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Electrical Engineering

Power Electronics

100 Days
Important Formula Notes

POWER ELECTRONICS (FORMULA NOTES)

Charge stored in depletion region:

Let Q_R be the charge stored in depletion region of power diode.

$$Q_R = \frac{1}{2}.I_{RM}.t_{rr}$$

$$I_{RM} = \frac{2Q_R}{t_{rr}} = t_a.\frac{di}{dt} = \sqrt{2Q_R.\left(\frac{di}{dt}\right)}$$

$$\text{If } t_a \approx t_{rr},\ t_{rr} = \sqrt{\frac{2Q_R}{di/dt}}$$

$$I_{RR} = t_{rr}.\frac{di}{dt} = \sqrt{2Q_R\left(\frac{di}{dt}\right)}$$

$$t_{rr} \propto \sqrt{Q_R}$$

$$Q_R \propto I_f,$$

$$t_{rr} \propto \sqrt{I_f}$$

$$I_{RM} \propto \sqrt{I_f}$$

Relation Between α and β:

$$\boxed{\alpha = \frac{I_C}{I_E}} \quad \boxed{\beta = \frac{I_C}{I_B}}$$

$$\frac{I_E}{I_C} = 1 + \frac{I_B}{I_C}$$

$$\frac{1}{\alpha} = 1 + \frac{1}{\beta}$$

$$\boxed{\alpha = \frac{\beta}{\beta + 1}} \quad \boxed{\beta = \frac{\alpha}{1 + \alpha}}$$

Design of Snubber circuit:

For Inductor (L):

$$\left(\frac{di}{dt}\right)_{max} = \frac{V_s}{L}$$

$$L = \frac{V_s}{(di/dt)_{max}}$$

For resistor (R_s):

$$\left(\frac{dv_a}{dt}\right)_{max} = R_s\left(\frac{di}{dt}\right)_{max}$$

$$\text{or } R_s = \frac{L}{V_s}\left(\frac{dv_a}{dt}\right)_{max}$$

For Capacitor (C_s):

$$C_s = \left(\frac{2\xi}{R_s}\right)^2 L \qquad \text{where } 0.5 < \xi < 1$$

Design of Snubber circuit:

$$\text{String efficiency} = \frac{\text{Actual voltage/current rating of string.}}{n \times \text{individual voltage/current rating of SCR}}$$

where n is the number of SCR in string.

Derating factor, DRF=1- string efficiency.

Series Operation of Thyristors:

Consider n thyristor connected in series as shown in figure. Let SCR1 has minimum leakage current I_{bmn}. SCR with lower leakage current blocks more voltage.

Remaining (n-1) SCRs have the same leakage current I_{bmx}

$I_{bmx} > I_{bmn}$

Here V_{bm} is the maximum permissible blocking voltage as SCR1.

$I = I_1 + I_{bmn}$

$I_1 = I - I_{bmn}$

$I = I_2 + I_{bmx}$

$I_2 = I - I_{bmx}$

Where, I = total string current

Voltage across SCR1 is $V_{bm} = I_1R$

Voltage across (n-1) SCRs $=(n-1)\, I_2\, R$

For a string voltage (V_s), the voltage equation for the series circuit is

$V_s = I_1\, R + (n-1)I_2R = V_{bm} + (n-1)R(I - I_{bmx})$

$= V_{bm} + (n-1)R(I_1 + I_{bmn} - I_{bmx})$

$= V_{bm} + (n-1)R[(I_1 - (I_{bmx} - I_{bmn})]$

$$V_s = V_{bm} + (n-1)R1_1 - (n-1)R\Delta I_b \quad [\therefore \Delta I_b = I_{bmx} -_{bmn}]$$

AS, $RI_1 = V_{bm}$

$$V_s = V_{bm} + (n-1)V_{bm} - (n-1)R\Delta I_b$$

$$V_s = nV_{bm} - (n-1)R.\Delta I_b$$

$$\boxed{R_s = \frac{nV_{bm} - V_s}{(n-1) - \Delta I_b}}$$

'R_s' is the static equalizing resistance.

Parallel Operation of Thyristors:

When current required by the load is more than the rated current of a single thyristor, SCR's are connected in parallel in a string.

Class A Commutation (Load Commutation)

$I(t) = V_s\sqrt{\frac{C}{L}}$ sinω_0t Conduction time of thyristor, $t_0 = \frac{\pi}{\omega_0} = \pi\sqrt{LC}$

where, ω_0 = resonant frequency $\frac{1}{\sqrt{LC}}$

Class B Commutation:

- Resonant current $dI_C = -V_s\sqrt{\frac{C}{L}}\sin\omega_0 t$
- Peak resonant current $I_p = V_s\sqrt{\frac{C}{L}}$
- Voltage across capacitor $V_c = V_s \cos \omega_0 t$
- Circuit turn-off time for the main thyristor (T_1); $tc = C\frac{V_{ab}}{I_0}$
- Reverse voltage across the main thyristor (T_1)

  $V_{sb} = V_s \cos \omega_0(t_3 - t_2)$

  Where t_3 = time when the main thyristor is turned off

  t_2= time when auxiliary thyristor is burned off

  $\omega_0(t_3 + -_{t2}) = \sin^{-1}\left(\frac{I_0}{I_p}\right)$ and

Class C Commutation:

When T_1 is turned on at t=0

- The charging current $I_s = \frac{V_s}{R_z}.\, e^{-t/R_2 C}$
- Voltage across capacitor

$$V_c(t) = V_s(1 - e^{-t/R_2 C})$$

When T₁ is to be turned-off, T₂ is turned-on at T₁

- The charging current $I_c(t) = -\frac{2V_S}{R_1} . e^{-t/R_1 C}$
- The Voltage across capacitor

$$V_c(t) = V_s[2e^{-t/R_1 C} - 1]$$

- Maximum current though thyrisor T₁

$$I_{T_1(max)} = V_s\left[\frac{1}{R_1} - \frac{2}{R_2}\right]$$

- Maximum current though thyrisor T₂,

$$I_{T_2(max)} = V_s\left[\frac{2}{R_1} + \frac{1}{R_2}\right]$$

Circuit turn-off time t_{c_1} for thyristor T₁

$$t_{c_1} = R_1 C \text{ In}(2)$$

Circuit turn-off time t_{c_2} for thyristor T₂

$$t_{c_2} = R_2 C \text{ In}(2)$$

Class D Commutation:

- **Capacitor current**

$$I_c = V_s\sqrt{\frac{C}{L}} \sin \omega_0 t = I_p \sin \omega_0 t$$

- Circuit turn-off time for main thyristor (T₁)

$$t_c = C\frac{V_s}{I_0}$$

- Circuit turn-off time for main thyristor (TA)

$$t_{c_1} = \frac{\pi}{2\omega_0}$$

Single Phase Half Wave Diode Rectifier:

With R Load:

- **RMS value of output voltage**

$$V_{0\ rms} = \frac{V_m}{2}$$

V_m**=** Maximum value of source voltage (V_s)

- Average value of output voltage,

$$V_{0\ DC} = \frac{V_m}{\pi}$$

- Average value of output voltage,
- Power delivered $P = I^2_{0\,rmz} . R$

  I_0 rms = rms value of load current = $\dfrac{\text{Power delivered to load}}{\text{Input } V_A}$

- Input power Factor = $\dfrac{V_{0\,rms} . I_{0\,rms}}{V_s - I_{0\,rms}} = 0.707$

With L load:

- **Output current** $I_0 = \dfrac{V_m}{\omega L}(1 - \cos \omega t)$
- Maximum value of current $I_0 = \dfrac{2X_m}{\omega L}$
- Average value of current $I_0 = \dfrac{I_{max}}{2}$
- RMS value of fundamental current

  $I_{1\,rms} = \dfrac{I_0}{\sqrt{2}}$

- Output voltage $V_0 = V_m \sin \omega t = V_s$
- Average value of current voltage $V_0 = 0$

With C Load:

- Output voltage $V_0 = V_m \sin \omega t = V_s = V_c$
- Diode voltage $V_D = V_m (\sin \omega t - 1)$
- Output current $I_0 = \omega c\, V_m \cos \omega t$
- Average value of diode voltage $V_D = V_m$
- RMS value of diode voltage

  $V_{rms\,D} = 1.225\, V_m$

Single-Phase Full wave Mid-point Diode Rectifier:

Average output voltage,

$$V_0 = \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \, d(\omega t)$$

$$\boxed{V_0 = \frac{2V_m}{\pi}}$$

Average output current,

$$i_0 = \frac{V_0}{\pi}$$

Rms value of load voltage,

$$V_{0(rms)} = \sqrt{\left[\frac{1}{2}\int_0^{\pi} V_m^2 \sin\omega t \; d(\omega t)\right]}$$

$$\boxed{V_{0(rms)} = \frac{V_m}{\sqrt{2}} = V_s}$$

Rms value of load current,

$$I_{0(rms)} = \frac{V_s}{R}$$

Power delivered to load = $V_{0(rms)}\ I_{0(rms)}$

$$\therefore \text{ Input power factor } = \frac{V_{0(rms)} I_{0(rms)}}{V_s I_{0(rms)}}$$

Single-Phase Full wave Diode Bridge Rectifier:

Average value of diode current,

$$I_D = \frac{1}{2\pi}\int_0^{\pi} I_m \sin\omega t.d(\omega t) = \frac{I_m}{\pi}$$

Rms value of diode current,

$$I_{D(rms)} = \sqrt{\left[\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t.d(\omega t)\right]}$$

$$\boxed{I_{D(rms)} = \frac{I_m}{2}}$$

Single phase half wave-controlled rectifier with R Load

Average output voltage,

$$V_0 = \frac{V_m}{2\pi}(1 + \cos\alpha)$$

Average output current,

$$I_0 = \frac{V_m}{2\pi R}(1 + \cos\alpha)$$

Rms value of output voltage,

$$V_{or} = \frac{V_m}{2\sqrt{\pi}}\left[(\pi - \alpha) + \frac{1}{2}\sin 2\alpha\right]^{1/2}$$

Commutation time or turn off time of the thyristor,

$$t_c = \frac{\pi}{\omega} \text{ sec}$$

Input power factor of the converter,

$$\text{p.f.} = \frac{V_{or}.I_{or}}{V_s.I_{or}} = \frac{1}{\sqrt{2\pi}}\left[\pi - \alpha + \frac{1}{2}\sin 2\alpha\right]^{1/2}$$

Single phase half wave-controlled rectifier with RL load:

Average output voltage,

$$V_0 = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

Average output current,

$$I_0 = \frac{V_m}{2\pi R}(\cos\alpha - \cos\beta)$$

Rms value of output voltage.

$$V_{or} = \frac{V_m}{2\sqrt{\pi}}\left[(\beta-\alpha) - \frac{1}{2}(\sin 2\beta - \sin 2\alpha)\right]^{\frac{1}{2}}$$

Commutation time or turn off time for the thyristor,

$$t_c = \frac{2\pi - \beta}{\omega}\text{sec}$$

Single phase half wave-controlled rectifier with RLE load:

The minimum value of firing angle at which thyristor can be triggered is

$$\theta_1 = \sin^{-1}\left(\frac{E}{V_m}\right)$$

Average output current,

$$I_0 = \frac{1}{2\pi R}\left[V_m(\cos\alpha - \cos(\gamma + a)) - E\gamma\right]$$

Average output voltage,

$$V_0 = \frac{1}{2\pi}\left[V_m(\cos\alpha - \cos\beta) + E(2\pi + \alpha - \beta)\right]$$

Input power factor,

$$\text{p.f.} = \frac{(I_{or}^2 R + EI_0)}{V_s I_{or}}$$

Commutation time or turn off time of thyristor,

$$t_c = \frac{2\pi - \beta}{\omega}\text{sec}$$

Three Phase Half Wave Diode Rectifier:

The peak inverse voltage (PIV) = $\sqrt{3}\,V_{mp}$ for each of the three diode D_1, D_2 and D_3. The average output voltage.

$$V_0 = \frac{1}{2\pi/3}\int_{\pi/6}^{5\pi/6} V_{mp}\sin\omega t\, d(\omega t)$$

$$= \frac{3\sqrt{3}}{2\pi}V_{mp}$$

$$= \frac{3\sqrt{6}}{2\pi}V_{ph} \qquad \left(\because V_{mp} = \sqrt{2}\,V_{ph}\right)$$

$$= \frac{3}{2\pi}V_{ml} \qquad \left(\because V_{ml} = \sqrt{3}\,V_{mp} = \sqrt{6}\,V_{ph}\right)$$

Rms value of output voltage,

$$V_{0(rms)} = \sqrt{\left[\frac{3}{2\pi}\int_{\pi6}^{5\pi/6} V_{mp}^2 \sin^2\omega t\, d(\omega t)\right]}$$

$$\boxed{V_{0(rms)} = 0.84068\ V_{mp}}$$

Three-Phase Mid-point 6-Pulse Diode Rectifier:

Average output voltage,

$$V_0 = \frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} V_{mp}\sin\omega t\, d(\omega t)$$

$$\boxed{V_0 = \frac{3V_{mp}}{\pi}}$$

Rms value of output voltage,

$$V_{0(rms)} = \sqrt{\left[\frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} (V_{mp}\sin\omega t)^2 d(\omega t)\right]}$$

$$\boxed{V_{0(rms)} = 0.9558\ V_{mp}}$$

THREE PHASE DIODE BRIDGE RECTIFIER

Average Value of load voltage,

$$V_0 = \frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} V_{ml}\sin\omega t\ (d\omega t)$$

$$\boxed{V_0 = \frac{3V_{ml}}{\pi}}$$

Rms value of output voltage,

$$V_{0(rms)} = \sqrt{\left[\frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} V_{ml}^2 \sin^2\omega t\ d(\omega t)\right]}$$

$$\boxed{V_{0(rms)} = 0.9558\ V_{ml}}$$

THREE PHASE HALF-WAVE-CONTROLLED RECTIFIER:

If V_{mp} is the peak value of phase voltage, the average value of the output voltage,

$$V_o = \begin{cases} \frac{3\sqrt{3}}{2\pi} V_{mp} \cos\alpha & ;\text{for } 0 < \alpha < 30^\circ \\ \frac{3}{2\pi} V_{mp} \left(1 + \cos\left(\alpha + 30^\circ\right)\right) & ;\text{for } 30^\circ < \alpha < 150^\circ \end{cases}$$

If V_{ml} is the peak value of line voltage, average output voltage,

$$V_0 = \frac{3V_{ml}}{\pi} \cos\alpha$$

If I_0 is the load current, average value of source current,

$$I_s = I_0\sqrt{\frac{2}{3}}$$

Average value of thyristor current,

$$I_s = I_0\sqrt{\frac{1}{3}}$$

EFFECT OF SOURCE INDUCTANCE IN SINGLE PHASE RECTIFIER

1. It reduces the average output voltage.
2. It limits the maximum firing angle,

$$\boxed{\alpha_{max} = 180^0 - (\omega t_q + \mu_0)}$$

where, ωt_q = device turn-off time (in degrees)

μ_0=overlap angle at α =0

3. Displacement angle, $\theta_1=-(\alpha+\mu/2)$
4. Fundamental displacement factor,

$$\boxed{\text{Fundamental Displacement Factor} = \cos\left(\alpha + \frac{\mu}{2}\right)}$$

5. Current distortion factor (g) increase because the waveform is smoother than without L_s waveform

$$\text{Total Harmonic Distortion} = \left(\frac{1}{g^2} - 1\right)^{1/2}$$

6. Power factor (F.F.) = current distortion factor x fundamental displacement factor
7. Average reduction voltage $= \frac{V_m}{2\pi}[\cos\alpha - \cos(\alpha + \mu)] = \frac{\omega L_s}{2\pi} I_0$

DC DRIVES

$$E_a = \frac{Z\phi NP}{60A} = Z\phi n\left(\frac{P}{A}\right) \qquad \omega_m = 2\pi n$$

$$E_a = Z\phi\left(\frac{\omega_m}{2\pi}\right)\left(\frac{P}{A}\right) = \left(\frac{Z}{2\pi}.\frac{P}{A}\right)\phi\omega_m \qquad \Rightarrow \boxed{E_a = K_a\phi\omega_m}$$

Where, $\boxed{K_a = \frac{Z}{2\pi}\left(\frac{P}{A}\right)}$

E_aI_a (Electrical power) = $T_e\omega_m$ (Mechanical power)

$$T_e = \frac{E_aI_a}{\omega_m} \qquad \Rightarrow \boxed{T_e = K_a\phi I_a}$$

Buck Converter:

In Buck regulator, the average output voltage V_0 is less than the input voltage V_S.

$$\Delta I = \frac{(V_S - V_0)T_{ON}}{L}$$

$$\Delta I = \frac{V_0T_{OFF}}{L} \quad V_0 = V_S\frac{T_{ON}}{T} = V_S\alpha$$

Where $\Delta I = I_2 - I_1$ is the peak to peak current ripple of the inductor L.

The peak to peak ripple current is $\Delta I = \frac{V_S\alpha(1-\alpha)}{fL}$

The peak to ripple voltage of the capacitor is $\Delta V_C = \frac{V_S\alpha(1-\alpha)}{8LCf^2}$

Condition for continuous inductor current and capacitor voltage:

If I_L is average inductor current, the inductor ripple current $\Delta I = 2I_L$, which gives the critical value of the inductor L_C as $L_C = L = \frac{(1-\alpha)R}{2f}$

If V_C is the average capacitor voltage, the capacitor ripple voltage $\Delta V_C = 2V_0$, which gives the critical value of capacitor C_C as $C_C = C = \frac{1-\alpha}{16Lf^2}$

Boost Converter:

$$\Delta I = \frac{V_ST_{ON}}{L} = \frac{(V_0 - V_S)T_{OFF}}{L}$$

where $\Delta I = I_2 - I_1$ is peak to peak ripple current of the inductor L.

The average output voltage,

$$V_0 = V_S\frac{T}{T_{OFF}} = \left(\frac{1}{1-\alpha}\right)V_S$$

The peak to peak current ripple is, $\Delta I = \frac{V_S\alpha}{fL}$

The peak to peak ripple voltage of capacitor, $\Delta V_C = \frac{I_0\alpha}{fC}$

Condition of continuous inductor current and capacitor voltage:

If V_C is the average capacitor voltage, the capacitor ripple voltage $\Delta V_C = 2V_0$, which gives the critical value of the capacitor C_C as $C_C = \frac{\alpha}{2fR}$

Buck Boost Converter:

$$\Delta I = \frac{V_S T_{ON}}{L} = \frac{-V_0 T_{OFF}}{L}$$

where $\Delta I = I_2 - I_1$ is the peak to peak ripple current of inductor L.

The average output voltage is, $V_0 = -\frac{V_S \alpha}{1-\alpha}$

The peak to peak current ripple is, $\Delta I = \frac{V_S \alpha}{fL}$

peak to peak ripple voltage of the capacitor is, $\Delta V_C = \frac{I_0 \alpha}{fC}$

Condition of continuous inductor current and capacitor voltage:

If V_C is the average capacitor voltage, the capacitor ripple voltage, $\Delta V_C = 2V_0$, which gives the critical value of the capacitor C_C as $C_C = \frac{\alpha}{2fR}$.

Inverters

Series Inverters: In a series inverter, the commutating elements L and C are connected in series with the load resistance R. The load resistance R can also be in parallel with C. The value of L and C are such that those form an underdamped circuit i.e.

$$R^2 < \frac{4L}{C}$$

$$f = \left[\frac{1}{2\left(\frac{T}{2} + T_{off}\right)}\right]$$

The frequency of output voltage.

Where, $\frac{T}{2}$ is the time period of oscillations.

T_{off} is the time gap between turn-off one thyristor and turn-on of the second thyristor.

$$\frac{T}{2} = \frac{\pi}{\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}}$$

The period of oscillation

Parallel Inverter: During the working of this inverter, capacitor C comes in parallel with the load

$$V_c = 2V_s\left[2\exp\left(-\frac{n^2r}{2RC}\right)-1\right]$$

Where, $n = \frac{N_2}{N_1}$ (turns ratio)

$$C = \frac{n^2.t_c}{4R\ \text{In}\ 2}$$

Commutating capacitance,

Where, t_c =Circuit turn-off time

Bridge Inverter: Bridge circuits are commonly used in DC-AC conversion. Moreover, an output transformer is not essential in a bridge circuit.

1φ Half Bridge Inverter - The output voltage volt $V_0 = \sum_{n-1,3,5...}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t$

1φ Full Bridge Inverter- The output voltage

$$V_0 = \sum_{n-1,3,5,...}^{\infty} \frac{4V_s}{n\pi} \sin n\omega t$$

Where, n = order of harmonic

ω=2nf, is frequency of the output voltage in red/sec

Key points:

- The load impedance (Z_n) at frequency

  $$n_{f,}Z_n = \left[R^{2+}\left(n\omega L - \frac{1}{n\omega C}\right)^2\right]^{-1/2}$$

- Phase angle, (ϕ_n), $\phi_n = \tan^{-1}\frac{\left[n\omega L - \frac{1}{n\omega C}\right]}{R}$ red

- Output current or load current at the instant of commutation $I_0 = I_0 = \frac{V_0}{Z_n}$ or $\omega t = \pi$ rad

- Fundamental load power

  $(P_{01})\ P_{01} = I^2{}_{01}\ R = V_{01}.\ I_{01}\ \cos\phi_1$

Amplitude Modulation Depth:

$$m_0 = \frac{\hat{V}_m}{\hat{V}_c}$$

Where V_m, V_c are the modulating and carrier signal voltage, respectively.

For sinusoidal PWM, the amplitude modulation depth must be less than 1.0

Output Voltages:

$$\hat{V}_{kg1} = m_a \frac{V_{DC}}{2}$$

$$V_{kg1} = \frac{1}{2\sqrt{2}} m_a V_{DC}$$

the fundamental line-line voltage is given by

$$V_{LLO_1} = \frac{\sqrt{3}}{2\sqrt{2}} m_a V_{DC}$$

Available output voltage:

Assuming that the DC voltage is created using a diode rectifier and capacitor dc link, the maximum available DC voltage is given by

$$V_{DC} = \sqrt{2} V_{LL_S}$$

where V_{LLS} is the line-line supply voltage. The maximum output using sinusoidal PWM (m_a=1) is

$$V_{LLO_1} = \frac{\sqrt{3}}{2\sqrt{2}} \sqrt{2} V_{LLS} = \frac{\sqrt{3}}{2} V_{LLS}.$$

Resistive Load:

$$I(t) = \frac{V_s}{R}$$

where, V_s =DV voltage source

R=Load resistance

R-C (Resistive-Capacitive Load):

$$I(t) = \frac{V_s}{R} e^{-t/RC}$$

$$V_c(t) = V_s(1 - e^{-t/\tau})$$

Where, $V_c(t)$=Voltage across capacitor at time t

Where, $V_c(t)$=Voltage across capacitor at time t

Initial rate of rise of capacitor voltage:

$$\left(\frac{dV_c}{dt}\right)_{t=0} = \frac{V_s}{RC} = \frac{V_s}{\tau} \text{ and time constant } \tau = \frac{V_s}{\left(\frac{dV_c}{dt}\right)_{t=0}}$$

R-L Load:

$$I(t) = \frac{V_s}{R}(1 - e^{Rt/L})$$

$$V_L = V_s e^{-Rt/L}; V_L = \text{Voltage accros inductor}$$

Initial rate of rise of current $\left(\frac{dI}{dt}\right)_{t=0} = \frac{V_s}{L}$

L-C Load:

$I(t) = I_P \sin \omega_0 t$

Where, $I_P = V_s\sqrt{\frac{C}{L}}$ and

$\omega_0 = \frac{1}{\sqrt{LC}}$ = resonant frequency

$\Rightarrow I(t) = V_s\sqrt{\frac{C}{L}} \sin \omega_0 t$

Key Points

- Voltage across inductor, $V_L = V_s (\cos \omega_0 t)$
- Voltage across inductor, $V_L = V_s (1 - \cos \omega_0 t)$

  $t_1 = \pi\sqrt{LC} = \frac{\pi}{\omega_0}$
- Conduction time,

R-L-Load:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Damping factor, $\xi = \frac{R}{2L}$

Resonant frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/s

Ringing frequency,

(or damped resonant frequency)

$\omega_0 = \sqrt{\omega_0^{\,2} - \xi^2}$

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