dots i \\ a_2x + b_2y &= c_2 \dots\dots\dots ii \end{aligned}$$

$$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

What are the Different Types of Matrix Methods?

In the case of the matrix method, unknown parameters of structures are solved with the help of matrix concepts. Based on the parameters of the matrix element, it is classified as the flexibility matrix method and displacement matrix method. Various matrix methods are mentioned in the [GATE ME syllabus](#). These methods are described below:

Flexibility Matrix Method

The flexibility matrix method is also a method of analysis of indeterminate structures. This method comes under the force method of analysis. In this method, forces are taken as unknown, and equations are expressed in terms of these forces. An additional compatibility condition equation is developed to find all the unknown forces. This method is suitable when the static indeterminacy is less than kinematic indeterminacy.

Flexibility (δ)

Flexibility is defined as the displacement produced due to unit force. It is the inverse of stiffness. Flexibility for various cases is as follows

(1) Axial flexibility= L/AE

(2) Transverse flexibility= $L^3/12AE$

(3) Flexural flexibility= $L/4AE$

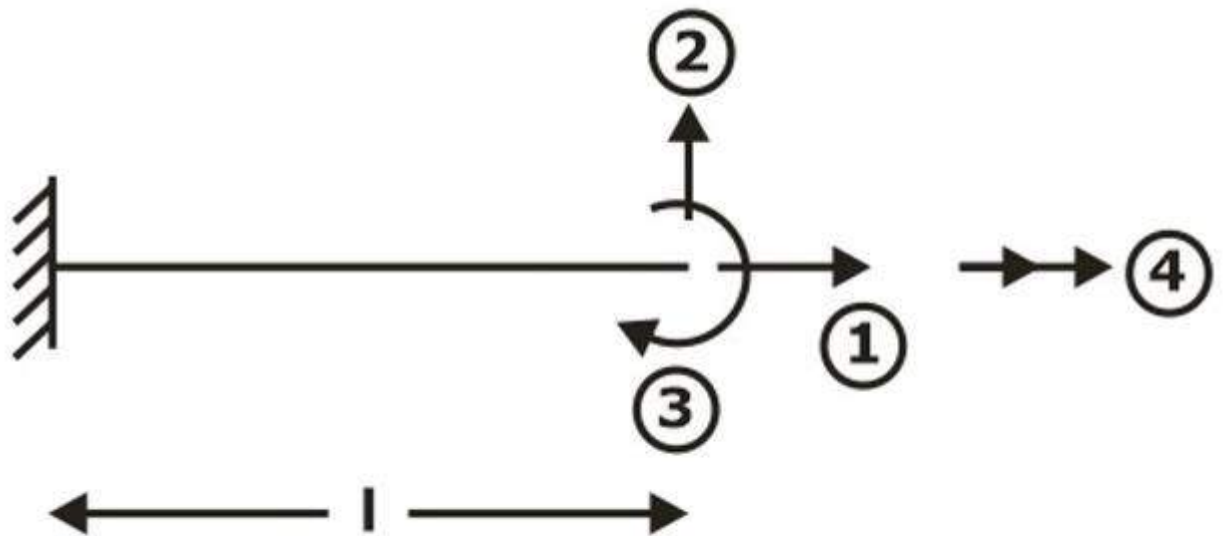
(4) Torsional flexibility= L/GJ

Displacement or Stiffness Matrix Method

The stiffness matrix method comes under the displacement method of analysis of indeterminate structures. In this method, displacements at the joints are taken as unknowns, and equations are expressed in terms of these unknown displacements. Additional joint equilibrium equations are developed to find the unknown displacement. This method is suitable when the Kinematic indeterminacy is less than the static indeterminacy.

Stiffness (k)

It is the load required to produce unit displacement. The stiffness for various cases is as follows.



(1) Axial stiffness (k_{11}) = AE/L

(2) Transverse stiffness (k_{22}) = $12EI/L^3$

(3) Flexural stiffness (k_{33}) = $4EI/L$

(4) Torsional stiffness (k_{44}) = GJ/L

Advantages of Matrix Methods of Analysis

As we know, the concept of matrix algebra can be used in the matrix method of analysis of indeterminate structures. Hence it provides a well-defined approach to solving the unknowns of equations used in the [GATE exam](#). In the case of indeterminate structures, compatibility equations are also used along with the equilibrium equations for the analysis. Some advantages of this method are listed below:

- These methods are based on the matrix, giving a simpler procedure for analysing indeterminate structures.
- Computer programming for this method can be written, so these methods can solve complex equations.

Matrix Method Example

As we have discussed, the matrix method for analysing indeterminate structures can be used for solving complex equations. So, it is very important to understand this method. Analysis of indeterminate structures by matrix method is based on finding out the flexibility or stiffness coefficients. Here an example is shown that illustrates the concept of these coefficients.

2° INDETERMINATE BEAM



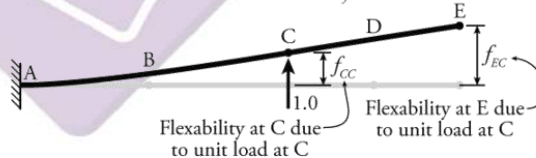
PRIMARY SYSTEM (USING C_y AND E_y AS REDUNDANTS)



PRIMARY SYSTEM WITH EXTERNAL LOADS



PRIMARY SYSTEM WITH UNIT REDUNDANT AT C_y



PRIMARY SYSTEM WITH UNIT REDUNDANT AT E_y

