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Quadratic Equations Formulae

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Quadratic equations:

A polynomial of the second degree is generally called a quadratic polynomial. In elementary algebra, the quadratic formula is the solution of the quadratic equation. There are other ways to solve the quadratic equation instead of using the quadratic formula, such as factoring, completing the square, or graphing. Using the quadratic formula is often the most convenient way.

If f(x) is a quadratic polynomial, then f(x) = 0 is called a quadratic equation.

The general form of a quadratic equation is $ax^2 + bx + c = 0$ where a, b, c are real numbers (constants) and $a \neq 0$, while b and c may be zero.

- The Discriminant of a quadratic equation is $ax^2 + bx + c = 0$ $(a \neq 0)$ is $\Delta = b^2 - 4ac$
  1. When a, b and c are real numbers, $a \neq 0$ and discriminant is positive (i.e., $b^2 - 4ac > 0$), then the roots α and β of the quadratic equation $ax^2 + bx + c = 0$ are real and unequal.

  2. When a, b and c are real numbers, $a \neq 0$ and discriminant is zero (i.e., $b^2 - 4ac = 0$), then the roots α and β of the quadratic equation $ax^2 + bx + c = 0$ are real and equal.

  3. When a, b and c are real numbers, $a \neq 0$ and discriminant is negative (i.e., $b^2 - 4ac < 0$), then the roots α and β of the quadratic equation $ax^2 + bx + c = 0$ are unequal and imaginary. Here the roots α and β are a pair of the complex conjugates.

  4. When a, b and c are real numbers, $a \neq 0$ and discriminant is positive and perfect square, then the roots α and β of the quadratic equation $ax^2 + bx + c = 0$ are real, rational unequal.

  5. When a, b and c are real numbers, $a \neq 0$ and discriminant is positive but not a perfect square then the roots of the quadratic equation $ax^2 + bx + c = 0$ are real, irrational and unequal.

6. When a, b and c are real numbers, $a \neq 0$ and the discriminant is a perfect square but any one of a or b is irrational then the roots of the quadratic equation $ax^2 + bx + c = 0$ are irrational.

- If α and β be the roots of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$ then

    1. Sum of roots: $\boldsymbol{\alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of x}}{\text{coefficient of } x^2}}$

    2. Product of roots: $\boldsymbol{\alpha\beta} = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

    3. The value of roots, **α** and **β** $= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

    4. If c and a are equal then the roots are reciprocal to each other

    5. If b = 0, then the roots are equal and are opposite in sign

- The formula for the formation of the quadratic equation whose roots are given: x^2 - (sum of the roots)x + product of the roots = 0.

- In a quadratic equation with real coefficients has a complex root $\alpha + i\beta$ then it has also the conjugate complex root $\alpha - i\beta$.

- In a quadratic equation with rational coefficients has a irrational or surd root $\alpha + \sqrt{\beta}$, where α and β are rational and β is not a perfect square, then it has also a conjugate root $\alpha - \sqrt{\beta}$.

- Signs of the roots: Let P be product of roots and S be their sum
    1. $P > 0, S > 0$: Both roots are positive
    2. $P > 0, S < 0$: Both roots are negative
    3. $P < 0, S > 0$: Numerical smaller root is negative and the other root is positive
    4. $P < 0, S < 0$: Numerical larger root is negative and the other root is positive

- Minimum and maximum values of $ax^2+bx+c = 0$

  If $a > 0$: minimum value $= \dfrac{4ac - b^2}{4a}$ and occurs at $x = -\dfrac{b}{2a}$

  If $a < 0$: maximum value $= \dfrac{4ac - b^2}{4a}$ and occurs at $x = -\dfrac{b}{2a}$

- If roots are reciprocals of roots of equation $ax^2 + bx + c = 0$, then equation is $cx^2 + bx + a = 0$

- If roots are k more than roots of $ax^2 + bx + c = 0$ then equation is $a(x-k)^2 + b(x-k) + c = 0$

- If roots are k times roots of $ax^2 + bx + c = 0$ then equation is $a(x/k)^2 + b(x/k) + c = 0$

Common roots:

Let the two quadratic equations are $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$, then

Condition for one common root: $(c_1a_2 - c_2a_1)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$, which is the required condition for one root to be common of two quadratic equations.

Condition for both roots common: $a_1/a_2 = b_1/b_2 = c_1/c_2$

Descartes Rules:

A polynomial equation with n sign changes can have a maximum of n positive roots. To find the maximum possible number of negative roots, find the number of positive roots of f(-x).

Note: An equation where highest power is odd must have at least one real root.

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