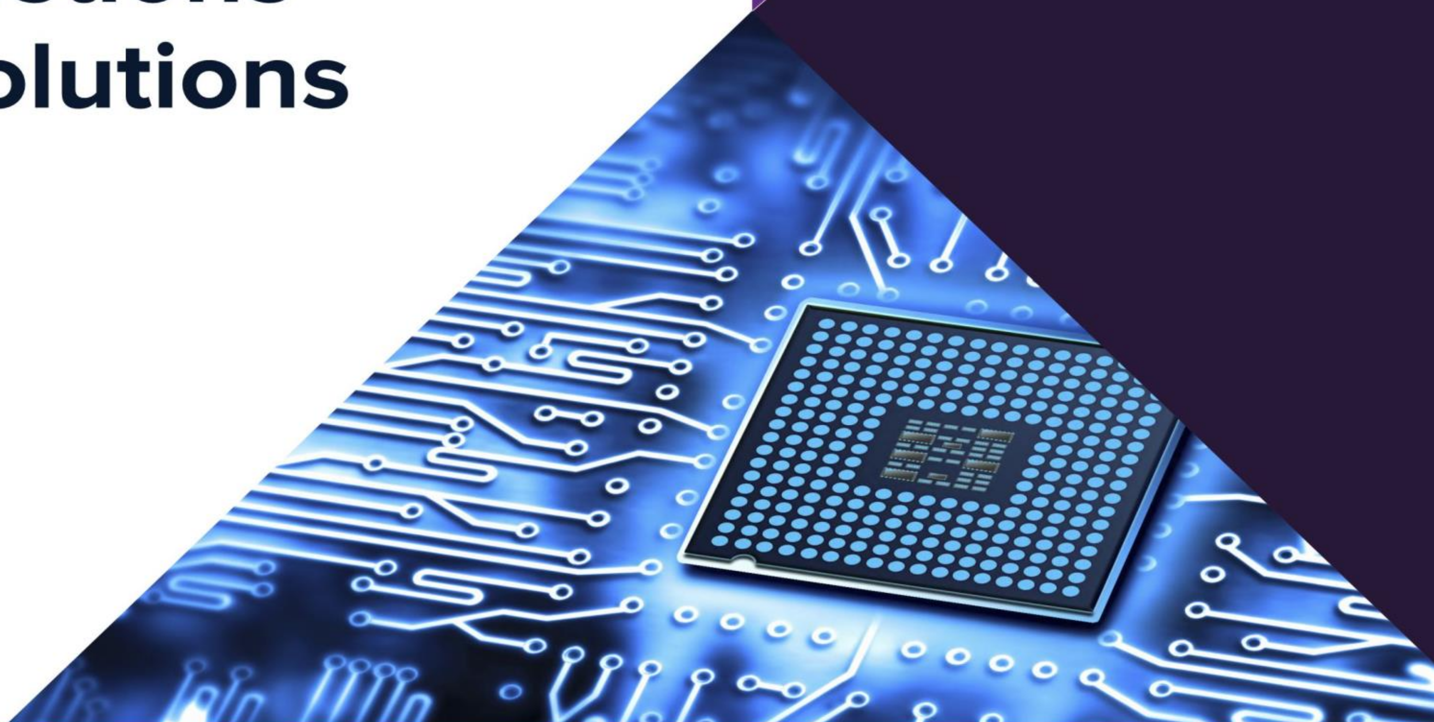
GATE 2018

Electronics & Communication Engineering

Questions & Solutions



1. A solar cell of area 1.0 cm^2 , operating at 1.0 sun intensity, has a short circuit current of 20 mA, and an open circuit voltage of 0.65 V. Assuming room temperature operation and thermal equivalent voltage of 26 mV, the open circuit voltage (in volts, correct to two decimal places) at 0.2 sun intensity is __________.

Ans. 0.608

Sol. For solar cell open circuit voltage is given by,

$$V_{OC} = V_T \ln\left(\frac{I_{SC}}{I_o}\right)$$

Since, the Current through the solar cell is directly proportional to intensity of light,

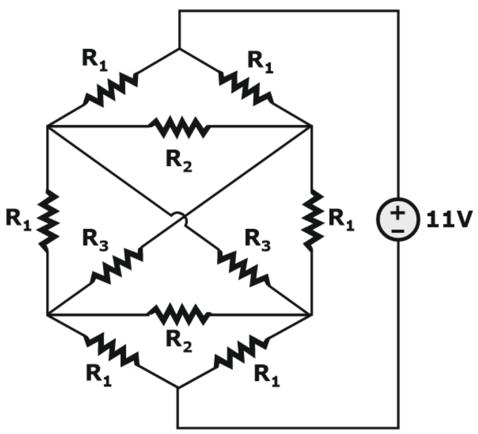
$$V_{OC2} - V_{OC1} = V_T \ln\left(\frac{I_{SC2}}{I_{SC1}}\right) = V_T \ln\left(\frac{0.20}{1.0}\right)$$

$$V_{OC2} = V_{OC1} - 0.026 \ln(5)$$

$$= 0.65 - 0.041845 = 0.608 \text{ V}$$

V_{OC2} = 0.608 V.

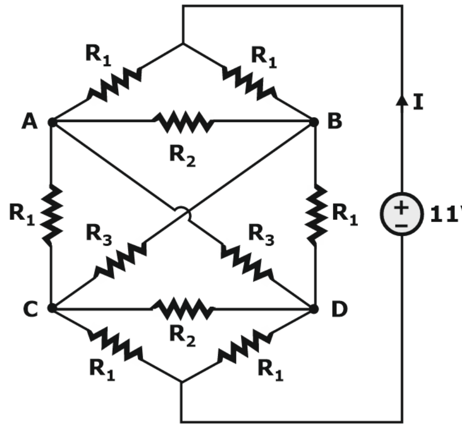
2. Consider the network shown below with R_1 = 1Ω, R_2 = 2Ω, and R_3 = 3Ω. The network is connected to a constant voltage source of 11 V.



The magnitude of the current (in amperes, accurate to two decimal places) through the source is ____________.

Ans. 8

Sol. Redrawing the circuit by renaming the nodes as A, B, C and D.

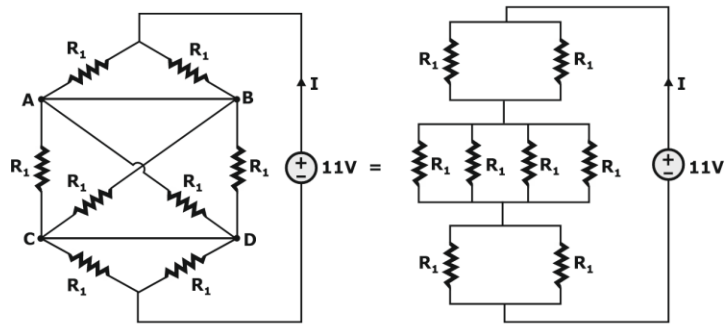


The given network is symmetric,

So, $V_A = V_B$ and $V_C = V_D$

Current through resistors R_2 is zero and as $V_A = V_B$ and $V_C = V_D$.

Electrically this circuit can be reduced as,



Total resistance R_T is resultant of following combination,

$$R_\tau = 2(R_1 \parallel R_1) + (R_1 \parallel R_1 \parallel R_3 \parallel R_3)$$

$$= R_1 + \left(\frac{R_1}{2} \parallel \frac{R_3}{2}\right)$$

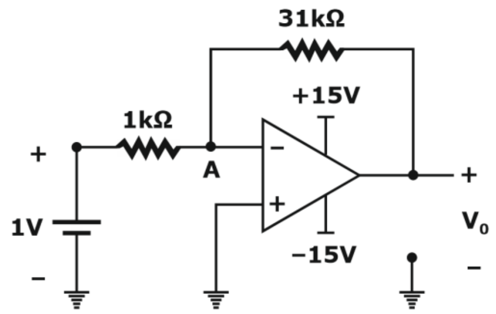
We have values for R_1 and R_3, R_1 = 1Ω and R_3 = 3 Ω

$$\text{So, } R_\tau = 1 + \left(\frac{1}{2} \parallel \frac{3}{2}\right)\Omega = 1 + \frac{3/2}{4} = \frac{11}{8}\Omega$$

Thus, current through 11 V voltage source is,

$$I \frac{11\text{V}}{R_\tau} = \frac{11}{(11/8)} = 8\text{A}$$

3. An op-amp based circuit is implemented as shown below.



In the above circuit, assume the op-amp to be ideal. The voltage (in volts, correct to one decimal place) at node A, connected to the negative input of the op-amp as indicated in the figure is ____________.

Ans. 0.5 V

Sol. Since we have an Op-Amp, Applying the concept of virtual ground,

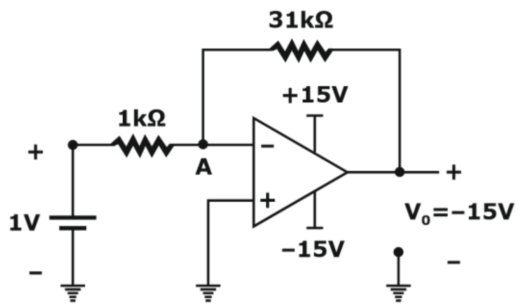
$$V_0 = \frac{-R_2}{R_1} V_{in}$$

$$V_0 = \frac{-31\,k\Omega}{1\,k\Omega} \times 1V$$

$$V_0 = -31V < -15V$$

Which is not possible.

So, the output voltage of the op-amp is equal to -15V.



Now applying KCL of node 'A', we get,

$$\frac{V_A - (-15)}{31\,k\Omega} + \frac{V_A - 1}{1\,k\Omega} = 0$$

$$\frac{V_A}{31\,k\Omega} + \frac{V_A}{1\,k\Omega} = \frac{-15}{31\,k\Omega} + \frac{1}{1\,k\Omega}$$

$$V_A\left[\frac{1}{31} + \frac{1}{1}\right] = \frac{15}{31} + 1$$

$$V_A = 0.5V$$

4. The cut-off frequency of TE_{01} mode of an air filled rectangular waveguide having inner dimensions a cm × b cm (a > b) is twice that of the dominant. TE_{10} mode. When the waveguide is operated at a frequency which is 25% higher than the cut-off frequency of the dominant mode, the guide wavelength is found to be 4 cm. The value of b (in cm, correct to two decimal places) is ____________.

Ans. 0.75

Sol. We have the relation between the cutoff frequencies of two different modes as follows,

$$f_{c(01)} = 2f_{c(10)} = \frac{2c}{2a} = \frac{c}{a}$$

$$\frac{c}{2b} = \frac{C}{a} \Rightarrow a = 2b \Rightarrow b = \frac{a}{2}$$

Given operating frequency,

$f = 1.25\ f_{c(10)}$

$$f_{c(10)} < 1.25\ f_{c(10)} < \left[f_{c(10)} = 2f_{c(10)}\right]$$

According to the given frequency, the waveguide will work in TE_{10} mode clearly.

$$\text{So, } \lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{f_{c(10)}}{f}\right)^2}} = \frac{c/f}{\sqrt{1-\left(\frac{1}{1.25}\right)^2}} = \frac{c/f}{0.6}$$

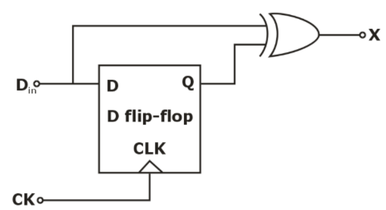
$$\frac{c}{(1.25)\,f_{c(10)(0.6)}} = \lambda_g = 4\,\text{cm}$$

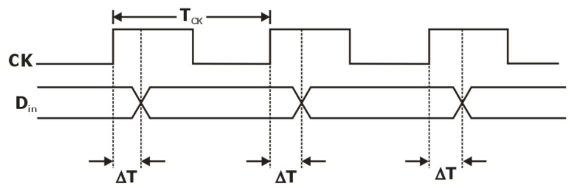
$$\frac{c}{f_{c(10)}} = 3 \times 10^{-2} = 2a$$

$$a = 1.5\,\text{cm}$$

$$b = \frac{a}{2} = 0.75\text{cm}$$

5. In the circuit shown below, a positive edge-triggered D flip-flop is used for sampling input data D_{in} using clock CK. The XOR gate outputs 3.3 volts for logic HIGH and 0 volts for logic LOW levels. The data bit and clock periods are equal and the value of $\Delta T/T_{CK}$ = 0.15, where the parameters ΔT and T_{CK} are shown in the figure. Assume that the Flip-Flop and the XOR gate are ideal.

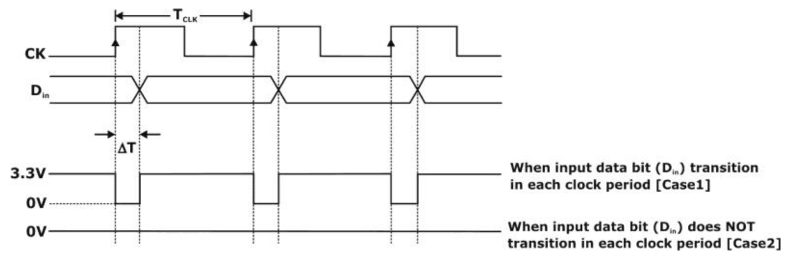




If the probability of input data (D_{in}) bit transition in each clock period is 0.3, the average value (in volts, accurate to two decimal places) of the voltages at node X, is _____________.

Ans. 0.84

Sol. The timing diagram for the circuit can be drawn as follows,



Now average voltage at node X can be calculated according to the timing diagram,

$$V_{avg} = p \times V_{High}\left(1 - \frac{\Delta T}{T_{CK}}\right) + (1-p) \times V_{Low}\left(\frac{\Delta T}{T_{CK}}\right)$$

Where, p = probability of input data bit (D_{in}) transition in each clock period

$$V_{High} = 3.3V$$

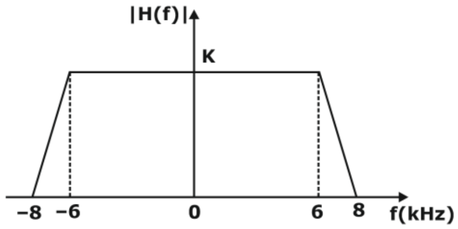
$$V_{Low} = 0V$$

$$\frac{\Delta T}{T_{CK}} = 0.15$$

$$V_{avg} = 0.3 \times 3.3(1-0.15) + (1-0.3) \times 0(0.15)$$

$$= 0.3 \times 3.3 \times 0.85 = 0.8415$$

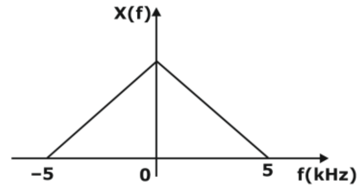
6. A band limited low-pass signal x(t) of bandwidth 5 kHz is sampled at a sampling rate f_s. The signal x(t) is reconstructed using the reconstruction filter H(f) whose magnitude response is shown below:



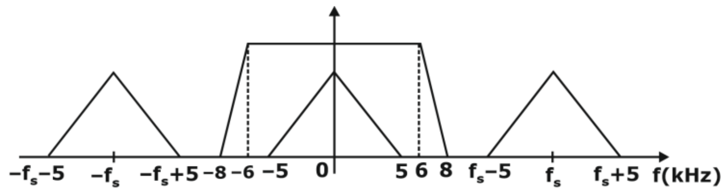
The minimum sampling rate f_s (in kHz) for perfect reconstruction of x(t) is __________.

Ans. 13

Sol. We have to assume an arbitrary spectrum for x(t) as shown below:



Then we would obtain spectrum of the sampled signal can be given as,



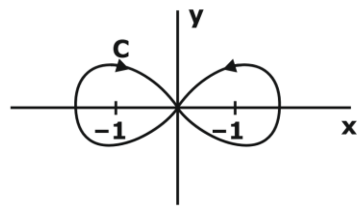
For proper reconstruction of the signal, the next sample must not overlap with previous sample,

$f_s - 5 \geq 8$

$f_s \geq 8 + 5 = 13$ kHz

So, $f_{s(min)} = 13$ kHz

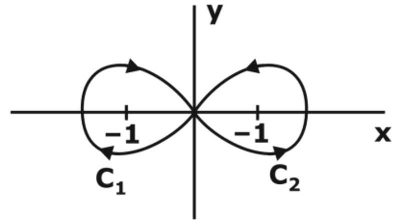
7. The contour C given below is on the complex plane $z = X + jy$, where $j = \sqrt{-1}$.



The value of the integral $\frac{1}{\pi j}\oint_C \frac{dz}{z^2 - 1}$ is ___________.

Ans. 2

Sol. Given complex integral in the question can be solved as follows after denoting the encirclement properly,



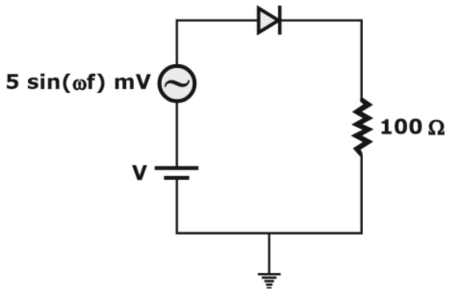
$$\frac{1}{\pi j}\oint_C \frac{dz}{z^2-1}$$

$$= 2\left[\frac{1}{\pi j}\oint_{C_1} \frac{dz}{}\,\frac{1}{(z+1)(z-1)} + \frac{1}{\pi j}\oint_{C_2} \frac{dz}{(z+1)(z-1)}\right]$$

$$= 2\left[-\left(\frac{1}{z-1}\right)\bigg|_{z-1} + \left(\frac{1}{z+1}\right)\bigg|_{z-1}\right]$$

$$= 2\left[-\left(-\frac{1}{2}\right)+\left(\frac{1}{2}\right)\right] = 2$$

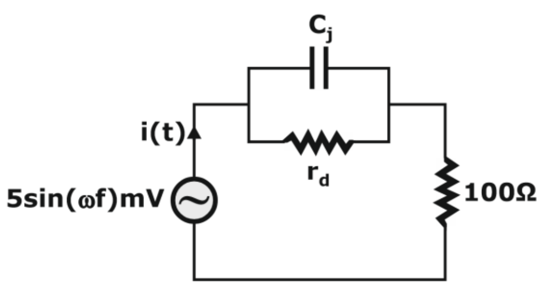
8. A DC current of 26 μA flows through the circuit shown. The diode in the circuit is forward biased and it has an ideality factor of one. At the quiescent point, the diode has a junction capacitance of 0.5 nF. Its neutral region resistances can be neglected. Assume that the room temperature thermal equivalent voltage is 26 mV.



For $\omega = 2 \times 10^6$ rad/s, the amplitude of the small-signal component of diode current (in μA, correct to one decimal place) is ____________.

Ans. 6.4

Sol. Small-signal equivalent model of the given circuit needs to be realized as follows,



Given information in question,

$\omega = 2 \times 10^6$ rad/sec

$C_j = 0.5$ nF

$I_{DC} = 26$ μA

$V_T = 26$ mV

$\eta = 1$

So we can obtain impedances,

$$r_d = \frac{\eta V_T}{I_{DC}} = \frac{26\text{ mV}}{26\,\mu\text{A}} = 1\text{ k}\Omega$$

$$\frac{1}{\omega C_j} = \frac{1}{2\times 10^6 \times 0.5 \times 10^{-9}}\Omega = 1\text{ k}\Omega$$

Now, total impedance of the circuit will be,

$$Z = \left(r_d \,||\, \frac{1}{j\omega C_j}\right) + 100\Omega$$

$$\left(r_d \,||\, \frac{1}{j\omega C_j}\right) = \frac{(1000)(-j1000)}{1000-j1000}\Omega = \frac{-j(1+j)}{2}\text{k}\Omega$$

$$= \frac{1}{2}(1-j)\text{k}\Omega = (500-j500)\Omega$$

$$Z = 600 - j500\Omega$$

$$|Z| = 100\sqrt{36+25} = 100\sqrt{61}\Omega$$

$$I_m = \frac{V_m}{|Z|} = \frac{5\text{mV}}{100\sqrt{61}\Omega} = \frac{50}{\sqrt{61}}\mu\text{A} = 6.40\mu\text{A}$$

9. The position of a particle y(t) is described by the differential equation:

$$\frac{d^2y}{dt^2} = -\frac{dy}{dt} - \frac{5y}{4}$$

The initial conditions are y(0) = 1 and $\left.\frac{dy}{dt}\right|_{t=0}$ 0. The position (accurate to two decimal places) of the particle at t = π is _____________.

Ans. -0.21

Sol. Given condition,

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + \frac{5y}{4} = 0$$

$$y(0) = 1$$

$$y'(0) = 0$$

This can be solved easily in laplace domain,

$$s^2Y(s) - s(1) + sY(s) - 1 + \frac{5}{4}Y(s) = 0$$

$$Y(s) = \frac{s+1}{s^2+s+\frac{5}{4}} = \frac{s+1}{\left(s+\frac{1}{2}\right)^2+1}$$

$$= \frac{\left(s+\frac{1}{2}\right)}{\left(s+\frac{1}{2}\right)^2+1} + \frac{\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+1}$$

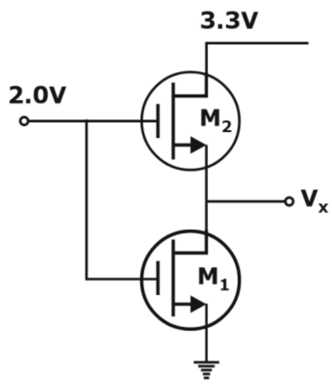
By taking inverse Laplace transform we get y(t),

$$y(t) = e^{-t/2}\left[\cos(t)\frac{1}{2}\sin(t)\right]; t > 0$$

Now its value at t = π,

$$y(t=\pi) = e^{-\pi/2}[(-1)+(0)] = e^{-\pi/2}$$
$$= -0.2078 = -0.21$$

10. In the circuit shown below, the (W/L) value for M_2 is twice that for M_1. The two NMOS transistors are otherwise identical. The threshold voltage V_T for both transistors is 1.0 V. Note that V_{GS} for M_2 must be > 1.0 V.



Current through the nMOS transistors can be modeled as

$$I_{DS} = \mu C_{ox}\left(\frac{W}{L}\right)\left((V_{GS}-V_T)V_{DS} - \frac{1}{2}V_{DS}^2\right)$$

for $V_{DS} \le V_{GS} - V_T$

$$I_{DS} = \mu C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2 / 2$$

for $V_{DS} \ge V_{GS} - V_T$

The voltage (in volts, accurate to two decimal places) at V_x is ________.

Ans. 0.42

Sol. The device constant K_n,

$$K_n = \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right)$$

Given that, $\left(\frac{W}{L}\right)_2 = 2\left(\frac{W}{L}\right)_1$

Then, $K_{n2} = 2K_{n1}$

For M_1,

$V_{GS1} - V_T = 2 - 1 = 1V$

Now, for M_2,

$$V_{GS2} - V_T = 2 - V_x - 1 = 1V - V_x < 1V$$
$$V_{DS2} = (3.3 - V_x) > (V_{GS2} - V_T)$$

Here, clearly M_1 will be in linear region and M_2 will be in saturation region. But current across them would be same,

$$I_{D_1} = I_{D_2}$$

$$K_{n1}\left[2(V_{GS1}-V_T)V_{DS1} - V_{DS1}^2\right] = K_{n2}(V_{GS2}-V_T)^2$$

$$K_{n1}\left[2(2-1)V_x - V_x^2\right] = 2K_{n1}(2 - V_x - 1)^2$$

$$2V_x - V_x^2 = 2\left(1 + V_x^2 - 2V_x\right) = 2V_x^2 - 4V_x + 2$$

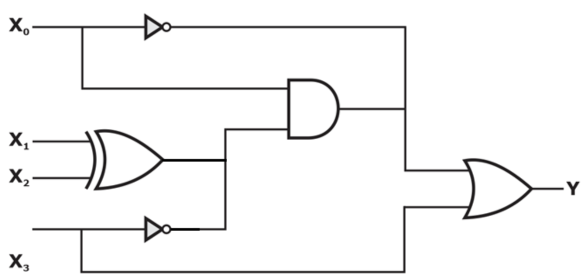
$$3V_x^2 - 6V_x + 2 = 0; V_x^2 - 2V_x + \frac{2}{3} = 0$$

$$V_x = 1 \pm \sqrt{\frac{4 - \frac{8}{3}}{43}} = 1 \pm \sqrt{\frac{1}{3}}V$$

$$V_{GS2} = (2 - V_x) \ge V_T \Rightarrow (1 - V_x) \ge 0$$

So, the only valid value, $V_x = 1 - \sqrt{\frac{1}{3}} = 0.4226V$

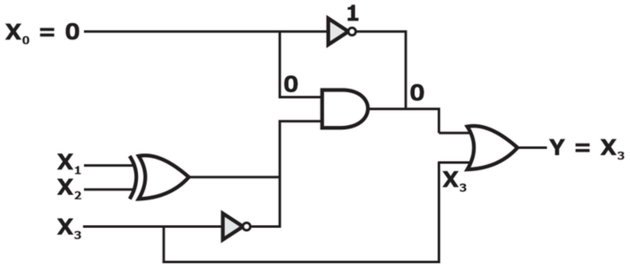
11. The logic gates shown in the digital circuit below use strong pull-down nMOS transistors for LOW logic level at the outputs. When the pull-downs are off, high-value resistors set the output logic levels to HIGH (i.e. the pull-ups are weak). Note that some nodes are intentionally shorted to implement "wired logic". Such shorted nodes will be HIGH only if the outputs of all the gates whose outputs are shorted are HIGH.

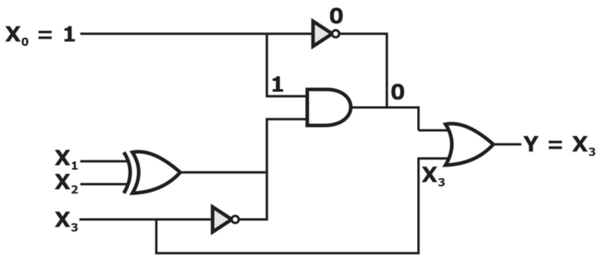


The number of distinct values of $X_3X_2X_1X_0$ (out of the 16 possible values) that given Y=1 is ___________.

Ans. 8

Sol.





From above two circuit, always Y = x_3

X_3	X_2	X_1	X_0	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

The number of distinct values of $X_3X_2X_1X_0$ (out of the 16 possible values) that give Y=1 is 8.

12. A junction is made between p^- Si with doping density N_{A1} = 10^{15} cm^{-3} and p Si with doping density N_{A2} = 10^{17} cm^{-3}.

Given: Boltzmann constant k= 1.38 x 10^{-23} J.K^{-1}, electronic charge q= 1.6 x 10^{-19} C.

Assume 100% acceptor ionization.

At room temperature (T= 300K), the magnitude of the built-in potential (in volts, correct to two decimal places) across this junction will be ____________.

Ans. 0.12

Sol. The standard formula for built-in potential is,

$$V_0 = \frac{kT}{q} \ln\left(\frac{N_A\, N_D}{n_i^2}\right)$$

since in the given semiconductor two doping are there so overall built in potential will be equal to difference of V_{01} and V_{02}

$$V_{01} = \frac{kT}{q}\left(\frac{N_{A1} \times N_{D1}}{n_{i1}^2}\right)$$

$$\text{and } V_{02} = \frac{kT}{q}\left(\frac{N_{A2} \times N_{D2}}{n_{i2}^2}\right)$$

since some material is doped $n_{i1} = n_{i2}$ and $N_{D1} = N_{D2}$

so $V_{bi} = V_{02} - V_{01}$

$$= \frac{kT}{q}\left(\frac{N_{A2} \times N_{D2}}{n_{i2}^{\,2}} \times \frac{n_{i1}^{\,2}}{N_{A1} \times N_{D1}}\right)$$

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_{A2}}{N_{A1}}\right) = \frac{1.38 \times 3}{1.6 \times 100} \ln(100)V$$

$$= 0.1192V$$

13. Let r = $X^2 + y - z$ and $Z^3 - XY + YZ + y^3 = 1$. Assume that x and y are independent variables.

At (x, y, z) = (2, −1, 1) the value (correct to two decimal places) of $\frac{\partial r}{\partial x}$ is ___________.

Ans. 4.5

Sol. Given that x and y are independent variables. From the relations given in the problem statement,

$$r = x^2 + y - z \qquad \ldots \text{(i)}$$

$$z^3 - xy + yz + y^3 = 1 \qquad \ldots \text{(ii)}$$

$$\frac{\partial r}{\partial x} = 2x - \frac{\partial z}{\partial x} \qquad \ldots \text{(iii)}$$

$$3z^2 \frac{\partial z}{\partial x} - y + y\frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x} = \frac{y}{3z^2 + y}$$

By substituting $\frac{\partial Z}{\partial X}$ in equation (iii), we get,

$$\frac{\partial r}{\partial x} = 2x - \frac{y}{3z^2 + y}$$

At given point (2, -1, 1),

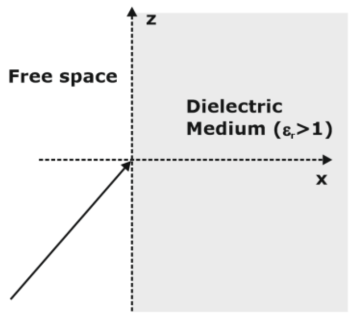
$$\frac{\partial r}{\partial x} = 2(2) - \frac{(-1)}{3(1)^2 + (-1)} = 4 + \frac{1}{2} = 4.50$$

14. A uniform plane wave traveling in free space and having the electric field

$$\bar{E} = \left(\sqrt{2}\hat{a}_x - \hat{a}_z\right)$$

$$\cos\left[6\sqrt{3}\pi \times 10^8 t - 2\pi(x + \sqrt{2}z)\right] V / m$$

is incident on a dielectric medium (relative permittivity > 1, relative permeability = 1) as shown in the figure and there is no reflected wave.



The relative permittivity (correct to two decimal places) of the dielectric medium is ______________.

Ans. 2

Sol. From the given equation of the electric field, we can get the following phase relation,

$$\bar{K}_i = 2\pi(\hat{x} + \sqrt{2}\hat{z}) = 2\pi\sqrt{3}\left(\frac{1}{\sqrt{3}}\hat{x} + \sqrt{\frac{2}{3}}\hat{z}\right)$$

$$\cos\theta_{ix} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan\theta_{ix} = \sqrt{2}$$

Since there is no reflected wave,

$\theta_{ix} = \theta_B$ = Brewester angle

And as the wave is parallel polarized,

$$\tan\theta_B = \sqrt{\frac{\epsilon_r \epsilon_0}{\epsilon_0}} = \sqrt{\epsilon_r} = \sqrt{2}$$

So, $\epsilon_r = 2$

15. Let X[k] = k + 1, 0 ≤ k ≤ 7 be 8-point DFT of a sequence x[n], where $x[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi/k/N}$

The value (correct to two decimal places) of $\sum_{n=0}^{3} X[2n]$ is __________.

Ans. 3

Sol. We have x(k) given and we need to find the sum of downsampled version of x(n). So,

$$x(k) = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

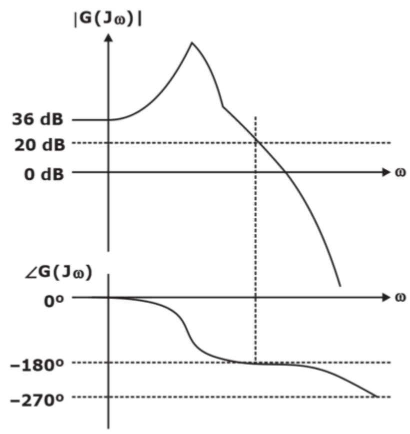
$$\sum_{n=0}^{3} x[2n] = x[0] + x[2] + x[4] + x[6]$$

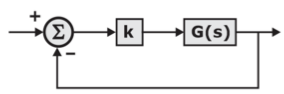
$$= 4.5 - 0.5 - 0.5j - 0.5 - 0.5 + 0.5j$$

$$= 4.5 - 1.5 = 3$$

16. The figure below shows the Bode magnitude and phase plots of a stable function

$$G(s) = \frac{n_0}{S^3 + d_2S^2 + d_1s + d_0}$$





Consider the negative unity feedback configuration with gain k in the feedforward path. The closed loop is stable for k < k_0. The maximum value of k_0 is ____________.

Ans. 0.1

Sol. At phase crossover frequency G(s),

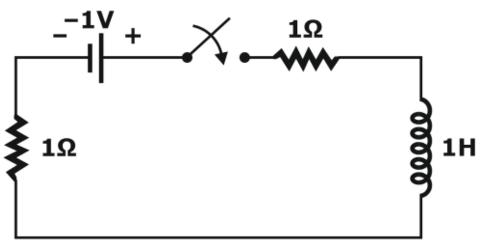
$$M_{dB}\left(\omega_{pc}\right) = 20 \text{ dB}$$

When cascaded with k,

$$GM_{dB} = -20dB - 20\log_{10}(k) > 0dB$$
$$20 + 20\log_{10}(k) < 0$$
$$20\log_{10}(k) < -20$$
$$k < 10^{-1} = 0.10$$
$$k_0 = 0.10$$

Thus, $k_0 = 0.10$

17. For the circuit given in the figure, the magnitude of the loop current (in amperes, correct to three decimal places) 0.5 second after closing the switch is ___________.(upto 3 decimal places)



Ans. 0.316

Sol. The circuit given here has no initial conditions as no energy is stored in inductor prior to switching.

Loop current, $i(t) = \frac{1}{1+1}\left(1 - e^{-t/z}\right)A; t > 0$

$$\tau = \frac{L}{R_{eq}} = \frac{1}{1+1} = \frac{1}{2}\text{sec}$$

$$i(t) = \frac{1}{2}\left(1 - e^{-2t}\right)A; t > 0$$

So current at t = 0.5 sec,

$$i(t) = \frac{1}{2}\left(1 - e^{-1}\right)A = 0.316\ A$$

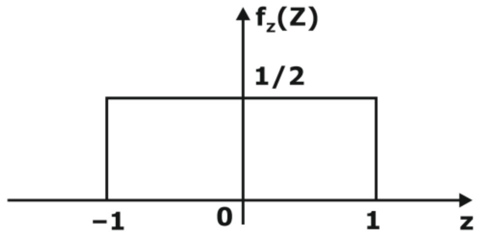
18. A random variable X takes values -0.5 and 0.5 with probabilities $\frac{1}{4}$ and $\frac{3}{4}$, respectively. The noisy observation of X is Y = X + Z, where Z has uniform probability density over the interval (-1, 1). X and Z are independent. If the MAP rule-based detector outputs $\hat{X}$ as

$$\hat{X} = \begin{cases} -0.5, & Y < \alpha \\ 0.5, & Y \ge \alpha' \end{cases}$$

then the value of α (accurate to two decimal places) is ___________.

Ans. -0.5

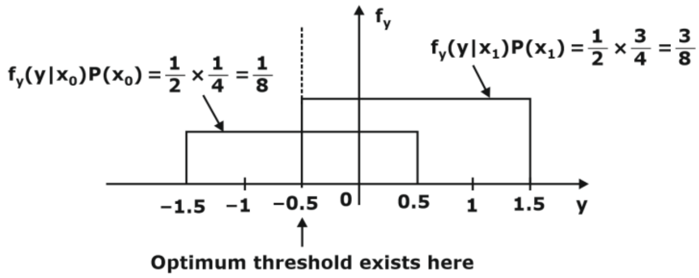
Sol. The noise profile can be drawn as follows,



And we know the probabilities of two signals X_0 and X_1,

$$P(x_0) = \frac{1}{4}$$
$$P(x_1) = \frac{3}{4}$$

MAP criteria, $f_y(y|x_0)P(x_0)f_y(y|x_1)p(x_1)$



So, α = −0.50

19. For a unity feedback control system with the forward path transfer function $G(s) = \frac{K}{s(s+2)}$. The peak resonant magnitude M_r of the closed-loop frequency response is 2.The corresponding value of the gain K (correct to two decimal places) is _____________.

Ans. 14.93

Sol. Maximum resonant peak is given as (In terms of damping factor),

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 2$$
$$2\xi\sqrt{1-\xi^2} = \frac{1}{2}$$
$$\xi^2\left(1-\xi^2\right) = \frac{1}{16}$$
$$\xi^4 - \xi^2 + \frac{1}{16} = 0$$
$$\xi^2 = \frac{1}{2} \pm \sqrt{\frac{1-\frac{1}{4}}{4}} = \frac{1}{2} \pm \frac{\sqrt{3}}{4}$$

But we know, $M_r = 2 > 1,\ \xi < \frac{1}{\sqrt{2}}$ and $\xi^2 < \frac{1}{2}$

Then, $\xi^2 = \frac{1}{2} - \frac{\sqrt{3}}{4}$

Also given in question,

$$G(s) = \frac{K}{s(s+2)} = \frac{\omega_n^2}{s\left(s+2\xi\omega_n\right)}$$

So, $\omega_n = \sqrt{K}$

$2\xi\sqrt{K} = 2$

$\sqrt{K} = \frac{1}{\xi}$

$$K = \frac{1}{\xi^2} = \frac{1}{\left(\frac{1}{2} - \frac{\sqrt{3}}{4}\right)} = \frac{4}{2-\sqrt{3}} = 14.928$$

20. The input 4 sin c (2t) is fed to a Hilbert transformer to obtain y(t), as shown in the figure below:

Here $\sin c(x) = \frac{\sin(\pi x)}{\pi x}$. The value (accurate to two decimal places) of $\int_{-\infty}^{\infty} |y(t)|^2\, dt$ is

__________.

Ans. 8

Sol. Hilbert transform does not alter the amplitude spectrum of the signal and using CTFT to determine the amplitude,

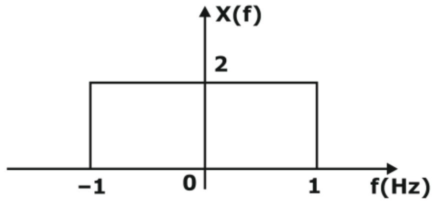
So,

$$\int_{-\infty}^{\infty} |\,y(t)\,|^2\, dt = \int_{-\infty}^{\infty} |\,x(t)\,|^2\, dt = \int_{-\infty}^{\infty} |\,x(t)\,|^2\, dt$$

$$\sin c(t) \xleftrightarrow{\text{CTFT}} \text{rect}(f)$$

$$4\sin c(2t) \xleftrightarrow{\text{CTFT}} \frac{4}{2}\text{rect}\left(\frac{f}{2}\right) = 2\ \text{rect}\left(\frac{f}{2}\right)$$

$$\int_{-\infty}^{\infty} |X(f)|^2\, dt = 2 \times 2(2)^2 = 8$$



Then, $\int_{-\infty}^{\infty} |y(t)|^2\, dt = 8$

21. Consider matrix $A = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$ and vector $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. The number of distinct real values of k for which the equation AX = 0 has infinitely many solution is __________.

Ans. 2

Sol. If AX = 0 has infinitely many solutions and X is non-zero, then |A| = 0

$$\begin{vmatrix} k & 2 \\ k^2 - k & k^2 \end{vmatrix} = 0$$

$k^3 - 2k^3 + 2k^2 = 0$

$k^2(2 - k) = 0$

k = 0, 2 ⇒ "two" distinct values of k

22. A p-n step junction diode with a contact potential of 0.65 V has a depletion width of 1μm at equilibrium. The forward voltage (in volts, correct to two decimal places) at which this width reduces to 0.6μm is __________.

Ans. 0.42

Sol. We have the formula for Width of depletion region in a pn junction, and from that relation between width and voltage can be directly applied to get the answer as follows,

$$W_{dep} = \sqrt{\frac{2\varepsilon}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\left(V_{bi} - V_{AK}\right)}$$

$$\frac{\sqrt{(0.65 - V_{AK})}}{\sqrt{0.65}} = \frac{0.6\mu m}{1\mu m} = 0.6$$

$$1 - \frac{V_{AK}}{0.65} = 0.36$$

$$V_{AK} = 0.65(1 - 0.36)$$

$$= 0.65 \times 0.64 = 0.416V$$

23. A lossy transmission line has resistance per unit length R = 0.05 Ω/m. The line is distortionless and has characteristic impendace of 50 Ω. The attenuation constant (in Np/m, correct to three decimal places) of the line is ______________.

Ans. 0.001

Sol. The following condition is true for a distortionless transmission line,

$$\frac{L}{R} = \frac{C}{G}$$

Propagation constant is given by,

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{RG}\left(1 + j\omega\frac{L}{R}\right)$$

And the attenuation constant, which is real part of the propagation constant,

$$\alpha = \sqrt{RG}$$

Characteristic impedance,

$$Z_o = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} = \sqrt{\frac{R}{G}}$$

$$\sqrt{G} = \frac{\sqrt{R}}{Z_o}$$

$$\alpha = \sqrt{R}.\frac{\sqrt{R}}{Z_o} = \frac{R}{Z_o} = \frac{0.05}{50} = \frac{0.01}{10}$$

So, = 0.001 Np/m

24. Consider a binary channel code in which each codeword has a fixed length of 5 bits. The Hamming distance between any pair of distinct codewords in this code is at least 2. The maximum number of codewords such a code can contain is ____________.

Ans. 16

Sol. Given that, n = 5 and d_{min} = 2

Without any constraint, 2^5 = 32 codewords can be formed.

By maintaining d_{min} = 2, the codewords can be formed as follows :

0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

⇒ Total 16 codewords are possible

Thus, 16 codewords are possible.

25. Let X_1,X_2,X_3 and X_4 be independent normal random variables with zero mean and unit variance. The probability that X_4 is the smallest among the four is ____________.

Ans. 0.25

Sol. Probability of X_4 being smallest is given as follows,

$$P(X_4 \text{ is smallest}) = \frac{3!}{4!} = \frac{1}{4} = 0.25$$

Note that here all four are similar random variables, so the probability of any one of them being smallest is same.

26. Taylor series expansion of $f(x) = \int_0^x e^{-\left(\frac{t^2}{2}\right)} dt$

around x = 0 has the form

$f(x) = a_0 + a_1x + a_2x^2 + \ldots$

The coefficient a_2 (correct to two decimal places) is equal to _____________.

Ans. 0

Sol. Given function, a_2 would appear alongwith 2^{nd} differential of f(x). We can calculate a_2 as follows,

$$f(x) = \int_0^x e^{-\left(\frac{t^2}{2}\right)} dt$$

$$f'(x) = e^{-x^2/2} - 1$$

$$\text{and } f''(x) = e^{-x^2/2}(-x)$$

$$f''(0) = 0$$

$$a_2 = \frac{f''(0)}{2!} = 0$$

27. There are two photolithography systems: one with light source of wavelength λ_1 = 156 nm (System 1) and another with a light source of wavelength λ_2 = 325 nm (System 2). Both photolithography systems are otherwise identical. If the minimum feature sizes that can be realized using System 1 and System 2 are L_{min1} and L_{min2} respectively, the ratio L_{min1}/L_{min2} (correct to two decimal places) is ____________.

Ans. 0.48

Sol. We know the dependence of feature size on wavelength as follows,

$$L_{min} \propto \lambda$$

$$\frac{L_{min1}}{L_{min2}} = \frac{\lambda_1}{\lambda_2} = \frac{156\text{ nm}}{325\text{ nm}} = 0.48$$

28. Consider the following amplitude modulated signal:

$$s(t) = \cos(2000\pi t) + 4\cos(2400\pi t) + \cos(2800\pi t)$$

The value of amplitude sensitivity of modulator is K_a

The ratio (accurate to three decimal places) of the power of the message signal to the power of the carrier signal is _______________.

Ans. 0.125

Sol. Given signal,

$$s(t) = \cos(2000\pi t) + 4\cos(2400\pi t) + \cos(2800\pi t)$$

It can be compared with the standard form of the AM signal,

$$s(t) = \frac{\mu A_c}{2}\cos\left[2\pi\left(f_c - f_m\right)t\right] + A_c\cos\left(2\pi f_c t\right) + \frac{\mu A_c}{2}\cos\left[2\pi\left(f_c + f_m\right)t\right]$$

By comparison, we get, A_c = 4 and with further manipulation,

$$\because \mu = \frac{A_m}{A_c}$$

& comparing standard equation

$$\frac{\mu A_c}{2} = 1$$

$$\mu = \frac{2}{A_c}$$

$$\frac{A_m}{A_c} = \frac{2}{A_c} \Rightarrow A_m = 2$$

Therefore, A_m=2 A_c=4 & μ= ½

$$\frac{P_m}{P_c} = \frac{\frac{1}{2} + \frac{1}{2}}{8} = 0.125$$

29. A traffic signal cycles from GREEN to YELLOW, YELLOW to RED and RED to GREEN. In each cycle, GREEN is turned on for 70 seconds, YELLOW is turned on for 5 seconds and the RED is turned on for 75 seconds. This traffic light has to be implemented using a finite state machine (FSM). The only to this FSM is a clock of 5 second period. The minimum number of flip-flops required to implement this FSM is____________.

Ans. 5

Sol. According to the given data,

$$\text{GREEN} \rightarrow 70\text{ seconds}$$
$$\text{YELLOW} \rightarrow 5\text{ seconds}$$
$$\text{RED} \rightarrow 75\text{ seconds}$$
$$\text{Clock period} \rightarrow 5\text{ seconds}$$

Total number of unique states required

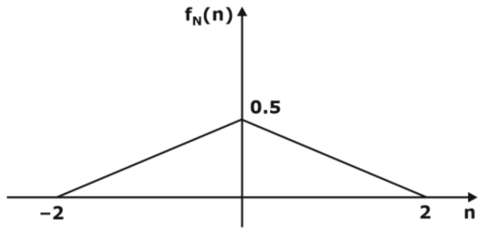
$$= \frac{70 + 5 + 75}{5} = 30$$

Minimum number of flip-flops required is,

n = [$\log_2$ (30)] = [4.91] = 5

flip flops are required for the stable output to make transition error zero and false triggering of output.

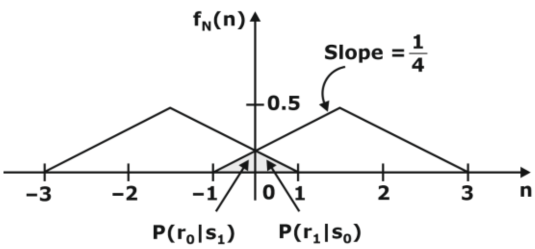
30. A binary source generates symbols X ϵ{−1, 1} which are transmitted over a noisy channel. The probability of transmitting the both symbols is equal.Input to the threshold detector is R = X + N. The probability density function $f_N(n)$ of the noise is shown below.



If the detection threshold is zero, then the probability of error (correct to two decimal places) is ____________.

Ans. 0.125

Sol. Let S_0 and S_1 be the transmitted symbols representing the transmitted value {-1, 1} respectively and let r_0 and r_1 be the received symbols.



Probability of error is given as,

$P_e = P(s_1)*P(r_0|s_1) + p(s_0) * P(r_1|s_0)$

Where $P(r_0|s_1)$ is probability of receiving r_0 when s_1 is transmitted and and $P(r_1|s_0)$ is probability of receiving r_1 when s_0 is transmitted.

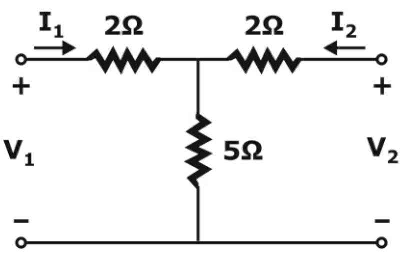
$P(r_0|s_1) = P(r_1|s_0) = 0.5 * 1 * 0.25 = 0.125$

Given that, $P(s_0) = P(s_1) = \frac{1}{2}$ (Probability of transmitting a signal)

So, $P_e = \frac{1}{2}\left(\frac{1}{8}+\frac{1}{8}\right) = \frac{1}{8} = 0.125$

31. The ABCD matrix for a two-port network is defined by:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$



The parameter B for the given two-port network (in ohms, correct to two decimal places) is ____________.

Ans. 4.8

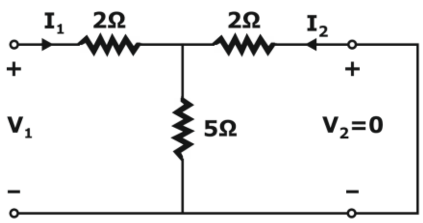
Sol. For ABCD Parameters, the general equations are,

$V_1 = AV_2 - BI_2$ ....(A)

$I_1 = CV_2 - DI_2$ ....(B)

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

When $V_2 = 0$ (i.e., when port-2 is short circuited),



Writing KVL equations in loops with I_1 and I_2 separately,

$V_1 = 2I_1 + 5(I_1 + I_2) = 7I_1 + 5I_2$ ....(1)

$V_2 = 2I_2 + 5(I_2 + I_1) = 5I_1 + 7I_2$ ....(2)

by equation (2) $I_1 = \frac{-7}{5}I_2 + \frac{1}{5}V_2$ ......(3)

by (1) & (3)

$V_1 = \frac{7}{5}V_2 - \frac{24}{5}I_2$ ....(4)

So, BY (A)and (4)

B = 4.8

32. The Cricket Board has long recognized John's potential as a leader of the team. However, his on-field Temper has always been a matter of concern for them since his junior days. While this aggression has filled stadia with die-hard fans, it has taken a toll on his own batting. Until recently, it appeared that he found it difficult to convert his aggression into big scores. Over the past three seasons though, that picture of John has been replaced by a cerebral, calculative and successful batsman-captain. After many years, it appears that the team has finally found a complete captain. Which of the following statements can be logically inferred from the above paragraph?

i. Even as a junior cricketer, John was considered a good captain.
ii. Finding a complete captain is a challenge.
iii. Fans and the Cricket Board have differing views on what they want in a captain.
iv. Over the past three seasons John was accumulated big scores.
A. (i), (ii) and (iii) only
B. (iii) and (iv) only
C. (ii) and (iv) only
D. (i), (ii), (iii) and (iv)

Ans. C

Sol. Statement (i) is not true as nowhere it is mentioned that John being a Captain at Junior level.
Statement (ii) can be concluded from the paragraph as the last line suggests.
Statement (iii) cannot be concluded from the given information as qualities seeked by selectors can be concluded but similar cannot be said about opinion of fans and viewers.
Statement (iv) can be concluded from the part in paragraph mentioning about last 3 seasons of John.

33. A coastal region with unparalleled beauty is home to many species of animals. It is dotted with coral reefs and unspoilt white sandy beaches. It has remained inaccessible to tourists due to poor connectivity and lack of accommodation. A company has spotted the opportunity and is planning to develop a luxury resort with helicopter service to the nearest major city airport. Environmentalists are upset that this would lead to the region becoming crowded and polluted like any other major beach resorts. Which one of the following statements can be logically inferred from the information given in the above paragraph?
A. The culture and tradition of the local people will be influenced by the tourists.
B. The region will become crowded and polluted due to tourism.
C. The coral reefs are on the decline and could soon vanish.
D. Helicopter connectivity would lead to an increase in tourists coming to the region.

Ans. D

Sol. (A) No such information given that supports this option.
Option (B) suggests only one part of the paragraph.
Similarly (C) can also be discarded because of no such information is given in paragraph.
While (D) option suggests helicopter as the connectivity means which is the crux of the paragraph.

34. A cab was involved in a hit and run accident at night.
You are given the following data about the cabs in the city and the accident.
i. 85% of cabs in the city are green and the remaining cabs are blue.
ii. A witness identified the cab involved in the accident as blue.
iii. It is known that a witness can correctly identify the cab colour only 80% of the time.
Which of the following options is closest to the probability that the accident was caused by a blue cab?

A. 12%	B. 15%
C. 41%	D. 80%

Ans. C

Sol. Probability that accident was caused by blue cab,
P(Blue Cab) = (P(Blue) * P(Correct)) + (P(Green) * P(Not Correct))
This gives total number of accidents being identified caused by a Blue cab.
P(correct) = 0.8
P(not correct) = 1 - 0.8 = 0.2
Actual probability that accident is caused by blue cab,
P(Actually Blue) = (P(Blue)) * P(Correct))/P(Blue cab)

P(Actually Blue) = (0.15 * 0.8)/ (0.15 * 0.8 + 0.85 * 0.2)

P(Actually Blue) = 0.4137

Thus, 41.37% is the probability.

35. Leila aspires to buy a car worth Rs. 10,00,000 after 5 years. What is the minimum amount in Rupees that she should deposit now in a bank which offers 10% annual rate of interest, if the interest was compounded annually?

A. 5,00,000 B. 6,21,000

C. 6,66,667 D. 7,50,000

Ans. B

Sol. We have the formula for compound interest as follows,

$$A = P * \left(1 + \frac{R}{100}\right)^n$$

Where, A is final amount, P is initial amount, R is rate of interest and n is the number of years the interest is compounded.

We have A = 1000000, R = 10 %, n = 5 years.

Then P can be found out,

P = 6,21,000.

$$1000000 = P^* \left(1 + \frac{10}{100}\right)^5$$

$$P = \frac{1000000}{\left(1 + \frac{10}{100}\right)^5} = 620921.32 \approx 621000$$

36. Two alloys A and B contain gold and copper in the ratios of 2 : 3 and 3 : 7 by mass, respectively. Equal masses of alloys A and B are melted to make an alloy C. The ratio of gold to copper in alloy C is ___________.

A. 5 : 10 B. 7 : 13

C. 6 : 11 D. 9 : 13

Ans. B

Sol. Alloy A contains Gold and Copper in ratio 2 : 3.

Let there be 10x mass of alloy A, so that we have Gold and Copper as 4x : 6x.

Alloy B contains Gold and Copper in ratio 3 : 7.

Let there be 10x mass of Alloy B, so that we have Gold and Copper as 3x : 7x.

As masses of Alloy A is equal to Alloy of mass B, Resultant ratio of Gold to Copper when equal masses of Alloy A and Alloy B are mixed would be

4x + 3x : 6x + 7x

7x : 13x

7 :13.

37. A curve passes through the point (x = 1, y = 0) and satisfies the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{2y} + \frac{y}{x}$. The equation that describes the curve is

A. $\ln\left(1 + \frac{y^2}{X^2}\right) = X - 1$ B. $\frac{1}{2}\ln\left(1 + \frac{y^2}{X^2}\right) = x - 1$

C. $\ln\left(1 + \frac{Y}{X}\right) = X - 1$ D. $\frac{1}{2}\ln\left(1 + \frac{Y}{X}\right) = X - 1$

Ans. A

Sol. Given Differential equation,

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2y} + \frac{y}{x}$$

We need to use suitable substitution here,

Put, $\frac{y}{x} = t$

$$\frac{dy}{dx} = t + x\frac{dt}{dx}$$

$$1 + x\frac{dt}{dx} = \frac{x}{2t} + \frac{tx}{2} + t$$

$$x\frac{dt}{dx} = x\left(\frac{1}{2t} + \frac{t}{2}\right)$$

$$x\frac{dt}{dx} = x\left(\frac{1 + t^2}{2t}\right)$$

$$\int \frac{2t}{1 + t^2}\, dt = \int dx + C$$

ln (1 + t^2) = x + C

$$t = \frac{y}{x}$$

After simplification we obtain the following relation,

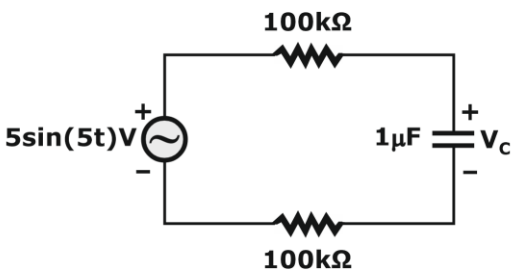
$$\ln\left(1 + \frac{y^2}{x^2}\right) = x + C$$

Given that the curve passes through points, x = 1, y = 0, we can obtain the value of constant C.

$$\ln\left(1+\frac{0}{1}\right)=\ln(1)=0=1+C$$

So, $\ln\left(1+\frac{y^2}{x^2}\right)=x-1$

38. For the circuit given in the figure, the voltage V_C (in volts) across the capacitor is



A. $1.25\sqrt{2}\sin(5t-0.25\pi)$

B. $1.25\sqrt{2}\sin(5t-0.125\pi)$

C. $2.5\sqrt{2}\sin(5t-0.25\pi)$

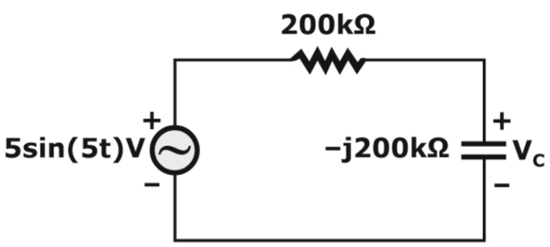
D. $2.5\sqrt{2}\sin(5t-0.125\pi)$

Ans. C

Sol. Reactance of capacitor with respect to given value of C and ω,

$$\frac{1}{\omega C}=\frac{1}{5\times10^{-6}}=200\text{ k}\Omega$$

Redrawing the simplified circuit,



Applying Voltage division rule to get voltage across Capacitor,

$$V_c=\frac{5\angle0^\circ}{200-j200}\times(-j200)$$

$$V=\frac{5\angle0^\circ\times1\angle-90^\circ}{\sqrt{2}\angle-45^\circ}V$$

$$=\frac{5}{\sqrt{2}}\angle-45^\circ V=2.5\sqrt{2}\sin\left(5t-\frac{\pi}{4}\right)V$$

$$=2.5\sqrt{2}\sin(5t-0.25\pi)$$

39. Let $c(t) = A_c\cos(2\pi f_ct)$ and $m(t) = \cos(2\pi f_mt)$. it is given that $f_c \gg 5f_m$. The signal $c(t) + m(t)$ is applied to the input of a non-linear device, whose output $v_0(t)$ is related to the input $v_i(t)$ as $v_0(t) = av_i(t) + bv_i^2(t)$, where a and b are positive constants. The output of the non-linear device is passed through an ideal band-pass filter with center frequency f_c and bandwidth $3f_m$, to produce an amplitude modulated (AM) wave. If it is desired to have the sideband power of the AM wave to be half of the carrier power, then a/b is

A. 0.25
B. 0.5
C. 1
D. 2

Ans. D

Sol. According to given input signal, we can obtain an output signal as follows,

$V_i(t) = A_c\cos(2\pi f_ct) + \cos(2\pi f_mt)$

$V_0(t) = av_i(t) + bv_i^2(t)$

$= [aA_c\cos(2\pi f_ct) + a\cos(2\pi f_mt)] + b$

$$\left[A_c^2\cos^2(2\pi f_ct)+\cos^2(2\pi f_mt)+2A_c\cos\right.$$

$$\left.(2\pi f_ct)\cos(2\pi f_mt)\right]$$

When the signal is passed through given Band Pass Filter,

$$y(t)=aA_c\cos2\pi f_ct+2bA_c\cos(2\pi f_ct)$$

$$\cos(2\pi f_mt)$$

$$=aA_c\left[1+\frac{2b}{a}\cos(2\pi f_mt)\right]\cos(2\pi f_ct)$$

The Modulation index can be obtained through output of the BPF,

$$\mu=\frac{2b}{a}$$

We have been given in the problem statement that Side Band contains half the carrier power,

$$P_{SB}=\frac{\mu^2}{2}P_c=\frac{1}{2}P_c$$

So, $\mu^2 = 1 \Rightarrow \mu = 1$

Comparing with the value obtained in form of a and b,

$$\frac{2b}{a} = 1$$

$$\frac{a}{b} = 2$$

40. The distance (in meters) a wave has to propagate in a medium having a skin depth of 0.1 m so that the amplitude of the wave attenuates by 20 dB, is

A. 0.12 B. 0.23

C. 0.46 D. 2.3

Ans. B

Sol. Attenuation constant is related with skin depth as follows, And according to given condition of 20 dB attenuation we can get required depth by following calculation,

$$\alpha = \frac{1}{\text{skin depth}} = 10 \text{ Np/m}$$

$$20 \log_{10}\left(\frac{E_o}{E_x}\right) = 20 \text{ dB}$$

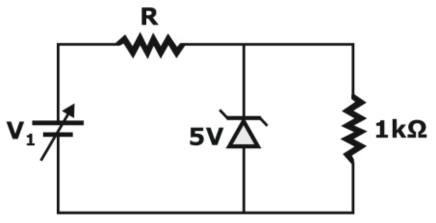
$$\frac{E_o}{E_x} = 10 \Rightarrow (E_x) = \frac{E_o}{10}$$

$$E_x = E_o e^{-\alpha x} = E_o e^{-10x} = \frac{E_o}{10}$$

$$e^{-10x} = \frac{1}{10}$$

$$x = \frac{1}{10} \ln(10) = 0.23 \text{ m}$$

41. The circuit shown in the figure is used to provide regulated (5 V) across the 1 kΩ resistor. Assume that the Zener diode has a constant reverse breakdown voltage for a current range, starting from a minimum required Zener current, I_{zmin} = 2 mA to its maximum allowable current. The input voltage V_1 may vary by 5% from its nomial value of 6 V. The resistance of the diode in the breakdown region is negligible.



The value of R and the minimum required power dissipation rating of the diode, respectively, are

A. 186 Ω and 10 mW B. 100 Ω and 40 mW

C. 100 Ω and 10 mW D. 186 Ω and 40 mW

Ans. B

Sol. According to data given in question, we can get the $I_{s(min)}$ as follows,

$$V_I = 6V \pm 5\% = 6V \pm 0.3V = 5.7V \text{to} 6.3V$$

$$I_L = \frac{5V}{1k\Omega} = 5\,mA$$

$$I_{s(min)} = I_L + I_{Z(min)} = 5\,mA + 2\,mA = 7\,mA$$

$$I_s = \frac{V_I - V_Z}{R}$$

$$I_{s(min)} = \frac{V_{I(min)} - V_z}{R_{max}} = 7\,mA$$

Now when $I_{s(min)}$ flows that means Resistance is maximum, $R_{max} = \frac{5.7-5}{7} k\Omega = \frac{700}{7}\Omega = 100\Omega$

Now obtaining Maximum current that could flow through Zener Diode due to fluctuation in Source and removal of Load while R = 100 Ω,

$$I_{s(max)} = \frac{6.3-5}{100} A = 13mA$$

$$I_{Z(max)} = I_{s(max)} - I_L = 13mA - 5mA = 8mA$$

$$P_{z(min)} = V_{z(max)} = (5 \times 8)mW = 40mW$$

42. Consider a white Gaussian noise process N(t) with two-sided power spectral density $S_N(f)$ = 0.5 W/Hz as input to a filter with impulse response 0.5 $e^{-t^2/2}$ (where t is in seconds) resulting in output Y(t). The power in Y(t) in watts is

A. 0.11 B. 0.22

C. 0.33 D. 0.44

Ans. B

Sol. Power Spectral Density of noise input,

$S_N(f)$ = 0.5 W/Hz

Power of y(t),

$$P_y = \int_{-\infty}^{\infty} S_N(f)|H(f)|^2 \, df$$

$$= 0.50 \int_{-\infty}^{\infty} |H(f)|^2 \, df = 0.50 \int_{-\infty}^{\infty} |h(f)|^2 \, dt$$

Given the impulse response of the filter being used,

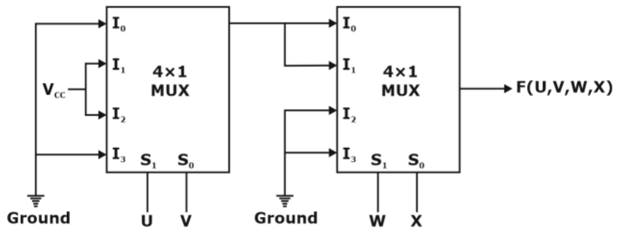
$$h(t) = \frac{1}{2} e^{-t^2/2}$$

So,

$$P_y = \frac{1}{2} \int_{-\infty}^{\infty} \left(\frac{1}{2} e^{-t^2/2} \right)^2 dt = \frac{1}{8} \int_{-\infty}^{\infty} e^{-t^2} dt$$

$$= \frac{\sqrt{\pi}}{8} = 0.22W$$

43. A four-variable Boolean function is realized using 4 × 1 multiplexers as shown in the figure.



The minimized expression for F(U, V, W, X) is

A. $(UV + \bar{U}\bar{V})\bar{W}$

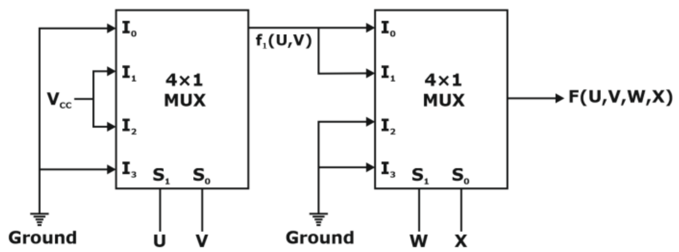
B. $(UV + \bar{U}\bar{V})(\bar{W}\bar{X} + \bar{W}X)$

C. $(U\bar{V} + \bar{U}V)\bar{W}$

D. $(U\bar{V} + \bar{U}\bar{V})(\bar{W}\bar{X} + \bar{W}X)$

Ans. C

Sol.



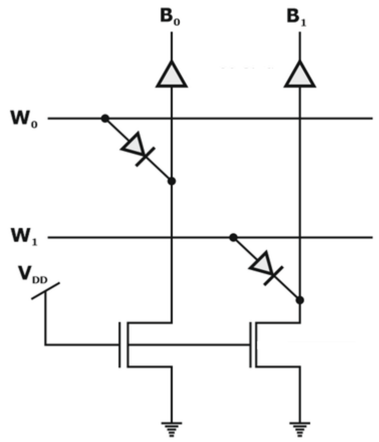
Output of the first 4x1 multiplexer can be expressed as,

$$F_1 = \bar{U}V + U\bar{V}$$

Output of the second 4x1 multiplexer can be expressed as,

$$F = \bar{W}\bar{X}F_1 + \bar{W}XF_1 = \bar{W}F_1 = (\bar{U}V + U\bar{V})\bar{W}$$

44. A 2 × 2 ROM array is built with the help of diodes as shown in the circuit below. Here W_0 and W_1 are signals that select the word lines and B_0 and B_1 are signals that are output of the sense amps based on the stored data corresponding to the bit lines during the read operation.



$$\begin{array}{ccc} & B_0 & B_1 \\ \begin{array}{c} W_0 \\ W_1 \end{array} & \multicolumn{2}{c}{\begin{bmatrix} D_{00} & D_{01} \\ D_{10} & D_{11} \end{bmatrix}} \end{array}$$

Bits stored in the ROM Array

During the read operation, the selected word line goes high and the other word line is in a high impedance state. As per the implementation shown in the circuit diagram above, what are the bits corresponding to D_{ij} (where i = 0 or 1 and j = 0 or 1) stored in the ROM?

A. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

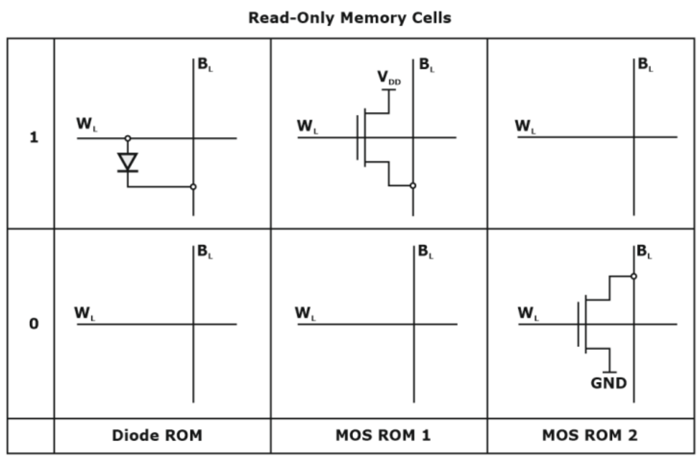
B. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

Ans. A

Sol. Given logic circuit,



The following can be observed,

When $W_0 = V_{DD}$, $B_0 = V_{DD}$; otherwise $B_0 = 0$

When $W_1 = V_{DD}$, $B_1 = V_{DD}$; otherwise $B_1 = 0$

So, $B_0 = W_0$ and $B_1 = W_1$

Hence, $\begin{array}{c} W_0 \\ W_1 \end{array}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

45. The state equation and the output equation of a control system are given below:

$$\dot{x} = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u,$$

$y = [1.5\ 0.625]x$

The transfer function representation of the system is

A. $\frac{3s+5}{s^2+4s+6}$ B. $\frac{3s+1.875}{s^2+4s+6}$

C. $\frac{4s+1.5}{s^2+4s+6}$ D. $\frac{6s+5}{s^2+4s+6}$

Ans. A

Sol. In terms of State Space form, the Transfer function is given as,

$$T(s) = \frac{Y(s)}{U(s)} = C[sI - A]^{-1} B$$

$$A = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$C = [1.5 \qquad 0.625]$

$$[sI - A] = \begin{bmatrix} s+4 & 1.5 \\ -4 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s^2+4s+6)} \begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix}$$

$$[sI - A]^{-1}B = \frac{1}{s^2+4s+6} \begin{bmatrix} 2s \\ 8 \end{bmatrix}$$

$$C[sI - A]^{-1}B = \frac{1}{s^2+4s+6} [1.5 \quad 0.625] \begin{bmatrix} 2s \\ 8 \end{bmatrix}$$

$$T(s) = \frac{3s+5}{s^2+4s+6}$$

46. Red (R), Green (G) and Blue (B) Light Emitting Diodes (LEDs) were fabricated using –n junctions of three different inorganic semiconductors having different band-gaps. The built-in voltages of red, green and blue diodes are V_R, V_G and V_B respectively. Assume donor and acceptor doping to be the same (N_A and N_D respectively) in the p and n sides of all the three diodes. Which one of the following relationships about the built-in voltages is TRUE?

A. $V_R > V_G > V_B$ B. $V_R < V_G < V_B$

C. $V_R = V_G = V_B$ D. $V_R > V_G < V_B$

Ans. B

Sol. We know the relation of wavelengths,

$\lambda_R > \lambda_G > \lambda_B$

Energy gap is related to wavelength as follows,

$$E_g \propto \frac{1}{\lambda}$$

So, $E_{gR} < E_{gG} < E_{gB}$

Materials with high energy gap will have high built-in voltages, when doping concentrations are same.

So, $V_R < V_G < V_B$

47. Let x(t) be a periodic function with period T = 10. The Fourier series coefficients for this series are denoted by a_k, that is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

The same function x(t) can also be considered as a periodic function with period T′ = 40. Let b_k be the Fourier series coefficients when period is taken as T′. If $\sum_{k=-\infty}^{\infty} |a_k| = 16$, then $\sum_{k=-\infty}^{\infty} |b_k|$ is equal to

A. 256 B. 64

C. 16 D. 4

Ans. C

Sol. There is no change in the value of Fourier series coefficients if there is a change in time period or frequency.

So, $b_k = a_k$

$$\sum_{k=-\infty}^{\infty}|b_k| = \sum_{k=-\infty}^{\infty}|a_k| = 16$$

48. Let the input be u and the output be y of a system, and the other parameters are real constants. Identify which among the following systems is not a linear system:

A. $\frac{d^3y}{dt^3}+a_1\frac{d^2y}{dt^2}+a_2\frac{dy}{dt}+a_3y=b_3u+b_2\frac{du}{dt}+b_1\frac{d^2u}{dt^2}$ (with initial rest conditions)

B. $y(t)=\int_0^t e^{\alpha(t-\tau)}\beta u(\tau)d\tau$

C. y = au + b, b ≠ 0

D. y = au

Ans. C

Sol. Presence of a constant term introduces non-linearity.

Thus, y = au + b, b ≠ 0 is a non-linear system.

49. Let M be a real 4 × 4 matrix. Consider the following statements:

S1: M has 4 linearly independent eigenvectors.

S2: M has 4 distinct eigenvalues.

S3: M is non-singular (invertible).

Which one among the following is TRUE?

A. S1 implies S2 B. S1 implies S3

C. S2 implies S1 D. S3 implies S2

Ans. C

Sol. Eigen vectors corresponding to distinct eigen values are linearly independent.

So, "S2 implies S1".

50. Let $f(x,y)=\frac{ax^2+by^2}{xy}$, where a and b areconstants. If $\frac{\partial f}{\partial x}=\frac{\partial f}{\partial y}$ at x = 1 and y = 2, then the relation between a and b is

A. $a=\frac{b}{4}$ B. $a=\frac{b}{2}$

C. a = 2b D. a = 4b

Ans. D

Sol. Given in question,

$$f(x,y)=\frac{ax^2+by^2}{xy}=a\left(\frac{x}{y}\right)+b\left(\frac{y}{x}\right)$$

$$\left.\frac{\partial f}{\partial x}\right|_{(1,2)}=\left[\frac{a}{y}-\frac{by}{x^2}\right]_{(1,2)}=\frac{a}{2}-2b$$

$$\left.\frac{\partial f}{\partial y}\right|_{(1,2)}=\left[-\frac{ax}{y^2}+\frac{b}{x}\right]_{(1,2)}=\frac{a}{4}+b$$

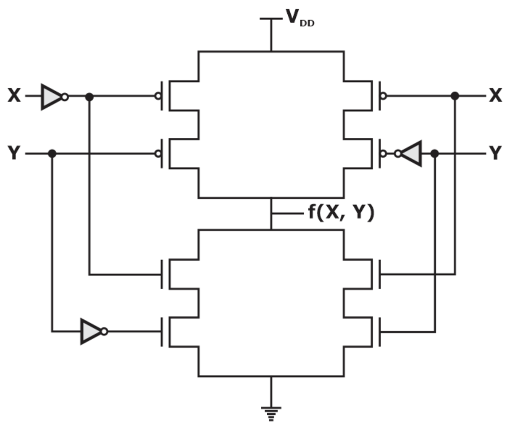
$$\frac{\partial f}{\partial x}=\frac{\partial f}{\partial y}$$

So, $\frac{a}{2}-2b=-\frac{a}{4}+b$

$$\frac{3a}{4}=3b$$

a = 4b

51. The logic f(X, Y) realized by the given circuit is

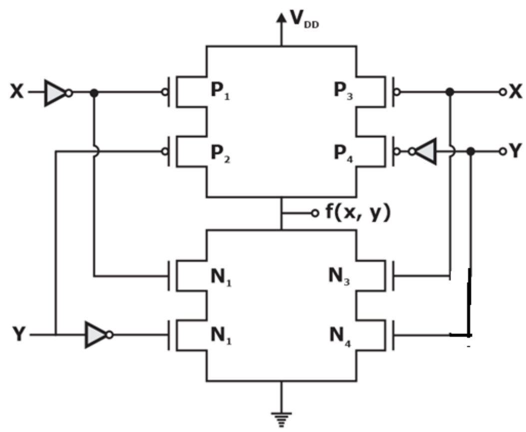


A. NOR B. AND

C. NAND D. XOR

Ans. D

Sol.



Truth table of the logic circuit,

X	Y	P_1	P_2	P_3	P_4	N_1	N_2	N_3	N_4	f(X, Y)
0	0	OFF	ON	ON	OFF	ON	ON	OFF	OFF	0
0	1	OFF	OFF	ON	ON	ON	OFF	OFF	ON	1
1	0	ON	ON	OFF	OFF	OFF	ON	ON	OFF	1
1	1	ON	OFF	OFF	ON	OFF	OFF	ON	ON	0

$f(X, Y) = \bar{X}Y + X\bar{Y} = X \oplus Y$

This denotes the XOR function.

52. A function F(A, B, C) defined by three Boolean variables A, B and C when expressed as sum of products is given by

$$F = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot\bar{C}$$

where, $\bar{A}$, $\bar{B}$, and $\bar{C}$ are the complements of the respective variables. The product of sums (POS) form of the function F is

A. $F = (A + B + C)\cdot(A + \bar{B} + C)\cdot(\bar{A} + B + C)$

B. $F = (\bar{A} + \bar{B} + \bar{C})\cdot(\bar{A} + B + \bar{C})\cdot(A + \bar{B} + \bar{C})$

C. $F = (A + B + \bar{C})\cdot(A + \bar{B} + \bar{C})\cdot(\bar{A} + B + \bar{C})\cdot(\bar{A} + \bar{B} + C)\cdot(\bar{A} + \bar{B} + \bar{C})$

D. $F = (\bar{A} + \bar{B} + C)\cdot(\bar{A} + B + C)\cdot(A + \bar{B} + C)\cdot(A + B + \bar{C})\cdot(A + B + C)$

Ans. C

Sol. We know that SOP and POS contain exhaustive terms in their expression. Any expression that is present in the SOP would not be present in the POS form. Thus,

$$F = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot\bar{C}$$

F(A,B,C) = Σm (000, 010, 100) = Σm (0,2,4)

$= \prod M$ (1,3,5,6,7)

$$F = (A + B + \bar{C})\cdot(A + \bar{B} + \bar{C})\cdot(\bar{A} + B + \bar{C})\cdot(\bar{A} + \bar{B} + C)\cdot(\bar{A} + \bar{B} + \bar{C})$$

53. A discrete-time all-pass system has two of its poles at $0.25\angle 0°$ and $2\angle 30°$ Which one of the following statements about the system is TRUE?

A. It has two more poles at $0.5\angle 30°$ and $4\angle 0°$

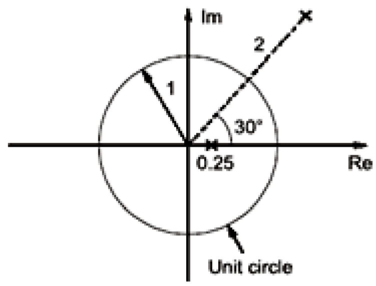
B. It is stable only when the impulse response is two-sided.

C. It has constant phase response over all frequencies.

D. It has constant phase response over the entire z-plane.

Ans. B

Sol. According to data given, we can draw the poles in z-domain as follows,



For the system to be stable, ROC should include the unit circle. From the given pole pattern, it is clear that to make the system stable, the ROC should be two-sided and hence the impulse response of the system should be also two-sided.

54. In a p-n junction diode at equilibrium, which one of the following statements is NOT TRUE?

A. The hole and electron diffusion current components are in the same direction.

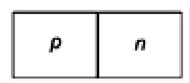
B. The hole and electron drift current components are in the opposite direction.

C. On an average, holes and electrons drift in opposite direction.

D. On an average, electrons drift and diffuse in the same direction.

Ans. D

Sol. The general direction of carrier movement can be denoted as follows for pn junction,



→ hole diffusion (→ hole diffusion current direction)

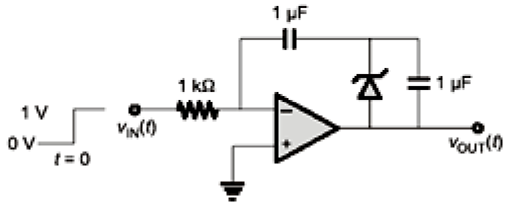
← electron diffusion (→ electron diffusion current direction)

→ hole drift direction (→ hole drift current direction)

→ electron drift direction (← electron drift current direction)

so only option D is incorrect

55. In the circuit shown below, the op-amp is ideal and Zener voltage of the diode is 2.5 volts. At the input, unit step voltage is applied i.e. $V_{IN}(t) = u(t)$ volts. Also at t = 0, the voltage across each of the capacitors is zero



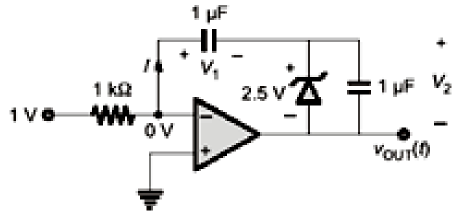
The time in milliseconds, at which the output voltage crosses – 10 V is

A. 2.5 B. 5

C. 7.5 D. 10

Ans. C

Sol. For t > 0,



$$I = \frac{1V}{1k\Omega} = 1\,mA$$

from t=0;both the capacitor charges and the zener is off (open circuited)

the capacitor across zener will charge upto 2.5V after that zener will behave as voltage regulator since it will go in breakdown region.

$v(t) = \frac{1}{C}\int_0^t i\,dt$ for v(t)=2.5 V t=2.5msec

Till t = 2.5 msec, both V_1 and V_2 will increase as C_1 and C_2 will get charged and after t = 2.5 msec, V_1 = 2.5 V and V_2 increases with time as C_1 is completely charged whereas C_2 is not completely charged.

So, when $V_{out}(t)$ = −10 V, V_1 = 7.5 V

So, $\frac{1}{1\mu F}\int_0^t (1\,mA)\,dt = 7.5$ V

$10^3 t = 7.5$

t = 7.5 m sec

56. A good transimpedance amplifier has

A. low input impedance and high output impedance

B. high input impedance and high output impedance

C. high input impedance and low output impedance

D. low input impedance and low output impedance

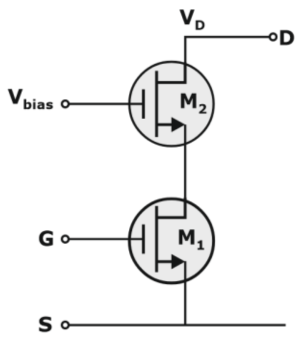
Ans. D

Sol. A good transimpedance amplifier should have low input impedance and low output impedance

57. Two identical nMOS transistors M_1 and M_2 are connected as shown below. The circuit is used as an amplifier with the input connected between G and S terminals and the output taken between D and S terminals, V_{bias} and V_D are so adjusted that both transistors are in saturation. The transconductance of this combination is defined as

$g_m = \frac{\partial i_D}{\partial V_{GS}}$ while the output resistance is

$r_0 = \frac{\partial V_{DS}}{\partial i_D}$, where i_D is the current flowing into the drain of M_2. Let g_{m1} , g_{m2} be the transconductances and r_{o1} , r_{o2} be the output resistance of transistors M_1 and M_2 respectively



Which of the following statements about estimates for g_m and r_o is correct?

A. $g_m \approx g_{m1} \cdot g_{m2} \cdot r_{o2}$ and $r_o \approx r_{o1} + r_{o2}$

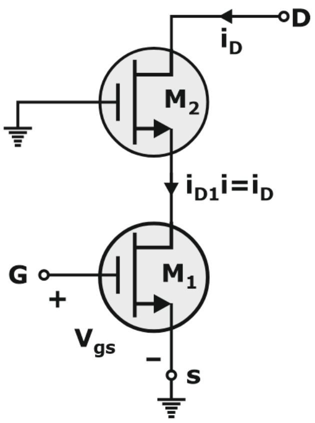
B. $g_m \approx g_{m1} + g_{m2}$ and $r_0 \approx r_{01} + r_{o2}$

C. $g_m \approx g_{m1}$ and $r_o \approx r_{01} \cdot g_{m2} \cdot r_{o2}$

D. $g_m \approx g_{m1}$ and $r_o \approx r_{o2}$

Ans. C

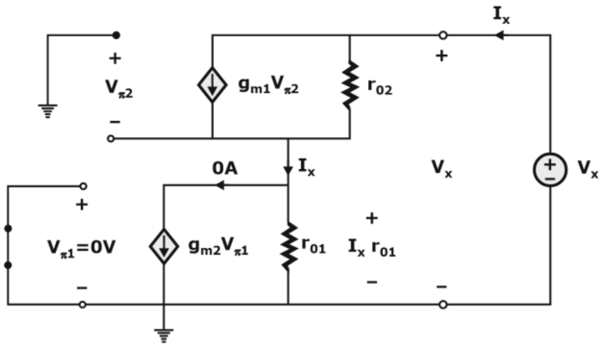
Sol.



Overall g_m can be calculated as follows,

$$g_m = \frac{\Delta I_D}{\Delta V_{in}} = \frac{i_D}{V_{gs}} = \frac{i_{D1}}{V_{gs}} = g_{m1}$$

To calculate r_0: we need to draw the small signal equivalent of the arrangement,



$V_{\pi 2} = -I_x r_{01}$

$$I_x = g_{m2}\ V_{\pi 2} + \frac{(V_x - I_x r_{01})}{r_{02}}$$

$$I_x = -g_{m2} r_{01}\ I_x + \frac{V_x}{r_{02}} - I_x \frac{r_{01}}{r_{02}}$$

$$V_x = r_{02}\left[1 + r_{01}\ g_{m2} + \frac{r_{01}}{r_{02}}\right] I_x$$

$$r_o = \frac{V_x}{I_x} = r_{01} + r_{02} + r_{01}r_{02}g_{m2} \approx r_{01}r_{02}g_{m2}$$

Thus, overall $g_m = g_{m1}$ and overall $r_o = r_{o1}r_{o2}g_{m2}$.

58. Consider $p(s) = s^3 + a_2s^2 + a_1s + a_0$ with all real coefficients. It is known that its derivative has no real roots. The number of real roots of p(s) is

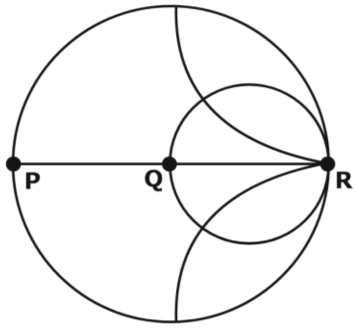
A. 0 B. 1

C. 2 D. 3

Ans. B

Sol. Here, p(s) has 3 roots whereas p′(s) has 2 roots and none of which are real. Thus, p(s) has to have 1 real and 2 complex roots.

59. The points P, Q and R shows on the Smith chart (normalized impedance chart) in the following figure represent:



A. P : Open Circuit, Q : Short Circuit, R : Matched Load

B. P : Open Circuit, Q : Matched Load, R : Short Circuit

C. P : Short Circuit, Q : Matched Load, R : Open Circuit

D. P : Short Circuit, Q : Open Circuit, R : Matched Load

Ans. C

Sol. For Short circuit, z = Re(z) = Im(z) = 0; Point "P"
For Open circuit, z = ∞; Point "R"
For Matched load, z = Re(z) = 1 and Im(z) = 0; Point "Q"

P : Short Circuit, Q : Matched Load, R : Open Circuit

60. The Nyquist stability criterion and the Routh criterion both are powerful analysis tools for determining the stability of feedback controllers. Identify which of the following statements is FALSE.

A. Both the criteria provide information relative to the stable gain range of the system.

B. The general shape of the Nyquist plot is readily obtained from the Bode magnitude plot for all minimum-phase systems.

C. The Routh criterion is not applicable in the condition of transport lag, which can be readily

handled by the Nyquist criterion.
D. The closed-loop frequency response for a unity feedback system cannot be obtained from the Nyquist plot.

Ans. D

Sol. (A) is true.
(B) is true as in a minimum-phase system, Bode magnitude plot is enough to obtain a general approximation of its Nyquist plot.
(C) Routh criterion can be applied to any system to check the stability of a system but a transport lag controller can only by explained using Nyquist Criterion.
(D) We can obtain closed-loop frequency response for Unity Feedback system easily by substituting s = jω, and draw the plot for different values of ω. Usually this is not done as it is not necessary as OLTF is enough to comment on the stability. Thus, (D) is false.

61. If the number 715?423 is divisible by 3 (? denotes the missing digit in the thousandths place), then the smallest whole number in the place of ? is ________.

A. 0 B. 2
C. 5 D. 6

Ans. B

Sol. A number is divisible by 3 if the sum of all digits is be divisible by 3.
7 + 1 + 5 + ? + 4 + 2 + 3 = 22 + ?
Next numbers after 22 which are divisible by 3 are 24, 27, 30 etc.
Minimum value of ? that would make the given number divisible by 3 is 2 as 24 is divisible by 3.

62. "Even though there is a vast scope for its ___________, tourism has remained a/an ___________ area."
The words that best fill the blanks in the above sentence are

A. improvement, neglected
B. rejection, approved
C. fame, glum
D. interest, disinterested

Ans. A

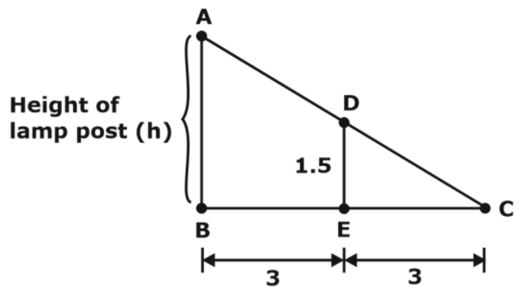
Sol. Even though there is a vast scope for its improvement, tourism has remained a neglected area.
Only these set of words give a meaning to the given sentence.

63. A 1.5 m tall person is standing at a distance of 3 m from a lamp post. The light from the lamp at the top of the post casts her shadow. The length of the shadow is twice her height. What is the height of the lamp post in meters?

A. 1.5 B. 3
C. 4.5 D. 6

Ans. B

Sol. If we make a figure using the information provided in the question,



Here, L (ACB) = L (DCE), thus

$$\tan L(ACB) = \tan L(DCE)$$
$$\frac{AB}{BC} = \frac{DE}{EC}$$
$$\frac{h}{6} = \frac{1.5}{3}$$
$$h = 3\text{meters}$$

64. "By giving him the last __________ of the cake, you will ensure lasting ___________ in our house today."
The words that best fill the blanks in the above sentence are

A. peas, piece B. piece, peace
C. peace, piece D. peace, peas

Ans. B

Sol. Piece means 'slice' and Peace means 'silence'.

65. What is the value of $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+\ldots$?

A. 2 B. $\frac{7}{4}$

C. $\frac{3}{2}$ D. $\frac{4}{3}$

Ans. D

Sol. This is an infinite GP.

Sum of infinite G.P is given by,

$S = \frac{a}{1-r}$, where first term (a) = 1 and common ratio $(r) = \frac{1}{4}$

$$\frac{1}{1-\frac{1}{4}} = \frac{1}{3/4} = \frac{4}{3}$$

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