## GATE 2020

## Electronics

\& Communication Engineering

## Questions \& Solutions

## SECTION: GENERAL APTITUDE

1. The untimely loss of life is a cause of serious global concern as thousands of people get killed $\qquad$ accidents every year while many other die $\qquad$ diseases like cardiovascular
A. from, from
B. from, of
C. in, of
D. during, from

Ans. C
Sol. The untimely loss of life is a cause of serious global concern as thousands of people get killed in accidents every year while many other die of diseases like cardiovascular
2. He was not only accused of theft $\qquad$ of conspiracy.
A. but even
B. rather than
C. rather
D. but also

Ans. D
Sol. He was not only accused of theft but also of conspiracy.
3. Select the word that fits the analogy:

Explicit: Implicit:: Express: $\qquad$
A. Impress
B. Compress
C. Suppress
D. Repress

Ans. D
Sol. Explicit: Implicit:: Express: Repress
4. The Canadian constitution requires that equal importance be given to English and French. Last year. Air Canada lost a lawsuit, and had to pay a six-figure fine to a French-speaking couple after they filed complaints about formal in-flight announcements in English lasting 15 seconds, as opposed to informal 5 second messages in French, The French-speaking couple were upset at
A. the English announcements being clearer than the French ones.
B. the in-flight announcements being made in English.
C. the English announcements being longer than the French ones.
D. equal importance being given to English and French.

Ans. C
Sol.
The French-speaking couple were upset at the English announcements being longer than the French ones.
5. A super additive function $n f$ (.) satisfies the following property
$f\left(x_{1}+x_{2}\right) \geq f\left(x_{1}\right)+f\left(x_{2}\right)$
Which of the following functions is a super additive function for $x>1$ ?
A. $e^{x}$
B. $e^{-x}$
C. $1 / \mathrm{x}$
D. $\sqrt{x}$

Ans. A
Sol. Let $x_{2}>x_{1}$


Check
Let $x_{2}=3$
$X_{1}=2$
$\mathrm{e}^{\mathrm{x} 1}+^{\mathrm{x} 2}=\mathrm{e}^{5}$
$e^{x 1}=e^{2}$
$\mathrm{e}^{\mathrm{x} 2}=\mathrm{e}^{3}$.
Then $e^{5}>e^{2}+e^{3}$. Which is true
6. The global financial crisis in 2008 is considered to be the most serious world-wide financial crisis, which started with the sub-prime lending crisis in USA in 2007. The subprime lending crisis led to the banking crisis in 2008 with the collapse of Lehman Brothers in 2008. The sub-prime lending refers to the provision of loans to those borrowers who may have difficulties in repaying loans, and it arises because of excess liquidity following the East Asian crisis.

Which one of the following sequences shows the correct precedence as per the given passage?
A. Subprime lending crisis $\rightarrow$ global financial crisis $\rightarrow$ banking crisis $\rightarrow$ East Asian crisis.
B. East Asian crisis $\rightarrow$ subprime lending crisis $\rightarrow$ banking crisis $\rightarrow$ global financial crisis.
C. Banking crisis $\rightarrow$ subprime lending crisis $\rightarrow$ global financial crisis $\rightarrow$ East Asian crisis.
D. Global financial crisis $\rightarrow$ East Asian crisis $\rightarrow$ banking crisis $\rightarrow$ subprime lending crisis.
Ans. B
Sol. East Asian crisis $\rightarrow$ subprime lending crisis $\rightarrow$ banking crisis $\rightarrow$ global financial crisis.
7. It is quarter past three in your watch. The angle between the hour hand and the minute hand
A. $0^{\circ}$
B. $22.5^{\circ}$
C. $15^{\circ}$
D. $7.5^{\circ}$

Ans. D
Sol. 60 units of min hand $=5$ units of hour hand.
$\therefore 15$ units of min hand $=\frac{5 \times 15}{50}$ units of hour hand
$=1.25$ units of hour hand
60 units $=360^{\circ}$
1.25 units $=6 \times 1.25^{\circ}$
$=7.5^{\circ}$.
8. A circle with centre $O$ is shown in the figure. A rectangle PQRS of maximum possible area is inscribed in the circle. If the radius of the circle is a, then the area of the shaded portion is $\qquad$ . .

A. $\pi a^{2}-3 a^{2}$
B. $\pi a^{2}-2 a^{2}$
C. $\pi \mathrm{a}^{2}-\mathrm{a}^{2}$
D. $\pi a^{2}-\sqrt{2} a^{2}$

Ans. B

## Sol.


$r^{2}=\frac{a^{2}}{4}+\frac{b^{2}}{4}$
$\because \frac{\mathrm{b}^{2}}{4}=\mathrm{r}^{2}-\frac{\mathrm{a}^{2}}{4}$
$b= \pm \sqrt{4 r^{2}-a^{2}}$
$b^{2}=4 r^{2}-a^{2}$
Area $\mathrm{A}=\mathrm{ab}$
$= \pm a \sqrt{4 r^{2}-a^{2}}$
$A^{2}=a^{2}\left(4 r^{2}-a^{2}\right)$
$\frac{d A^{2}}{d a} 4 r^{2} \times 2 a-4 a^{3}=0$
$4 a\left(2 r^{2}-a^{2}\right)=0$
$\Rightarrow a^{2}=2 r^{2}$
$\therefore \mathrm{b}^{2}=4 \mathrm{r}^{2}-2 \mathrm{r}^{2}$
$=2 r^{2}$.
$\therefore$ area of rectangle $=(r \sqrt{2})^{2}$
$=2 r^{2}$.
area of circle $=\pi r^{2}$.
$\therefore$ Required area $=(\pi-2) r^{2}$
9. $a, b, c$ are real numbers. The quadratic equation $a \times 2$
$-b x+c=0$ has equal roots, which is $\beta$, then
A. $b^{2} \neq 4 a c$
B. $\beta^{3}=b c /\left(2 a^{2}\right)$
C. $\beta=b / a$
D. $\beta^{2}=a c$

Ans. B
Sol. $A x^{2}-b x+c=0$
$2 \beta=\frac{b}{a} \Rightarrow \beta=\frac{b}{2 a}-----(1)$
$\beta^{2}=\frac{c}{a}-----$ (ii)
(i) $\times$ (ii)given $\beta^{3}=\frac{\mathrm{bc}}{2 \mathrm{a}^{2}}$
10. The following figure shows the data of students enrolled in 5 years (2014 to 2018) for two schools P and Q . During this period, the ratio of the average number of the students enrolled in schools $P$ to the average of the difference of the number of students enrolled in schools $P$ and $Q$ is $\qquad$

A. $23: 8$
B. $8: 23$
C. $23: 31$
D. $31: 23$

Ans.
Sol. No of students enrolled in $\mathrm{P}=3+5+5+6+4=$ 23

No of students enrolled in $\mathrm{Q}=4+7+8+7+5$
$=31$
$\therefore$ ratio $=\frac{23 / 5}{(31-23) 15}=\frac{23}{8}=2.875$

## TECHNICAL

1. A transmission line of length $3 \lambda / 4$ and having $a$ characteristic impedance of $50 \Omega$ is terminated with a load of $400 \Omega$. The impedance (rounded off to two decimal places) seen at the input end of the transmission line is $\qquad$ $\Omega$.
Ans. (6.25-6.25)
Sol. Given, characteristic impedance $Z_{0}=50 \Omega$
Load impedance $Z_{L}=400$
And input impedance $Z_{\text {in }}=\left(Z_{0}\right)^{2} / Z_{L}$

$$
\begin{aligned}
& =50^{2} / 400 \\
= & 6.25 \Omega
\end{aligned}
$$

2. In the given circuit, the two-port network has the impedance matrix $[Z]=\left[\begin{array}{ll}40 & 60 \\ 60 & 120\end{array}\right]$. The value of $Z_{L}$ for which maximum power is transferred to the load is. $\qquad$


Ans. (48-48)
Sol. Old Parameters -

$$
\begin{align*}
& \mathrm{V}_{1}=40 \mathrm{i}_{1}+60 \mathrm{i}_{2}  \tag{i}\\
& \mathrm{~V}_{2}=60 \mathrm{i}_{1}+120 \mathrm{i}_{2} \tag{ii}
\end{align*}
$$

$Z_{\text {th }}$ by testing method


KVL in mesh (i)
$10 i_{1}+V_{1}=0$
$V_{1}=-10 i_{1}$
And $\mathrm{i}_{2}=1 \mathrm{~A}$


From eq. (i)
$\mathrm{V}_{1}=40 \mathrm{i}_{1}+60 \times 1$
$-10 i_{1}=40 i_{1}+60$
$i_{1}=-6 / 5$
$V_{2}=60 \times\left(-\frac{6}{5}\right)+120 \times 1$
$=-72+120=48 \mathrm{~V}$
$\therefore$ to deliver max. power to load $\mathrm{z}_{\mathrm{L}}$
$Z_{L}=R_{t h}$
$\therefore \mathrm{z}_{\mathrm{L}}=\frac{\mathrm{V}_{2}}{\mathrm{i}_{2}}=\frac{48}{1}=48 \Omega$
3. Consider the recombination process via bulk traps in a forward biased pn homojunction diode. The maximum recombination rate is $U_{\max }$. If the electron and the hole capture cross-sections are equal, which one of the following is FALSE?
A. $U_{\max }$ depends exponentially on the applied bias.
B. $U_{\text {max }}$ occurs at the edges of the depletion region in the device.
C. With all other parameters unchanged, $U_{\max }$ increases if the thermal velocity of the carriers increases.
D. With all other parameters unchanged, $U_{\text {max }}$ decreases if the intrinsic carrier density is reduced.
Ans. B
Sol. $U_{\max }$ occurs at the edges of the depletion region in the device..
4. In the circuit shown below, the Thevenin voltage $\mathrm{V}_{\mathrm{TH}}$ is

A. 2.8 V
B. 3.6 V
C. 4.5 V
D. 2.4 V

Ans. B
Sol.


KVL in Loop (i)
$(i \times 1)-2(3-i)-2(1-i)+2=0$
$i-6+2 i-2+2 i+2=0$
$5 i=6$
$\mathrm{i}=1.2 \mathrm{~A}$
$\therefore \mathrm{V}_{\mathrm{th}}=2 \times(3-\mathrm{i})$
$=2 \times(3-1.2)$
$=3.6$ volts
5. A single crystal intrinsic semiconductor is at temp of 300 K with effective density of states for holes twice that of electrons. $\mathrm{V}_{\mathrm{T}}=26 \mathrm{mV}$. The intrinsic Fermi level is shifted from mid bond gap energy level by
A. 9.01 meV
B. 13.45 mev
C. 18.02 meV
D. 26.90 meV

Ans. A
Sol. $E_{f_{i}}=\frac{E_{C}+E_{V}}{2}-\frac{k T}{2} \ln \frac{N_{C}}{N_{V}}$
$=\frac{\mathrm{E}_{\mathrm{g}}}{2}-\frac{\mathrm{kT}}{2} \ln \frac{\mathrm{~N}_{\mathrm{C}}}{2 \mathrm{~N}_{\mathrm{C}}}\left(\because \mathrm{N}_{\mathrm{v}}=2 \mathrm{~N}_{\mathrm{c}}\right)$
$E_{F_{1}}=\frac{E_{g}}{2}-\frac{k T}{2} \ln \left(\frac{1}{2}\right)$
$E_{F_{1}}=\frac{E_{g}}{2}+9.01 \mathrm{meV}$
6. The random variable
$\mathrm{Y}=\int_{-\infty}^{\infty} \mathrm{W}(\mathrm{t}) \phi(\mathrm{t}) \mathrm{dt}$
Where $\phi(t)=\left\{\begin{array}{l}1 ; 5 \leq t \leq 7 \\ 0 ; \text { otherwise }\end{array}\right.$
And $W(t)$ is a real white Gaussian noise process with two-sided power spectral density $\mathrm{SW}(\mathrm{f})=3 \mathrm{~W} / \mathrm{Hz}$, for all $f$. The variance of $Y$ is
Ans. (6-6)
Sol. Given
$\mathrm{Y}=\int_{-\infty}^{\infty} \mathrm{w}(\mathrm{t}) \mathrm{s}(\mathrm{t}) \mathrm{dt}$, where $\phi(\mathrm{t})=\left\{\begin{array}{cc}1 & 5 \leq \mathrm{t} \leq 7 \\ 0 & \text { otherwise }\end{array}\right.$
$S_{w}(f)=3 W / H z$
$E(Y)=\int_{-\infty}^{\infty} E(w(t)) \phi(t) d t=0$
$E\left[Y^{2}\right]=S_{w}(f) \quad$ energy $\varphi(t)$
$=6$
$\operatorname{Var}[\mathrm{Y}]=6-0=6$
7. The two sides of a fair coin are labelled as 0 and 1 . The coin is tossed two times independently. Let $M$ and N denote the labels corresponding to the outcomes of those tosses. For a random variable $X$, defined as $X=\min (M, N)$, the expected value $E(X)$ (rounded off to two decimal places) is $\qquad$ -

Ans. (0.25-0.25)
Sol. There can be 4 out comes.
$\{\mathrm{HH}\},\{\mathrm{HT}\},\{\mathrm{TH}\},\{\mathrm{TH}\}$.
$\therefore$ Let 1 is denoted by head
$\therefore$ Let 0 is denoted by Tail.
$\therefore M=\left\{\begin{array}{llll}1 & 1 & 0\end{array}\right\}$
$N=\left\{\begin{array}{llll}1 & 0 & 1 & 0\end{array}\right\}$
$X=\min (M, N)=1000$.
$P(X)=\frac{1}{4} \frac{1}{4} \frac{1}{4} 1 / 4$
Now, $X=1$
When $\{\mathrm{HH}\}$ comes up
$\therefore \mathrm{P}(\mathrm{X}=1)=\mathrm{P}[\{\mathrm{HH}\}]=\frac{1}{4}$
Now $X=0$ when $\{H T\},\{T H\}$ or $\{T T\}$ come up When
$\therefore P(X=0)=\frac{1}{4}, \quad \therefore E(x)=\frac{1}{4} \times 1=0.25$
8. The figure below shows a multiplexer where $S_{1}$ and $S_{0}$ are the select lines, $I 0$ to $I_{3}$ are the input data lines. $E N$ is the enable line, and $F(P, Q, R)$ is the output. $F$ is

A. $P+Q \bar{R}$
B. $P \bar{Q} R+\bar{P} Q$
C. $P Q+\bar{Q} R$
D. $\bar{Q}+P R$

Ans. C
Sol.
$F=\bar{P} \bar{Q} R+P \bar{Q} R+P Q$
$=\bar{P} \bar{Q} R+P \bar{Q} R+P Q \bar{R}+P Q R$
$=\Sigma \mathrm{m}(1,5,6,7)$

9. A digital communication system transmits a block of $N$ bits. The probability of error in decoding a bit is a a. The error event of each bit is independent of error events of other bits. The received block is declared erroneous if at least one of the bits is decoded wrongly. The probability that the received block is erroneous is
A. $N(1-a)$
B. $1-a^{N}$
C. $a^{N}$
D. $1-(1-a)^{N}$

Ans. D
Sol. error probability $=a$
correct probability $=1-a$
' N ' Bits So
Correct probability $=(1-a)(1-a) \ldots$. N ' times $=$ $(1-a)^{N}$
Erroneous probability $=1$ - correct probability $=$ [1 - $\left.(1-a)^{N}\right]$.
10. The loop transfer function of a negative feedback system is
$G(s) H(s)=\frac{K(s+11)}{s(s+2)(s+8)}$
The value of $K$, for which the system is marginally stable, is $\qquad$
Ans. (160-160)
Sol. Closed loop characteristic equations
$1+G(s) H(s)=0$
$1+\frac{K(s+11)}{s(s+2)(s+8)}=0$
$s^{3}+10 s^{2}+16 s+K s+11 K=0$
$s^{3}+10 s^{2}+(16+k) s+11 K=0$

| $s^{3}$ | 1 | $16+K$ |
| :--- | :--- | :--- |
| $s^{2}$ | 10 | 11 K |
| $s^{1}$ | $\frac{10(16+\mathrm{K})-11 \mathrm{~K}}{10}$ | 0 |
| $s^{0}$ | 11 K | 0 |

For marginal stable system
$\frac{10(16+\mathrm{K})-11 \mathrm{~K}}{10}=0$
$160+10 K-11 K=0$
$K=160$
11. In the circuit shown below all component ideal, input voltage is sinusoidal. Magnitude of steady state output $V_{0}$ is $\qquad$ V.


Ans. (644-657)
Sol. The circuit shown is a voltage doubler. So $\mathrm{V}_{\mathrm{o}}=2 \mathrm{~V}_{\mathrm{m}}$


So the peak value at ' $P$ ' $m 2 \mathrm{Vm}$, Then the voltage across $\mathrm{C}_{2}$ which is $2 \mathrm{~V}_{\mathrm{m}}$.
$\therefore \mathrm{Vo}=2 \mathrm{Vm}$ where $\mathrm{V}_{\mathrm{m}}=230 \sqrt{2}=325.27 \mathrm{~V}$
$\mathrm{V}_{\mathrm{o}}=650.5 \mathrm{~V}$
12. The output $y[n]$ of a discrete-time system for an input $x[n]$ is
$y[n]=\max _{-\infty \leq k \leq n}|x[k]|$
The unit impulse response of the system is
A. 0 for all $n$
B. unit impulse signal $\delta$ [ n$]$
C. unit step signal $u[n]$
D. 1 for all $n$

Ans. C
Sol. $Y(n)=\max _{-\infty \leq K \leq n}[X(K)]$
$\mathrm{Y}(\mathrm{n})=\max [\delta(\mathrm{K})]=1 \quad-\infty \leq \mathrm{K} \leq \mathrm{n}$
$Y(n)$ is 1 for all $n$.
13. Which one of the following pole-zero plots corresponds to the function of an LTI system characterized by the input-output difference equation given below?
$y[n]=\sum_{k=0}^{3}(-1)^{k} x[n-k]$
A.

B.

C.

D.


Ans. D
Sol. $Y(n)=\sum_{K=0}^{3}(-1)^{K} X(n-K)$

$$
\begin{aligned}
& Y(n)=X(n)-X(n-1)+X(n-2)-X(n-3) \\
& Y(z)=1-z^{-1}+z^{-2}-z^{-3} \\
& y(z)=\frac{z^{3}-z^{2}+z^{1}-1}{z^{3}}
\end{aligned}
$$

3 poles at $z=0$ and number of zeros is 4
So the option (D) is correct
14. If $v_{1}, v_{2}, \ldots, v_{6}$ are six vectors in $R^{4}$, which one of the following statements is FALSE?
A. If $\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{5}, \mathrm{v}_{6}\right\}$ spans $\mathrm{R}^{4}$, then it forms a basis for $\mathrm{R}^{4}$
B. These vectors are not linearly independent.
C. It is not necessary that these vectors span $R^{4}$
D. Any four of these vectors form a basis for $\mathrm{R}^{4}$

Ans. (d)
Sol. Given $\mathrm{V}_{1}, \mathrm{~V}_{2}, \ldots \mathrm{~V} 6$ are six vectors in $\mathrm{IR}^{4}$.
As the dimension of $I R^{4}$ is 4 , any four vectors that spans $I R^{4}$ forms a basis for $I R^{4}$. So, choice $A$ is not FALSE
If the dimension of a vector space is $n$ then any set of $n+1$ or more vectors in that vector space are NOT linearly independent. So, choice $B$ is NOT FALSE
A collection of four vectors of IR $^{4}$ forms a basis for $\mathrm{IR}^{4}$ only when they span $\mathrm{IR}^{4}$ or they are linearly independent
So, any four of the given vectors need not form a basis for IR $^{4}$
15. The general solution of $\frac{d^{2} y}{d x^{2}}-6 \frac{d y}{d x}+9 y=0$ is
A. $y=C_{1} e^{3 x}$
B. $y=\left(C_{1}+C_{2} x\right) e^{-3 x}$
C. $C_{1} e^{3 x}+C_{2} e^{-3 x}$
D. $y=\left(C_{1}+C_{2} x\right) e^{3 x}$

Ans. D
Sol. $D^{2}-6 D+9=0$
$\Rightarrow(D-3)^{2}=0$
$D=3,3 \rightarrow$ equal roots
$\therefore \mathrm{y}=\left(\mathrm{c}_{1}+\mathrm{c}_{2}\right) \mathrm{e}^{3 \mathrm{x}}$.
16. The current in the RL-circuit shown below is $\mathrm{i}(\mathrm{t})=$ $10 \cos (5 t-\pi / 4) A$
The value of the inductor (rounded off to two decimal places) is $\qquad$ H.


Ans. 2.80-2.85
Sol. Given
$\mathrm{V}_{\mathrm{s}}=200 \cos 5 \mathrm{t}$
$i(t)=10 \cos \left(5 t-\frac{\pi}{4}\right)$
By KVL
$V_{s}(\mathrm{t})=\mathrm{i}(\mathrm{t}) \mathrm{Z}$
And $z=R+j \omega L=R+j X_{L}$
$|Z|=\sqrt{R^{2}+X_{L}^{2}}=\frac{V_{m}}{i_{m}}=\frac{200}{10}$
$\therefore|Z|=20$
Or simply $\sqrt{R^{2}+X_{L}^{2}}=20$

Given,

$$
\begin{align*}
& \theta=\tan ^{-1}\left(\frac{\omega \mathrm{~L}}{\mathrm{R}}\right)=\tan ^{-1}\left(\frac{\mathrm{X}_{\mathrm{L}}}{\mathrm{R}}\right) \\
& =45^{\circ} \\
& \therefore \frac{X_{\mathrm{L}}}{\mathrm{R}}=\tan \left(45^{\circ}\right)=1 \\
& \therefore X_{\mathrm{L}}=\mathrm{R} \tag{ii}
\end{align*}
$$

From equation (i) and (ii)
$\sqrt{R^{2}+X_{L}^{2}}=20$
$\sqrt{X_{L}^{2}+X_{L}^{2}}=20$
$X_{L} \sqrt{2}=20$
$X_{L}=14.14 \Omega$
Or $\omega \mathrm{L}=14.14$
Given $\omega=5 \mathrm{rad} / \mathrm{sec}$
$\therefore \mathrm{L}=2.828 \mathrm{H}$
17. In the circuit shown below, all the components are ideal. If Vi is +2 V , the current $\mathrm{I}_{0}$ sourced by the opamp is $\qquad$ mA .


Ans. (6-6)
Sol. Applying virtual ground

$\mathrm{I}_{1}=\mathrm{I}_{\mathrm{f}}$
$\frac{2-0}{1 K}=\frac{V_{0}-2 V}{1 K}$
$2 m A=\frac{V_{0}-2 V}{1 K}$
$V_{0}=4 \mathrm{~V}$
$\therefore \mathrm{I}_{\mathrm{C}}=\frac{\mathrm{V}_{\mathrm{O}}-0}{1 \mathrm{~K}}=\frac{4-0}{1 \mathrm{~K}}=4 \mathrm{~mA}$
$\therefore \mathrm{I}_{\mathrm{o}}=\mathrm{I}_{\mathrm{L}}+\mathrm{I}_{\mathrm{f}}=4 \mathrm{~mA}+2 \mathrm{~mA}\left(\because \mathrm{I}_{\mathrm{f}}=\mathrm{I}_{1}=2 \mathrm{~mA}\right)$
$=6 \mathrm{~mA}$
18. A 10 -bit $D / A$ converter is calibrated over full range from 0 to 10 V . If input to $\mathrm{D} / \mathrm{A}$ converter is 13 A (in hexadecimal number). Then output voltage is
$\qquad$ V.

Sol. 3.050-3.080
$(13 \mathrm{~A})_{16}=(?)_{10}$
$=1 \times(16)^{2}+3 \times(16)^{1}+10 \times(16)^{0}$
$\{A=10\}$
$=256+48+10$
$=314$
$(13 A)_{16}=(314)_{10}$
Output voltage $=$ Resolution $\times$ Decimal equivalent
$=\frac{10}{2^{10}} \times 314=\frac{10}{2^{10}} \times 314=\frac{10}{1024} \times 314=3.065$
19. In 8085 microprocessor, number of address lines required to access 16 K byte memory bank?
Ans. 14-14
Sol.


16K
$16 \times 2^{10} \times 8$ Bit
$2^{4} \times 2^{10} \times 8$ bits
[8 data line for 8085 microprocessor]
$2^{14} \times 8$ Bits
$\mathrm{n}=14$
So, required address line $=14$
20. The pole-zero map of a rational function $G(s)$ is shown below. When the closed contour $\Gamma$ is mapped into the $\mathrm{G}(\mathrm{s})$-plane, then the mapping encircles.

A. the origin of the $G(s)$-plane once in the counter-clockwise direction.
B. the point $-1+j 0$ of the $G(s)$-plane once in the counter-clockwise direction.
C. the origin of the $G(s)$-plane once in the clockwise direction.
D. the point $-1+j 0$ of the $G(s)$-plane once in the clockwise direction.

Ans. C
Sol. Given number of closed loop poles 2 in contour and number of closed loop 3 zero in contour. $\mathrm{P}=2$ and
$Z=3$. So effective number of encirclements to the origin is once in clockwise direction.
21. A binary random variable $X$ takes the value +2 or 2. The probability $P(X=+2)=a$. The value of $a$ (rounded off to one decimal place), for which the entropy of $X$ is maximum, is

Ans. (0.5-0.5)
Sol. There are only two symbols
$X=-2$
$X=2$
Maximum entropy occurs for equal probability
$H(X)_{\text {max }}=\log _{2}^{2}=1$
$P(X=2)=\frac{1}{2}$
$P(X=-2)=\frac{1}{2}$
22. The impedances $Z=j X$, for all $X$ in the range ( $\infty, \infty)$, map to the Smith chart as
A. a circle of radius 0.5 with centre at $(0.5,0)$.
B. a circle of radius 1 with centre at $(0,0)$.
C. a point at the centre of the chart.
D. a line passing through the centre of the chart.

Ans. B
Sol. $Z=j X$
$\mathrm{R}=0$ (constant)
Hence, its mapping to the smith chart will represent a circle which has centre $\left(\frac{R}{R+1}, 0\right) \equiv(0,0)$
23. The partial derivative of the function
$F(x, y, z)=e^{1-x \cos y}+x z e^{-1 /(1+y 2)}$
With respect to $x$ at the point $(1,0, e)$ is
A. 1
B. $\frac{1}{\mathrm{e}}$
C. 0
D. -1

Ans. C
Sol. $\frac{\partial f}{\partial x}=e^{(1-x \cos y)}(-\cos y)+z e^{\left(-\frac{1}{1+y 2}\right)}$

$$
\begin{aligned}
& \left(\frac{\partial f}{\partial x}\right)_{(1,0, e)}=e^{0}(-1)+e e^{-1} \\
& =-1+1=0
\end{aligned}
$$

24. The components in the circuit shown below are ideal.


If the Op-amp is in positive feedback and the input voltage $\mathrm{V}_{\mathrm{i}}$ is a sine wave of amplitude $1 \mathrm{~V}, \mathrm{~V}_{0}=$ ?
A. a constant of either +5 V or -5 V
B. A square wave of 5 V amplitude
C. A non-inverted sine wave of 2 V amplitude
D. An inverted sine wave of 1 V amplitude

Ans. A
Sol. $V_{N}=V_{i} \frac{1 k}{2 k}+V_{0} \frac{1 k}{2 k}$
$V_{N}=\frac{V_{i}+V_{0}}{2}$
$\mathrm{V}_{\mathrm{N}}>0 \Rightarrow \mathrm{~V}_{\mathrm{o}}=+\mathrm{V}_{\text {sat }}$
Where $V_{N}=\frac{V_{i}+V_{0}}{2}$
If $\mathrm{V}_{\mathrm{o}}=+\mathrm{V}_{\text {sat }} \Rightarrow \mathrm{V}_{\mathrm{N}}=\frac{1+5}{2}=3 \mathrm{~V}$ if $\mathrm{V}_{\mathrm{i}}=1 \mathrm{~V}$ peak
$\mathrm{V}_{\mathrm{N}}=\frac{-1+5}{2}=2 \mathrm{~V}$ if $\mathrm{V}_{\mathrm{i}}=-1 \mathrm{~V}$ peak
If $V_{o}=-V_{\text {sat }} \Rightarrow V_{N}=\frac{1-5}{2}=-2$ if $V i=+1 V$ peak
$\Rightarrow V_{N}=\frac{-1-5}{2}=-3$ if $\mathrm{V}_{\mathrm{i}}=-1 \mathrm{~V}$ peak
So the output is either $+\mathrm{V}_{\text {sat }}$ or $-\mathrm{V}_{\text {sat }}$ as $\mathrm{V}_{\mathrm{N}}$ is not crossing ' 0 '.
25. For a vector field $\vec{A}$, Which one of the following is false
A. $\nabla \times \overrightarrow{\mathrm{A}}$ in another vector field
B. $\nabla \times(\nabla \times \overrightarrow{\mathrm{A}})=\nabla(\nabla \cdot \overrightarrow{\mathrm{A}})-\nabla^{2} \mathrm{~A}^{2}$
C. $\overrightarrow{\mathrm{A}}$ is irrotational if $\nabla^{2} \times \overrightarrow{\mathrm{A}}=0$
D. $\vec{A}$ is sinusoidal if $\vec{A}$ is sinusoidalif $\nabla \cdot \vec{A}=0$

Ans. C
Sol. For a vector field $\vec{A}$ checking from options, we can see
(a) $\overrightarrow{\mathrm{A}}$ is said to be solenoidal if $\nabla \cdot \overrightarrow{\mathrm{A}}=0$
(b) the curl of a vector $\overrightarrow{\mathrm{A}}$ is another vector field, i.e., $\nabla \times \overrightarrow{\mathrm{A}}$ is another vector field.
(c) $\overrightarrow{\mathrm{A}}$ is irrotational/conservative only if $\nabla \times \overrightarrow{\mathrm{A}}=0$
(d) $\nabla \times(\nabla \times \overrightarrow{\mathrm{A}})=\nabla(\nabla \cdot \overrightarrow{\mathrm{A}})-\nabla^{2} \overrightarrow{\mathrm{~A}}$ gives EM wave equation.
26. In a digital communication system, a symbol $S$ randomly chosen from the set $\left\{\mathrm{s}_{1}, \mathrm{~s}_{2}, \mathrm{~s}_{3}, \mathrm{~S}_{4}\right\}$ is transmitted. It is given that $\mathrm{s}_{1}=-3, \mathrm{~s}_{2}=-1, \mathrm{~s}_{3}=$ +1 and $\mathrm{S}_{4}=+2$. The received symbol is $\mathrm{Y}=\mathrm{S}+\mathrm{W}$. W is a zero-mean unit-variance Gaussian random variable and is independent of $S$. $P$, is the conditional probability of symbol error for the maximum likelihood (ML) decoding when the transmitted symbol $S=s_{i}$. The index $i$ for which the conditional symbol error probability $P_{i}$ is the highest is $\qquad$
Ans. (3-3)
Sol. as ML detector is used, the decision boundary between two adjacent signal points will be their arithmetic mean. $\therefore$
for $s_{1}=-3$, the probability of error $\left(p_{1}\right)$ :

$P_{1}=1$ - (shaded area)
for $s_{2}$ : The probability of error $\left(P_{2}\right)$

$P_{2}=1-($ shaded area)
for $s_{3}$ : the probability of error $P_{3}$.

$P_{3}=1$ - (shaded area)
for $s_{4}$ : The probability of error $\left(P_{4}\right)$

$P_{4}=1$ - (shaded area) By concluding above graph $P_{3}$ i.e. probability of error when s3 is transmitted is larger among the four.
$\therefore \mathrm{I}=3$
27. The characteristic equation of a system is $s^{3}+3 s^{2}+(K+2) s+3 K=0$

In the root locus plot for the given system, as $K$ varies from 0 to $\infty$, the break-away or break-in point(s) lie within
A. $(-\infty,-3)$
B. $(-2,-1)$
C. $(-3,-2)$
D. $(-1,0)$

Ans. D
Sol. $s^{3}+3 s^{2}+2 s+K(s+3)=0$
$1+\frac{K(s+3)}{s+\left(s^{2}+3 s+2\right)}=0$
$1+\frac{\mathrm{K}(\mathrm{s}+3)}{\mathrm{s}(\mathrm{s}+1)(\mathrm{s}+2)}=0$
Compare it with $1+\mathrm{G}(\mathrm{s}) \mathrm{H}(\mathrm{s})=0$
$G(s) H(s)=\frac{K(s+3)}{s(s+1)(s+2)}$


Breakaway point is in between $(0,-1)$
28. For the modulated signal $x(t)=m(t) \cos \left(2 \pi f_{\mathrm{c}} \mathrm{t}\right)$, the message signal $m(t)=4 \cos (1000 \pi t)$ and the carrier frequency $f_{c}$ is 1 MHz . The signal $x(t)$ is passed through a demodulator, as shown in the figure below. The output $y(t)$ of the demodulator is

A. $\cos (460 \pi \mathrm{t})$
B. $\cos (1000 \pi t)$
C. $\cos (920 \pi t)$
D. $\cos (540 \pi t)$

Ans. C
Sol. $x(t)=m(t) \cos (2 \pi f c t)$
$m^{\prime}(\mathrm{t})=4 \cos (1000 \pi \mathrm{n})$
$\mathrm{f}_{\mathrm{c}}=1 \mathrm{MHz}$
$Z(t)=m(t) \cos \left(2 \pi f_{c} t\right) \cos \left[2 \pi\left(f_{c}+40\right) t\right]$
$=\frac{\mathrm{m}(\mathrm{t})}{2} \cos \left[2 \pi 2 \mathrm{f}_{\mathrm{c}}+40\right]++\cos (\pi \times 40) \mathrm{t}$
$=\cos (1080 \pi \mathrm{t})+\cos (920 \pi \mathrm{t})$
$=f_{m 1}=540 \mathrm{~Hz}, \quad f_{m 2}=460 \mathrm{~Hz}$
So $y(t)=\cos 920 n t$
29. The band diagram of a $p$-type semiconductor with a band-gap of 1 eV is shown. Using this semiconductor, a MOS capacitor having $\mathrm{V}_{\mathrm{TH}}$ of -0.16 $\mathrm{V}, \mathrm{C}^{\prime}{ }^{\prime}$ of $100 \mathrm{nF} / \mathrm{cm}^{2}$ and a metal work function of
3.87 eV is fabricated. There is no charge within the oxide. If the voltage across the capacitor is $\mathrm{V}_{\mathrm{TH}}$, the magnitude of depletion charge per unit area (in $\mathrm{C} / \mathrm{cm}^{2}$ ) is

A. $1.41 \times 10^{-8}$
B. $1.70 \times 10^{-8}$
C. $0.93 \times 10^{-8}$
D. $0.52 \times 10^{-8}$

Ans. B
Sol. From the figure, $\mathrm{E}_{\mathrm{C}}-\mathrm{E}_{\mathrm{V}}=1 \mathrm{eV}=0.5 \mathrm{Ev}+\mathrm{q} \varphi_{\mathrm{B}}+$
0.2 eV
$\Rightarrow \mathrm{q} \varphi_{\mathrm{B}}=0.3 \mathrm{eV}$
$\Rightarrow \varphi_{\mathrm{B}}=0.3 \mathrm{~V}$, where $\mathrm{q} \varphi_{\mathrm{B}}=\mathrm{E}_{\mathrm{i}}-\mathrm{E}_{\mathrm{FS}}$
The magnitude of depletion charge density
$\rho_{S}=\sqrt{2 \epsilon_{S} N_{A} \psi_{S}}$
where, $\psi_{\mathrm{s}}=2 \varphi_{\mathrm{B}}=2 \times 0.3 \mathrm{~V}=0.6 \mathrm{~V}$
Voltage across capacitor,
$V_{T H}=V_{F B}+\frac{\sqrt{2 \epsilon_{S} N_{A} \psi_{S}}}{C_{o x}^{\prime}}+\psi_{S}$
where, $\mathrm{V}_{\mathrm{FB}}=\varphi_{\mathrm{ms}}=\varphi_{\mathrm{m}}-\varphi_{\mathrm{S}}$

$$
\begin{equation*}
=3.87-4.8 \tag{4}
\end{equation*}
$$

$\mathrm{V}_{\mathrm{FB}}=\varphi_{\mathrm{mS}}=-0.93 \mathrm{~V}$
From (1), (2), (3) \& (4), $\rho_{\mathrm{s}}=1.7 \times 10^{-8} \mathrm{C} / \mathrm{cm}^{2}$
30. The transfer function of a stable discrete time LTI system is $H(z)=\frac{K(z-\alpha)}{Z+0.5}$, where $K$ and a real no. The value of $a=$ ? with $|a|>1$, for which magnitude
rest one of the system to constant over all frequency.
Ans. (-2--2)
Sol. Magnitude will become constant for all pass system
$H(z)=\frac{Z-a}{Z+b}$
$\mathrm{b}=\frac{1}{\alpha^{*}}$
$\alpha=\frac{1}{0.5}$
$a=2$
31. A finite duration discrete time signal $x(n)$ is obtained by sampling the continuous time signal,
$x(t)=\cos (200 \quad \pi t)$ at sampling instants, $\mathrm{t}=\frac{\mathrm{n}}{400}, \mathrm{n}=0, \ldots, 7$. The 8 -point DFT and $\mathrm{x}[\mathrm{n}]$ is defined as
$X[k]=\sum_{n=0}^{7} x[n] e^{\frac{-j \pi k n}{4}} \quad ; k=0,1 \ldots, 7$
Which is true?
A. Only $\mathrm{X}[4]$ is non zero.
B. All $X[K]$ are non-zero
C. Only X[3] and X[5] are non-zero
D. Only $X[2]$ and $X[6]$ are non-zero.

Ans. D
Sol. $X(t)=\cos (200 \varphi t)$

$$
\mathrm{t}=\frac{\mathrm{n}}{400} \mathrm{n}=0,1, \ldots 7
$$

$$
x(n)=\cos \left(200 \pi \times \frac{n}{400}\right)=\cos \left(\frac{\pi}{2} \cdot n\right)
$$

$x[0]=1$
$x[1]=0$
$x[2]=-1$
$x[3]=0$
$x[4]=1$
$x[5]=0$
$x[6]=-1$
$x[7]=0$
$x[0]=\{1,0,-1,0,1,0,-1,0\}$
$X(K)=\sum_{n=0}^{7} x[n] e^{-j \frac{\pi}{4} K \cdot n}$
$X(3)=\sum_{n=0}^{7} x[n] e^{-j \frac{\pi}{4} 7 \cdot n}$
$\left(\frac{1}{\sqrt{2}}-j \frac{1}{2}\right)^{n} \neq 0$
$e^{-j \frac{7 \pi}{4}}=\cos \frac{7 \times 180}{4}-\frac{j \sin 7 \times 180}{4}=\frac{1}{\sqrt{2}}-j \frac{1}{\sqrt{2}}$
32. The state diagram of a sequence detector is shown below. State S0 is the initial state of the sequence detector. If the output is 1 , then

A. the sequence 01010 is detected.
B. the sequence 01011 is detected.
C. the sequence 01110 is detected.
D. the sequence 01001 is detected.

Ans. A
Sol. If output of sequence is 1 , then it is transversed.
So,

| $\mathrm{S}_{0}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 0 | 1 | 0 |

33. For the given circuit, which one of the following is the correct state question?

A. $\frac{d}{d t}\left[\begin{array}{l}v \\ i\end{array}\right]=\left[\begin{array}{cc}-4 & 4 \\ -2 & -4\end{array}\right]\left[\begin{array}{l}v \\ i\end{array}\right]+\left[\begin{array}{ll}0 & 4 \\ 4 & 0\end{array}\right]\left[\begin{array}{l}i_{1} \\ i_{2}\end{array}\right]$
B. $\frac{d}{d t}\left[\begin{array}{l}v \\ i\end{array}\right]=\left[\begin{array}{ll}-4 & -4 \\ -2 & -4\end{array}\right]\left[\begin{array}{l}v \\ i\end{array}\right]+\left[\begin{array}{ll}4 & 0 \\ 0 & 4\end{array}\right]\left[\begin{array}{l}i_{1} \\ i_{2}\end{array}\right]$
C. $\frac{d}{d t}\left[\begin{array}{l}v \\ i\end{array}\right]=\left[\begin{array}{rr}4 & -4 \\ -2 & -4\end{array}\right]\left[\begin{array}{l}v \\ i\end{array}\right]+\left[\begin{array}{ll}0 & 4 \\ 4 & 4\end{array}\right]\left[\begin{array}{l}i_{1} \\ i_{2}\end{array}\right]$
D. $\frac{d}{d t}\left[\begin{array}{l}v \\ i\end{array}\right]=\left[\begin{array}{cc}-4 & -4 \\ -2 & 4\end{array}\right]\left[\begin{array}{l}v \\ i\end{array}\right]+\left[\begin{array}{ll}4 & 4 \\ 4 & 0\end{array}\right]\left[\begin{array}{l}i_{1} \\ i_{2}\end{array}\right]$

Ans. A
Sol.


KVL in Loop
$-0.5 \frac{d i}{d t}-V+\left(i_{1}-i\right) 2=0$
$\frac{d i}{d t}=-2 V+(-4 i)+4 i_{1} \ldots$ (i)
KCL at node $V$
$-i+C \frac{d V}{d t}+\frac{V}{1}-i_{2}=0$
$-i+0.25 \frac{d V}{d t}+V-i_{2}=0$
$0.25 \frac{\mathrm{dV}}{\mathrm{dt}}=-\mathrm{V}+\mathrm{i}+\mathrm{i}_{2}$
$\frac{\mathrm{dV}}{\mathrm{dt}}=-4 \mathrm{~V}+4 \mathrm{i}+4 \mathrm{i}_{2}$
write eqn. (i) and (ii) in matrix from
$\frac{d}{d t}\left[\begin{array}{l}V \\ i\end{array}\right]=\left[\begin{array}{cc}-4 & 4 \\ -2 & -4\end{array}\right]\left[\begin{array}{l}V \\ i\end{array}\right]+\left[\begin{array}{ll}0 & 4 \\ 4 & 0\end{array}\right]\left[\begin{array}{l}i_{1} \\ i_{2}\end{array}\right]$
option (A) is correct.
34. Consider the following system of linear equation
$x_{1}+2 x_{2}=b_{1} ; 2 x_{1}+4 x_{2}=b_{2} ; \quad 3 x_{1}+7 x_{2}=b_{3} ;$
$3 x_{1}+9 x_{2}=b_{4}$

Which one of the following conditions ensures that a solution exists for the above system?
A. $b_{2}=2 b_{1}$ and $3 b_{1}-6 b_{3}+b_{4}=0$
B. $b_{3}=2 b_{1}$ and $3 b_{1}-6 b_{3}+b_{4}=0$
C. $b_{2}=2 b_{1}$ and $6 b_{1}-3 b_{3}+b_{4}=0$
D. $b_{3}=2 b_{1}$ and $6 b_{1}-3 b_{3}+b_{4}=0$

Ans. C
Sol.
$\left.\begin{array}{rl}x_{1}+2 X_{2} & =b_{1} \\ 2 X_{1}+4 X_{2} & =b_{2}\end{array}\right\} 2 b_{1}=b_{2}$
In eqn. (i) we can write as
$3 X_{1}+6 X_{2}+X_{2}=b_{3}$
$3 b_{1}+x_{2}=b_{3}$
$x_{2}=b_{3}-3 b_{1}$
and in eqn. (ii)
$3 X_{1}+6 X_{2}+3 X_{2}=b_{4}$
$3 b_{1}+3\left[b_{3}-3 b_{1}\right)=b_{4}$
$-6 b_{1}+3 b_{3}-b_{4}=0$
$6 b_{1}-3 b_{3}+b_{4}=0$
option (B) is correct
35. An enhancement MOSFET of threshold voltage 3 V is being used in the sample and hold circuit given below. Assume that the substrate of MOS device is connected to -10 V . If the input voltage $\mathrm{V}_{1}$ lies between $\pm 10 \mathrm{~V}$, the minimum and the maximum values of $V_{G}$ required of proper sampling and holding respectively, are

A. 10 V and -10 V .
B. 10 V and -13 V .
C. 13 V and -7 V .
D. 3 V and -3 V .

Ans. C
Sol. During sampling, MOSFET must be as ON switch.
$\Rightarrow V_{G S}>V_{T H}$
$\Rightarrow\left(\mathrm{V}_{\mathrm{G}}-\mathrm{V}_{\mathrm{S}}\right)>\mathrm{V}_{\mathrm{TH}}$
$\Rightarrow \mathrm{V}_{\mathrm{G}}>\mathrm{V}_{\mathrm{S}}+\mathrm{V}_{\mathrm{TH}}$
$\Rightarrow \mathrm{V}_{\mathrm{G}}>10+3 \mathrm{~V}$

$$
\begin{equation*}
\because \mathrm{V}_{\mathrm{S}}=\mathrm{V}_{\mathrm{I}, \max }=10 \mathrm{~V} \tag{1}
\end{equation*}
$$

$\Rightarrow \mathrm{V}_{\mathrm{G}}>13 \mathrm{~V}$
During hold, MOSFET must be as OFF switch.
$\Rightarrow \mathrm{V}_{\mathrm{GS}}<\mathrm{V}_{\mathrm{TH}}$
$\Rightarrow\left(\mathrm{V}_{\mathrm{G}}-\mathrm{V}_{\mathrm{S}}\right)<\mathrm{V}_{\mathrm{TH}}$
$\Rightarrow \mathrm{V}_{\mathrm{G}}<\left(\mathrm{V}_{\mathrm{S}}+\mathrm{V}_{\mathrm{TH}}\right)$
$\Rightarrow \mathrm{V}_{\mathrm{G}}<-7 \mathrm{~V}$
$\therefore \mathrm{V}_{\mathrm{S}}=\mathrm{V}_{\mathrm{I}, \min }=-10 \mathrm{~V}$
36. Which one of the following options contains two solutions of the differential equation $\frac{d y}{d x}=(y-1) x$ ?
A. $\ln |y-1|=2 x^{2}+C$ and $y=1$
B. $\ln |y-1|=2 x^{2}+C$ and $y=-1$
C. $\ln |y-1|=0.5 x^{2}+C$ and $y=-1$
D. $\ln |y-1|=0.5 x^{2}+C$ and $y=1$

Ans. D
Sol. $\frac{d y}{y-1}=x d x$
$\int \frac{d y}{y-1}=\int x d x$
Such that $y \neq 1$.
37. $X(\omega)$ is the Fourier transform of $x(t)$ shown below. The value of $\int_{-\infty}^{\infty}|X(\omega)|^{2} d \omega$ (rounded off to two decimal places) is $\qquad$


Ans. (58.50-58.80)
Sol.

$E=\frac{1}{2 \pi} \int_{-\infty}^{\infty}|X(\omega)|^{2} d \omega=\int_{-\infty}^{\infty} x^{2}(t) d t$
$\int_{-\infty}^{\infty}|X(\omega)|^{2} d \omega=2 \pi \int_{-\infty}^{\infty} X^{2}(t) d t$
$\int_{-\infty}^{\infty} \mathrm{x}^{2}(\mathrm{t}) \mathrm{dt}=\int_{-1}^{0}(\mathrm{t}+1)^{2} \mathrm{dt}+\int_{0}^{1}(2 \mathrm{t}+1)^{2} \mathrm{dt}$ $+\int_{1}^{2}(-2 \mathrm{t}+5)^{2} \mathrm{dt}+\int_{2}^{3}(-\mathrm{t}+3)^{2} \mathrm{dt}$
$=9.33$

$$
\begin{aligned}
\int_{-\infty}^{\infty}|X(\omega)|^{2} & =2 \pi \times 9.33 \\
& =2 \times 3.14 \times 9.33=58.5924
\end{aligned}
$$

38. The current I in the given network is

A. $2.38 \angle 143.63^{\circ} \mathrm{A}$.
B. $2.38 \angle-96.37^{\circ} \mathrm{A}$.
C. $2.38 \angle-23.63^{\circ} \mathrm{A}$.
D. 0 A .

Ans. D
Sol.

$i_{2}=\frac{120 \angle-90^{\circ}}{Z}=\frac{120 \angle-90^{\circ}}{(80-35 j)}$
$=1.3742 \angle-66.37^{\circ}$
And
$+120 \angle-30^{\circ}+Z i_{4}=0$
$\mathrm{i}_{4}=\frac{-\left(120 \angle-30^{\circ}\right)}{Z}$
$=\frac{120 \angle 150}{Z}$
$=\frac{120 \angle 150}{(80-35 j)}$
$=1.3742 \angle 173.62^{\circ}$
$\mathrm{i}_{4}=\mathrm{i}+\mathrm{i}_{2}$
$\therefore \mathrm{i}=\mathrm{i}_{4}-\mathrm{i}_{2}$
$=(1.3742 \angle 173.62)-1.3742 \angle-66.37^{\circ}$
$=2.38 \angle 143.625^{\circ}$
39. The magnetic field of a uniform plane wave in vacuum is given by
$\vec{H}(x, y, z, t)=\left(\hat{a}_{x}+2 \hat{a}_{y}+b \hat{a}_{z}\right) \cos (\omega t+3 x-y-z)$
The value of $b$ is $\qquad$
Ans. (1-1)
Sol. Given,

$$
\begin{aligned}
& \vec{H}(x, y, z, t)=\left(a_{x}+2 a y+b+a_{z}\right) \\
& \cdot \cos (\omega t+3 x-y-z) A / m
\end{aligned}
$$

For a uniform wave,
$\overrightarrow{\mathrm{k}} \cdot \overrightarrow{\mathrm{H}}_{\mathrm{o}}=0, \overrightarrow{\mathrm{k}} \cdot \overrightarrow{\mathrm{E}}_{0}=0, \overrightarrow{\mathrm{E}}_{0} \cdot \overrightarrow{\mathrm{H}}_{0}=0$
i.e., $\vec{E}, \vec{H}$ and $\vec{k}$ are mutually perpendicular to each other.
( $\vec{k}$ is the vector along the direction of wave propagation)
Comparing the given expression of $\vec{H}$ with the standard expression.
$\vec{k}=3 a_{x}-a_{y}-a_{z}$
And, $\vec{H}_{o}=\left(a_{x}+2 a_{y}+b a_{z}\right)$
Then, $\overrightarrow{\mathrm{k}} \cdot \overrightarrow{\mathrm{H}}_{\mathrm{o}}=3-2-\mathrm{b}=0$
$\Rightarrow b=1$
40. $S_{P M}(t)$ and $S_{F M}(t)$ as defined below, are the phase modulated and frequency modulated waveforms, respectively, corresponding to the message signal $m(t)$ shown in the figure.
$S_{P M}(t)=\cos \left(1000 \pi t+K_{p} m(t)\right.$
and $\mathrm{S}_{\mathrm{FM}}(\mathrm{t})=\cos \left(1000 \pi \mathrm{t}+\mathrm{K}_{\mathrm{f}} \int_{-\infty}^{\mathrm{t}} \mathrm{m}(\tau) \mathrm{d} \tau\right)$
Where $K_{p}$ is the phase deviation constant in radians/volt and $K_{f}$ is the frequency deviation constant in radians/second/volt. If the highest instantaneous frequencies of $S_{P M}(t)$ and $S_{F M}(t)$ are same, then the value of the ratio $\frac{\mathrm{K}_{\mathrm{p}}}{\mathrm{K}_{\mathrm{f}}}$ is $\qquad$ seconds.


Ans. - (2-2)
Sol. $S_{p m}(t)=\cos \left[1000 п t+K_{p m}(t)\right]$
$S_{F m}(t)=\cos \left[1000 \pi t+K_{P} \int_{\infty}^{t} m(\tau) d \tau\right]$
Maximum instantaneous frequency in FM.
$\mathrm{f}_{\mathrm{i}}=\frac{1}{2 \pi}\left[\frac{\mathrm{~d}}{\mathrm{dt}} \theta_{\mathrm{i}}(\mathrm{t})\right]$
$f_{i}=\frac{1}{2 \pi}\left[1000 \pi t+K_{f} m(t)\right]$
And maximum instantaneous frequency in PM
$\mathrm{f}_{\mathrm{i}}=\frac{1}{2 \pi}\left[1000 \pi+\frac{\mathrm{d}}{\mathrm{dt}} \mathrm{k}_{\mathrm{p}} \mathrm{m}(\mathrm{t})\right]$
$\frac{d}{d t} m(t)$

$f_{i}=\frac{1}{2 \pi}\left[1000 \pi+K_{p} \times 5\right]$
Given maximum instantaneous frequency is same
$\frac{1}{2 \pi}\left[1000 \pi+\mathrm{K}_{\mathrm{f}} \mathrm{m}(\mathrm{t})\right]=\frac{1000 \pi}{2 \pi}+\frac{5 \mathrm{~K}_{\mathrm{P}}}{2 \pi}$
$\mathrm{K}_{\mathrm{f}} \times 10=5 \mathrm{~K}_{\mathrm{p}}$
$\frac{K_{p}}{K_{f}}=2$
41. For a 2-port network consisting an ideal lossless transformer, the parameter $\mathrm{S}_{21}$, (rounded off to two decimal places) for a reference impedance of $10 \Omega$, is $\qquad$ .


Ans. (0.8 to 0.8)
Sol. $\mathrm{S}_{12}=\dot{\mathrm{S}}_{21}=\frac{2 \mathrm{n}}{\mathrm{n}^{2}+1}=\frac{4}{4+1}=0.8$
42. $P, Q$ and $R$ are the decimal integers corresponding to the 4-bit binary number 1100 considered in signed magnitude, 1's complement, and 2's complement representations, respectively. The 6-bit 2's complement representation of $(P+Q+R)$ is
A. 110010
B. 111101
C. 110101
D. 111001

Ans. B
Sol.

$P+Q+R=-11$
43. A pn junction solar cell of area $1.0 \mathrm{~cm}^{2}$, illuminated uniformly with $100 \mathrm{~mW} \mathrm{~cm}{ }^{-2}$, has the following parameters: Efficiency = 15\%, open circuit voltage $=0.7 \mathrm{~V}$, fill factor $=0.8$, and thickness $=200 \mu \mathrm{~m}$.

The charge of an electron $1.6 \times 10^{-19} \mathrm{C}$. The average optical generation rate (in $\mathrm{cm}^{-3} \mathrm{~s}^{-1}$ ) is
A. $1.04 \times 10^{19}$.
B. $0.84 \times 10^{19}$.
C. $5.57 \times 10^{19}$.
D. $83.60 \times 10^{19}$.

Ans. B
Sol. Fill factor, $F F=\frac{P_{0}}{V_{O C} I_{S C}}$...
Efficiency, $\eta=\frac{P_{0}}{P_{\text {in }}}$
where $P_{\text {in }}=100 \frac{\mathrm{~mW}}{\mathrm{~cm}^{2}} \times$ Area
$=100 \frac{\mathrm{~mW}}{\mathrm{~cm}^{2}} \times 1 \mathrm{~cm}^{2}$
$=100 \mathrm{~mW}$.
$0.15=\frac{P_{0}}{100 \mathrm{~mW}}$
$\therefore \mathrm{P}_{0}=15 \mathrm{~mW}$
(1) $\Rightarrow 0.8=\frac{15 \mathrm{~mW}}{0.7 \times \mathrm{I}_{\mathrm{SC}}}$
$\mathrm{I}_{\mathrm{sc}}=0.027 \mathrm{~A}$
Optical generation rate,
$G_{a v g}=\frac{I_{S C}}{q \times \text { Area } \times \text { thickness }}$

$$
\begin{aligned}
& =\frac{0.027}{1.6 \times 10^{-19} \times 1 \times 200 \times 10^{-4}} \\
& =0.837 \times 10^{19} / \mathrm{cm}^{3} / \mathrm{S}
\end{aligned}
$$

44. Using the incremental low frequency small-signal model of the MOS device, the Norton equivalent resistance of the following circuit is

A. $r_{d s}+R+g_{m} r_{d s} R$
B. $\frac{r_{d s}+R}{1+g_{m} r_{d s}}$
C. $r_{d s}+R$
D. $r_{d s}+\frac{1}{g_{m}}+R$

Ans. D
Sol.

$\mathrm{V}_{\text {test }}=-\mathrm{V}_{\mathrm{gs}}$
$=r_{\text {ds }}\left(I_{\text {test }}-g_{m} V_{\text {test }}\right)+I_{\text {test }} R$
$V_{\text {test }}\left(1+g_{m} r_{\text {ds }}\right)=I_{\text {test }}\left(r_{d s}+R\right)$
$\frac{V_{\text {test }}}{I_{\text {test }}}=\operatorname{Req}=\frac{r_{d s}+R}{1+g_{m} r_{d s}}$
45. For the BJT in the amplifier shown below, $\mathrm{V}_{\mathrm{BE}}=0.7$ $\mathrm{V}, \mathrm{kT} / \mathrm{q}=26 \mathrm{mV}$. Assume that BJT output resistance $\left(r_{0}\right)$ is very high and the base current is negligible. The capacitors are also assumed to be short circuited at signal frequencies. The input $v_{i}$ is direct coupled. The low frequency voltage gain $v_{0} / v_{i}$ of the amplifier is

A. -89.42
B. -178.85
C. -128.21
D. -256.42

Ans. A
Sol. $A_{V}=\frac{-\left[R_{C}| | R_{L}\right]}{r_{e}}$
$r_{e}=\frac{26 m V}{I_{E}}$
DC analysis of the circuit gives,
$\mathrm{I}_{\mathrm{E}}=\frac{10-0.7}{20 \mathrm{k}}=0.465 \mathrm{~mA}$
$\therefore \mathrm{r}_{\mathrm{e}}=55.9 \Omega$
$\therefore A_{V}=\frac{-(10 k| | 10 k)}{55.9}=-89.4$
46. For an infinitesimally small dipole in free space, the electric field $E_{\theta}$ in the far field proportional to ( $e^{-}$ $j \mathrm{kr} / \mathrm{r}) \sin \theta$, where $\mathrm{k}=2 \pi / \lambda$. A vertical infinitesimally small electric dipole $(\delta \mid \ll \lambda)$ is placed at a distance $h(h>0)$ above an infinite ideal conducting plane, as shown in the figure. The minimum value of $h$, for which one of the maxima in the far field radiation pattern occurs at $\theta=60^{\circ}$, is

A. $0.75 \lambda$
B. $\lambda$
C. $0.25 \lambda$
D. $0.5 \lambda$

Ans. B
Sol. As the plane is conducting, i.e., from image theory, the image of a small electric dipole will be formed at the same distance under the plane.

as we know that

$$
\begin{aligned}
& \mid \text { A.F } \left\lvert\,=\frac{\sin (N \psi / 2)}{\sin (\psi / 2)}=\frac{\sin 2(\psi / 2)}{\sin (\psi / 2)}\right. \\
& \because \sin 2 a=2 \sin (a) \cos (a) \\
& \therefore \text { | A.F| }=\frac{2 \sin (\psi / 2) \cdot \cos (\psi / 2)}{\sin (\psi / 2)}
\end{aligned}
$$

| A.F|=2 $\cos (\psi / 2)$

$$
\therefore\left|A \cdot F_{N}\right|=\frac{(A \cdot F)}{\left(A \cdot F_{\max }\right)}=\frac{2 \cos \psi / 2}{2}
$$

$$
\left|A \cdot F_{N}\right|=\cos (\psi / 2)
$$

$$
\because \psi=\beta \mathrm{d} \cos \theta, \beta=\frac{2 \pi}{\lambda}
$$

$$
\because \quad \psi=\frac{2 \pi}{\lambda} \cdot(2 h) \cos \theta
$$

$$
\because \quad \theta=60^{\circ} \text { (given) }
$$

$$
\therefore \quad \psi=\frac{2 \pi}{\lambda} \cdot 2 h \cdot \frac{1}{2}
$$

$$
\therefore\left|\mathrm{AF}_{\mathrm{N}}\right|_{\theta=60^{\circ}}=\cos \left(\frac{2 \pi \cdot \mathrm{~h}}{\lambda \cdot 2}\right)
$$

$$
\left|\mathrm{AF}_{\mathrm{N}}\right|_{\theta=60^{\circ}}=\cos \left(\frac{\pi \cdot \mathrm{h}}{\lambda}\right)
$$

If $\frac{\pi h}{\lambda}=n \pi$, where $n=0,1,2 \ldots$
$\left|A F_{N}\right|$ will be maximum

$$
\text { For } h_{\min }, n=1
$$

$$
\frac{\mathrm{h}_{\min }}{\lambda}=1 \quad \mathrm{~h}_{\min }=\lambda
$$

47. For the solid $S$ shown below, the of $\iiint_{S} x d x d y d z$ (rounded off to two decimal places ) is $\qquad$


Ans. 2.25 (2.25-2.25)
Sol. From the figure


$$
\begin{aligned}
& X=0 \text { to } 3 . \\
& Y=0 \text { to } 1 \\
& Z=0 \text { to } 1-Y \\
& =\int_{0}^{3} \int_{0}^{1} \int_{0}^{Y-1} X d Z d Y d X \\
& =\int_{0}^{3} \int_{0}^{1} X(Z) d Y d X \\
& =\int_{0}^{3} \int_{0}^{1} X(1-Y) d Y d X \\
& =\left.\int_{0}^{3} X\left(Y-\frac{Y^{2}}{2}\right)\right|_{0} ^{1} d X \\
& =\int_{0}^{3} x \frac{1}{2} d x \\
& =\left.\frac{x^{2}}{4}\right|_{0} ^{3}=\frac{9}{4}=2.25
\end{aligned}
$$

48. A system with transfer function $\mathrm{G}(\mathrm{s})=\frac{1}{(\mathrm{~s}+1)(\mathrm{s}+\mathrm{a})}, \mathrm{a}>0$ is subjected to an input $5 \cos 3 t$. The steady state output of the system is $\frac{1}{\sqrt{10}} \cos (3 t-1.892)$. The value of $a$ is $\qquad$
Ans. (4-4)
Sol.


Where $M=|G(j \omega)|_{\omega=\omega}$
$G(j \omega)=\frac{1}{(1+j \omega)+(a+j \omega)}$
$|G(j \omega)|=\frac{1}{\sqrt{\left(\omega^{2}+1\right)\left(\omega^{2}+a^{2}\right)}}$
$M=|G(j \omega)|_{\omega=3}=\frac{1}{\sqrt{(10)\left(a^{2}+9\right)}}$
$A M=\frac{1}{\sqrt{10}}=\frac{5}{\sqrt{10} \sqrt{\mathrm{a}^{2}+9}}$
$a^{2}+9=25$
$a^{2}=16$
$a=4$
$\because a>0$
49. The base of an npn BJT T1 has a linear doping profile $N_{B}(x)$ as shown below, The base of another npn BJT T2 has a uniform doping $N_{B}$ of $10^{17} \mathrm{~cm}^{-3}$. All other parameters are identical for both the devices. Assuming that the hole density profile is the same as that of doping, the common-emitter current gain of T2 is

A. approximately 2.0 times that of T 1
B. approximately 0.7 times that of T1
C. approximately 0.3 times that of T 1
D. approximately 2.5 times that of T1

Ans. (*)
Sol. $\frac{\beta_{1}}{\beta_{2}}=\frac{\int_{0}^{w} N_{A 2} d x}{\int_{0}^{w} N_{A 1} d x}=\frac{w \times 10^{17}}{\frac{1}{2} \times w \times\left(10^{17}-10^{14}\right)} \simeq 2$
$\beta 2=0.5 \beta 1$ Hence no option is matching
50. For the components in the sequential circuit shown below, $\mathrm{t}_{\text {pd }}$ is the propagation delay, $\mathrm{t}_{\text {setup }}$ is the setup time, and thold is the hold time. The maximum clock frequency (rounded off to the nearest integer), at which the given circuit can operate reliably, is .... MHz.


Ans. 76.92 (76-78)
Sol. Total maximum propagation delay $=\left(T_{p d}+\right.$
$\left.\mathrm{T}_{\text {setup }}\right)_{\max }=8 \mathrm{~ns}+5 \mathrm{~ns}=13 \mathrm{~ns}$
frequency of operation $=\left(\frac{1000}{13}\right) \mathrm{MHz}=76.92 \mathrm{MHz}$
51. In the voltage regulator shown below, $\mathrm{V}_{1}$ is the unregulated at 15 V . Assume $\mathrm{V}_{\mathrm{BE}}=0.7 \mathrm{~V}$ and the base current is negligible for both the BJTs. If the regulated output $V_{0}$ is 9 V , the value of $R_{2}$ is $\Omega$


Ans. 800 (800-800)
Sol.


Voltage $V_{B}=V_{z}+V_{B E}$

$$
=3.3+0.7
$$

$V_{B}=4 V \ldots(i)$
$\therefore \quad \mathrm{I}=\frac{9-4}{1 \mathrm{~K}}$
$\mathrm{I}=5 \mathrm{~mA}$
Since base cement is negligible,
$V_{B}=9 \times \frac{R_{2}}{R_{1}+R_{2}}$
$4=\frac{9 R_{2}}{1 K+R_{2}} \Rightarrow R_{2}=800 \Omega$
52. The components in the circuit given below are ideal. If $R=2 \mathrm{k} \Omega$ and $\mathrm{C}=1 \mu \mathrm{~F}$, the -3 dB cut-off frequency of the circuit in Hz is

A. 14.92
B. 79.58
C. 34.46
D. 59.68

Ans. B
Sol. The circuit show is LPF
Applying virtual ground,

$\mathrm{I}=\mathrm{I}_{1}+\mathrm{I}_{2}$
$\frac{V_{i}-0}{R}=\frac{0-V_{0}}{R}+\frac{0-V_{0}}{1 / C S}$
$=-V_{0}\left[\frac{1}{R}+C S\right]$
$\frac{V_{0}}{V_{i}}=\left[\frac{1}{1+R C S}\right]$
$\therefore \omega=\frac{1}{\mathrm{RC}}$
$\Rightarrow f=\frac{1}{2 \pi R C}=\frac{1}{2 \pi\left(2 \times 10^{3}\right)\left(1 \times 10^{-6}\right)}=79.58 \mathrm{~Hz}$
53. A one-sided abrupt pn junction diode has a depletion capacitance $C_{D}$ of 50 PF at reverse bias of 0.2 V . The plot of $1 / C_{D}{ }^{2}$ versus the applied voltage $V$ for this diode is a straight line as shown in the below. The slope of the plot is $\qquad$ $\times 10^{20} \mathrm{~F}^{-2} \mathrm{~V}^{-1}$.

A. -0.47
B. -5.7
C. -1.2
D. -3.8

Ans. $B^{*}$
Sol. We know that

$$
\begin{aligned}
& C_{d} \propto \frac{1}{\sqrt{V_{0}+V_{R}}} \\
& \frac{1}{\left(C_{d}\right)^{2}}=K\left(V_{0}+V_{R}\right)
\end{aligned}
$$

$$
\frac{1}{\left(50 \times 10^{-12}\right)^{2}}=\mathrm{K}\left[\mathrm{~V}_{0}+\mathrm{V}_{\mathrm{R}}\right]
$$

Reverse bias voltage given $\mathrm{V}_{\mathrm{R}}=0.2 \mathrm{~V}$
$\mathrm{V}_{0}=$ applied voltage.

$$
K=\frac{1}{2500\left(V_{0}+V_{R}\right)} \times 10^{24}
$$


value of $V_{0}$ is not given so slope will not to be calculated
54. $X$ is random variable with uniform probability density function in the interval $[-2,10]$. For $Y=2 X$ -6, the conditional probability $P(Y \leq 7 \mid X \geq 5)$ (rounded off to three decimal places) is ..... .
ANS. ( $0.3-0.3$ )
Sol. Given (0.3 to 0.3)
$f_{x}(x)=\left\{\begin{array}{cc}1 / 12 & -2 \leq x \leq 10 \\ 0 & \text { otherwise }\end{array}\right.$
As $\mathrm{y}=2 \mathrm{x}-6$
So, $f_{y}(y)=\left\{\begin{array}{cc}1 / 24 & -10 \leq x \leq 14 \\ 0 & \text { otherwise }\end{array}\right.$
If $x \geq 5$ then $y \geq 4$
So, $P(y \leq 7 / x \geq 5)=P(Y \leq 7 / y \geq 4)$
$=\frac{P(4 \leq y \leq 7)}{P(4 \leq y \leq 14)}=\frac{3}{10}=0.3$

55. Consider the following closed loop control system


Where $G(s)=\frac{1}{s(s+1)}$ and $C(s)=K \frac{s+1}{s+3}$. If the steady state error for a unit ramp is 0.1 , then the value of $K$ is $\qquad$
Ans. (30-30)
Sol. $\quad$ OLTF $=\frac{K(s+1)}{(s+3)} \times \frac{1}{s(s+1)}$
OLTF $=\frac{K}{s(s+3)}$
It is type ' 1 ' system
Steady state error for unit ramp input
$e_{s s}=\frac{1}{K_{V}}$
Where
$K_{p}=\lim _{s \rightarrow \infty} s \times \frac{K}{s(s+3)}=\frac{K}{3}$
$e_{s s}=\frac{1}{K / 3}=\frac{3}{K}$
According to question, $\mathrm{e}_{\mathrm{ss}}=0.1$
$0.1=\frac{3}{\mathrm{~K}} \Rightarrow \mathrm{~K}=30$

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