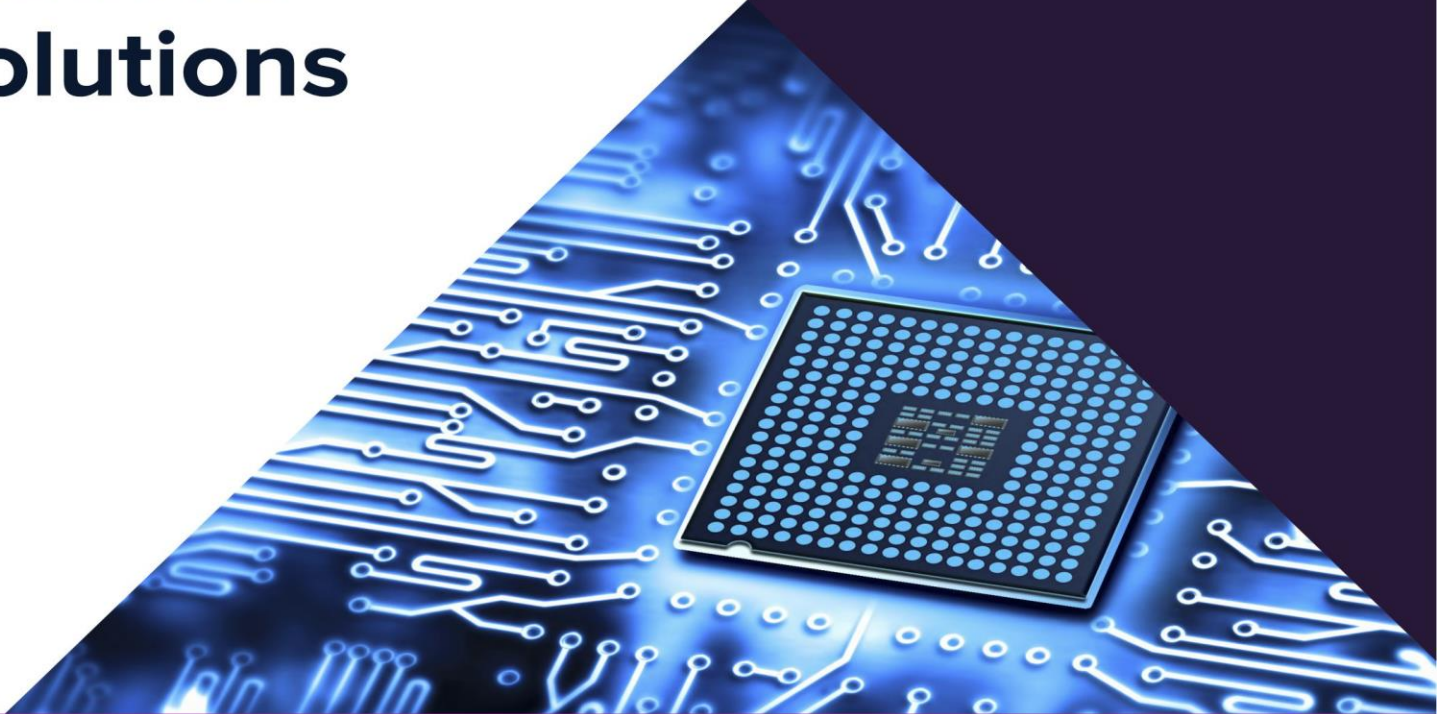


# GATE 2019

Electronics  
& Communication  
Engineering

Questions  
& Solutions



**SECTION: GENERAL APTITUDE**

1. The strategies that the company \_\_\_\_\_ to sell its products \_\_\_\_\_ house-to-house marketing.

- A. uses, include                      B. used, includes  
C. uses, including                    D. use, includes

**Ans.** A

**Sol.** The strategies that the company uses to sell its products include house to house marketing.

2. The boat arrived \_\_\_\_\_ dawn.

- A. under                                  B. in  
C. on                                        D. at

**Ans.** D

**Sol.** The boat arrived at dawn

3. Five different books (P, Q, S, R, T) are to be arranged on a shelf. The books R and S are to be arranged first and second, respectively from the right side of the shelf. The number of different orders in which P, Q and T may be arranged is \_\_\_\_\_.

- A. 12                                        B. 120  
C. 6                                         D. 2

**Ans.** C

**Sol.** As the positions of book R & S are fixed. The books P, Q and T can be arranged in  $3! = 6$  ways

4. When he did not come home, she \_\_\_\_\_ him lying dead on the roadside somewhere.

- A. concluded                            B. looked  
C. notice                                 D. pictured

**Ans.** D

**Sol.** When he did not come home, she pictured him lying dead on the roadside somewhere.

5. It would take one machine 4 hours to complete a production order and another machine 2 hours to complete the same order. If both machines work simultaneously at their respective constant rates, the time taken to complete the same order is \_\_\_\_\_ hours.

- A. 2/3                                        B. 3/4  
C. 4/3                                        D. 7/3

**Ans.** C

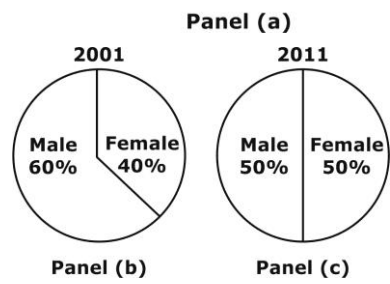
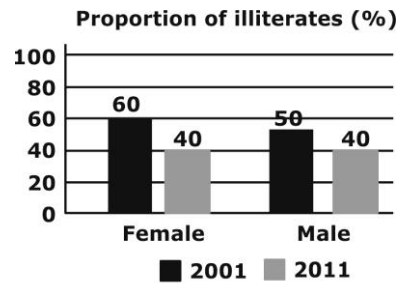
**Sol.** Let t be the time taken by the machines when they work simultaneously.

$$\therefore \frac{1}{t} = \frac{1}{4} + \frac{1}{2}$$

$$\therefore \frac{1}{t} = \frac{3}{4}$$

$$\therefore t = \frac{4}{3}$$

6. The bar graph in Panel (a) shows the proportion of male and female illiterates in 2001 and 2011. The proportions of males and females in 2001 and 2011 are given in Panel (b) and (c), respectively. The total population did not change during this period. The percentage increase in the total number of literates from 2001 to 2011 is \_\_\_\_\_.



- A. 34.43                                    B. 35.43  
C. 30.43                                    D. 33.43

**Ans.** C

**Sol.** Given is the % of illiterates

So % of literates will be

	F	M
2001	40%	50%
2011	60%	60%

And population distribution is

	F	M
2001	40%	60%



next to the garden. R, who is currently sharing an office with T wants to move to the adjacent office with S, the handsome new intern. Given the floor plan, what is the current location of Q, R and T?  
(O = Office, WR = Washroom)

A.

WR	O1 P,Q	O2	O3 R,T	O4 S
Manager	Entry		Teller 1	Teller 2
Garden				

B.

WR	O1 P,Q	O2	O3 R	O4 S
Manager T	Entry		Teller 1	Teller 2
Garden				

C.

WR	O1 P	O2 Q	O3 R	O4 S
Manager	Entry		Teller 1	Teller 2
Garden				

D.

WR	O1 P,Q	O2	O3 T	O4 R,S
Manager	Entry		Teller 1	Teller 2
Garden				

**Ans.** D

**Sol.** As it is given that R is sharing an office with T. So only option (D) is correct.

**TECHNICAL**

1. Which one of the following functions is analytic over the entire complex plane?

- A.  $e^{1/z}$
- B.  $\ln(z)$
- C.  $\frac{1}{1-z}$
- D.  $\cos(z)$

Ans. D

Sol. A function  $F(z)$  is said to be analytic at a point  $z = a$  then  $F(z)$  has a derivative at  $z = a$  and derivative exists at each neighbouring point of  $z = a$  in domain

D.

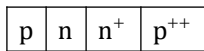
$\frac{1}{e^z}$  at  $z=0 \rightarrow e^\infty \rightarrow$  No derivative

$\ln z$  at  $z=0 \rightarrow \ln(0) = -\infty \rightarrow$  does not exist

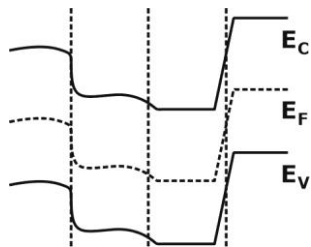
$\frac{1}{1-z}$  at  $z=1 \rightarrow \frac{1}{0} = \infty \rightarrow$  does not exist

But  $\cos z$  exists for all values of  $z$  so it is analytic over the entire complex plane.

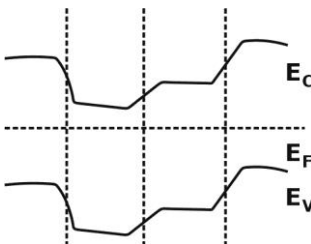
2. Which one of the following options describes correctly the equilibrium band diagram at  $T = 300$  K of a Silicon pnn+p++ configuration shown in the figure?



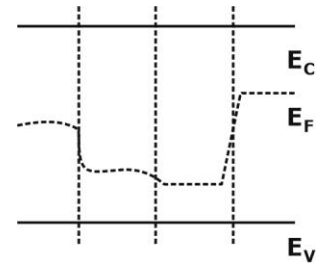
A.



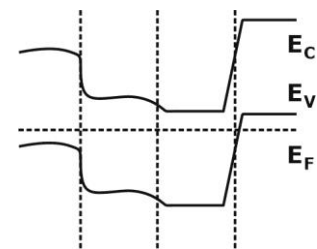
B.



C.



D.



Ans. B

Sol. As no supply is connected hence fermi level will be constant.

In P type semiconductor Fermi level should be closer to EV.

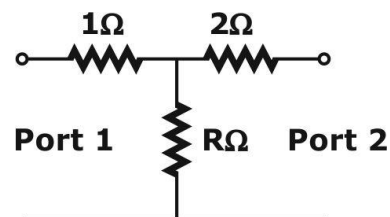
In N type semiconductor Fermi level should be closer to EC.

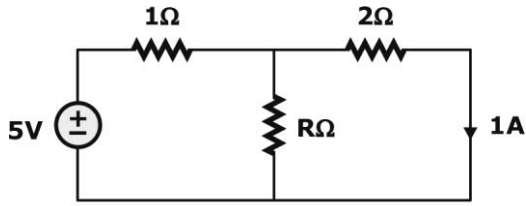
In P++ type semiconductor due to large doping Fermi level enters into valance band.

Hence answer is (B).

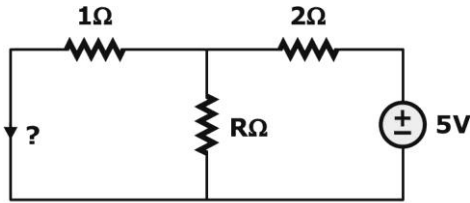
3. Consider the two-port resistive network shown in the figure. When an excitation of 5 V is applied across Port 1, and Port 2 is shorted, the current through the short circuit at Port 2 is measured to be 1 A (see (a) in the figure).

Now, if an excitation of 5 V is applied across Port 2, and Port 1 is shorted (see (b) in the figure), what is the current through the short circuit at Port 1?





(a)



(b)

- A. 0.5 A
- B. 1 A
- C. 2 A
- D. 2.5 A

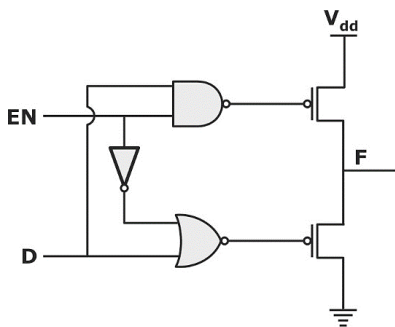
**Ans.** B

**Sol.** By reciprocity theorem,

$$\frac{1}{5} = \frac{I}{5}$$

$$\therefore I = 1A$$

4. In the circuit shown, what are the values of F for  $E_N = 0$  and  $E_N = 1$ , respectively?



- A. Hi-Z and D
- B. Hi-Z and  $\bar{D}$
- C. 0 and 1
- D. 0 and D

**Ans.** A

**Sol.** let output of NAND gate is M and output of NOR gate is N

$$\therefore M = \overline{E_N \cdot D}$$

$$\text{And } N = \overline{E_N + D}$$

$$\therefore N = \overline{E_N \cdot \bar{D}}$$

When  $E_N = 0$

$M = 1$  and  $N = 0$

So both PMOS and NMOS will be OFF

So F will be at high impedance

When  $E_N = 1$

$$M = \bar{D} \text{ \& } N = \bar{D}$$

So this CMOS will act as not gate

$\therefore$  F will be D

$\therefore$  Option (A) is correct.

5. The number of distinct eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

is equal to \_\_\_\_\_.

**Ans.** 3

**Sol.** Since it is a upper triangular matrix eigen values will be 2, 1, 3, 2

$\therefore$  distinct eigen values are three

6. The families of curves represented by the solution of the equation

$$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^n$$

for  $n = -1$  and  $n = +1$ , respectively, are

- A. Hyperbolas and Circles
- B. Circles and Hyperbolas
- C. Parabolas and Circles
- D. Hyperbolas and Parabolas

**Ans.** A

**Sol.** 
$$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^n$$

When  $n = -1$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\therefore \frac{dy}{y} = -\frac{dx}{x}$$

$$\therefore \ln y = -\ln(x) + \ln(c)$$

$$\therefore \ln(xy) = \ln(c)$$

$$\therefore xy = c$$

This represents rectangular hyperbola.

Now for  $n = +1$

$$\frac{dy}{dx} = -\frac{x}{y}$$

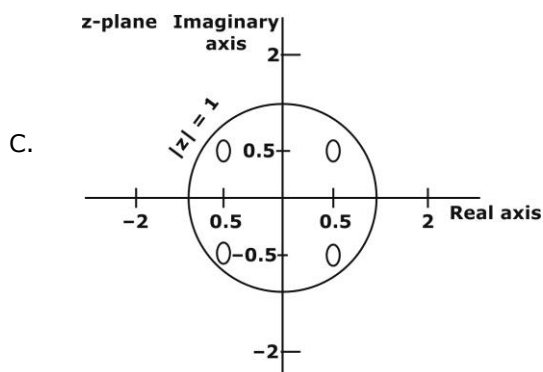
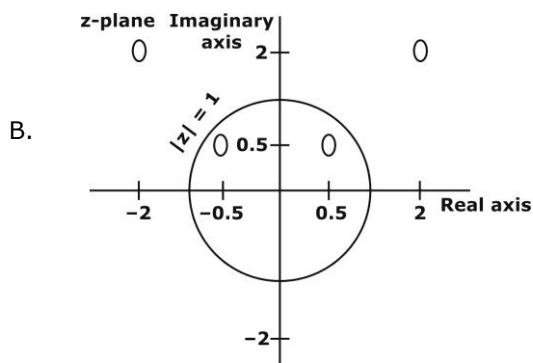
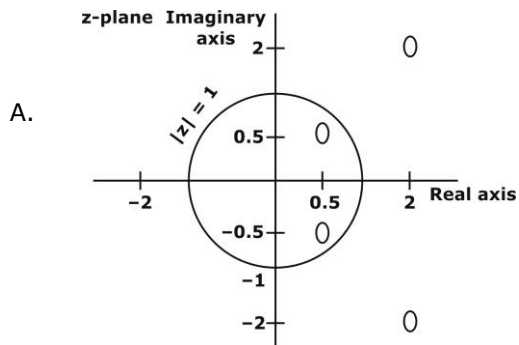
$$\therefore ydy = -x dx$$

$$\therefore \frac{y^2}{2} = \frac{-x^2}{2} + c$$

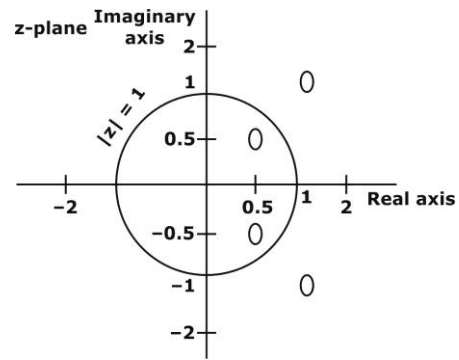
$$\therefore x^2 + y^2 = 2c$$

This represents family of circles.

7. Let  $H(z)$  be the z-transform of a real-valued discrete-time signal  $h[n]$ . If  $P(z) = H(z) H\left(\frac{1}{z}\right)$  has a zero at  $z = \frac{1}{2} + \frac{1}{2}j$ , and  $P(z)$  has a total of four zeros, which one of the following plots represents all the zeros correctly?



D.



Ans. B

Sol. let  $H(z) = \frac{(z-a)(z-b)}{(z-c)(z-d)}$

$$\therefore H\left(\frac{1}{z}\right) = \frac{\left(\frac{1}{z}-a\right)\left(\frac{1}{z}-b\right)}{\left(\frac{1}{z}-c\right)\left(\frac{1}{z}-d\right)}$$

$$\therefore H\left(\frac{1}{z}\right) = \frac{\left(z-\frac{1}{a}\right)\left(z-\frac{1}{b}\right)}{\left(z-\frac{1}{c}\right)\left(z-\frac{1}{d}\right)}$$

$$\therefore H(z) \cdot H\left(\frac{1}{z}\right) = \frac{(z-a)(z-b)\left(z-\frac{1}{a}\right)\left(z-\frac{1}{b}\right)}{(z-c)(z-d)\left(z-\frac{1}{c}\right)\left(z-\frac{1}{d}\right)}$$

$$\therefore \text{zeros are } a, b, \frac{1}{a}, \frac{1}{b}$$

$$\text{given zero is } a = \frac{1}{2} + \frac{1}{2}j$$

as  $h(n)$  is real valued signal another zero must be complex conjugate of this

$$\therefore b = \frac{1}{2} - \frac{1}{2}j$$

$$\text{Now } z_3 = \frac{1}{a} = \frac{1}{\frac{1}{2} + \frac{1}{2}j}$$

$$= \frac{2}{1+j} = \frac{2(1-j)}{2}$$

$$z_3 = 1 - j$$

as  $h(n)$  is real valued signal another zero must be complex conjugate of this

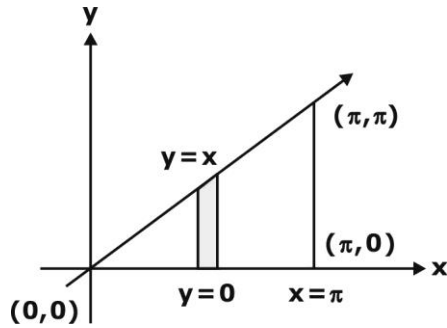
$$z_4 = 1 + j$$

$$\therefore z_1 = \frac{1}{2} + \frac{1}{2}j \quad z_2 = \frac{1}{2} - \frac{1}{2}j \quad z_3 = 1 - j \quad z_4 = 1 + j$$

8. The value of the integral  $\int_0^\pi \int_y^\pi \frac{\sin x}{x} dx dy$ , is equal to \_\_\_\_\_.

Ans. 2

Sol.



By changing order of integration

$$\int_{x=0}^{\pi} \left( \int_{y=0}^{y=x} dy \right) \frac{\sin x}{x} dx$$

$$\therefore \int_{x=0}^{\pi} x \frac{\sin x}{x} dx$$

$$\therefore \int_{x=0}^{\pi} x \sin x dx$$

$$\therefore [-\cos x]_0^\pi = 2$$

9. Radiation resistance of a small dipole current element of length  $l$  at a frequency of 3 GHz is 3 ohms. If the length is changed by 1%, then the percentage change in the radiation resistance, rounded off to two decimal places, is \_\_\_\_\_%.

Ans. 2

Sol.  $R_{rad} = 80\pi^2 \left( \frac{dl}{\lambda} \right)^2$

$$80\pi^2 \left( \frac{dlf}{C} \right)^2$$

$$\therefore R_{rad} \propto l^2 f^2$$

Now frequency is constant

$$\therefore R_{rad} \propto f^2$$

$$\therefore \frac{\Delta R}{R} = 2 \frac{\Delta l}{l}$$

$$= 2 \times 1\%$$

$$\therefore \frac{\Delta R}{R} = 2\%$$

10. Let  $Y(s)$  be the unit-step response of a causal system having a transfer function

$$G(s) = \frac{3-s}{(s+1)(s+3)}$$

that is,  $Y(s) = \frac{G(s)}{s}$ . The forced response of the

system is

A.  $u(t) - 2e^{-t}u(t) + e^{-3t}u(t)$

B.  $u(t)$

C.  $2u(t) - 2e^{-t}u(t) + e^{-3t}u(t)$

D.  $2u(t)$

Ans. A

Sol.  $y(s)$  is unit step response

$$\therefore y(s) = G(s) \times \frac{1}{s}$$

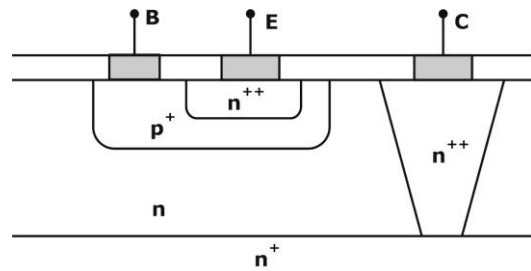
$$= \frac{3-s}{s(s+1)(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

$$\therefore y(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+3}$$

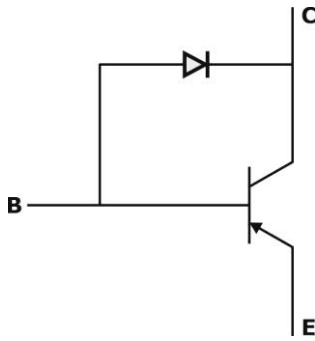
$$\therefore y(t) = u(t) - 2e^{-t}u(t) + e^{-3t}u(t)$$

11. The correct circuit representation of the structure shown in the figure is

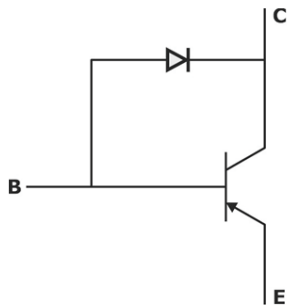




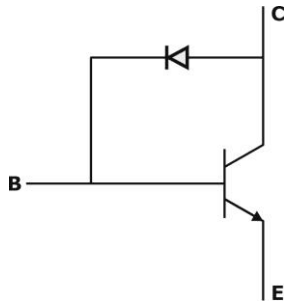
A.



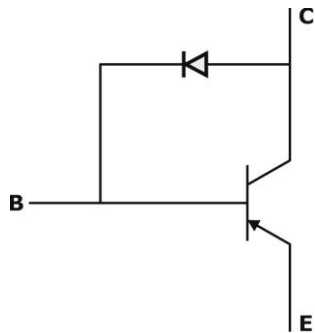
B.



C.

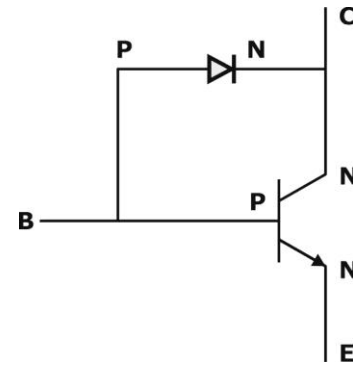
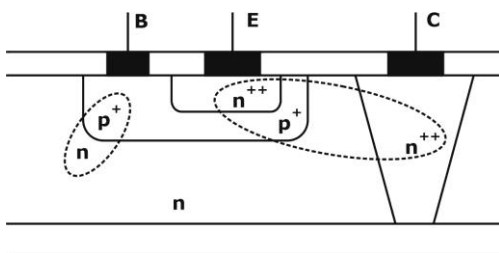


D.

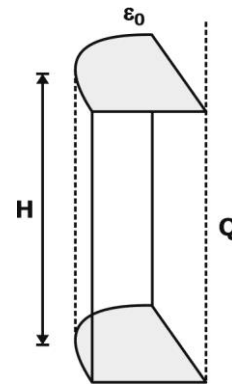


Ans. A

Sol.



12. What is the electric flux ( $\int \vec{E} \cdot d\hat{a}$ ) through a quarter-cylinder of height H (as shown in the figure) due to an infinitely long line charge along the axis of the cylinder with a charge density of Q?



A.  $\frac{H \epsilon_0}{4Q}$

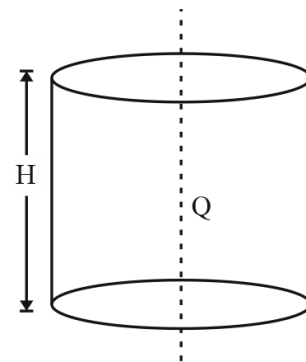
B.  $\frac{HQ}{\epsilon_0}$

C.  $\frac{HQ}{4\epsilon_0}$

D.  $\frac{4H}{Q\epsilon_0}$

Ans. C

Sol.



If we consider a total cylinder then by gauss law

$$\oint \vec{D} \cdot d\vec{s} = Q_{\text{enclosed}}$$

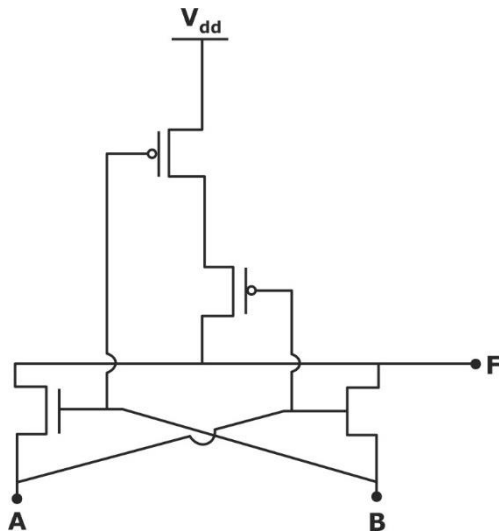
But  $Q_{\text{enclosed}} = Q \cdot H$

And we are considering only  $\frac{1}{4}$  th of the cylinder

$$\therefore D = \frac{Q \cdot H}{4}$$

$$\therefore E = \frac{Q \cdot H}{4 \epsilon_0}$$

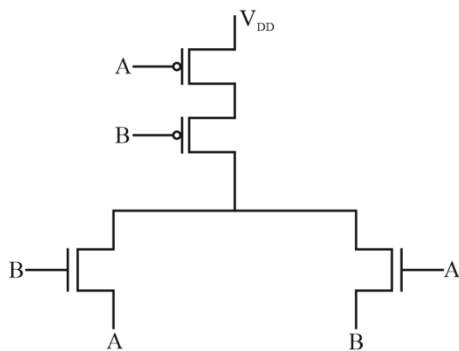
13. In the circuit shown, A and B are the inputs and F is the output. What is the functionality of the circuit?



- A. XOR
- B. SRAM Cell
- C. Latch
- D. XNOR

Ans. D

Sol. By rearranging the circuit,



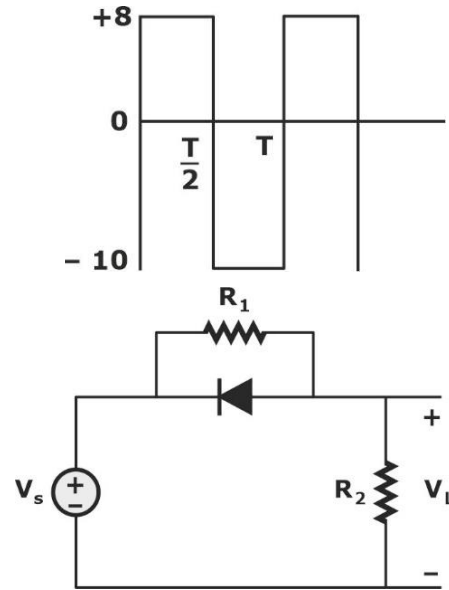
Truth table:

A	B	F
0	0	1
0	1	0
1	0	0
1	1	1

So it is XNOR gate.

14. In the circuit shown,  $V_s$  is a square wave of period  $T$  with maximum and minimum values of 8 V and -10 V, respectively. Assume that the diode is ideal and  $R_1 = R_2 = 50 \Omega$ .

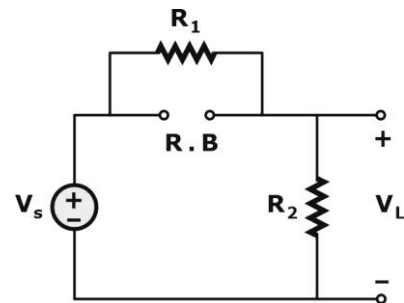
The average value of  $V_L$  is \_\_\_\_\_ volts (rounded off to 1 decimal place).



Ans. -3

Sol. When  $V_s$  is +ve

Diode will be reverse biased



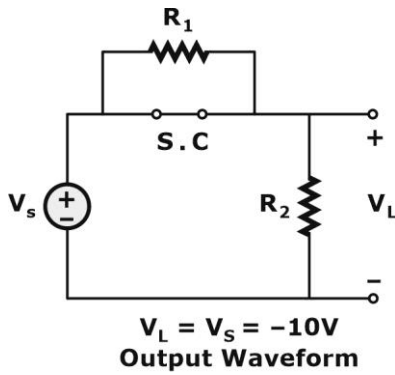
$$V_L = \frac{R_2}{R_1 + R_2} V_s$$

$$\therefore V_L = \frac{50}{50 + 50} \times 8$$

$$\therefore V_L = 4V \quad \dots(i)$$

When  $V_s$  is -ve

Diode will be forward biased



$\therefore V_L = V_S = -10V \quad \dots(ii)$

From (i) and (ii)

$$\text{Average value} = \frac{4 + (-10)}{2} = -3$$

$\therefore$  Average value = -3

- 15.** If X and Y are random variables such that  $E[2X + Y] = 0$  and  $E[X + 2Y] = 33$ , then  $E[X] + E[Y] =$  \_\_\_\_\_.

**Ans.** 11

**Sol.** We know that

$$E[AX + BY] = AE[X] + BE[Y]$$

$$\therefore E[2X + Y] = 2E[X] + E[Y] = 0 \quad \dots(i)$$

$$\text{And } E[X + 2Y] = E[X] + 2E[Y] = 33 \quad \dots(ii)$$

Adding (i) and (ii)

$$3E[X] + 3E[Y] = 33$$

$$\therefore E[X] + E[Y] = 11$$

- 16.** A standard CMOS inverter is designed with equal rise and fall times ( $\beta_n = \beta_p$ ). If the width of the pMOS transistor in the inverter is increased, what would be the effect on the LOW noise margin ( $NM_L$ ) and the HIGH noise margin  $NM_H$ ?

- A.  $NM_L$  increases and  $NM_H$  decreases.
- B. Both  $NM_L$  and  $NM_H$  increase.
- C.  $NM_L$  decreases and  $NM_H$  increase.
- D. No change in the noise margins.

**Ans.** A

**Sol.** We know that

$$NM_L = V_{L_L} - V_{O_L}$$

$$NM_H = V_{O_H} - V_{I_H}$$

$$\text{Now, } V_{IL} = \frac{2V_0 - |V_{TP}| - V_{DD} + kV_{Tn}}{1+k}$$

$$V_{OL} = V_{in} - V_{TP} + \sqrt{(V_{in} - V_{DD} - V_{TP})^2 + k(V_{in} - V_{TP})^2}$$

$$V_{OH} = V_{in} - V_{Tn} + \sqrt{(V_{in} - V_{Tn})^2 + \frac{1}{k}(V_{in} - V_{DD} - V_{TP})^2}$$

$$V_{IH} = \frac{V_{DD} + V_{TP} + k(2V_0 + V_{TP})}{1+k}$$

$$\text{Where } k = \frac{(w/L)_n}{(w/L)_p}$$

$\therefore$  as  $W_p \uparrow \rightarrow NM_L \uparrow$  and  $NM_H \downarrow$

- 17.** In the table shown, List I and List II, respectively, contain terms appearing on the left-hand side and the right-hand side of Maxwell's equations (in their standard form). Match the left-hand side with the corresponding right-hand side.

List I		List II	
1	$\nabla \cdot D$	P	0
2	$\nabla \times E$	Q	$\rho$
3	$\nabla \cdot B$	R	$-\frac{\partial B}{\partial t}$
4	$\nabla \times H$	S	$J + \frac{\partial D}{\partial t}$

- A. 1 - Q, 2 - R, 3 - P, 4 - S
- B. 1 - Q, 2 - S, 3 - P, 4 - R
- C. 1 - R, 2 - Q, 3 - S, 4 - P
- D. 1 - P, 2 - R, 3 - Q, 4 - S

**Ans.** A

**Sol.**  $\nabla \cdot \bar{D} = \rho_V$

This is Gauss law

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

This is faraday law of electromagnetic induction

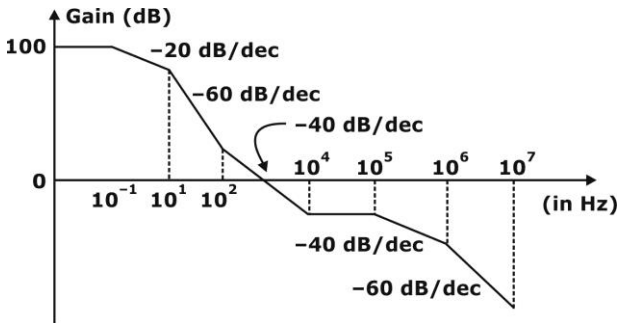
$$\nabla \times \bar{B} = 0$$

This is Gauss law in magnetostatics which states magnetic monopole does not exist.

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

This is modified form of ampere's circuital law.

18. For an LTI system, the Bode plot for its gain is as illustrated in the figure shown. The number of system poles  $N_p$  and the number of system zeros  $N_z$  in the frequency range  $1 \text{ Hz} \leq f \leq 10^7 \text{ Hz}$  is

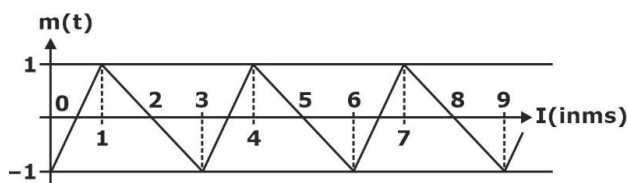


- A.  $N_p = 6, N_z = 3$       B.  $N_p = 5, N_z = 2$   
 C.  $N_p = 4, N_z = 2$       D.  $N_p = 7, N_z = 4$

Ans. A

- Sol. at  $F = 10 \text{ Hz}$  we have one pole  
 At  $F = 10^2 \text{ Hz}$  we can see two more poles are added as slope is decreased by 40 dB/decade  
 At  $F = 10^3 \text{ Hz}$  we have a zero  
 At  $F = 10^4 \text{ Hz}$  we have two zero's  
 At  $F = 10^5 \text{ Hz}$  we have two pole's  
 At  $F = 10^6$  we have one pole  
 $\therefore$  Total poles  $N_p = 6$   
 And total zeros  $N_z = 3$

19. The baseband signal  $m(t)$  shown in the figure is phase-modulated to generate the PM signal  $\phi(t) = \cos(2\pi f_c t + k m(t))$ . The time  $t$  on the x-axis in the figure is in milliseconds. If the carrier frequency is  $f_c = 50 \text{ kHz}$  and  $k = 10\pi$ , then the ratio of the minimum instantaneous frequency (in kHz) to the maximum instantaneous frequency (in kHz) is \_\_\_\_\_ (rounded off to 2 decimal places).



Ans. 0.75

- Sol.  $x(t) = \cos(2\pi f_c t + k m(t))$   
 $\therefore Q(t) = 2\pi f_c t + k m(t)$

$$\text{And } f_i = \frac{1}{2\pi} \frac{\partial}{\partial t} (Q(t))$$

$$= \frac{1}{2\pi} \frac{\partial}{\partial t} [2\pi f_c t + k m(t)]$$

$$f_i = f_c + \frac{k}{2\pi} \frac{\partial}{\partial t} m(t)$$

$$\therefore f_{i_{\max}} = f_c + \frac{k}{2\pi} \left[ \frac{\partial}{\partial t} m(t) \right]_{\max}$$

$$\therefore f_{i_{\max}} = 50 \text{ kHz} + 5 \times \frac{1 - (-1)}{(7-6) \times 10^{-3}}$$

$$\therefore f_{i_{\max}} = 50 \text{ kHz} + 10 \text{ kHz}$$

$$\therefore f_{i_{\max}} = 60 \text{ kHz}$$

$$\text{And } f_{i_{\min}} = f_c + \frac{k}{2\pi} \left[ \frac{\partial}{\partial t} (m(t)) \right]_{\min}$$

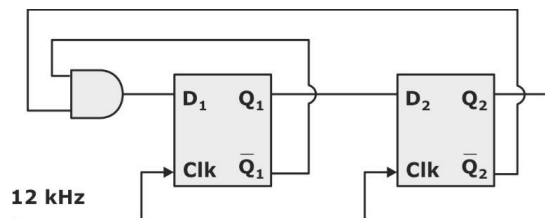
$$50 \text{ kHz} + 5 \times \frac{-1 - 1}{(9-7) \times 10^{-3}}$$

$$= 50 \text{ kHz} - 5 \text{ kHz}$$

$$f_{i_{\min}} = 45 \text{ kHz}$$

$$\therefore \frac{f_{i_{\min}}}{f_{i_{\max}}} = \frac{45}{60} = 0.75$$

20. In the circuit shown, the clock frequency, i.e., the frequency of the Clk signal, is 12 kHz. The frequency of the signal at  $Q_2$  is \_\_\_\_\_ kHz.



Ans. 4

Sol.  $D_1 = \bar{Q}_1 \cdot \bar{Q}_2$

$$D_2 = Q_1$$

Present State		Excitation		Next state	
$Q_1$	$Q_2$	$D_1$	$D_2$	$Q_1^+$	$Q_2^+$

0	0	1	0	1	0
1	0	0	1	0	1
0	1	0	0	0	0

As three states are there

$$\text{Frequency of output} = \text{Frequency of } Q_2 = \frac{12 \text{ kHz}}{3}$$

= 4 kHz

**21.** A linear Hamming code is used to map 4-bit messages to 7-bit codewords. The encoder mapping is linear. If the message 0001 is mapped to the codeword 0000111, and the message 0011 is mapped to the codeword 100110, then the message 0010 is mapped to

- A. 1111000                      B. 0010011  
C. 1111111                      D. 1100001

**Ans.** D

**Sol.** As it is given that it is linear hamming code addition of two codes will produce another code.

(Here we are talking about mod 2 addition)

$$\begin{array}{r} 0001 \rightarrow 0000111 \\ 0011 \rightarrow 1100110 \\ \hline 0010 \rightarrow 1100001 \end{array}$$

**22.** Let Z be an exponential random variable with mean 1. That is, the cumulative distribution function of Z is given by

$$F_z(x) = \begin{cases} 1 - e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then  $\Pr(Z > 2 | Z > 1)$ , rounded off to two decimal places, is equal to \_\_\_\_\_.

**Ans.** 0367

**Sol.** Probability density function (Pdf) =  $\frac{d}{dx}(\text{CDF})$

$$\therefore \text{Pdf} = \begin{cases} e^{-x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

$$\text{Now } \Pr(z > 2 | z > 1) = \frac{\Pr[(z > 2) \cap (z > 1)]}{\Pr(z > 1)}$$

$$= \frac{\Pr(z > 2)}{\Pr(z > 1)}$$

$$= \frac{\int_2^{\infty} e^{-x} dx}{\int_1^{\infty} e^{-x} dx}$$

$$= \frac{-1(e^{-\infty} - e^{-2})}{-1(e^{-\infty} - e^{-1})}$$

$$= \frac{e^{-2}}{e^{-1}} = \frac{1}{e}$$

$$\therefore \Pr(Z > 2 | Z > 1) = 0.367$$

**23.** Consider the signal  $f(t) = 1 + 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$ , where t is in seconds.

Its fundamental time period, in seconds, is \_\_\_\_\_.

**Ans.** 12

**Sol.** DC value and phase shift does not affect time period of a signal.

So it is equivalent to find time period of

$$x(t) = 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t\right)$$

$$\therefore \omega_1 = \pi \quad T_1 = \frac{2\pi}{\omega_1} = 2 \text{ second}$$

$$\omega_2 = \frac{2\pi}{3} \quad T_2 = \frac{2\pi}{\omega_2} = 3 \text{ second}$$

$$\omega_3 = \frac{\pi}{2} \quad T_3 = \frac{2\pi}{\omega_3} = 4 \text{ second}$$

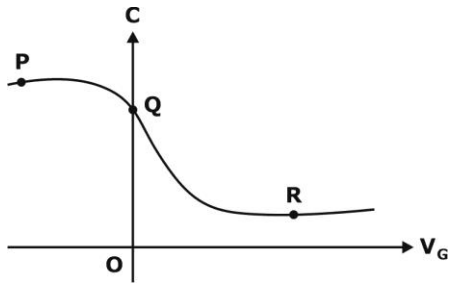
Now overall T = LCM ( $T_1, T_2, T_3$ )

$$= \text{LCM}(2, 3, 4)$$

$\therefore$  overall T = 12 seconds

**24.** The figure shows the high-frequency C-V curve of a MOS capacitor (at T = 300 K) with  $\Phi_{ms} = 0$  V and no oxide charges. The flat-band, inversion, and

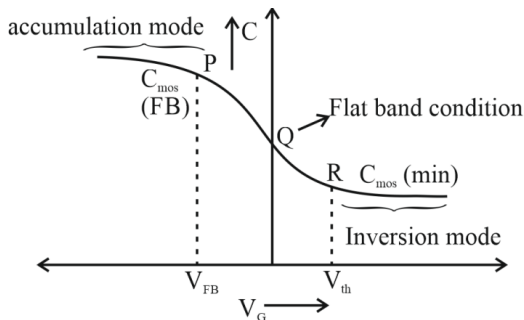
accumulation conditions are represented, respectively, by the points



- A. Q, R, P
- B. Q, P, R
- C. R, P, Q
- D. P, Q, R

Ans. A

Sol.



25. The value of the contour integral

$$\frac{1}{2\pi j} \oint \left( z + \frac{1}{z} \right)^2 dz$$

evaluated over the unit circle  $|z| = 1$  is \_\_\_\_\_.

Ans. 0

Sol. 
$$\frac{1}{2\pi i} \oint_{|z|=1} \frac{(z^2 + 1)^2}{z^2} dz$$

For poles :

Consider  $z^2 = 0 \Rightarrow z = 0, 0$

Now  $f(z) = (z^2 + 1)^2$

$$\oint_C \frac{f(z)}{(z-a)^n} dz = \frac{2\pi i}{(n-1)!} f^{(n-1)}(a)$$

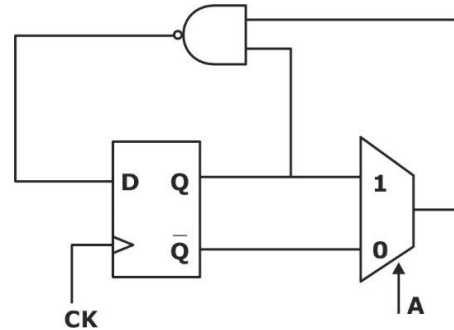
$$= \frac{1}{2\pi i} \left[ \frac{2\pi i}{(2-1)!} f^{2-1}(a) \right] = f'(a) = f'(0)$$

Now  $f'(z) = 2(z^2 + 1)(2z)$

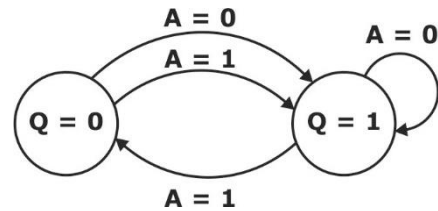
$f'(0) = 2(0 + 1)(0) = 0$

∴ So answer is zero.

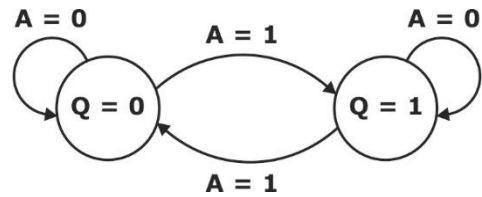
26. The state transition diagram for the circuit shown is



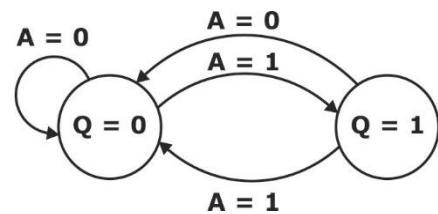
A.



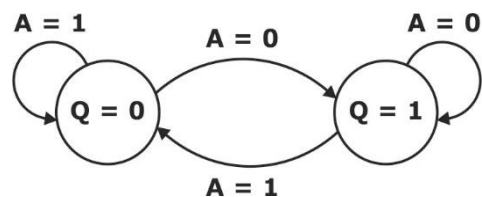
B.



C.



D.



Ans. A

Sol. Let output of MUX is M

So  $M = \bar{A}\bar{Q} + AQ$

∴  $M = A \odot Q$

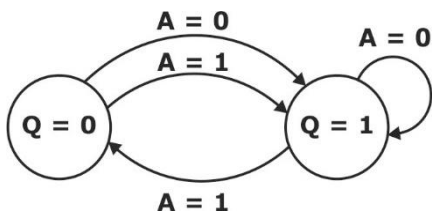
And  $D = \overline{MQ}$

$= \overline{M} + \overline{Q}$

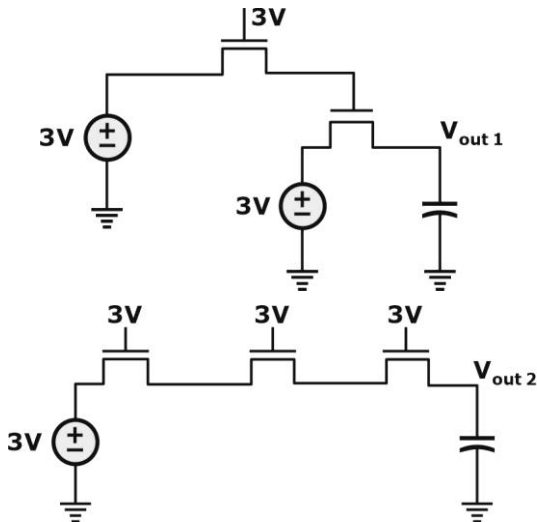
$D = A \oplus Q + \overline{Q}$

Present State	Input	Next State
Q	A	$Q^+ = D$
0	0	1
0	1	1
1	0	1
1	1	0

State Diagram:-



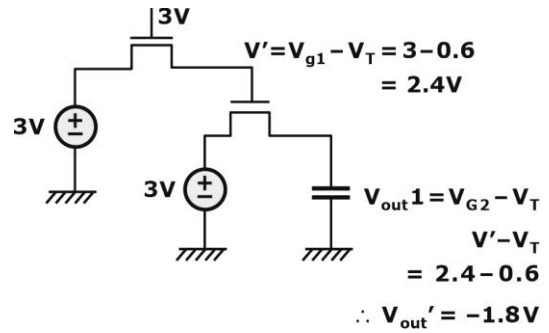
27. In the circuits shown, the threshold voltage of each nMOS transistor is 0.6 V. Ignoring the effect of channel length modulation and body bias, the values of  $V_{out 1}$  and  $V_{out 2}$ , respectively, in volts, are



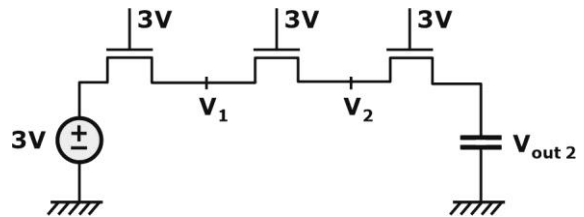
- A. 2.4 and 2.4
- B. 1.8 and 1.2
- C. 1.8 and 2.4
- D. 2.4 and 1.2

Ans. C

Sol. Given  $V_{TN} = 0.6V$ ,  $V_{SB} = 0$  and  $\lambda = 0$   
In figure (i)

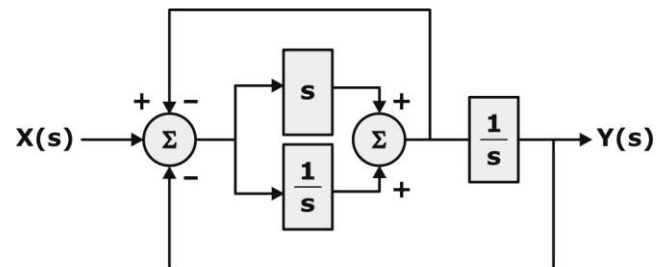


In figure (ii)



Every MOS transistor has same  $V_G = 3V$   
 $\therefore V_1 = V_2 = V_{out 2} = V_G - V_T$   
 $= 3 - 0.6$   
 $\therefore V_{out 2} = 2.4 V$

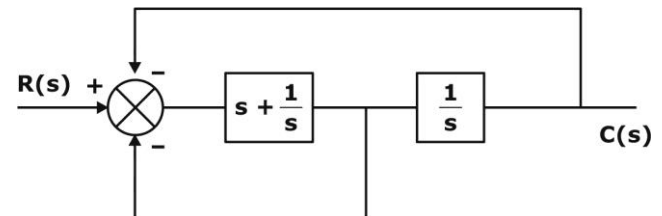
28. The block diagram of a system is illustrated in the figure shown, where  $X(s)$  is the input and  $Y(s)$  is the output. The transfer function  $H(s) = \frac{Y(s)}{X(s)}$  is



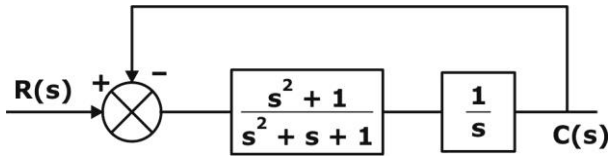
- A.  $H(s) = \frac{s^2 + 1}{2s^2 + 1}$
- B.  $H(s) = \frac{s^2 + 1}{s^3 + 2s^2 + s + 1}$
- C.  $H(s) = \frac{s + 1}{s^2 + s + 1}$
- D.  $H(s) = \frac{s^2 + 1}{s^3 + s^2 + s + 1}$

Ans. B

Sol.



$$\frac{\frac{s^2 + 1}{s}}{1 + \frac{s^2 + 1}{s}} = \frac{s^2 + 1}{s^2 + s + 1}$$



$$TF = \frac{\frac{s^2 + 1}{s(s^2 + s + 1)}}{1 + \frac{s^2 + 1}{s(s^2 + s + 1)}}$$

$$\therefore TF = \frac{s^2 + 1}{s^3 + 2s^2 + s + 1}$$

**29.** A random variable  $X$  takes  $-1$  and  $+1$  with probabilities  $0.2$  and  $0.8$ , respectively. It is transmitted across a channel which adds noise  $N$ , so that the random variable at the channel output is  $Y = X + N$ . The noise  $n$  is independent of  $X$ , and is uniformly distributed over the interval  $[-2, 2]$ . The receiver makes a decision

$$\hat{X} = \begin{cases} -1, & \text{if } Y \leq \theta \\ +1, & \text{if } Y > \theta \end{cases}$$

Where the threshold  $\theta \in [-1, 1]$  is chosen so as to minimize the probability of error  $\Pr[\hat{X} \neq X]$ . The minimum probability of error, rounded off to 1 decimal place, is \_\_\_\_\_.

**Ans.** 0.1

**Sol.**  $P_o(-1 + N > V_{th})$

$$P_o(N > V_{th} + 1) = \int_{V_{th} + 1}^2 \frac{1}{4} dx = \frac{1}{4}[2 - V_{th} - 1] = \frac{1}{4}(1 - V_{th})$$

$$P_1(1 + N < V_{th})$$

$$P_1(N < V_{th} - 1) = \int_{-2}^{V_{th} - 1} \frac{1}{4} dx = \frac{1}{4}[V_{th} - 1 + 2] = \frac{1}{4}(V_{th} + 1)$$

$$P_e = P(0)P_o(N > V_{th} + 1) + P(1)P_1(N < V_{th} - 1)$$

$$P_e = 0.2 \times \frac{1}{4}(1 - V_{th}) + 0.8 \times \frac{1}{4}(V_{th} + 1)$$

$$= 0.05 - 0.5V_{th} + 0.2V_{th} + 0.2$$

$$P_e = 0.25 + 0.15V_{th}$$

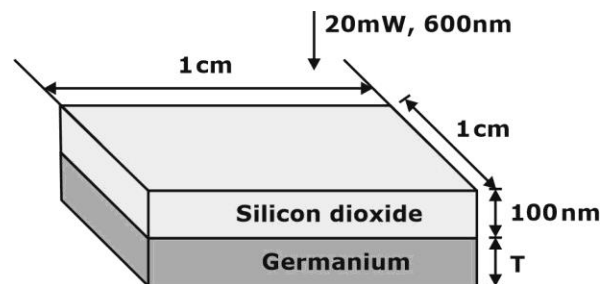
$$\text{For } V_{th} = 0 \rightarrow P_e = 0.25$$

$$\text{For } V_{th} = 1 \rightarrow P_e = 0.4$$

$$\text{For } V_{th} = -1 \rightarrow P_e = 0.1$$

$$\therefore \text{Minimum probability of error} = 0.1$$

**30.** A germanium sample of dimensions  $1 \text{ cm} \times 1 \text{ cm}$  is illuminated with a  $20 \text{ mW}$ ,  $600 \text{ nm}$  laser light source as shown in the figure. The illuminated sample surface has a  $100 \text{ nm}$  of loss-less Silicon dioxide layer that reflects one-fourth of the incident light. From the remaining light, one-third of the power is reflected from the Silicon dioxide-Germanium interface, one-third is absorbed in the Germanium layer, and one-third is transmitted through the other side of the sample. If the absorption coefficient of Germanium at  $600 \text{ nm}$  is  $3 \times 10^4 \text{ cm}^{-1}$  and the bandgap is  $0.66 \text{ eV}$ , the thickness of the Germanium layer, rounded off to 3 decimal places, is \_\_\_\_\_  $\mu\text{m}$ .



**Ans.** 0.231

**Sol.**  $1 - e^{-\alpha x} = 0.5$

$$e^{-\alpha x} = 0.5$$

$$\text{now } \alpha = 3 \times 10^4 \text{ cm}^{-1}$$

$$\therefore x = \frac{-\ln(0.5)}{3 \times 10^4}$$

$$\therefore x = 0.231 \mu\text{m}$$

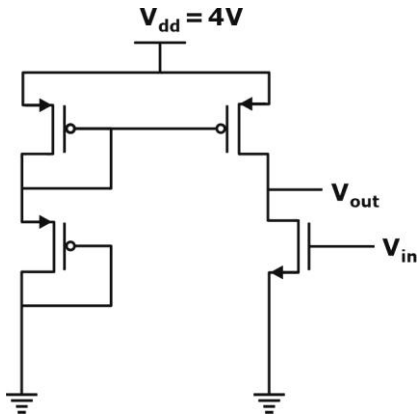
**31.** In the circuit shown, the threshold voltages of the pMOS ( $|V_{tp}|$ ) and nMOS ( $V_{tn}$ ) transistors are both equal to  $1 \text{ V}$ . All the transistors have the same output resistance  $r_{ds}$  of  $6 \text{ M}\Omega$ . The other parameters are listed below.



$$\mu_n C_{ox} = 60 \mu A/V^2; \left(\frac{W}{L}\right)_{nMOS} = 5$$

$$\mu_p C_{ox} = 30 \mu A/V^2; \left(\frac{W}{L}\right)_{pMOS} = 10$$

$\mu_n$  and  $\mu_p$  are the carrier mobilities, and  $C_{ox}$  is the oxide capacitance per unit area. Ignoring the effect of channel length modulation and body bias, the gain of the circuit is \_\_\_\_\_ (rounded off to 1 decimal place).



Ans. -900

Sol.  $I_D = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_{GSP} - |V_{TP}|)^2$

$$= \frac{1}{2} \times 30 \times 10^{-6} \times 10 \times (2-1)^2$$

$$I_D = 150 \mu A$$

Now,  $g_m = \sqrt{2I_D \mu_n C_{ox} \left(\frac{W}{L}\right)_n}$

$$g_m = \sqrt{2 \times 150 \times 10^{-6} \times 60 \times 10^{-6} \times 5}$$

$$\therefore g_m = 300 \times 10^{-6} \text{ s}$$

Now  $A_v = -g_m (r_{ds} || r_{ds})$

$$= -300 \times 10^{-6} \left( (6 \times 10^6) || (6 \times 10^6) \right)$$

$$= -300 \times 10^{-6} \times 3 \times 10^6$$

$$\therefore A_v = -900$$

32. It is desired to find a three-tap causal filter which gives zero signal as an output to an input of the form

$$x[n] = c_1 \exp\left(-\frac{j\pi n}{2}\right) + c_2 \exp\left(\frac{j\pi n}{2}\right),$$

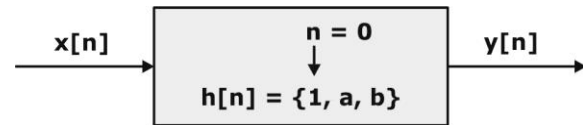
where  $c_1$  and  $c_2$  are arbitrary real numbers. The desired three-tap filter is given by

$$h[0] = 1, h[1] = a, h[2] = b$$

and

$$h[n] = 0 \text{ for } n < 0 \text{ or } n > 2.$$

What are the values of the filter taps  $a$  and  $b$  if the output is  $y[n] = 0$  for all  $n$ , when  $x[n]$  is as given above?



A.  $a = 0, b = -1$

B.  $a = 1, b = 1$

C.  $a = -1, b = 1$

D.  $a = 0, b = 1$

Ans. D

Sol. Given that

$$h(0) = 1, h(1) = a, h(2) = b \text{ and } h(n) = 0 \text{ otherwise}$$

$$\therefore H(e^{j\omega}) = 1 + ae^{-j\omega} + be^{-j2\omega}$$

Now  $y(n) = 0$  for all  $n$

$$\text{Now } x(n) = C_1 e^{\left(\frac{-j\pi n}{2}\right)} + C_2 e^{\left(\frac{j\pi n}{2}\right)}$$

If we consider  $C_1 e^{\left(\frac{-j\pi n}{2}\right)}$  as input then

$$\text{Output} = C_1 \left[ 1 + ae^{+j\frac{\pi}{2}} + be^{-j2\left(\frac{-\pi}{2}\right)} \right]$$

$$\text{Output} = C_1 \left[ 1 + ae^{j\frac{\pi}{2}} + be^{j\pi} \right] \quad \dots(i)$$

If we consider  $C_2 e^{\left(\frac{j\pi n}{2}\right)}$  as input then

$$\text{Output} = C_2 \left[ 1 + ae^{-j\frac{\pi}{2}} + be^{-j2\left(\frac{\pi}{2}\right)} \right]$$

$$= C_2 \left[ 1 + ae^{-j\frac{\pi}{2}} + be^{-j\pi} \right] \quad \dots(ii)$$

Both output (i) and (ii) will be zero if  $a = 0, b = 1$

**33.** Consider a long-channel MOSFET with a channel length 1  $\mu\text{m}$  and width 10  $\mu\text{m}$ . The device parameters are acceptor concentration  $N_A = 5 \times 10^{16} \text{ cm}^{-3}$ , electron mobility  $\mu_n = 800 \text{ cm}^2/\text{V}\cdot\text{s}$ , oxide capacitance/area  $C_{ox} = 3.45 \times 10^{-7} \text{ F/cm}^2$ , threshold voltage  $V_T = 0.7 \text{ V}$ . The drain saturation current ( $I_{Dsat}$ ) for a gate voltage of 5 V is \_\_\_\_\_ mA (rounded off to two decimal places).  
[ $\epsilon_0 = 8.854 \times 10^{-14} \text{ F/cm}$ ,  $\epsilon_{si} = 11.9$ ]

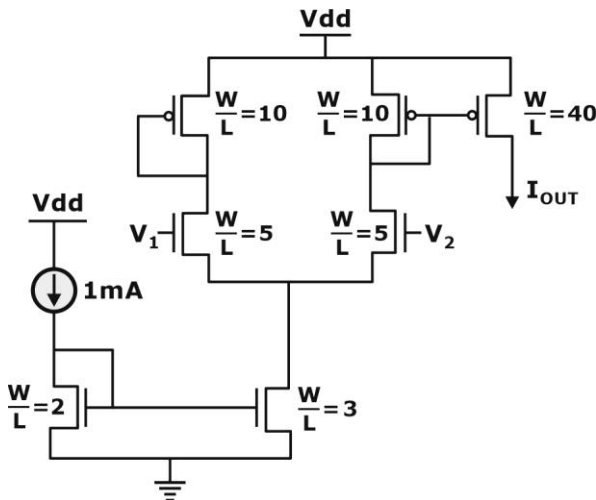
**Ans.** 25.5

**Sol.** 
$$I_D = \frac{\mu_n C_{ox}}{2} \cdot \left(\frac{W}{L}\right) \cdot (V_{gs} - V_T)^2$$

$$= \frac{300 \times 3.45 \times 10^{-7}}{2} \times \left(\frac{10}{1}\right) \times (5 - 0.7)^2$$

$\therefore I_D = 25.5 \text{ mA}$

**34.** In the circuit shown,  $V_1 = 0$  and  $V_2 = V_{dd}$ . The other relevant parameters are mentioned in the figure. Ignoring the effect of channel length modulation and the body effect, the value of  $I_{out}$  is \_\_\_\_\_ mA (rounded off to 1 decimal place).



**Ans.** 6

**Sol.** Current through FET having  $\left(\frac{W}{L}\right) = 3$  will be  $I_1$

$$\therefore I_1 = \frac{(W/L)_2}{(W/L)_1} \times 1\text{mA}$$

$$\therefore I_1 = \frac{3}{2} \text{mA}$$

Now,

$$I_{out} = \frac{(W/L)_4}{(W/L)_3} \times I_1$$

$$= \frac{40}{10} \times \frac{3}{2} \text{mA}$$

$\therefore I_{out} = 6\text{mA}$

**35.** The quantum efficiency ( $\eta$ ) and responsivity ( $R$ ) at wavelength  $\lambda$  (in  $\mu\text{m}$ ) in a p-i-n photodetector are related by

A.  $R = \frac{1.24 \times \lambda}{\eta}$       B.  $R = \frac{\eta \times \lambda}{1.24}$

C.  $R = \frac{1.24}{\eta \times \lambda}$       D.  $R = \frac{\lambda}{\eta \times 1.24}$

**Ans.** B

**Sol.** Quantum Efficiency  $\eta = \frac{R_e}{R_p}$

$R_e$  = Corresponding Electron Rate (electrons/sec)  
 $R_p$  = Incident Photon Rate (Photons/sec)

$$R_e = \frac{I_p}{q}, R_p = \frac{P_{in}}{h\nu}, R = \frac{I_p}{P_{in}}$$

$$\eta = \frac{I_p/q}{P_{in}/h\nu}$$

Now

$$\eta = \frac{I_p/q}{P_{in}/h\nu} = \frac{I_p h\nu}{q P_{in}} = \frac{h\nu R}{q}$$

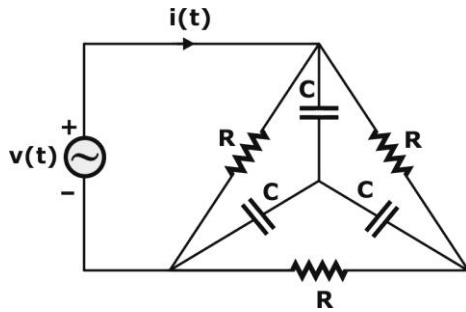
So

$$\Rightarrow R = \frac{q\eta}{h\nu} = \frac{q\eta\lambda}{hc} = \eta \times \left(\frac{q}{hc}\right)$$

$q = 1.6 \times 10^{-19} \text{ C}$ ,  $h = 6.63 \times 10^{-34} \text{ Js}$ ,  $C = 3 \times 10^8 \text{ m/s}$

$$R = \frac{\eta\lambda}{1.24}$$

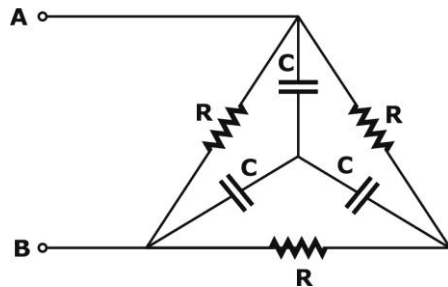
**36.** In the circuit shown, if  $v(t) = 2 \sin(1000t)$  volts,  $R = 1 \text{ k}\Omega$  and  $C = 1 \mu\text{F}$ , then the steady-state current  $i(t)$ , in milliamperes (mA), is



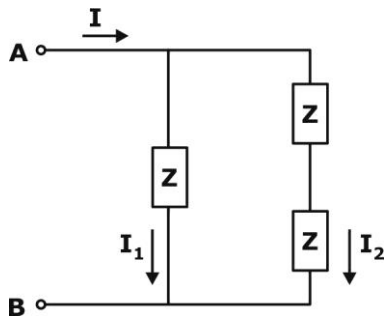
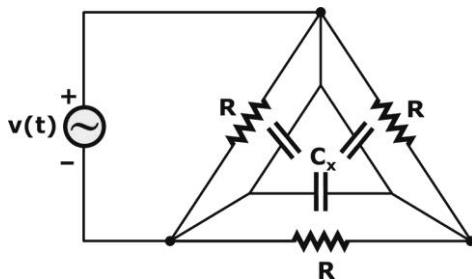
- A.  $\sin(1000 t) + \cos(1000 t)$
- B.  $2 \sin(1000 t) + 2 \cos(1000 t)$
- C.  $3 \sin(1000 t) + \cos(1000 t)$
- D.  $\sin(1000 t) + 3 \cos(1000 t)$

**Ans.** C

**Sol.**



Performing star to delta conversion



Where  $Z_1 = 2 \left[ \frac{R}{1 + \frac{jWCR}{3}} \right]$

$$\therefore Z_{eq} = Z_1 \parallel \left( \frac{R}{1 + \frac{jWCR}{3}} \right)$$

$$\therefore Z_{eq} = \frac{2}{3} \left( \frac{R}{1 + \frac{jWCR}{3}} \right)$$

Now  $R = 1k\Omega$ ,  $C = 1\mu F$  and  $W = 1000$  rad/sec

$$\therefore Z_{eq} = 0.66 - 0.2178j$$

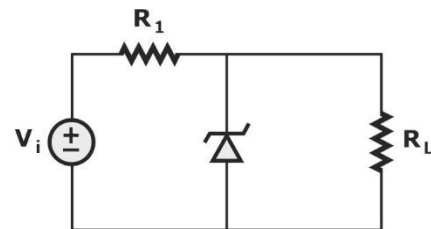
$$\therefore I = \frac{V}{Z_{eq}} = \frac{2 \sin(1000t)}{0.66 - 0.2178j}$$

$$= \frac{2}{\sqrt{0.66^2 + 0.2178^2}} \cdot \sin \left( 1000t - \tan^{-1} \left( \frac{1}{3} \right) \right)$$

$$= 3.16 \sin(1000t + 18.43^\circ)$$

$$\therefore I \approx 3 \sin(1000t) + \cos(1000t)$$

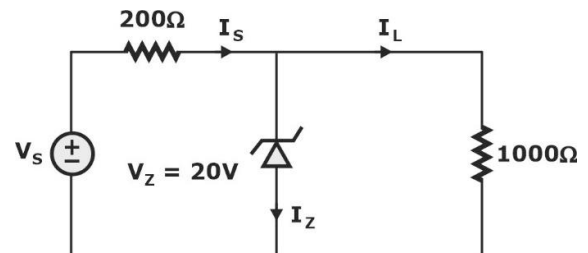
- 37.** In the circuit shown, the breakdown voltage and the maximum current of the Zener diode are 20 V and 60 mA, respectively. The values of  $R_1$  and  $R_L$  are 200  $\Omega$  and 1 k $\Omega$ , respectively. What is the range of  $V_i$  that will maintain the Zener diode in the 'on' state?



- A. 18 V to 24 V
- B. 20 V to 28 V
- C. 24 V to 36 V
- D. 22 V to 34 V

**Ans.** C

**Sol.**



$$I_{Zmax} = 60 \text{ mA}$$

$$I_L = \frac{20}{1000} = 20 \text{ mA}$$

As  $I_{Zmin}$  not given,

$$I_{Zmin} = 0 \text{ mA}$$

$$\text{Now } I_S = I_Z + I_L$$

$$\therefore I_{Smin} = I_{Zmin} + I_L = 0 + 20 \text{ mA}$$

$$\therefore I_{Smin} = 20 \text{ mA}$$

$$\text{Now } I_S = \frac{V_S - V_Z}{200}$$

$$\therefore 20\text{mA} = \frac{V_S - 20}{200}$$

$$\therefore V_S = 24\text{V}$$

$$\text{Now } I_{Smax} = I_{Zmax} + I_L = 60 + 20$$

$$I_{Smax} = 80 \text{ mA}$$

$$\therefore I_S = \frac{V_S - V_Z}{200}$$

$$\therefore 80\text{mA} = \frac{V_S - 20}{200}$$

$$\therefore V_S = 36 \text{ V}$$

- 38.** Consider a differentiable function  $f(x)$  on the set of real numbers such that  $f(-1) = 0$  and  $|f'(x)| \leq 2$ . Given these conditions, which one of the following inequalities is necessarily true for all  $x \in [-2, 2]$ ?

A.  $f(x) \leq \frac{1}{2}|x|$                       B.  $f(x) \leq 2|x+1|$

C.  $f(x) \leq \frac{1}{2}|x+1|$                       D.  $f(x) \leq 2|x|$

**Ans.** B

**Sol.**  $f(-1) = 0$

So only option (B) and (C) are possible

Let's try option (B)

$$f(x) = 2|x+1|$$

$$\therefore f(x) = \begin{cases} 2(x+1) & \text{for } x+1 > 0 \\ -2(x+1) & \text{for } x+1 < 0 \end{cases}$$

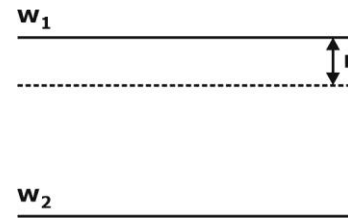
$$\therefore f(x) = \begin{cases} 2(x+1) & \text{for } x > -1 \\ -2(x+1) & \text{for } x < -1 \end{cases}$$

$$\therefore f'(x) = \begin{cases} 2 & \text{for } x > -1 \\ -2 & \text{for } x < -1 \end{cases}$$

$$\therefore |f'(x)| \leq 2$$

$\therefore$  option (B) is correct.

- 39.** Two identical copper wires W1 and W2, placed in parallel as shown in the figure, carry currents  $I$  and  $2I$ , respectively, in opposite directions. If the two wires are separated by a distance of  $4r$ , then the magnitude of the magnetic field  $\vec{B}$  between the wires at a distance  $r$  from W1 is



- A.  $\frac{\mu_0 I}{6\pi r}$                       B.  $\frac{\mu_0^2 I^2}{2\pi r^2}$   
 C.  $\frac{5\mu_0 I}{6\pi r}$                       D.  $\frac{6\mu_0 I}{5\pi r}$

**Ans.** C

**Sol.**  $H = \frac{I}{2\pi\rho}ap$

For wire  $\omega_1$

$$H_1 = \frac{I}{2\pi r}$$

For wire  $\omega_2$

$$H_2 = \frac{2I}{2\pi 3r}$$

Magnetic field will be circular and can be find out by right hand rule

Both fields will add at middle region

$\therefore$  at dotted line

$$H = H_1 + H_2$$

$$\therefore H = \frac{5I}{6\pi r}$$

Now  $B = \mu_0 H$

$$B = \frac{\mu_0 5I}{6\pi r}$$

- 40.** The dispersion equation of a waveguide, which relates the wavenumber  $k$  to the frequency  $\omega$ , is

$$k(\omega) = (1/c)\sqrt{\omega^2 - \omega_0^2}$$

where the speed of light  $c = 3 \times 10^8$  m/s, and  $\omega_0$  is a constant. If the group velocity is  $2 \times 10^8$  m/s, then the phase velocity is

- A.  $1.5 \times 10^8$  m/s      B.  $2 \times 10^8$  m/s  
 C.  $4.5 \times 10^8$  m/s      D.  $3 \times 10^8$  m/s

**Ans.** C

**Sol.**  $V_g = \frac{d\omega}{d\beta}$

Now,

$$\frac{d\beta}{d\omega} = \frac{dk(\omega)}{d\omega} = \frac{d}{d\omega} \cdot \frac{1}{c} \sqrt{\omega^2 - \omega_0^2} = \frac{1}{2c\sqrt{\omega^2 - \omega_0^2}} \times 2\omega$$

$$\boxed{\frac{d\beta}{d\omega} = \frac{\omega}{c\sqrt{\omega^2 - \omega_0^2}}}$$

$$V_g = \frac{1}{\frac{d\beta}{d\omega}} = 2 \times 10^8 \Rightarrow \frac{c\sqrt{\omega^2 - \omega_0^2}}{\omega} = 2 \times 10^8$$

$$\Rightarrow \boxed{\sqrt{\omega^2 - \omega_0^2} = \frac{2\omega}{3}}$$

$$\text{Now, } V_p = \frac{\omega}{\beta} = \frac{\omega}{k} = \frac{\omega}{\frac{1}{c}\sqrt{\omega^2 - \omega_0^2}} = \frac{\omega c}{\frac{2\omega}{3}} = \frac{3c}{2}$$

$$V_p = \frac{3}{2} \times 3 \times 10^8 = 4.5 \times 10^8 \text{ m/s}$$

$$\boxed{V_p = 4.5 \times 10^8 \text{ m/s}}$$

**41.** Consider a causal second-order system with the transfer function

$$G(s) = \frac{1}{1 + 2s + s^2}$$

with a unit-step  $R(s) = \frac{1}{s}$  as an input. Let  $C(s)$  be the corresponding output. The time taken by the system output  $c(t)$  to reach 94% of its steady-state value  $\lim_{t \rightarrow \infty} c(t)$  rounded off to two decimal places, is

- A. 4.50      B. 2.81  
 C. 5.25      D. 3.89

**Ans.** A

**Sol.**  $G(s) = \frac{C(s)}{R(s)}$

$$\therefore C(s) = G(s) \cdot R(s)$$

$$= \frac{1}{s(s^2 + 2s + 1)}$$

$$\therefore C(s) = \frac{1}{s(s+1)^2}$$

$$\therefore C(s) = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+1)^2}$$

$$\therefore A(s+1)^2 + Bs(s+1) + Cs = 1$$

$$\therefore As^2 + 2As + A + Bs^2 + Bs + Cs = 1$$

$$\therefore A + B = 0$$

$$\therefore 2A + B + C = 0$$

$$\therefore A = 1$$

$$\text{So } B = -1$$

$$\text{And } C = -1$$

$$\therefore C(s) = \frac{1}{s} + \frac{-1}{s+1} + \frac{-1}{(s+1)^2}$$

$$\therefore C(t) = (1 - e^{-t} - te^{-t}) u(t)$$

At  $t \rightarrow \infty$  steady state will occur

$$\therefore C(\infty) = 1$$

Now we are asked to find time at which 94% of the steady state value reached.

$$\therefore C(t) = 1 - e^{-t} - te^{-t} = 0.94$$

$$\therefore e^{-t} + te^{-t} = 0.06$$

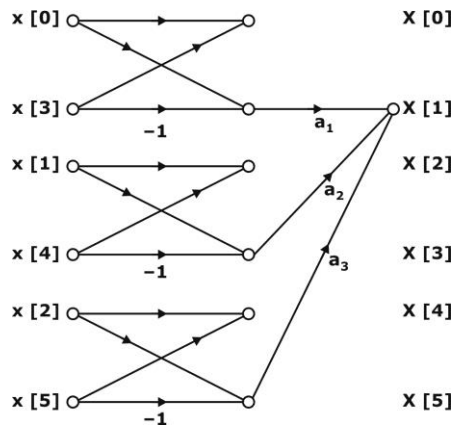
$$\therefore e^{-t}(1 + t) = 0.06$$

Now from the given options try all option you will get  $t = 4.50$  sec.

**42.** Consider a six-point decimation-in-time Fast Fourier Transform (FFT) algorithm, for which the signal-flow graph corresponding to  $X[1]$  is shown in the figure.

Let  $W_6 = \exp\left(-\frac{j2\pi}{6}\right)$ . In the figure, what should be

the values of the coefficients  $a_1, a_2, a_3$  in terms of  $W_6$  so that  $X[1]$  is obtained correctly?



- A.  $a_1 = 1, a_2 = W_6, a_3 = W_6^2$
- B.  $a_1 = -1, a_2 = W_6, a_3 = W_6^2$
- C.  $a_1 = 1, a_2 = W_6^2, a_3 = W_6$
- D.  $a_1 = -1, a_2 = W_6^2, a_3 = W_6$

Ans. A

Sol.  $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$

We are obtaining  $X(1)$  correctly

$\therefore k = 1$

$\therefore x(1) = x(0) + x(1)W_6^1 + x(2)W_6^2 + x(3)W_6^3 + x(4)W_6^4 + x(5)W_6^5$

We know that

$W_N^{k+\frac{N}{2}} = -W_N^k$

$\therefore W_6^3 = -W_6^0 = -1$

$W_6^4 = -W_6^1$

$W_6^5 = -W_6^2$

$\therefore$  comparing with given graph

$a_1 = 1, a_2 = W_6, a_3 = W_6^2$

43. Let the state-space representation of an LTI system be  $\dot{x}(t) = Ax(t) + Bu(t), y(t) = C x(t) + d u(t)$  where A, B, C are matrices, d is a scalar, u(t) is the input to the system, and y(t) is its output. Let  $B = [0 \ 0 \ 1]^T$  and  $d = 0$ . Which one of the following

options for A and C will ensure that the transfer function of this LTI system is

$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1}$  ?

A.  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$  and  $C = [1 \ 0 \ 0]$

B.  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$  and  $C = [0 \ 0 \ 1]$

C.  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$  and  $C = [0 \ 0 \ 1]$

D.  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$  and  $C = [1 \ 0 \ 0]$

Ans. A

Sol.  $H(s) = \frac{1}{s^2 + 3s^2 + 2s + 1}$

$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [u]$

&  $[y] = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0][u]$

$\therefore A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$  and  $C = [1 \ 0 \ 0]$

44. A CMOS inverter, designed to have a mid-point voltage  $V_I$  equal to half of  $V_{dd}$ , as shown in the figure, has the following parameters :

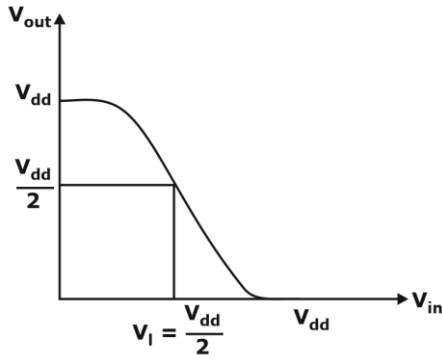
$V_{dd} = 3 \text{ V}$

$\mu_n C_{ox} = 100 \mu\text{A}/\text{V}^2 ; V_{tn} = 0.7 \text{ V}$  for nMOS

$\mu_n C_{ox} = 40 \mu\text{A}/\text{V}^2 ; |V_{tp}| = 0.9 \text{ V}$  for pMOS

The ratio of  $\left(\frac{W}{L}\right)_n$  to  $\left(\frac{W}{L}\right)_p$  is equal to \_\_\_\_\_

(rounded off to 3 decimal places).



**Ans.** 0.225

**Sol.** Same current will flow through both NMOS & PMOS

$$\therefore I_{D1} = I_{D2}$$

$$\begin{aligned} \therefore \frac{\mu_n C_{OX}}{2} \cdot \left(\frac{\omega}{L}\right)_N (V_{G_{SN}} - V_{TN})^2 \\ = \frac{\mu_p C_{OX}}{2} \cdot \left(\frac{\omega}{L}\right)_p (V_{G_{SP}} - |V_{TP}|)^2 \end{aligned}$$

$$\begin{aligned} \therefore 100 \times \left(\frac{\omega}{L}\right)_N \cdot (1.5 - 0.7)^2 \\ = 400 \times \left(\frac{\omega}{L}\right)_p (1.5 - 0.9)^2 \end{aligned}$$

$$\therefore \frac{(\omega/L)_N}{(\omega/L)_p} = \frac{9}{16} \times \frac{4}{10}$$

$$= 0.225$$

$$\left( \because V_{G_{SN}} = V_{G_{SP}} = \frac{V_{dd}}{2} = 1.5V \right)$$

**45.** A rectangular waveguide of width  $w$  and height  $h$  has cut-off frequencies for  $TE_{10}$  and  $TE_{11}$  modes in the ratio 1 : 2. The aspect ratio  $w/h$ , rounded off to two decimal places, is \_\_\_\_\_.

**Ans.** 1.732

**Sol.**

$$f_c = \frac{V}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\text{For } TE_{10}, \quad m=1, \quad n=0$$

$$f_{c1} = \frac{V}{2} \sqrt{\left(\frac{1}{a}\right)^2} = \frac{V}{2a}$$

$$\text{For } TE_{11}, \quad m=1, \quad n=1$$

$$f_{c2} = \frac{V}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$$

$$\text{Given } \frac{f_{c1}}{f_{c2}} = \frac{1}{2}$$

$$\frac{V/2a}{\frac{V}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \frac{1}{2}$$

$$\frac{\frac{1}{a}}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \frac{1}{2} \Rightarrow \frac{\frac{1}{a}}{\frac{1}{ab} \sqrt{a^2 + b^2}} = \frac{1}{2}$$

$$\frac{b}{\sqrt{a^2 + b^2}} = \frac{1}{2}$$

$$\Rightarrow 4b^2 = a^2 + b^2$$

$$\Rightarrow 3b^2 = a^2$$

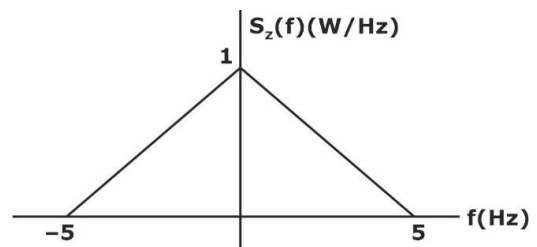
$$\Rightarrow \frac{b^2}{a^2} = \frac{1}{3}$$

$$\Rightarrow \frac{b}{a} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{a}{b} = \sqrt{3}$$

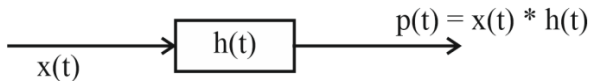
$$\frac{\text{width}}{\text{height}} = \sqrt{3} = 1.732$$

**46.** Let a random process  $Y(t)$  be described as  $Y(t) = h(t) * X(t) + Z(t)$ , where  $X(t)$  is a white noise process with power spectral density  $S_X(f) = 5 \text{ W/Hz}$ . The filter  $h(t)$  has a magnitude response given by  $|H(f)| = 0.5$  for  $-5 \leq f \leq 5$ , and zero elsewhere.  $Z(t)$  is a stationary random process, uncorrelated with  $X(t)$ , with power spectral density as shown in the figure. The power in  $Y(t)$ , in watts, is equal to \_\_\_\_\_ W (rounded off to two decimal places).



**Ans.** 17.5

**Sol.**



and  $y(t) = z(t) + p(t)$

$\therefore R_{yy}(\tau) = R_{zz}(\tau) + R_{pp}(\tau) + R_{pz}(\tau) + R_{zp}(\tau)$

now  $x(t)$  &  $z(t)$  are uncorrelated.

$\therefore R_{pz}(\tau) = R_{zp}(\tau) = 0$

$\therefore R_{yy}(\tau) = R_{zz}(\tau) + R_{pp}(\tau)$

So the power spectral relation can be given by

Fourier transform of the above relation.

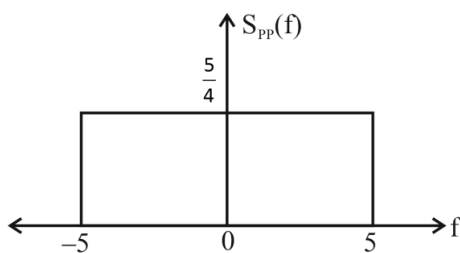
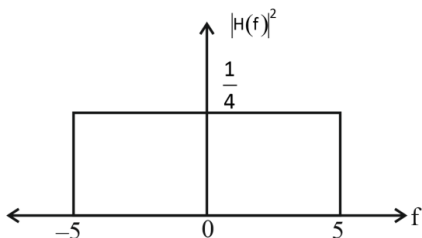
$\therefore S_{yy}(f) = S_{zz}(f) + S_{pp}(f)$

now power of  $y(t) =$

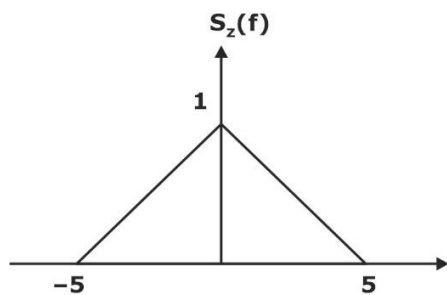
$$\int_{-\infty}^{\infty} s_{yy}(f) df$$

$$\therefore P = \int_{-\infty}^{\infty} s_{zz}(f) df + \int_{-\infty}^{\infty} s_{pp}(f) df$$

now  $S_{pp}(f) = |H(f)|^2 \times S_{xx}(f)$



&



$$\therefore P = \frac{5}{4} \times 10 + \frac{1}{2} \times 10 \times 1$$

$$\therefore P = 17.5 \text{ watt}$$

- 47.** Let  $h[n]$  be a length-7 discrete-time finite impulse response filter, given by  $h[0] = 4, h[1] = 3, h[2] = 2, h[3] = 1, h[-1] = -3, h[-2] = -2, h[-3] = -1$ , and  $h[n]$  is zero for  $[n] \geq 4$ . A length-3 finite impulse response approximation  $g[n]$  of  $h[n]$  has to be obtained such that

$$E(h, g) = \int_{-\pi}^{\pi} |H(e^{j\omega}) - G(e^{j\omega})|^2 d\omega$$

is minimized, where  $H(e^{j\omega})$  and  $G(e^{j\omega})$  are the discrete-time Fourier transforms of  $h[n]$  and  $g[n]$ , respectively. For the filter that minimizes  $E(h, g)$ , the value of  $10g[-1] + g[1]$ , rounded off to 2 decimal places, is \_\_\_\_\_.

**Ans.** -27

**Sol.** For the minimization of the energy in the error signal there are different approaches like, Prony's method, Pade approximation. As  $g(n)$  has three samples.

Consider them as  $g(-1), g(0), g(1)$  we can minimise  $E(h, g)$  by making  $h(n) = g(n)$  using rectangular window and Parseval's there of OTFT.

Based on which  $10g(-1) + g(1) = 10(-3) + 3 = -27$

- 48.** In an ideal pn junction with an ideality factor of 1 at  $T = 300 \text{ K}$  the magnitude of the reverse-bias voltage required to reach 75% of its reverse saturation current, rounded off to 2 decimal places, is \_\_\_\_\_ mV.

$[k = 1.38 \times 10^{-23} \text{ JK}^{-1}, h = 6.625 \times 10^{-34} \text{ J-s}, q = 1.602 \times 10^{-19} \text{ C}]$

**Ans.** 35.87

**Sol.**  $I_f = 0.75 I_s$

$\therefore$  Forward current  $= I_D = -0.75 I_s$

$\therefore I_s(e^{V_0/nVT} - 1) = -0.75 I_s$

Now Take  $n = 1$

$\therefore e^{V_0/VT} = 0.25$

$\therefore V_D = VT \ln(0.25)$

$\therefore V_R = -VT \ln(0.25)$



$$= -\frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \times -1.386$$

$$\therefore V_R = 35.87 \text{ mv}$$

49. Consider the homogeneous ordinary differential equation

$$x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 3y = 0, x > 0$$

with  $y(x)$  as a general solution. Given that  $y(1) = 1$  and  $y(2) = 14$  the value of  $y(1.5)$ , rounded off to two decimal places, is \_\_\_\_\_.

Ans. 5.25

- Sol. Given differential equation is of Cauchy - Euler differential equation type.

$$\text{So let } x = e^z \quad \therefore z = \ln x$$

The differential equation can be written as,

$$D(D-1) - 3D + 3 = 0$$

$$\therefore D^2 - 4D + 3 = 0$$

$$\therefore D = 1, 3$$

$$\therefore y = C_1 e^z + C_2 e^{3z}$$

$$\therefore y = C_1 x + C_2 x^3$$

$$\text{Now } y(1) = 1$$

$$\therefore C_1 + C_2 = 1 \quad \dots(i)$$

$$\text{And } y(2) = 14$$

$$\therefore 2C_1 + 8C_2 = 14 \quad \dots(ii)$$

From (i) and (ii)

$$C_1 = -1, C_2 = 2$$

$$\therefore y = -x + 2x^3$$

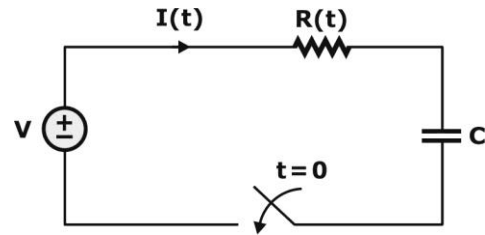
$$\therefore y(1.5) = -1.5 + 2(1.5)^3$$

$$\therefore y(1.5) = 5.25$$

50. The RC circuit shown below has a variable resistance  $R(t)$  given by the following expression :

$$R(t) = R_0 \left( 1 - \frac{t}{T} \right) \text{ for } 0 \leq t < T$$

where  $R_0 = 1 \Omega$ , and  $C = 1 \text{ F}$ . We are also given that  $T = 3 R_0 C$  and the source voltage is  $V_s = 1 \text{ V}$ . If the current at time  $t = 0$  is 1 A, then the current  $I(t)$ , in amperes, at time  $t = T/2$  is \_\_\_\_\_ (rounded off to 2 decimal places).



Ans. 0.1

Sol. We know that,

$$I_C(t) = C \frac{dV_C(t)}{dt}$$

And capacitor will be charged by the following equation

$$V_C(t) = V_s(1 - e^{-t/\tau})$$

$$I_C(t) = C \cdot \frac{d}{dt} [V_s(1 - e^{-t/\tau})]$$

$$\therefore I_C(t) = \frac{V_s}{R(t)} e^{-t/R(t)C}$$

$$\text{Given } R(t) = R_0 \left[ 1 - \frac{t}{T} \right]$$

Now  $R_0 = 1$  and  $C=1$

$$\therefore T = 3R_0C = 3$$

$$\therefore R(t) = \left[ 1 - \frac{t}{3} \right]$$

$$\& I_C(t) = \frac{1}{\left( 1 - \frac{t}{3} \right)} \times e^{\left( \frac{-t}{1 - \frac{t}{3}} \right)}$$

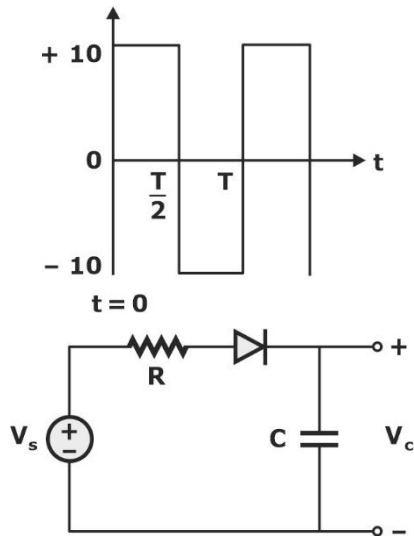
$$\text{At } t = \frac{T}{2} = \frac{3}{2} \text{ sec}$$

$$I_C(t) = 2e^{-3}$$

$$= 0.099$$

$$I_C(t) \approx 0.1 \text{ mA}$$

51. In the circuit shown,  $V_s$  is a 10 V square wave of period,  $T = 4 \text{ ms}$  with  $R = 500 \Omega$  and  $C = 10 \mu\text{F}$ . The capacitor is initially uncharged at  $t = 0$ , and the diode is assumed to be ideal. The voltage across the capacitor ( $V_c$ ) at 3 ms is equal to \_\_\_\_\_ volts (rounded off to one decimal place).



**Ans.** 3.3

**Sol.**  $V_s = 10\text{ V}$

Voltage across capacitor will be

$$V_C(t) = 10(1 - e^{-t/RC})$$

$$R_C = 500 \times 10 \times 10^{-6} = 5 \times 10^{-3} \text{ sec}$$

$$\text{At } t = 2 \text{ ms} = 2 \times 10^{-3} \text{ sec}$$

$$V_C(2 \text{ ms}) = 10 \left( 1 - e^{-\frac{2}{5}} \right)$$

$$V_C(2 \text{ ms}) = 3.3 \text{ V}$$

For  $\frac{T}{2}$  to  $T$  diode will be off so capacitor will not

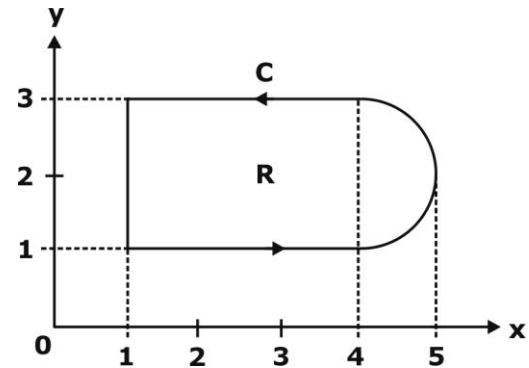
charge further

$$\therefore V_C(3 \text{ msec}) = 3.3\text{V}$$

**52.** Consider the line integral

$$\int_C (x dy - y dx)$$

the integral being taken in a counterclock-wise direction over the closed curve C that forms the boundary of the region R shown in figure below. The region R is the area enclosed by the union of a  $2 \times 3$  rectangle and a semi-circle of radius 1. The line integral evaluates to



- A.  $16 + 2\pi$
- B.  $8 + \pi$
- C.  $12 + 8\pi$
- D.  $6 + \pi/2$

**Ans.** C

**Sol.** By greens theorem

$$\int_C x dy - y dx = \iint_R \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy$$

$$\int_C (x dy - y dx) = \iint_R (1 + 1) dx dy$$

$$2 \iint_R dx dy$$

$$\iint_R dx dy = \text{area of the region}$$

$$= \left[ 2 \times 3 + \frac{\pi(1)^2}{2} \right]$$

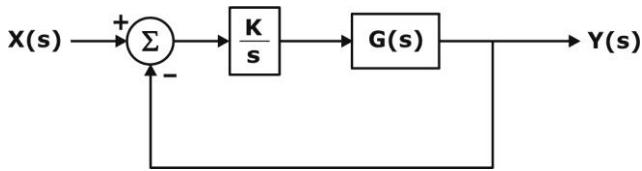
$$\left[ 6 + \frac{\pi}{2} \right]$$

$$\therefore \int_C (x dy - y dx) = 12 + \pi$$

**53.** Consider a unity feedback system, as in the figure shown, with an integral compensator and open-loop transfer function

$$G(s) = \frac{1}{s^2 + 3s + 2}$$

Where  $K > 0$ . The positive value of K for which there are exactly two poles of the unity feedback system on the  $j\omega$  axis is equal to \_\_\_\_\_ (rounded off to two decimal places).



**Ans.** 6

**Sol.** Overall  $G_C(s) = \frac{K}{s(s^2 + 3s + 2)}$

$\therefore q(s) = s^3 + 3s^2 + 2s + k = 0$

$$\begin{array}{r|l} s^3 & 1 & 2 \\ s^2 & 3 & k \\ s^1 & \frac{6-k}{3} & \\ s^0 & k & \end{array}$$

Auxiliary equation is  $3s^2 + k = 0$

And for roots on imaginary axis  $s^1$  row = 0

$\therefore \frac{6-k}{3} = 0$

$\therefore k = 6$

**54.** A voice signal  $m(t)$  is in the frequency range 5 kHz to 15 kHz. The signal is amplitude-modulated to generate an AM signal  $f(t) = A(1 + m(t)) \cos 2\pi f_c t$ , where  $f_c = 600$  kHz. The AM signal  $f(t)$  is to be digitized and archived. This is done by first sampling  $f(t)$  at 1.2 times the Nyquist frequency, and then quantizing each sample using a 256-level quantizer. Finally, each quantized sample is binary coded using  $K$  bits, where  $K$  is the minimum number of bits required for the encoding. The rate, in Megabits per second (rounded off to 2 decimal places), of the resulting stream of coded bits is \_\_\_\_\_ Mbps.

**Ans.** 0.59

**Sol.**  $m(t)$  has frequency range 5 kHz to 15 kHz

Now it is amplitude modulated

$f(t) = A(1 + m(t)) \cos 2\pi f_c t$  where  $f_c = 600$  kHz

$\therefore$  AM signal will have highest frequency =  $f_c + f_m$  (max)

=  $600 + 15 = 615$  kHz

And AM signal will have lowest frequency =  $f_c - f_m$  (max)

=  $600 - 15 = 585$  kHz

It is a band pass signal so we use bandpass sampling

$f_s = 1.2 \times \frac{2f_H}{k}$

$K = \frac{f_H}{f_H - f_L} = \frac{615}{615 - 585}$

$K = 20.5$

We select  $K = 20$

$\therefore f_s = 1.2 \times \frac{2 \times 615}{20}$

$\therefore f_s = 73.8$  kHz

Now  $L = 256$

And  $2^n = L = 256$

$\therefore n = 8$

Bitrate =  $R_b = n f_s$

$\therefore R_b = 8 \times 73.8 \times 10^3$

$\therefore R_b = 0.59$  Mbps

**55.** A single bit, equally likely to be 0 and 1, is to be sent across an additive white Gaussian noise (AWGN) channel with power spectral density  $N_0/2$ . Binary signaling, with  $0 \rightarrow p(t)$  and  $1 \rightarrow q(t)$ , is used for the transmission, along with an optimal receiver that minimizes the bit-error probability.

Let  $\phi_1(t), \phi_2(t)$  form an orthonormal signal set.

If we choose  $p(t) = \phi_1(t)$  and  $q(t) = -\phi_1(t)$ , we would obtain a certain bit-error probability  $P_b$ .

If we keep  $p(t) = \phi_1(t)$ , but take  $q(t) = \sqrt{E} \phi_2(t)$ , for what value of  $E$  would we obtain the same bit-error probability  $P_b$ ?

- A. 3
- B. 2
- C. 0
- D. 1

**Ans.** A

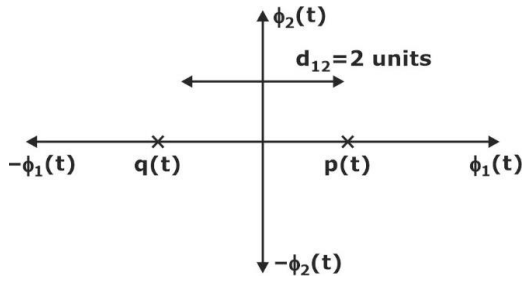
**Sol.** 0 is represented by  $p(t)$

And 1 is represented by  $q(t)$

And  $\psi_1(t)$  and  $\psi_2(t)$  are orthogonal signal set

(i)  $p(t) = \psi_1(t)$  and  $q(t) = -\psi_1(t)$

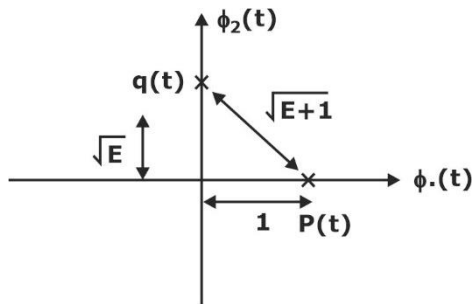
So signal space diagram will be,



$\therefore d_{min_1} = 2$

(ii)  $p(t) = \psi_1(t)$  and  $q(t) = \sqrt{E}\psi_2(t)$

So signal space diagram will be



$\therefore d_{min_2} = \sqrt{E+1}$

Now bit error probability is same in both cases

$\therefore d_{min_1} = d_{min_2}$

$\sqrt{1+E} = 2$

$\therefore E = 3$

\*\*\*\*

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