Triangles Formulae Download PDF

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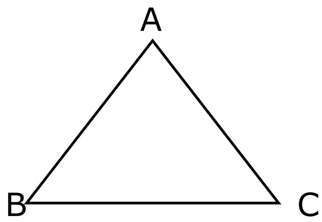
Triangle

Triangle is a polygon, which has three sides, three angles and three vertices.



Basic properties of a triangle:

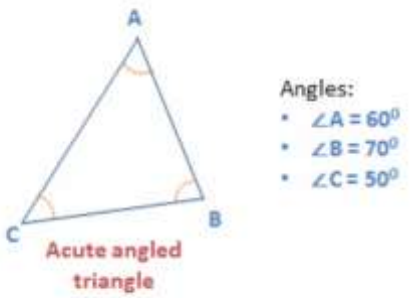
1. Sum of all three angles of any triangle equals 180°.
2. The sum of the length of any two sides of a triangle is always greater than the length of the third side.
3. The side opposite to the largest angle of a triangle is the largest side.
4. The angle opposite to the largest side of a triangle is the largest angle.
5. The side opposite to the smallest angle of a triangle is the smallest side.
6. The angle opposite to the smallest side of a triangle is the smallest angle.
7. Any exterior angle of the triangle is equal to the sum of its interior opposite angles.

Types of triangles:

I. Based on the angle measurement:

1. Acute angle triangle:

A triangle that has all three angles less than 90° is an acute angle triangle.

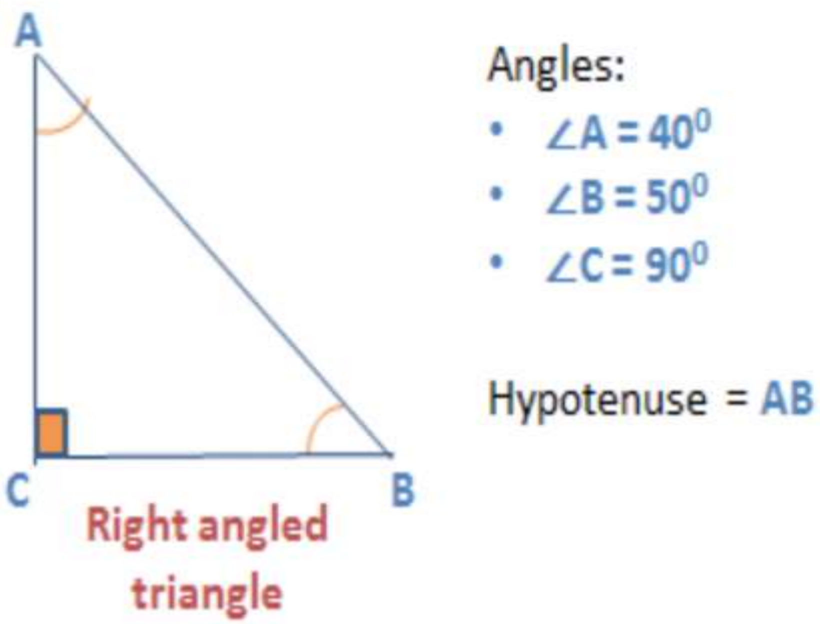


2. Right angle triangle:

A triangle that has one angle that measures exactly 90° is a right-angle triangle.

Note:

a. The other two angles of a right-angle triangle are acute angles.
b. The side opposite to the right angle is the largest side of the triangle and is called the hypotenuse.



c. Pythagoras theorem:

In a right-angled triangle, the sum of squares of the perpendicular sides is equal to the square of the hypotenuse.

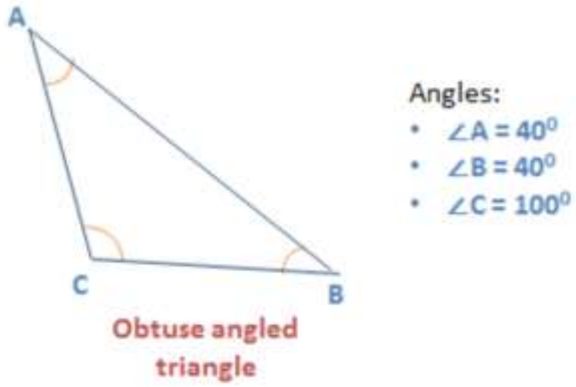
For example: In above right-angled triangle ACB,

$(AC)^2 + (CB)^2 = (AB)^2$

Vice versa, we can say that if a triangle satisfies the Pythagoras condition, then it is a right-angled triangle.

3. Obtuse Angle Triangle:

A triangle that has one angle that measures more than 90° is an obtuse angle triangle.

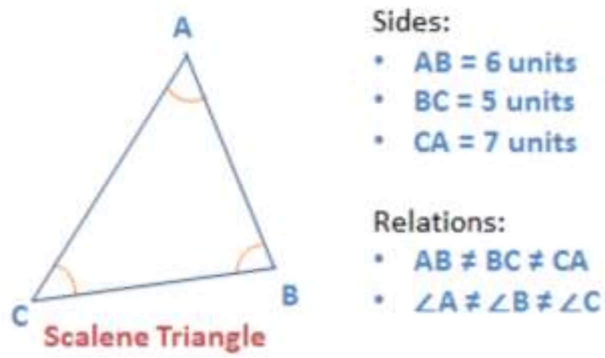


I. Based on the length of the sides:

1. Scalene triangle:

A triangle that has all three sides of different lengths is a scalene triangle.

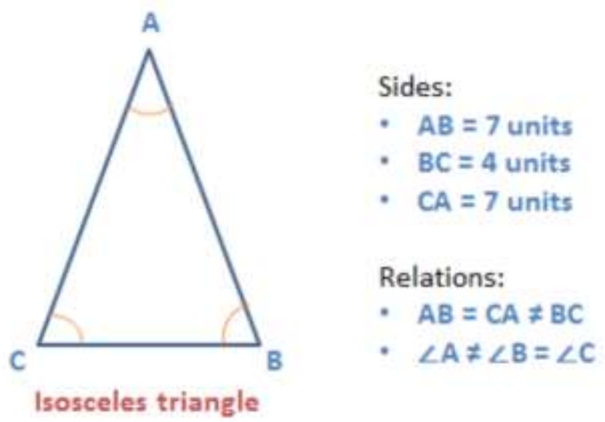
Note: Since all the three sides are of different lengths, the three angles will also be different.



Note: Since all the three sides are of different lengths, the three angles will also be different.

2. Isosceles triangle:

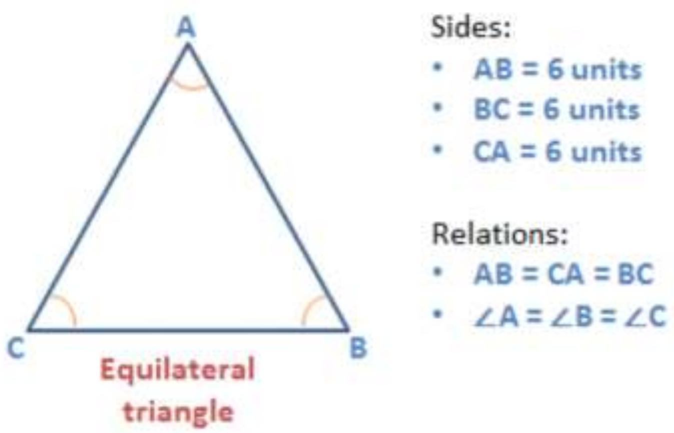
A triangle that has two sides of the same length and the third side of a different length is an isosceles triangle.



Note: The angles opposite the equal sides measure the same.

3. Equilateral triangle:

A triangle which has all the three sides of the same length is an equilateral triangle.



Note: Since all the three sides are of the same length, all the three angles will also be equal and each interior angle of an equilateral triangle = 60°

Important lines in a triangle:

1. **Median:** A line joining the mid-point of a side with the opposite vertex is called a median.
2. **Altitude:** A perpendicular drawn from a vertex to the opposite side is called the altitude.
3. **Perpendicular bisector:** A line that bisects and also makes right angle with the same side of the triangle is called perpendicular bisector.
4. **Angle bisector:** A line that divides the angle at one of the vertices into two parts is called angular bisector.

   Note:

   a. All points on an angular bisector are equidistant from both arms of the angle.
   b. All points on a perpendicular bisector of a line are equidistant from both ends of the line.
   c. In an equilateral triangle, the perpendicular bisector, median, angle bisector and altitude (drawn from a vertex to a side) coincide.

Geometric centres:

1. Orthocentre:

The point of intersection of the three altitudes is the Orthocentre.

2. Centroid:

The point of intersection of the three medians is the centroid.

3. Circumcentre:

The three perpendicular bisectors of a triangle meet at a point called the Circumcentre. A circle drawn from this point with the circumradius would pass through all the vertices of the triangle.

4. Incentre:

The three angle bisectors of a triangle meet at a point called the incentre of a triangle. The incentre is equidistant from the three sides and a circle drawn from this point with the inradius would touch all the sides of the triangle.

Other properties of a triangle:

1. Apollonius theorem:

In a triangle ABC, if AD is the median to side BC then by Apollonius theorem,

$2 \times (AD^2 + BD^2) = AC^2 + AB^2$

2. Mid-Point Theorem:

The line joining the midpoint of any two sides in a triangle is parallel to the third side and is half the length of the third side. If X is the midpoint of CA and Y is the midpoint of CB Then XY will be parallel to AB and XY = ½ × AB

3. Basic proportionality theorem:

If a line is drawn parallel to one side of a triangle which intersects the other two sides at two distinct points, then the line divides the two sides in the same ratio.

For example: In a triangle ABC, D and E are the points lying on AB and BC respectively and DE is parallel to AC then

$$\frac{AD}{DB} = \frac{CE}{BE}$$

4. Interior Angular Bisector theorem:

In a triangle the angular bisector of an angle divides the side opposite to the angle, in the ratio of the remaining two sides.

For example: In a triangle ABC, if AD is the angle bisector of angle A, then AD divides the side BC in the same ratio as the other two sides of the triangle. i.e.

$$\frac{BD}{CD} = \frac{AB}{AC}$$

5. Exterior Angular Bisector theorem:

The angular bisector of exterior angle of a triangle divides the opposite side externally in the ratio of the sides containing the angle.

For example: In a triangle ABC, if AD is the angle bisector of exterior angle CAB of a triangle i.e.

$$\frac{BD}{CD} = \frac{AB}{AC}$$

6. Equilateral triangle:

If P is the side of an equilateral triangle, then

a. Altitude (h) = $\frac{\sqrt{3}}{2}$ P

b. Area = $\frac{\sqrt{3}}{2}$ P^2

c. Inradius = $\frac{h}{3} = \frac{1}{3} \times \frac{\sqrt{3}}{2}$ P = $\frac{P}{2\sqrt{3}}$

d. Circumradius = $\frac{2h}{3} = \frac{2}{3} \times \frac{\sqrt{3}}{2}$ P = $\frac{P}{\sqrt{3}}$

7. Area of an isosceles triangle = $\frac{a}{4}\sqrt{4c - a^2}$

(where a, b and c are the length of the sides of BC, AC and AB respectively and b = c)

8. Similar triangles:

If two triangles are similar then their corresponding angles are equal and the corresponding sides will be in proportion.
For any two similar triangles:

a. Ratio of sides = Ratio of medians = Ratio of heights = Ratio of circumradii = Ratio of Angular bisectors
b. Ratio of areas = Ratio of the square of the sides.
c. Tests of similarity: (AA / SSS / SAS)

9. Congruent triangles:

If two triangles are congruent then their corresponding angles and their corresponding sides are equal.

Tests of congruence: (SSS / SAS / AAS / ASA)

Area of a triangle:

a. Area = ½ × base × height
b. A = $\sqrt{s\,(s-a)\,(s-b)\,(s-c)}$, where s = (a + b + c)/2
c. A = ½ × ab × Sin C, where C is the angle formed between sides a and b
d. A = $\frac{abc}{4R}$, where R = circumradius
e. A = r × s, where r = in-radius and s = $\frac{a+b+c}{2}$, where a, b and c are the lengths of the sides BC, AC and AB.

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