Roll No. | | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilator

Question Booklet Series **X**

PAPER–II

Question Booklet No.
(Identical with OMR Answer Sheet Number)

Subject Code : 16

PHYSICAL SCIENCES

Time : 2 Hours *Maximum Marks: 200*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) Verify whether the Question Booklet No. is identical with OMR Answer Sheet No.; if not, the full set is to be replaced.
   (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of One hundred (100) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D) . You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D) , where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except in the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Black Ball point pen.**
11. **Use of any calculator, mobile phone, electronic devices/gadgets etc. is strictly prohibited.**
12. **There is no negative marks for incorrect answer.**

PHYSICAL SCIENCES

Paper II

1. The value of the integral $I = \int \frac{6x^2 + \cos x}{x^3 + \sin x} dx$, by assuming that the integration constant is zero, is

(A) $2\, ln\, (x + \cos x)$

(B) $3\, ln\, (x^3 + \sin x)$

(C) $3\, ln\, (x^3 + \cos x)$

(D) $2\, ln\, (x^3 + \sin x)$

2. What is the derivative of the function $y = g^x$ with respect to x?

(A) $x\, g^{x-1}$

(B) $x \ln g$

(C) $g^x \ln g$

(D) $x^g \ln g$

3. If $u = \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$, then θ must be equal to

(A) $i \log \tanh\left(\frac{\pi}{4} + \frac{iu}{2}\right)$

(B) $-i \log \tan\left(\frac{\pi}{4} + \frac{iu}{2}\right)$

(C) $\frac{i}{4} \log \tan\left(\frac{\pi}{4} + \frac{u}{2}\right)$

(D) $-i \log \tan\left(\frac{\pi}{4} + iu\right)$

4. If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, then e^{Ax} will be

(A) $\begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$

(B) $\begin{pmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{pmatrix}$

(C) $\begin{pmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{pmatrix}$

(D) $\begin{pmatrix} \cos x & \sin x \\ \sin x & -\cos x \end{pmatrix}$

5. The signal $f(t)$ is defined by,

$$f(t) = \begin{cases} 1, & -1 \le t \le 1 \\ 0, & \text{Otherwise} \end{cases}.$$

Two of the angular frequencies at which its Fourier transform becomes zero are,

(A) $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

(B) π and 2π

(C) 0 and π

(D) 2π and $\frac{5\pi}{2}$

6. Probability density of a random variable x is given by

$\rho(x) = \frac{1}{10}\left[\delta(x+1) + 2\,\delta(x) + 3\,\delta(x-1) + 4\,\delta(x-2)\right]$, what is the mean of x?

(A) 1·5

(B) 0·8

(C) 1·2

(D) 1·0

7. Consider the two functions of real variables x, y

$$f(x,y) = \left[x^3 - 3y^2 x + iy\left(3x^2 - y^2\right)\right]^{12}$$

$$g(x,y) = \left(x^2 - y^2 - 2ixy\right)^2 \left(x + iy - 4\right)^6$$

If $z = x + iy$ is a complex variable, then

(A) $f(x, y)$ is non-analytic while $g(x, y)$ is analytic.

(B) $g(x, y)$ is non-analytic while $f(x, y)$ is analytic.

(C) neither of $f(x, y)$ and $g(x, y)$ is analytic.

(D) Both are analytic

8. The solution of the differential equation $\frac{dy}{dx} = 3y + 1$, with $y = 0$ when $x = 0$ is

(A) $\frac{1}{3}\left(1 - e^{3x}\right)$

(B) $\frac{1}{3}\left(1 - e^{-3x}\right)$

(C) $\frac{1}{3}\left(e^{-3x} - 1\right)$

(D) $\frac{1}{3}\left(e^{3x} - 1\right)$

9. The complex number $N_1 = 1 + \sqrt{3}\,i$, and the complex number $N_2 = 1 - \sqrt{3}\,i$. Then the value of $\left(\frac{N_1}{N_2}\right)^{10}$ is

(A) $-\frac{\sqrt{3}}{2} + \frac{3}{\sqrt{2}}\,i$

(B) $-\frac{1}{2} + \frac{\sqrt{3}}{2}\,i$

(C) $-\frac{1}{4} - \frac{\sqrt{2}}{3}\,i$

(D) $-\frac{1}{2} + \frac{\sqrt{2}}{3}\,i$

10. For the given vectors $\vec{V}_1 = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{V}_2 = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{V}_3 = 3\hat{i} - 4\hat{j} + 2\hat{k}$, what is the projection of $\vec{V}_1 + \vec{V}_3$ in the direction of $\vec{V}_2$?

(A) $\frac{3}{8}$

(B) $\frac{17}{3}$

(C) $\frac{-3}{8}$

(D) $\frac{-17}{3}$

11. If a 2×2 real matrix has a positive value of the determinant and negative value of trace, then which of the following statements is always true?

(A) One of the eigenvalue is positive and the other is negative.

(B) Both the eigenvalues are real and negative.

(C) Real parts of both the eigenvalues are positive.

(D) Real parts of both the eigenvalues are negative.

12. $\hat{A}$ is a hermitian operator in an n-dimensional vector space V. $\{\lambda_i\}$ is the set of eigenvalues of this operator, where $\lambda_i < \lambda_{i+1}$ for all i. Then for any $|\Psi\rangle \in V$

$\lim_{k\to\infty} \frac{\langle\Psi|\hat{A}^{k+1}|\Psi\rangle}{\langle\Psi|\hat{A}^{k}|\Psi\rangle}$ is

(A) λ_n

(B) λ_1

(C) $\frac{\lambda_1 + \lambda_2}{2}$

(D) $\frac{\sum_{i=1}^{n} \lambda_i}{n}$

13. A person in a jet plane is flying along the Equator due East with a speed of 450 m/s. The magnitude of the Coriolis acceleration is

(A) $0{\cdot}036\,\text{m/s}^2$

(B) $0{\cdot}048\,\text{m/s}^2$

(C) $1{\cdot}148\,\text{m/s}^2$

(D) $0{\cdot}0648\,\text{m/s}^2$

14. A particle of mass m and charge q moves in the equatorial plane of a magnetic dipole μ. It is given that the dipole's vector potential is $\vec{A} = \frac{\mu \sin\theta}{4\pi r^2}\hat{\varphi}$, where (r, θ, φ) denote the spherical polar coordinates. If, at $t = 0$, the particle starts from infinity with a speed v_0 and an impact parameter b, the instantaneous speed $v(t)$ is given by

(A) $v(t) = v_0$

(B) $v^2(t) = v_0^2 + \frac{2}{m}q\left|\vec{A}(\vec{r}(t))\right|$

(C) $v^2(t) = v_0^2 + \frac{2}{m}q\int_0^t \vec{A}(\vec{r}'(t'))\cdot\hat{r}(t')\,dt'$

(D) $v^2(t) = v_0^2 + \frac{2}{m}q\int_0^t \left[\vec{\nabla}\times\vec{A}(\vec{r}(t'))\right]\cdot\hat{r}(t')\,dt'$

15. A child of mass $3m$ sits on a swing, the base of which has mass m, while the rope (of length l) has a negligible mass. An older child pulls back the swing (with the child) until the rope makes an angle of half-a-radian with the vertical, and, then, pushes with a force mg along the arc of a circle until the rope is exactly vertical, and releases it. If the rope was always taut, for how long did the child push?

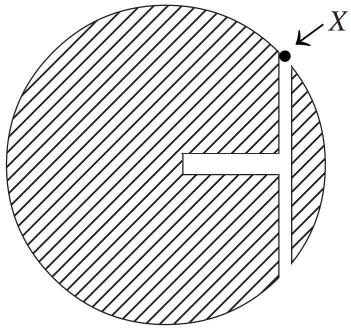
(A) $\frac{\pi}{3}\sqrt{\frac{l}{g}}$

(B) $\sqrt{\frac{l}{g}}$

(C) $\pi\sqrt{\frac{2l}{3g}}$

(D) $\sqrt{\frac{2l}{3g}}$

16. A narrow T-shaped tunnel is excavated inside the Earth as shown the figure below. A man X falls into the tunnel at one end as shown. Subsequently



(A) he will shoot out of the opposite end of the tunnel.

(B) he will execute simple harmonic motion in the tunnel and never come out of it.

(C) he will fall through the T-junction and reach the centre of the Earth.

(D) he will reach the opposite end of the tunnel and remain there.

17. The Sun is approximately 25,000 light-years away from the centre of the Milky Way and moves around it, in an approximately circular path, in roughly 170 million years. Given that sunlight takes approximately 8 min to reach the Earth, what is the ratio of the masses of the galaxy and the Sun?

(A) $1{\cdot}5\times10^{11}$

(B) 5×10^{11}

(C) $1{\cdot}5\times10^{12}$

(D) 5×10^{10}

18. If the gravitational force is assumed to vary inversely as the nth power of distance r, then the time period of a planet depends upon r as

(A) r^n

(B) r^{-n}

(C) $r^{\frac{n+1}{2}}$

(D) $r^{\frac{n-1}{2}}$

19. Which of the following quantities is invariant under Lorentz transformation?

(A) Charge density

(B) Charge

(C) Current

(D) Electric field

20. A relativistic particle of rest mass m_0 moving with velocity $\dot{x}_i$, its Lagrangian is $\left[\text{where } \gamma = \frac{1}{\sqrt{1-\frac{\dot{x}_i \dot{x}^i}{c^2}}}\right]$

(A) $L = \frac{1}{2} m_0 \dot{x}_i \dot{x}^i - \gamma(x_i)$

(B) $L = m_0 c^2 (\gamma - 1) - V(x_i)$

(C) $L = -\frac{1}{\gamma} m_0 c^2 - V(x_i)$

(D) $L = \frac{1}{2} m_0 \dot{x}_i \dot{x}^i \gamma - V(x_i)$

21. Which one is the expression of velocity (v) of a relativistic particle?

(A) $v = \frac{pc}{\sqrt{p^2 - m_0^2 c^2}}$

(B) $v = \frac{pc}{p^2 + m_0^2 c^2}$

(C) $v = \frac{2pc}{\sqrt{p^2 + m_0^2 c^2}}$

(D) $v = \frac{pc}{\sqrt{p^2 + m_0^2 c^2}}$

22. A particle leaving a cyclotron has a total relativistic energy of 10 GeV and momentum of 8 GeV/c. What is the rest mass of the particle?

(A) 0·25 GeV/c^2

(B) 1·20 GeV/c^2

(C) 2·00 GeV/c^2

(D) 6·00 GeV/c^2

23. The Lagrangian of a system is given by $L = \frac{1}{2}\dot{q}^2 + q\dot{q} - \frac{1}{2}q^2$. It corresponds to motion of a

(A) harmonic oscillator

(B) damped harmonic oscillator

(C) anharmonic oscillator

(D) system with unbound motion

24. The Lagrangian of a particle of mass m moving along the x-axis is given by $L = \frac{1}{2}m\dot{x}^2 - qx^2 - \alpha x$ with $q, \alpha > 0$. What type of motion the particle will execute?

(A) A harmonic oscillator with equilibrium point at (0, 0).

(B) A harmonic oscillator with equilibrium point on positive x-axis.

(C) A harmonic oscillator with equilibrium point on negative x-axis.

(D) An anharmonic oscillator with equilibrium point at (0, 0).

25. The Hamiltonian for a particle in one-dimension is given by $H(x,p) = \frac{p^2}{2m} + \lambda px + \frac{\lambda}{2}x^2$, where m, λ are constants. The corresponding Lagrangian is

(A) $L = \frac{m}{2}(\dot{x} - \lambda x)^2 - \frac{\lambda}{2}x^2$

(B) $L = \frac{m}{2}\dot{x}^2 - \frac{\lambda}{2}x^2$

(C) $L = \frac{m}{2}\dot{x}^2 - \lambda m x\dot{x} - \frac{\lambda}{2}x^2$

(D) $L = \frac{m}{2}(\dot{x} - \lambda x)^2 - \lambda m x\dot{x} - \frac{\lambda}{2}x^2$

26. The value of α and β for the canonical transformation $Q = q^\alpha \cos\beta p$, $P = q^\alpha \sin\beta p$ are

(A) $\alpha = \frac{1}{2}$, $\beta = 1$

(B) $\alpha = 1$, $\beta = \frac{1}{2}$

(C) $\alpha = \frac{1}{2}$, $\beta = 2$

(D) $\alpha = 2$, $\beta = \frac{1}{2}$

27. A particle of mass m moving in 3 dimensions is characterized by the Lagrangian

$L = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - \sigma J_z$, where σ is a constant and $\vec{J} = m\vec{r} \times \dot{\vec{r}}$ is the angular momentum. The Poisson bracket $[\dot{x}, \dot{y}]$ is

(A) $\frac{-2\sigma}{m}$

(B) 0

(C) $\frac{2\sigma}{m}$

(D) $\frac{\sigma}{m}$

28. A particle is executing small oscillations and its Lagrangian is

$L = \frac{1}{2}\dot{q}_1^2 + \frac{1}{2}\dot{q}_2^2 + \frac{1}{2}\dot{q}_3^2 - q_1^2 - q_2^2 - q_3^2 + \alpha q_1 q_2$. Which of the following one is not a small oscillation frequency for normal modes?

(A) $\sqrt{2-\alpha}$

(B) $\sqrt{2+\alpha}$

(C) $\sqrt{2}$

(D) $\sqrt{\alpha - 2}$

29. A Lambda hyperon $\Lambda°\left(1115\ MeV/c^2\right)$ cannot have the decay $\Lambda° \rightarrow \pi° + 1°^{n}$, where the pion has mass $135\,MeV/c^2$, because of conservation of

(A) energy

(B) baryon number

(C) angular momentum

(D) isospin

30. The ground state spin and parity of ^{17}O nucleus, as predicted by the shell model, are respectively

(A) $\frac{1}{2}$, even

(B) $\frac{5}{2}$, odd

(C) $\frac{5}{2}$, even

(D) $\frac{1}{2}$, odd

31. In the table below, the elementary particles $P - S$ are identified by the interactions they participate in, which are marked by tic (✓) marks. Dashes indicate non-participation.

Particle	Gravity	Weak	Electromagnetic	Strong
P	✓	✓	✓	✓
Q	✓	–	✓	✓
R	✓	✓	–	✓
S	✓	✓	✓	–

The particle which is not found in Nature is

(A) P

(B) Q

(C) R

(D) S

32. The quark wavefunction of a Δ^{++} particle is antisymmetric because of

(A) spin

(B) isospin

(C) strangeness

(D) colour

33. The universality of weak interactions can be inferred from a comparison of the lifetimes of

(A) isolated neutron and isolated pion ($\pi°$).

(B) alpha decay and muon decay.

(C) neutrino and isolated neutron.

(D) muon and isolated neutron.

34. A nuclear transition with nuclear spin and parity follows $\frac{3^+}{2} \rightarrow \frac{5^+}{2}$. The electric and magnetic multipole radiation from the nuclear transition is

(A) E1, M2, E3, M4

(B) M1, E2, M3, E4

(C) E1, E2, E3, E4

(D) M1, M2, E1, E2

35. In a Cooper pair there is

(A) pairing of a proton and a neutron mediated by an electron.

(B) pairing of two electrons mediated by phonon.

(C) pairing of two electrons mediated by photon.

(D) pairing of two protons mediated by an electron.

36. The potential energy function for the force between two atoms separated by 'x', in a diatomic molecule can be approximated by $U(x)=\frac{a}{x^{12}}-\frac{b}{x^{6}}$, where a and b are positive constants. The minimum of $U(x)$ occurs at

(A) $\left(\frac{a}{b}\right)^{5/6}$

(B) $\left(\frac{2a}{b}\right)^{1/5}$

(C) $\left(\frac{2a}{b}\right)^{1/2}$

(D) $\left(\frac{2a}{b}\right)^{1/6}$

37. The primitive translation vectors for a periodic lattice in three dimensions are $\vec{a}=2\hat{i}, \vec{b}=\hat{i}+2\hat{j}$ and $\vec{c}=\hat{k}$. The corresponding reciprocal lattice vectors are:

(A) $\vec{A}=\pi\hat{i}-\frac{\pi}{2}\hat{j}, \vec{B}=\pi\hat{j}, \vec{C}=2\pi\hat{k}$

(B) $\vec{A}=\pi\hat{j}, \vec{B}=\pi\hat{i}+\frac{\pi}{2}\hat{j}, \vec{C}=\pi\hat{k}$

(C) $\vec{A}=\pi\hat{k}, \vec{B}=\pi\hat{i}-\frac{\pi}{2}\hat{j}, \vec{C}=\pi\hat{j}$

(D) $\vec{A}=\frac{\pi}{2}\hat{k}, \vec{B}=\pi\hat{j}-\pi\hat{k}, \vec{C}=2\pi\hat{i}$

38. If n is the number of atoms in the unit cell of a cubic system, N_A and M_A are the Avogadro number and atomic weight respectively and ρ is the density of the element, then the lattice constant is given by

(A) $\left[\frac{M_A\rho}{nN_A}\right]^{1/3}$

(B) $\left[\frac{nN_A}{M_A\rho}\right]^{1/3}$

(C) $\left[\frac{nM_A}{N_A\rho}\right]^{1/3}$

(D) $\left[\frac{\rho N_A}{M_A n}\right]^{1/3}$

39. A one dimensional "crystal-lattice" of lattice spacing a and length L contains N uniformly spaced lattice points (atomic sites). Consider periodic boundary condition imposed on the wave function of an electron travelling in such a lattice. Taking 'spin' of an electron into account, the energy-band will be completely 'filled' when the band contains

(A) no quantum states.

(B) $2N$ quantum states.

(C) N quantum states.

(D) $\frac{N}{2}$ quantum states.

40. An electron with energy E and wave-vector $\vec{k}$ is moving in a periodic lattice. The energy dispersion relation is given by $\left[E - f(E)\right]\left[E - g(E,\vec{k})\right]=0.$

Where $f(E)$ is a function of energy alone and $g(E, \vec{k})$ depends on E and $\vec{k}$ both. The effective mass of the electron will be infinite for energies obtained by solving the equation,

(A) $g(E, k)=0$

(B) $f(E)=0$

(C) $E-f(E)=0$

(D) $E-g(E, k)=0$

41. An atom with one outer electron having angular momentum l is placed in a weak magnetic field. The number of energy level into which the higher angular momentum state splits is

(A) $2l+2$

(B) $2l+1$

(C) $2l$

(D) $2l-2$

42. The coherence length of laser light is

(A) directly proportional to the length of the active lasing medium.

(B) directly proportional to the width of the spectral line.

(C) inversely proportional to the width of the spectral line.

(D) inversely proportional to the length of the active lasing medium.

43. Electric dipole approximation of electromagnetic (EM) wave induced atomic transition is applicable when the

(A) wavelength of the inducing EM wave is much smaller than the atomic size.

(B) wavelength of the inducing EM wave is equal to the average radius of the valence electron.

(C) wavelength of the inducing EM wave is twice the value of the average radius of the valence electron.

(D) wavelength of the inducing the inducing EM wave is much larger than the atomic size.

44. The interatomic potential between H-atoms has a range of approximately 4Å. If a gas of H-atoms is in thermal equilibrium, what is the temperature T below which the atom-atom scattering is overwhelmingly dominated by the *S*-wave amplitude?

(A) 1K

(B) 10K

(C) 20 K

(D) 50 K

45. If the co-efficient of stimulated emission for a particular transition is $2{\cdot}1{\times}10^{19}\ m^3w^{-1}s^{-3}$ and the emitted photon is at wavelength 3000 Å, then the lifetime of the excited sate is approximately

(A) 20 ns

(B) 40 ns

(C) 60 ns

(D) 80 ns

46. Which of the following excited states of a hydrogen atom has the highest lifetime?

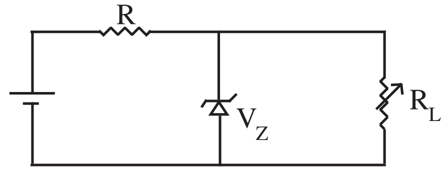
(A) 2p

(B) 2s

(C) 3s

(D) 3k

47. A 24V, 600mW, Zener diode is used for providing a 24V stabilized supply to a variable load.



Assuming for a proper zener action, a minimum of 10mA must flow through the Zener. If the input voltage is 32V. What would be the value of R and maximum load current?

(A) 320 Ω, 10 mA

(B) 400 Ω, 15 mA

(C) 400 Ω, 10 mA

(D) 320 Ω, 15 mA

48. If differential amplifier has a differential gain of 20,000, CMRR=80dB, then common mode gain is

(A) 2

(B) 1

(C) $\frac{1}{2}$

(D) 0

49. Consider the circuits shown in figures (a) & (b) below.

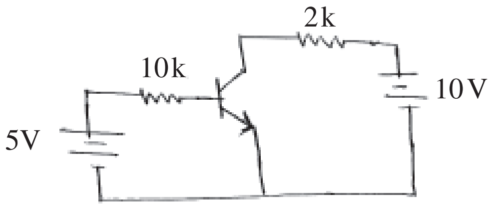


figure (a)

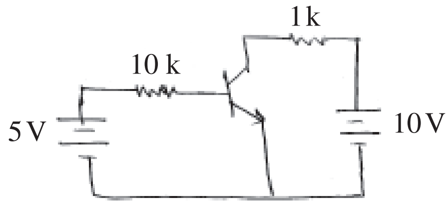


figure (b)

If the transistors in figures (a) and (b) have current gain (β_{dc}) of 100 and 10 respectively, then they operate in the

(A) active region and saturation region respectively.

(B) saturation region and active region respectively.

(C) saturation region in both cases.

(D) active region in both cases.

50. Consider the following Karnaugh map of four variables:

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$			1	1
$\bar{A}B$	1	1	1	1
AB	1	1	1	1
$A\bar{B}$		1		

The simplified expression of the above Boolean variables will be

(A) $B+\bar{A}C+A\bar{C}D$

(B) $A+\bar{B}C+A\bar{C}D$

(C) $B+A\bar{C}+\bar{A}CD$

(D) $B+\bar{A}C+AC\bar{D}$

51. A 4-bit analog to digital converter is designed to digitize analog signals ranging from 0V to 10V. For this converter, the binary output corresponding to an input of 9·33V is

(A) 1000
(B) 1100
(C) 1110
(D) 1111

52. Leakage current approximately doubles for every 10°C increase in the temperature of a transistor. If a Si transistor has its leakage current 45·25 μA at 90°, find I_{CBO} at 25°C.

(A) 500 μA
(B) 50 μA
(C) 500 nA
(D) 50 nA

53. Experimentally, it is observed that the heat capacity of crystalline EuO varies as $C_V = aT^{3/2} + bT^3$ for $0 \le T < 70K$. Then EuO is a/an

(A) paramagnet
(B) ferromagnet
(C) antiferromagnet
(D) ferrimagnet

54. You wish to pick a substance for a photocell operable with visible light. Which of the following will do (work function in parentheses)?

(A) Tungsten (4·5 eV)
(B) Barium (2·5 eV)
(C) Lithium (2·3 eV)
(D) Cesium (1·9 eV)

55. The largest analog output voltage from a 6-bit digital to analog converter (DAC) which produces 1·0 V output for a digital input of 010100 is

(A) 1·6 V
(B) 2·9 V
(C) 3·15 V
(D) 5·06 V

56. In order to obtain a solution of the differential equation $\frac{d^2v}{dt^2} - 2\frac{dv}{dt} + v_1 = 0$ involving voltages $v(t)$ and v_1, an operational amplifier circuit would require at least

(A) 2 OP-AMP integrators and one OP-AMP Adder.
(B) 2 OP-AMP differentiators and one OP-AMP Adder.
(C) 1 OP-AMP integrator and one OP-AMP Adder.
(D) 1 OP-AMP integrator, one OP-AMP differentiator and one OP-AMP Adder.

57. The correct matching pair of various types of measuring devices with its approximate measuring range is

(A) Pirani gauge .... 10^{-12}–10^{-3} torr
(B) Penning gauge .... 10^{-5}–10^{-1} torr
(C) Knudsen gauge .... 10^{-8}–10^{-2} torr
(D) Ionisation gauge .... 10^{-6}–10^{-3} torr

58. A wattmeter has a full scale range of 2500 W. It has an error ±1% of the true value. What would be the range of readings if true power is 1250 W?

(A) 1225 W – 1275 W
(B) 1245 W – 1255 W
(C) 1200 W – 1300 W
(D) 1237·5 W – 1262·5 W

59. Which of the followings is the correct statement(s) for neutron detection?

I. Neutron can cause flourescence directly.
II. Neutrons trigger Geiger counters.
III. Neutron counters are ionisation chambers with electrodes coated with boron.
IV. Neutrons can be detected with ^{3}He proportional counters.

(A) I & IV
(B) II & III
(C) III only
(D) III & IV

60. The mean and variance of a χ^2 distribution with 8 degrees of freedom are,

(A) 4, 8
(B) 8, 16
(C) 16, 8
(D) 8, 4

61. An electric field of 100 V/m is applied to a sample of n-type semiconductor whose Hall Coefficient is $-0{\cdot}0125\,\text{m}^3$. Assume electron mobility to be $0{\cdot}36\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$, the current density in the sample will be

(A) 2880 A/m^2
(B) 3280 A/m^2
(C) 3640 A/m^2
(D) 2460 A/m^2

62. For detection of neutron, the ionization chamber is filled with

(A) Phosphorus
(B) Benzene
(C) Sodium chloride
(D) Boron trifluoride

63. An LC circuit is of frequency of oscillation ω_0, uses a parallel plate capacitor with vacuum in between. The distance between the plates is now doubled and the gap filled with a material of dielectric constant k. The new frequency of oscillation is

(A) $\sqrt{\frac{2}{k}}\,\omega_0$

(B) $\frac{2}{k}\omega_0$

(C) $\frac{k}{2}\omega_0$

(D) $\sqrt{\frac{k}{2}}\,\omega_0$

64. A particle of charge q is released with a velocity $\vec{v}$ parallel to a current I flowing through a thin wire. The wire also carries a linear charge density λ. If the particle continues to travel parallel to the wire, then

(A) $\vec{v} = \frac{\lambda}{\mu_0 \in_0 I}$

(B) $\vec{v} = \frac{\lambda}{2\mu_0 \in_0 I}$

(C) $\vec{v} = \frac{2\lambda}{\mu_0 \in_0 I}$

(D) $\vec{v} = \frac{\pi\lambda}{\mu_0 \in_0 I}$

65. Consider a rectangular waveguide with transverse dimension 1m × 2m driven with an angular frequency $\omega = 10^9$ rad/sec. Which transverse electric (TE) modes will propagate in this waveguide?

(A) TE_{01}, TE_{11}, TE_{20}
(B) TE_{01}, TE_{10}, TE_{02}
(C) TE_{01}, TE_{10}, TE_{22}
(D) TE_{01}, TE_{10}, TE_{11}

66. Suppose the yz-plane forms a chargeless boundary between two media of permittivities E_{left} and E_{right}, where $E_{left} : E_{right} = 1 : 2$. If the uniform electric field on the left is $\vec{E}_{left} = C\left(\hat{i} + \hat{j} + \hat{k}\right)$; (where C is constant), the electric field on the right E_{right} is

(A) $\frac{1}{2}C\left(\hat{i} + \hat{j} + \hat{k}\right)$

(B) $2C\left(\hat{i} + \hat{j} + \hat{k}\right)$

(C) $\left(\frac{1}{2}C\,\hat{i} + \hat{j} + \hat{k}\right)$

(D) $C\hat{i} + \frac{1}{2}\left(\hat{j} + \hat{k}\right)$

67. For a plane wave of angular frequency ω and propagating vector $\vec{K}$ propagating in the medium of permittivity $\in$ and a permeability μ, Maxwell's equations reduce to

(A) $\vec{K}\cdot\vec{E} = 0; \vec{K}\cdot\vec{H} = 0; \vec{K}\times\vec{E} = \omega\varepsilon\,\vec{H}; \vec{K}\times\vec{H} = -\omega\mu\vec{E}$

(B) $\vec{K}\cdot\vec{E} = 0; \vec{K}\cdot\vec{H} = 0; \vec{K}\times\vec{E} = -\omega\,\varepsilon\,\vec{H}; \vec{K}\times\vec{H} = \omega\mu\vec{E}$

(C) $\vec{K}\cdot\vec{E} = 0; \vec{K}\cdot\vec{H} = 0; \vec{K}\times\vec{E} = -\omega\mu\,\vec{H}; \vec{K}\times\vec{H} = \omega\varepsilon\vec{E}$

(D) $\vec{K}\cdot\vec{E} = 0; \vec{K}\cdot\vec{H} = 0; \vec{K}\times\vec{E} = -\omega\mu\vec{H}; \vec{K}\times\vec{H} = -\omega\varepsilon\vec{E}$

68. Using the gauge function $\lambda = -\frac{1}{4\pi\varepsilon_0}\frac{qt}{r}$, transform the potential $V(r,t)=0;\ \vec{A}(r,t) = -\frac{1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r}$

(A) $V' = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r};\ \vec{A}' = 0$

(B) $V' = \frac{1}{4\pi\varepsilon_0}\frac{q}{r};\ \vec{A}' = 0$

(C) $V' = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r};\ \vec{A}' = \frac{1}{4\pi\varepsilon_0}\frac{2qt}{r^2}\hat{r}$

(D) $V' = \frac{1}{4\pi\varepsilon_0}\frac{q}{r};\ \vec{A}' = -\frac{1}{4\pi\varepsilon_0}\frac{2qt}{r^2}\hat{r}$

69. The magnetic field corresponding to the vector potential $\vec{A} = \vec{F}\times\vec{r} + \frac{\hat{r}}{r^2}$ (where $\vec{F}$ is constant vector) is

(A) $2\vec{F}$

(B) $-2\vec{F}$

(C) $2\vec{F} + \frac{30}{r^4}\vec{r}$

(D) $2\vec{F} - \frac{30}{r^4}\vec{r}$

70. A static, spherically symmetric charge distribution is given by $f(r) = \frac{A}{r}e^{-kr}$, where A and k are positive constants. The electrostatic potential corresponding to this charge distribution varies with r as

(A) re^{-kr}

(B) $\frac{1}{r}e^{-kr}$

(C) $\frac{1}{r^2}e^{-kr}$

(D) $\frac{1}{r}\left(1-e^{-kr}\right)$

71. A square loop of wire, with sides of length a, lies in the first quadrant of the xy - plane, with one corner at the origin. In this region there is a nonuniform time dependent magnetic field $B(y,t) = k\,y^4 t^3\,\hat{z}$ (where k is constant). The *e.m.f.* induced in the loop is

(A) $-\frac{1}{5}k\,t\,a^6$

(B) $-\frac{3}{5}k\,t^2 a^6$

(C) $\frac{3}{5}k\,t^2 a^6$

(D) $\frac{2}{5}k\,t^2 a^6$

72. A current of 10mA flows in an infinitely long straight wire along the axis of a hemisphere of radius a metre. The value of $\int_s (\bar{\nabla}\times\bar{B}).\,d\vec{s}$ over the hemispherical surface

(A) $\mu_0\ wb$

(B) $2{\cdot}5\mu_0\ wb$

(C) $10\mu_0\ wb$

(D) $0{\cdot}01\mu_0\ wb$

73. Four equal negative charges occupy the corners of a square. At the centre of this charge distribution a test positive charge is placed. Which of the following statements about the test charge is true?

(A) It is in stable equilibrium

(B) It is in unstable equilibrium

(C) It is in neutral equilibrium

(D) It is not in any equilibrium

74. A charge q is located at a point P at a distance $a(>R)$ from the origin O of a grounded spherical conductor of radius R. If a charge q' located at a distance b from the origin on the OP line acts as an image charge then the relation between these two charges are

(A) $q = -\sqrt{\frac{b}{a}}\,q'$

(B) $q = -q'$

(C) $q = -\frac{a}{b}q'$

(D) $q = -\sqrt{\frac{a}{b}}\,q'$

75. A particle of mass m moves in one-dimension and is subjected to a potential $V(x)=\frac{1}{2}m\omega^2x^2$. It is given that the particle is in a state $|\Psi\rangle$ such that $\langle\Psi|\hat{\Pi}|\Psi\rangle=0$, where $\hat{\Pi}$ is the party operator. The lowest possible esxpectation value of the energy for such a state is

(A) $\hbar\omega$

(B) $\frac{1}{2}\hbar\omega$

(C) $\frac{3}{2}\hbar\omega$

(D) $2\hbar\omega$

76. A system of three non-identical spin ½ particle (with spin operators $\vec{S}_1$, $\vec{S}_2$ and $\vec{S}_3$) are governed by the Hamiltonian $H=B\left(\vec{S}_1+\vec{S}_2\right)\cdot\vec{S}_3$. B is a constant. Largest eigenvalue of this system and the associated degeneracies are

(A) $\frac{3}{2}B$ and 4

(B) $\frac{3}{2}B$ and 3

(C) $\frac{1}{2}B$ and 4

(D) $\frac{1}{2}B$ and 2

77. The radial wave function of the electron in the ground state of a hydrogen atom is $R_{10}(r)=\frac{\exp(-r/a_0)}{\sqrt{\pi a_0^3}}$, where a_0 is the first Bohr radius. The radial distance r at which the electron is most likely to be found is given by

(A) $0{\cdot}5\ a_0$

(B) a_0

(C) $1{\cdot}5\ a_0$

(D) zero

78. Consider a one-dimensional potential $V(x)$ such that $V(x)<0$ for all x. This has a bound state if

(A) $V(x)$ has a unique minimum.

(B) $V'(x)$ vanishes at the minimum value of $V(x)$.

(C) the wave function $\Psi(x)\to 0$ in the limits $x\to\pm\infty$.

(D) the energy E is also negative, i.e. $E<0$.

79. A photon of energy 3 MeV strikes an electron at rest and is scattered with energy 2·8 MeV. The speed of the scattered photon with respect to the recoiling electron will be

(A) $0{\cdot}6c$

(B) $0{\cdot}8c$

(C) c

(D) $0{\cdot}2c$

80. Photons of wavelength 600 nm are emitted from the Sun and strike the Earth with an intensity of approximately $1{\cdot}4\ \text{kW/m}^2$. The radiation pressure exerted by these photons is approximately

(A) $0{\cdot}46\times10^{-5}\ \text{N/m}^3$

(B) $0{\cdot}93\times10^{-11}\ \text{N/m}^3$

(C) $0{\cdot}46\times10^{6}\ \text{N/m}^3$

(D) $0{\cdot}93\times10^{2}\ \text{N/m}^3$

81. $\Psi_{nlm}(r,\theta,\phi)$ denotes the stationary state wave function of a hydrogen atom. The atom is in a state $\chi=\frac{1}{\sqrt{5}}\left(\Psi_{2,1,1}-2\Psi_{2,1,-1}\right)$. Which of the following statements is true?

(A) χ is a energy eigenfunction.

(B) Expectation value of L_Z is zero.

(C) χ is not an eigenfunction of L^2.

(D) Expectation value of energy is zero.

82. A matrix M, written in a set of basis vectors $\{|1\rangle, |2\rangle\}$ is given by $M=\begin{pmatrix}\lambda & 1\\ 1 & \lambda\end{pmatrix}$. In a new basis, given by $|I\rangle=\frac{1}{\sqrt{2}}(|1\rangle+|2\rangle)$ and $|II\rangle=\frac{1}{\sqrt{2}}(|1\rangle-|2\rangle)$ the matrix will assume the form:

(A) $\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix}$

(B) $\begin{pmatrix}0 & \lambda\\ \lambda & 0\end{pmatrix}$

(C) $\begin{pmatrix}\lambda+1 & 0\\ 0 & \lambda-1\end{pmatrix}$

(D) $\begin{pmatrix}1 & \lambda\\ \lambda & 1\end{pmatrix}$

83. Two quantum particles of mass m each are confined to one-dimension and are subjected to a potential $V(x)=\frac{1}{2}kx^2$. If the inter-particle potential is given by $\delta V(x_1,x_2)=-\lambda k x_1 x_2$ when $x_{1,2}$ are their instantaneous positions and λ is a constant with $|\lambda|<1$, then the energy eigenvalues are $\left(\omega=\sqrt{k/m}\right)$

(A) $E(n_1,n_2)=\hbar\omega\left[\sqrt{1-\lambda}\left(n_1+\frac{1}{2}\right)+\sqrt{1+\lambda}\left(n_2+\frac{1}{2}\right)\right]$

(B) $E(n_1,n_2)=\hbar\omega\left[n_1\sqrt{1-\lambda}+n_2\sqrt{1+\lambda}+1\right]$

(C) $E(n_1,n_2)=\hbar\omega\left[n_1\sqrt{1-\lambda^2}+n_2\sqrt{1+\lambda^2}+1\right]$

(D) $E(n_1,n_2)=\hbar\omega\left[\sqrt{1-\lambda^2}\left(n_1+\frac{1}{2}\right)+\sqrt{1+\lambda^2}\left(n_2+\frac{1}{2}\right)\right]$

84. A particle restricted to $x\in[0,\infty)$ has the wave function $\Psi(x)=\left(\frac{x}{\lambda}\right)^n e^{-x/\lambda}$ where λ, n are constants. If the potential $V(x)$ satisfies $\underset{x\to\infty}{Lt}\,V(x)=0$, then

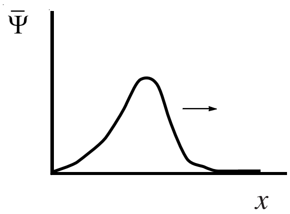
(A) $V(x)=\frac{\hbar^2}{2m}\left[\frac{n(n-1)}{x^2}-\frac{2n}{\lambda x}\right]$

(B) $V(x)=\frac{\hbar^2}{2m}\left[\frac{n(n+1)}{x^2}-\frac{n}{\lambda x}\right]$

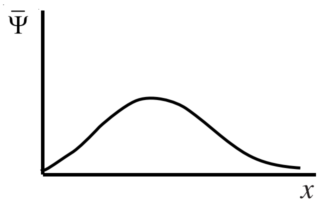
(C) $V(x)=\frac{\hbar^2}{2m}\left[\frac{n(n-1)}{x^2}+\frac{n+1}{\lambda x}\right]$

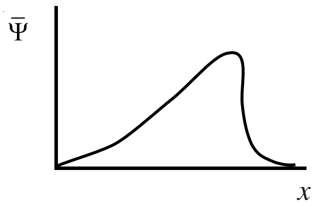
(D) $V(x)=\frac{\hbar^2}{2m}\left[\frac{n(n+1)}{x^2}+\frac{n-1}{\lambda x}\right]$

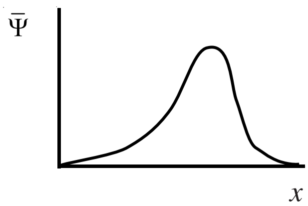
85. The diagram below indicates the profile of a wave packet propagating as shown in one-dimension, at time t=0

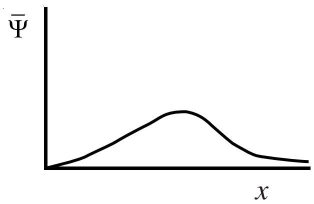


After some time, at $t=t$, the most likely profile of this wavepacket would be

(A)


(B)


(C)


(D)


86. A plane matter wave scatters from a small target. The Optical Theorem tells us that

(A) only the $l=0$ partial waves contribute to the scattering cross-section.

(B) the total scattering cross-section is proportional to the imaginary part of the scattering amplitude in the forward direction.

(C) the total scattering cross-section is proportional to the imaginary part of the differential scattering cross-section in the forward direction.

(D) All of the above

87. A quantum paricle is trapped in a one-dimensional well between $x=0$ and $x=L$. At $t=0$ the wave function is given by

$$\Psi(x,0) = \sqrt{\frac{30}{L^5}}\, x(L-x), \text{ for } 0 \le x \le a$$

$$= 0 \qquad\qquad , \text{ otherwise}$$

Which of the following statements is true?

(A) Measurement of energy of the system will yield a definite value.

(B) The expectation value of energy is independent of time.

(C) The expectation value of position is independent of time.

(D) The expectation value of momentum > $\frac{\pi\hbar}{L}$.

88. A beam of electrons in one-dimension is incident on a square barrier of width a and height V_0. If V_0 and a can be controlled externally, the tunnelling current will be more accurately controlled by the same per cent variation in

(A) V_0

(B) $V_0 a$

(C) a

(D) either V_0 or a

89. A flux of 10^{12} neutrons/m^2 emerges each second from a port in a nuclear reactor. If these neutrons obey MB (Maxwell-Boltzmann) distribution corresponding to T=300K, the density of neutrons in the beam will be approximately

(A) $10^8 m^{-3}$

(B) $10^6 m^{-3}$

(C) $10^4 m^{-3}$

(D) $10^{10} m^{-3}$

90. Assume that the heat capacity at constant volume of a metal varies as $aT + bT^3$ for low temperature. Then, the variation of entropy with temperature will be

(A) $aT + \frac{bT^2}{2}$

(B) $aT + bT^3$

(C) $aT + \frac{bT^3}{3}$

(D) $a + \frac{bT^2}{2}$

91. A system has three energy levels of energy 0, $100K_B$ and $200K_B$ with degeneracies of 1, 3 and 5 respectively. Then, the average energy at a temperature of 100K will be

(A) $78{\cdot}3K_B$

(B) $88{\cdot}3K_B$

(C) $100K_B$

(D) $68{\cdot}3K_B$

92. Consider a system of two atoms each having any 3 quantum states of energies 0, E and $2E$. The system is in contact with a heat reservoir at temperature T. If the particles obey Fermi-Dirac Statistics, then the partition function in terms of $A = 1 + e^{-\beta E} + e^{-2\beta E}$ will be

(A) $2A$

(B) A^2

(C) $\frac{A}{2}$

(D) $e^{-\beta E} A$

93. If the partition function of a harmonic oscillator with frequency ω at a temperature T is $\frac{kT}{\hbar\omega}$, the free energy of N such independent oscillator is

(A) $\frac{3}{2} NkT$

(B) $kT \ln \frac{\hbar\omega}{kT}$

(C) $NkT \ln \frac{\hbar\omega}{kT}$

(D) $NkT \ln \frac{\hbar\omega}{2kT}$

94. A heat pump working on the carnot cycle maintains the inside temperature of a house at 22°C by supplying power at 450 kJ/sec. If the outside temperature is 0°C, the heat taken in kJ/sec from the outside air is approximately

(A) 489
(B) 250
(C) 467
(D) 417

95. Temperature of star's surface is determined from its spectrum on the basis of

(A) Fraunhoffer lines
(B) Infrared emission bands
(C) Doppler shift
(D) Ultraviolet emission lines

96. A two-dimensional box in a uniform magnetic field B contains $\frac{N}{2}$ localized spin $\frac{1}{2}$ particles with magnetic moment μ, and $\frac{N}{2}$ spineless particles, which do not interact with each other. The average energy of the system at temperature T is

(A) $3NkT - \frac{1}{2}N\mu \text{B} \sinh\left(\frac{\mu \text{B}}{kT}\right)$
(B) $NkT - \frac{1}{2}N\mu \text{B} \sinh\left(\frac{\mu \text{B}}{kT}\right)$
(C) $\frac{1}{2}NkT - \frac{1}{2}N\mu \text{B} \sinh\left(\frac{\mu \text{B}}{kT}\right)$
(D) $\frac{3}{2}NkT + \frac{1}{2}N\mu \text{B} \sinh\left(\frac{\mu \text{B}}{kT}\right)$

97. For equation of state $P = \frac{RT}{V-b} \exp\left(-\frac{a}{RTV}\right)$, the critical volume and temperature are respectively

(A) $V_c = 3b, T_c = \frac{a}{2bR}$
(B) $V_c = 2b, T_c = \frac{a}{2bR}$
(C) $V_c = 2b, T_c = \frac{a}{4bR}$
(D) $V_c = 2b, T_c = \frac{3a}{2bR}$

98. Consider a system of 100 non-interacting spins in presence of a magnetic field B. Each spin has a probability of 0·51 of pointing parallel to B and the rest pointing antiparallel to B. If ΔM is the standard deviation to the above system and $\bar{M}$ is the average total magnetic moment, then the ratio $\frac{\Delta M}{\bar{M}}$ will be

(A) 5
(B) 10
(C) 0·01
(D) 5×10^{-2}

99. According to Debye theory, the heat capacity of a two-dimensional solid at low temperature is proportional to

(A) T^3
(B) T
(C) T^2
(D) T^6

100. In Bose-Einstein condensates, the particles

(A) have large and positive chemical potential.
(B) have overlapping wave functions.
(C) have strong inter-particle attraction.
(D) condense in real space.

ROUGH WORK

ROUGH WORK

ROUGH WORK

ROUGH WORK