Important Questions On Quantum Mechanics

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1. For a H-atom, how much energy is needed to remove an electron from the n = 8 state of a hydrogen atom?
   A. -0.32 eV
   B. -0.21 eV
   C. -4.32 eV
   D. None of the above

2. Calculate the maximum wavelength that hydrogen in its ground state can absorb. What would be the next smallest wavelength that would work?
   A. 122 nm ,103 nm
   B. 120 nm, 110 nm
   C. 130 nm, 103 nm
   D. 90 nm, 103 nm

3. A microscope using suitable photons is used to locate an electron in an atom within a distance of $0.1\mathring{A}$. Determine the uncertainty involved in the measurement of its velocity? (h = 6.626 × 10^{-34} Js, mass of electron = 9.1 × 10^{-31} kg).
   A. 5.79×10^{5} m/s
   B. 5.79×10^{4} m/s
   C. 5.79×10^{6} m/s
   D. 5.79×10^{7} m/s

4. A cricket ball has a weight of 100 g is located within 1 nm. Determine the uncertainty in the velocity.
   A. 5.27×10^{-23} m/s
   B. 5.27×10^{-25} m/s
   C. 5.27×10^{-24} m/s
   D. 5.27×10^{-27} m/s

5. Calculate the wavelength of light emitted when the electron in a hydrogen atom undergoes transition from an energy level with n = 4 to an energy level with n = 2.
   A. 500 nm
   B. 684 nm
   C. 486 nm
   D. 328 nm

6. The function $N_1(a^2 - x^2)$ is defined for the interval x = -a to x = +a. Determine the value of N_1.
   A. 3/2a
   B. $\sqrt{15}/4a^{5/2}$
   C. $\sqrt{5}/2a^{3/2}$
   D. None of the above.

7. An electron is confined in a 1- D box having a length of 1 Å. If electron falls from excited state to ground state, what energy it will emit?

   A. 120.8 eV
   B. 112.8 eV
   C. 210.7 eV
   D. 320.8 eV

8. An electron is confined to a one-dimensional box of length 0.14 nm has a ground-state energy corresponding to the radiation of wavelength about 70 nm. Benzene, as a rough approximation, may be a two-dimensional box that encompasses the regular hexagonal shape. The C — C bond length in benzene is 0.14 nm due to which side of the box would be about 0.28 nm. Estimate wavelength for transition from ground state to first excited state of benzene, assuming that it is π-bonding electron that are involved.

   A. 100 nm
   B. 93 nm
   C. 87 nm
   D. 78 nm

9. The wave function of a state of the hydrogen atom is:

   $$\psi = \psi_{200} + 2\psi_{211} + 3\psi_{210} + \sqrt{2}\psi_{211}$$

   Here, ψ_{nlm} denotes the normalized eigen function of the state with quantum numbers n, l and m in the usual notation. Calculate the expectation value of L_z in the state ψ.

   A. $\frac{15\hbar}{16}$ B. $\frac{11\hbar}{16}$ C. $\frac{3\hbar}{8}$ D. $\frac{\hbar}{8}$

10. Given the usual canonical commutation relations, the commutator [A, B] of $A = i(xp_y - yp_x)$ and $B = (yp_z + zp_y)$ is :

    A. $\hbar(xp_z - p_x z)$

    B. $-\hbar(xp_z - p_x z)$

    C. $\hbar(xp_z + p_x z)$

    D. $-\hbar(xp_z + p_x z)$

Answer Key:

1.B

2. A

3. C

4. B

5. C

6. B

7. B

8. B

9. D

10.D

Solutions:

Solution 1. For H-atom, the expression of energy in eV is:

$E=-13.6\ eV/n^2$

$=-13.6\ eV/64$

$=-0.21\ eV$

Solution 2. For hydrogen atom, the wavelength is given by

$$\frac{1}{\lambda}=R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

For ground state absorb, $n_1 = 1$

For maximum wavelength, $n_2 = 2$ so,

$$\frac{1}{\lambda_{max}}=R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=1.097\times10^7\times\frac{3}{4} \text{ or } \lambda_{max}=122nm$$

For next smaller wavelength, $n_2 = 3$,

Now,

$$\frac{1}{\lambda}=1.097\times10^7\left(1-\frac{1}{9}\right) \text{ or } \lambda=103nm$$

Solution 3. According to Heisenberg's Uncertainty principle:

$$\Delta x \Delta p = \frac{h}{2\pi}$$

m = 9.11 × 10^{-31} kg

h = 6.626 × 10^{-34} Js

So,

$$\Delta v \geq \frac{h}{4\pi m \Delta x}$$

$$\Delta v \geq \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 9.11 \times 10^{-31} \times 0.1 \times 10^{-10}}$$

$$\Delta v \geq 0.579 \times 10^{7}\ ms^{-1}$$

$$\Delta v \geq 5.79 \times 10^{6}\ ms^{-1}$$

Solution 4. According to uncertainty principle

$$\Delta x \,.\, m\Delta v = \frac{h}{4\pi} \text{ or } \Delta v = \frac{h}{4\pi \times m \times \Delta x}$$

$$h = 6.626 \times 10^{-34} kgm^2s^{-1}, m = 100g = 0.1kg, \Delta x = 1nm = 10^{-9}m, \pi = 3.143$$

$$\Delta v = \left[\frac{(6.626 \times 10^{-34} kgm^2s^{-1}}{4 \times 3.143 \times x(0.1kg)(10^{-9}m)}\right] = 5.27 \times 10^{-25} ms^{-1}.$$

Solution 5. The expression for the wavelength of radiation is,

$$\frac{1}{\lambda} = R(\frac{1}{n_1^2} - \frac{1}{n_2^2})$$

Substitute values in the above expression,

$$\frac{1}{\lambda} = 109677 \times (\frac{1}{2^2} - \frac{1}{4^2}) = 109677 \times (\frac{1}{4} - \frac{1}{16}) = 109677 \times (\frac{12}{64})$$

$$\lambda = 4.86 \times 10^{-5} cm = 486 \times 10^{-9} m = 486nm.$$

Solution 6. $\int_{-a}^{+a} N_1\left(a^2-x^2\right)N_1\left(a^2-x^2\right)dx=1$

$$\Rightarrow \quad N_1^2\int_{-a}^{+a}\left(a^4+x^4-2a^2x^2\right)dx=1$$

$$\Rightarrow \quad \left[\int_{-a}^{+a}a^4dx+\int_{-a}^{+a}x^4dx-\int_{-a}^{+a}2a^2x^2dx=1\right]$$

$$\Rightarrow \quad N_1^2\left[a^4\left[x\right]_{-a}^{+a}+\left[\frac{x^5}{5}\right]_{-a}^{+a}-2a^2\left[\frac{x^3}{3}\right]_{-a}^{+a}\right]=1$$

$$\Rightarrow \quad N_1^2\left[a^4(a+a)+\left(\frac{a^5}{5}+\frac{a^5}{5}\right)-2a^2\left[\frac{a^3}{3}+\frac{a^3}{3}\right]\right]=1$$

$$\Rightarrow \quad N_1^2\left[2a^5+\frac{2a^5}{5}-4\frac{a^5}{3}\right]=1 \qquad \Rightarrow N_1^2\left[\frac{30a^5+6a^5-20a^5}{15}\right]=1$$

$$N_1^2\left(\frac{16a^5}{15}\right)=1$$

$$N_1^2=\frac{15}{16a^5}$$

$$N_1=\frac{\sqrt{15}}{4a^{5/2}}$$

Solution 7. Length of box, $\ell=1Å=10^{-10}\,m$

Energy of 1st excited state, $E_1=\frac{1^2\,h^2}{8m\ell^2}=\frac{h^2}{8m\ell^2}$

Energy of 2nd excited state, $E_2=\frac{2^2\,h^2}{8m\ell^2}=\frac{4h^2}{8m\ell^2}$

$$\therefore\ \Delta E=E_2-E_1=(4-1)\frac{h^2}{8m\ell^2}=(3\times 37.6)eV = 112.8\text{ eV}$$

Solution 8. For the one-dimensional box, $E=\frac{h^2}{8m\ell^2}n^2$

Thus, the ground state energy E_1 in a one-dimensional box of length 0.14 nm is

$$E_1=\frac{h^2}{8m(0.14\,nm)^2}$$

For the two-dimensional square box, $E_2 = \frac{h^2}{8m\ell^2}(n_1^2 + n_2^2)$

Now, since l = 0.28 nm, we have

$$E_2 = \frac{h^2}{8m(2\times 0.14\text{nm})^2}(n_1^2 + n_2^2) = \frac{E_1}{4}(n_1^2 + n_2^2)$$

The various energy levels are as follows.

n_1	n_2	E_2	
1	1	$E_1/2$	
2	1	$(5/4)E_1$	degenerate
1	2	$(5/4)E_1$	degenerate
2	2	$2E_1$	

The first three energy levels will be doubly occupied in the ground state and hence the first state is obtained when the electron is promoted from $n_1 = 1$, $n_2 = 2$ state to $n_1 = 2$, $n_2 = 2$ state. Thus,

$$\Delta E = 2E_1 - \frac{5}{4}E_1 = \frac{3}{4}E_1$$

Since the wavelength is inversely proportional to energy, the corresponding wavelength would be (4/3) λ, i.e.

$$\frac{4}{3}\times 70\text{nm} = 93\text{nm}$$

Solution 9. The given wave function is $\psi = \psi_{200} + 2\psi_{211} + 3\psi_{210} + \sqrt{2}\psi_{21-1}$

The expectation value of L_z is given by:

$$\langle L_z \rangle = \frac{\langle \psi | L_z | \psi \rangle}{\langle \psi | \psi \rangle} = \frac{\sum_l |C_l|^2 m_l \hbar}{\sum_l |C_l|^2} = \frac{1\times 0\times\hbar + 4\times 2\times\hbar + 9\times 0\times\hbar + 2\times(-1)\times\hbar}{1+4+9+2}$$

$$= \frac{4\hbar - 2\hbar}{16} = \frac{2\hbar}{16} = \frac{\hbar}{8}$$

Solution 10. $A = i(xp_y - yp_x)$ & $B = (yp_z + zp_y)$

$[A, B] = [i(xp_y - yp_x), (yp_z + zp_y)]$

By using the distributive properties,

$[AB, C] = A[B, C] + [A, C]$ and $[A, BC] = B[A, C] + [A, B]C$

And associative properties [A, B + C] = [A, B] ± [A, C]

The above equation can be simplified as:

$[A, B] = i[(xp_y - yp_x), yp_z] + i[(xp_y - yp_x), zp_y]$

$= i[xp_y, yp_z] - i[yp_x, yp_z] + i[xp_y, zp_y] - i[yp_x, zp_y]$

[* out of 4 communication relations 2nd and 3rd will be zero as there will be no canonically conjugate pair term.

$= ix\,[p_y, yp_z] + i[x, yp_z]\,p_y - iy[p_x, zp_y] - i[y, zp_y]p_x$

[2nd and 3rd will be also zero, reason will be same as above]

$[A, B] = ix[p_y, yp_z] - i[y, zp_y]p_x$

$$= ixy\underset{\shortparallel}{[p_y, p_z]} + ix[p_y, y]p_z - iz[y, p_y]p_x - i\underset{\shortparallel}{[y, z]}p_y p_x$$

$$[A,B] = ix(-i\hbar)p_z - iz(i\hbar)p_x$$

$= \hbar(xp_z + zp_x)$, (as $zp_x = p_x z$)

$= \hbar(xp_z + p_x z)$