Ratio, Proportion and Variation Formulae

Ratio:

- A ratio can be represented as fraction a/b or using the notation a : b. In each of these representation 'a' is called the antecedent and 'b' is called the consequent.

  Example:

  The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

- For a ratio to be defined, the quantities of the items should be of same nature. We can't compare the Area of the square to the volume of a cube.

- However if these quantities are represented in numbers, i.e., Area of a square is a cm^2 and area of a cube is b m^3, we can still define the ratio of these numbers as a:b

Properties of ratios:

- A ratio need not be positive. However, if we are dealing with quantities of items, their ratios will be positive. In this concept we will consider only positive ratios.

- The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

$$\frac{\mathbf{a}}{\mathbf{b}} = \frac{\mathbf{pa}}{\mathbf{pb}} = \frac{\mathbf{a/p}}{\mathbf{b/p}}, \mathbf{p} \neq \mathbf{0}$$

  Example: $\frac{3}{5} = \frac{2 \times 3}{2 \times 5} = \frac{3/2}{5/2}$

- Two ratios in their fraction notation can be compared just as we compare real numbers.

$$\frac{\mathbf{a}}{\mathbf{b}} = \frac{\mathbf{p}}{\mathbf{q}} \Leftrightarrow \mathbf{aq} = \mathbf{bp}$$

$$\frac{\mathbf{a}}{\mathbf{b}} > \frac{\mathbf{p}}{\mathbf{q}} \Leftrightarrow \mathbf{aq} > \mathbf{bp}$$

$$\frac{\mathbf{a}}{\mathbf{b}} < \frac{\mathbf{p}}{\mathbf{q}} \Leftrightarrow \mathbf{aq} < \mathbf{bp}$$

- If antecedent > consequent, the ratio is said to be ratio of greater inequality.
- If antecedent < consequent, the ratio is said to be ratio of lesser inequality.
- If the antecedent = consequent, the ratio is said to be ratio of equality.
- Duplicate Ratio of a : b is $a^2 : b^2$
- Sub-duplicate ratio of a : b is $\sqrt{a} : \sqrt{b}$
- Triplicate Ratio of a : b is $a^3 : b^3$
- Sub-triplicate ratio of a : b is $a^{1/3} : b^{1/3}$

If a, b, x are positive, then

- If a > b, then $\frac{a+x}{b+x} < \frac{a}{b}$
- If a < b, then $\frac{a+x}{b+x} > \frac{a}{b}$
- If a > b, then $\frac{a-x}{b-x} > \frac{a}{b}$
- If a < b, then $\frac{a-x}{b-x} < \frac{a}{b}$
- If $\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{d}{s} = \cdots$, then a : b : c : d :...... = p : q : r : s :......

Proportions:

A proportion is an equality of ratios. Hence a : b = c : d is a proportion. The first and last terms are called extremes and the other two terms are called means.

If four terms a, b, c, d are said to be proportional, then a : b = c : d. If three terms a, b, c are said to be proportional, then a : b = b : c

Properties of proportions:

If a : b = c : d is a proportion, then

- Product of extremes = product of means i.e., ad = bc
- Denominator addition/subtraction: a : (a + b) = c : (c + d) and a : (a - b) = c : (c - d)
- a, b, c, d,.... are in continued proportion means, a : b = b : c = c : d = ...
- a : b = b : c then b is called mean proportional and $b^2 = ac$
- The third proportional of two numbers, a and b, is c, such that, a : b = b : c
- d is fourth proportional to numbers a, b, c if a : b = c : d

Variations:

- If x varies directly to y, then x is said to be in directly proportional with y and is written as $x \propto y$
  $x = ky$ (where k is direct proportionality constant)
  $x = ky + C$ (If x depends upon some other fixed constant C)

- If x varies inversely to y, then x is said to be in inversely proportional with y and is written as $x \propto \frac{1}{y}$
  $x = k\frac{1}{y}$ (where k is indirect proportionality constant)
  $x = k\frac{1}{y} + C$ (If x depends upon some other fixed constant C)

- If $x \propto y$ and $y \propto z$ then $x \propto z$
- If $x \propto y$ and $x \propto z$ then $x \propto (y \pm z)$
- If $a \propto b$ and $x \propto y$ then $ax \propto by$

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