Rajasthan RVUNL

Electrical Engineering

Electrical Machines

Important Formula Notes

ELECTRICAL MACHINES (FORMULA NOTES)

1. Transformer:

- Gross cross-sectional area = Area occupied by magnetic material + Insulation material.
- Net cross sectional area = Area occupied by only magnetic material excluding area of insulation material.
- Hence for all calculations, net cross-sectional area is taken since majority $\phi(\text{flux})$ flows in magnetic material.

  $\phi = BA_n$

- Specific weight of t/f = $\dfrac{\text{Weight of Transformer}}{\text{KVA rating of Transformer}}$
- Staking/iron factor:

  $(k_s) = \dfrac{\text{Net cross sectional area}}{\text{Gross cross sectional area}}$

- k_s is always less than 1
- Gross CS area = A_G = Length × Breadth
- Net CS area = A_n = $k_s \times A_G$
- Utilization factor of transformer core

  $= \dfrac{\text{Effective CS area}}{\text{Total CS area}}$ U.F. of cruciform core = 0.8 to 0.85.

- Flux = $\dfrac{\text{mmF}}{\text{Reluctance}} = \phi_m \sin\omega t$
- According to faradays second law,

  $e_1 = -N_1 \dfrac{d\phi}{dt} = -N_1 \dfrac{d}{dt}(\phi_m \sin\omega t)$

  Instantaneous value of emf in primary

  $e_1 = N_1\phi_m\omega \sin\left(\omega t - \dfrac{\pi}{2}\right)$

- Transformer emf equations:

  $E_1 = 4.44\ N_1\ B_{max}\ A_n f$ ... (i)

  $E_2 = 4.44\ N_2\ B_{max}\ A_n f$ ... (ii)

- Emf per turn in Primary:

  $I^{ry} = \dfrac{E_1}{N_1} = 4.44 B_{max} A_n f$

- Emf per turn in Secondary:

$II^{ry} = \frac{E_2}{N_2} = 4.44B_{max}A_nf$

- Emf per turn on both sides of the transformer is same

  $\frac{E_1}{N_1} = \frac{E_2}{N_2} \Rightarrow \frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{1}{k}$

  Transformation ratio = $K = \frac{E_2}{E_1} = \frac{N_2}{N_1}$

  Turn ratio = $\frac{1}{K} = N_1 : N_2$

- For an ideal two-winding transformer with primary voltage V_1 applied across N_1 primary turns and secondary voltage V_2 appearing across N_2 secondary turns:

  $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

- The primary current I_1 and secondary current I_2 are related by:

  $\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{V_2}{V_1}$

- For an ideal step-down autotransformer with primary voltage V_1 applied across (N_1 + N_2) primary turns and secondary voltage V_2 appearing across N_2 secondary turns.

  $\frac{V_1}{V_2} = \frac{(N_1 + N_2)}{N_2}$

- The primary (input) current I_1 and secondary (output) current I_2 are related by:

  $\frac{I_1}{I_2} = \frac{N_2}{(N_1 + N_2)} = \frac{V_2}{V_1}.$

- For a single-phase transformer with rated primary voltage V_1, rated primary current I_1, rated secondary voltage V_2 and rated secondary current I_2, the volt ampere rating S is:

  $S = V_1I_1 = V_2I_2$

- For a balanced m-phase with rated primary phase voltage V_1 rated primary current I_1, rated secondary phase voltage V_2 and rated secondary current I_2, the volt ampere rating S is:

  $S = mV_1I_1 = mV_2I_2$

- The primary circuit impedance Z_1 referred to the secondary circuit for an ideal transformer with N_1 primary turns and N_2 secondary turns is:

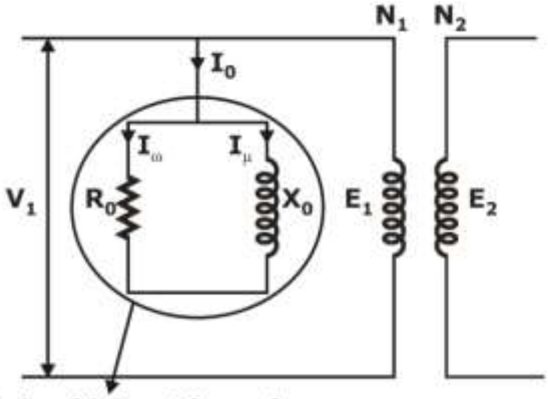
  $Z_{12} = Z_1\left(\frac{N_2}{N_1}\right)^2$

- During operation of transformer:

  $B_m \propto \frac{E_1}{f} \propto \frac{V_1}{f}$

$$B_{max} = \text{Constant} \Rightarrow \frac{V_1}{f} = \text{Constant}$$

Equivalent circuit of Transformer under No-Load condition:



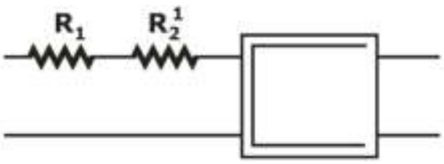
- No load current = $I_0 = \bar{I}_\mu + \bar{I}_w = I_0\angle -\phi_0$

$I_w = I_0 \cos\phi_0$
$I_\mu = I_0 \sin\phi_0$

- No load power = $V_1 I_0 \cos\phi_0 = V_1 I_w$ = Iron losses

$$R_0 = \frac{V_1}{I_{w1}}; X_0 = \frac{V_1}{I_\mu} \Rightarrow \quad I_w = \frac{\text{No load power}}{V_1}$$

Transferring from Secondary to Primary:



$I_2^2 R_2 = I_1^2 R_2^1$

$$R_2^1 = R_1\left(\frac{I_2}{I_1}\right)^2 = \frac{R_2}{K^2} \quad \because \qquad R_2^1 = \frac{R_2}{K^2}$$

From Primary to Secondary:

$I_1^2 R_1 = I_2^2 R_1^1$

$$R_1^1 = \frac{I_1^2}{I_2^2} \times R_1 = R_1 K_2$$

- Total resistance ref to primary = $R_1 + R_2^1$

  $$R_{01} = R_1 + \frac{R_2}{k^2}$$

- Total resistance ref to secondary = $R_2 + R_1^1$

  $R_{02} = R_2 + k^2 R_1$

- Total Cu loss = $I_1^2 R_{01}$ or $I_2^2 R_{02}$

Per unit resistance drops:

- P.U. primary resistance drop = $\frac{I_1R_1}{E_1}$
- P.U. secondary resistance drop = $\frac{I_2R_2}{E_2}$
- Total P.U. resistance drop ref to I^{ry} = $\frac{I_1R_{01}}{E_1}$
- Total P.U. resistance drop ref to II^{ry} = $\frac{I_2R_{02}}{E_2}$
- The P.U resistance drops on both sides of the t/f is same

  $\frac{I_1R_{01}}{E_1} = \frac{I_2R_{02}}{E_2}$

Losses present in transformer:

1. Copper losses: Transformer windings major losses
2. Iron losses: Transformer core
3. Stray load losses: Cu parts and Iron parts minor losses
4. Dielectric losses: Insulating materials.

1. Cu losses in transformer:

Total Cu loss = $I_1^2R_1 + I_2^2R_1 = I_1^2R_{01} = I_2^2R_{02}$

- Rated current on I^{ry} = $\frac{\text{VA rating of t / f}}{E_1}$
- Similarly, current on II^{ry} = $\frac{\text{VA rating of t / f}}{E_2}$
- Cu losses $\propto I_1^2$ or I_2^2. Hence these are called as variable losses.
- P.U. full load Cu loss = $\frac{\text{FL Cu loss in watts}}{\text{VA rating of t / f}} = \frac{I_1^2R_{01}}{E_1I_1}$
- If VA rating of transformer is taken as base then P.U Cu loss $\propto I_1^2$ as remaining terms are constant.
- P.U. Cu loss at x(Loading factor) of FL = x^2 × PU FL Cu loss
- $\left.\begin{array}{l}\text{PU resistance drop ref to } I^{ry} \\ \text{PU resistance ref to } I^{ry}\end{array}\right\} = \frac{I_1R_{01}}{E_1} \times \frac{I_1}{I_1} = \frac{I_1^2R_{01}}{E_1I_1}$

  $\therefore$ PU resistance drop = PU Full Load Cu loss

  % FL Cu loss = %R = %Resistance drop.

Iron (or) Core losses in Transformer:

1. Hysteresis loss:

Steinmetz formula:

$W_h = \eta B_{max}^x f V$

Area under one hysteresis loop

where,

η = Stienmetz coefficient

B_{max} = Maximum flux density in transformer core.

f = Frequency of magnetic reversal = supply frequency.

V = Volume of core material

x = Hysteresis coefficient (or) stienmetz exponent = 1.6 (Si or CRGO steel)

2. Eddy current loss:

Eddy current loss, $W_e \propto R_{ce} \times I_e^2$

As area decreases in laminated core resistance as a result conductivity decrease.

$$W_e = K(\text{Constant})B_{max}^2 f^2(\text{Supply freq.}) \times t^2(\text{Thickness of lamination})$$

(It is a function of σ)

During operation of transformer:

$$B_m \propto \frac{V_1}{f}$$

Case(i): $\frac{V_1}{f}$ = Constant, B_{max} = Constant

$$W_e \propto f^2$$

$$W_e = Bf^2$$

$$\therefore \left.\begin{aligned} W_i &= W_h + W_e \\ W_i &= Af + Bf^2 \end{aligned}\right\} \text{When } B_{max} = \text{Constant}$$

Case(ii): $\frac{V_1}{f} \neq$ Constant, $B_{max} \neq$ Constant

$$W_e \propto \left(\frac{V_1}{f}\right)^2 f^2$$

$$W_e \propto V_1^2$$

$$W_i = W_h + W_e$$

$$W_i = \frac{AV_1^{1.6}}{f^{0.6}} + BV_1^2$$

P.U. iron loss:

- P.U. iron loss = $\frac{\text{Iron loss in watts}}{\text{VA rating of t / f}}$
- As VA rating is chosen as base then the P.U. iron loss are also constant at all load conditions.

To find out constant losses:

- W_0 = Losses in t/f under no-load condition = Iron losses + Dielectric loss + no-load primary loss ($I_0^2R_1$)

- Constant losses = $W_0 - I_{02}^2 R_1$

  Where, R_1 = LV winding resistance.

To find out variable losses:

- W_{sc} = Loss in transformer under SC condition = Full Load Cu loss + stray load losses (Cu and Iron) + Iron losses in both windings.
- Variable losses = W_{sc} – Iron losses corresponding to V_{SC}.

OC Test:

$$V_1 \text{ rated} \rightarrow W_i$$

SC Test:

$$V_{sc} \rightarrow (W_i)_{sc}$$
$$W_i \propto V_1^2$$

$$\frac{W_i}{(W_i)_{sc}} = \left(\frac{V_1 \text{ rated}}{V_{sc}}\right)^2$$

$$(W_i)_{sc} = W_i\left(\frac{V_{sc}}{V_1 \text{ rated}}\right)^2$$

$$\therefore \text{ Variable losses} = W_{sc} - (W_i)_{sc}\left(\frac{V_{sc}}{V_1 \text{ rated}}\right)^2$$

- Under the assumption that small amount of iron losses corresponds to V_{sc} and stray load losses are neglected the wattmeter reading in SC test can be approximately taken as FL Cu losses in the transformer.
- $W_{sc} \simeq \text{FL Cu loss} \simeq I_{sc}^2 \times R_{01}$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2}$$

Efficiency:

- Efficiency of transformer is given by

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{input power} + \text{losses}}$$

$$= \frac{E_2 I_2 \cos\phi_2}{E_2 I_2 \cos\phi_2 + \text{FL Cu losses} + \text{Iron losses}}$$

$$\eta_{FL} = \frac{E_2 I_2 \cos\phi_2}{E_2 I_2 \cos\phi_2 + I_2^2 R_{02} + W_i}$$

$$\eta_{x \text{ of FL}} = \frac{x(E_2 I_2)\cos\phi_2}{x(E_2 I_2)\cos\phi_2 + x^2\left(I_2^2 R_{02}\right) + W_i}$$

- Transformer efficiency = $\frac{KVA \times \cos\phi}{KVA \times \cos\phi + W_i + Cu\ losses}$
- Voltage drop in t/f at a specific load, p.f. = $I_2R_{02}\cos\phi_2 \pm I_2X_{02}\sin\phi_2$
- % Voltage regulation = $\frac{I_2R_{02}\cos\phi_2 \pm I_2X_{02}\sin\phi_2}{V_2} \times 100 = \left(\frac{I_2R_{02}}{V_1}\right)\cos\phi_2 \pm \left(\frac{I_2R_{02}}{V_1}\right)\sin\phi_2$

  % Regulation = $\left[(R_{PU})\cos\phi_2 + (X_{PU})\sin\phi_2\right] \times 100$

Auto Transformer:

- K of auto transformer = $\frac{LV}{HV}$

  $(KVA)_{induction} = (V_1 - V_2) I_1$

  Input KVA = V_1I_1

  $\frac{(KVA)_{induction}}{input\ KVA} = \frac{(V_1 - V_2)I_1}{V_1I_1} = 1 - \frac{LV}{HV} = 1 - K$

  $\therefore$ $(KVA)_{induction}$ = (1 – K) input KVA

  $(KVA)_{induction}$ = Input KVA – $(KVA)_{ind}$

  $(KVA)_{conduction}$ = K × input KVA
- Wt. of conductor in section AB of auto $t/f \propto (N_1 - N_2)I_1$
- Wt. of conductor in section BC of auto $t/f \propto (I_1 - I_2)N_1$

  $\therefore$ Total Wt. of conductor in auto t/f is

  $\propto I_1(N_1 - N_2)N_1 + (N_2 - N_1)N_2$

  $\propto 2(N_1 - N_2)I_1$
- Total Wt. of conductor in 2 wdg transformer

  $\propto I_1N_1 + I_2N_2 \propto 2I_1N_1$
- $\frac{\text{Wt. of conductor in an auto } t/f}{\text{Wt. of conductor in 2 wdg } t/f} = \frac{2(N_1 - N_2)I_1}{2N_1I_1} = 1 - \frac{N_2}{N_1} = 1 - K$

  Wt. of conductor in auto t/f = (1 – K) (Wt. of conductor in 2 wdg t/f)
- Thus, saving of conductor material if auto-transformer is used} = K × (Conductor wt. in 2 wdg transformer)
- $(\%\ FL\ losses)_{AT} = (1 - K)(\%\ FL\ losses)_{2wdg\ t/f}$
- $(\%Z)_{AT} = (1 - K)(\%Z)_{2wdg\ t/f}$
- $(KVA)_{AT} = \frac{1}{1-K}(KVA)_{2wdg\ t/f}$

2. DC Machines:

S.No.		Lap winding	Wave winding
1.	Coil span:	$Y_{cs} = s/p$	$Y_{cs} = s/p$
2.	Back pitch	$Y_b = UY_{cs}$	$Y_b = UY_{cs}$
3.	Commutator pitch	$Y_c = 1$ for progressive winding $Yc = -1$ for retrogressive winding	$Y_c = \frac{2(c+1)}{p}$ for progressive winding $Y_c = \frac{2(c+1)}{p}$ for retrogressive winding (Y_c must be integer)
4.	Front pitch	$Y_f = Y_b + 2$ for progressive winding $Y_f = Y_b - 2$ for retrogressive winding	$Y_f = 2Y_b - Y_b$
5.	Parallel paths	A = P	A = 2
6.	Conductor current	$I_c = \frac{I_a}{A}$	$I_c = \frac{I_a}{A}$
7.	No. of brushes	No. of brushes = A = P	No. of brushes = 2

S = No. of commutator segments

P = No. of poles

U = No. of coil sides/No. of poles = $\frac{2c}{s}$

C = No. of coils on the rotor

A = No. of armature parallel paths

I_a = Armature current

- Distribution factor

$$(K_d) = \frac{\text{Phasor sum coil emf}}{\text{Arthematic sum of coil emf}} = \frac{\text{Chord}}{\text{arc}} = \frac{2}{\pi}$$

- Pitch factor

$$(K_p) = \frac{\text{Electrical angle of coil}}{180^\circ} \times 100\%$$

- $\theta^0_{\text{electrical}} = \frac{P}{2}\theta^0_{\text{mechanical}}$

- Armature mmf/Pole(Peak), $AT_a = \frac{zI_a}{2AP}$

- AT (compensating winding) $= \frac{zI_a}{2AP} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$
- At (Inter pole) $= AT_a + \frac{B_i}{\mu_o} I_{gi}$

  where B_i = flux density in inter pole air gap

  I_{gi} = Length of inter pole air gap,

  $\mu_0 = 4\pi \times 10^{-7}$
- No of turns in each interpole,

  $N_{interpole} = \frac{AT\,(\text{Inter pole})}{I_a}$
- The no. of compensating conductor per pole, $\frac{N_{cw}}{\text{Pole}} = \frac{Z}{AP}\left(\frac{\text{Pole arc}}{\text{Pole pitch}}\right)$
- The mechanical power that is converted is given by $P_{conv} = T_{ind}\,\omega_m$

  Where, T = Induced torque

  ω_m = Angular speed of the machine rotor
- The resulting electric power produced $P_{conv} = E_A I_A$
- The power balance equation of the DC machine is $T_{ind}\,\omega_m = E_A I_A$
- The induced emf in the armature is $E_a = \frac{\varnothing ZNP}{60A}$.
- Torque developed in DC machine, $T_e = \frac{PZ}{2\pi A}\varnothing I_a$.

  Where $\varnothing = \frac{\text{Flux}}{\text{Pole}}$, Z = Number of armature conductors, P = No. of poles, N = Speed in rpm,

  A = No. of armature parallel paths I_a = Armature current
- The terminal voltage of the DC generator is given by $V_t = E_a - I_a R_a$
- The terminal voltage of the DC motor is given by $V_t = E_a + I_a R_a$
- Speed regulation of DC machine is given by, $SR = \frac{\omega_{n1} - \omega_{f1}}{\omega_{f1}} \times 100\% = \frac{N_{n1} - N_{f1}}{N_{f1}} \times 100\%$
- Voltage regulation, $VR = \frac{V_{n1} - V_{f1}}{V_{f1}} \times 100\%$

Shunt Generator:

- For a shunt generator with armature induced voltage E_a, armature current I_a and armature resistance R_a, the terminal voltage V is:

  $V = E_a - I_a R_a$
- The field current I_f for a field resistance R_f is: $I_f = \frac{V}{R_f}$
- The armature induced voltage E_a and torque T with magnetic flux Φ at angular speed ω are:

$E_a = k_f \Phi \omega = k_m \omega \qquad T = k_f \Phi I_a = k_m I_a$

Where, k_f and k_m are design coefficients of the machine.

Note that for a shunt generator:

- Induced voltage is proportional to speed.
- Torque is proportional to armature current.

- The airgap power P_e for shunt generator is:

  $P_e = \omega T = E_a I_a = k_m \omega I_a$

Series Generator:

- For a series generator with armature induced voltage E_a, armature current I_a, armature resistance R_a and field resistance R_f, the terminal voltage V is:

  $V = E_a - (I_aR_a + I_aR_f) = E_a - I_a(R_a + R_f)$

  The field current is equal to the armature current.

- The armature induced voltage E_a and torque T with magnetic flux Φ at angular speed ω are:

  $E_a = k_f \Phi \omega I_a = k_m \omega I_a$

  $T = k_f \Phi I_a^2 = k_m I_a^2$

  Where k_f and k_m are design coefficient of the machine.

  Note that for a series generator:

  - Induced voltage is proportional to both speed and armature current,
  - Torque is proportional to the square of armature current.
  - Armature current is inversely proportional to speed for a constant E_a

- The airgaps power P_e for a series generator is:

  $P_e = \omega T = E_a I_a = k_m \omega I_a^2$

- Cumulatively compounded DC generator: (long shunt)

  (a) $I_a = I_f + I_L$

  (b) $V_t = E_a - I_a (R_a + R_s)$

  (c) $I_{sf} = \frac{V_x}{R_f} =$ Shunt field current

  (d) The equivalent effective shunt field current for this machine is given by

  $$I_{sf} = I_{sf} + \frac{N_{se}}{N_f} I_a - \left(\frac{\text{Armature reaction MMF}}{N_f} \right)$$

  Where, N_{se} = No. of series field turns

  N_f = No of shunt field turns

- Differentially compounded DC generator: (long shunt)

  (a) $I_a = I_f + I_L$

  (b) $V_t = E_a - I_a (R_a + R_s)$

(c) $I_{sf} = \frac{V_x}{R_f}$ = Shunt field current

(d) The equivalent effective shunt field current for this machine is given by

$$I_{sf} = I_{sf} - \frac{N_{se}}{N_f} I_a - \left(\frac{\text{Armature reaction MMF}}{N_f}\right)$$

Where, N_{se} = No. of series field turns

N_f = No of shunt field turns

Shunt Motor:

- For a shunt generator with armature induced voltage E_a, armature current I_a and armature resistance R_a, the terminal voltage V is:

  $V = E_a + I_aR_a$

  The field current I_f for a field resistance R_f is: $I_f = \frac{V}{R_f}$

- The armature induced voltage E_a and torque T with magnetic flux Φ at angular speed ω are:

  $$E_a = k_f\Phi\omega = k_m\omega$$
  $$T = k_f\Phi I_a = k_m I_a$$

  Where, k_f and k_m are design coefficients of the machine.

  Note that for a shunt motor:

  - Induced voltage is proportional to speed.
  - Torque is proportional to armature current.

- The airgap power P_e for shunt generator is:

  $$P_e = \omega T = E_a I_a = k_m \omega I_a$$

- The speed of the shunt motor, $\omega = \frac{V}{k\varnothing} - \frac{R_a}{(K\varnothing)^2}T$ where, $K = \frac{PZ}{2\pi A}$

Series Motor:

- For a series motor with armature induced voltage E_a, armature current I_a, armature resistance R_a and field resistance R_f, the terminal voltage V is:

  $V = E_a + (I_aR_a + I_aR_f) = E_a + I_a(R_a + R_f)$

  The field current is equal to the armature current.

- The armature induced voltage E_a and torque T with magnetic flux Φ at angular speed ω are:

  $$E_a = k_f\Phi\omega I_a = k_m\omega I_a \qquad T = k_f\Phi I_a^2 = k_m I_a^2$$

  Where k_f and k_m are design coefficient of the machine.

  Note that for a series motor:

  - Induced voltage is proportional to both speed and armature current.
  - Torque is proportional to the square of armature current.
  - Armature current is inversely proportional to speed for a constant E_a

- The airgaps power P_e for a series motor is:

$$P_e = \omega T = E_a I_a = k_m \omega I_a^2$$

Losses:

- Constant losses (P_k) = P_{wf} + P_{io}

  where, P_{io} = No of load core loss
- Variables losses (P_v) = P_c + P_{st} + P_b

  Where, P_c = Copper losses = $I_a^2 R_a$

  P_{st} = Stray load loss = αI_2

  P_b = Brush contact drop = $V_b I_a$, where, V_b = Brush voltage drop
- The total machine losses, $P_L = P_k + V_b I_a + K_v I_a^2$

Efficiency:

- The per unit efficiency η of an electrical machine with input power P_{in}, output power P_{out} and power loss P_{loss} is:

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{(P_{out} + P_{loss})} = \frac{(P_{in} - P_{loss})}{P_{in}}$$

- Rearranging the efficiency equations:

$$P_{in} = P_{out} + P_{loss} = \frac{P_{out}}{\eta} = \frac{P_{loss}}{(1-\eta)}$$

$$P_{out} = P_{in} - P_{loss} = \eta P_{in} = \frac{\eta P_{loss}}{(1-\eta)}$$

$$P_{loss} = P_{in} - P_{out} = (1-\eta) P_{in} = \frac{(1-\eta) P_{out}}{\eta}$$

Temperature Rise:

- The resistance of copper and aluminum windings increases with temperature, and the relationship is quite linear over the normal range of operating temperatures. For a linear relationship, if the winding resistance is R_1 at temperature θ_1 and R_2 at temperature θ_2, then:

$$\frac{R_1}{(\theta_1 - \theta_0)} = \frac{R_2}{(\theta_2 - \theta_0)} = \frac{(R_2 - R_1)}{(\theta_2 - \theta_1)}$$

  where θ_0 is the extrapolated temperature for zero resistance.
- The ratio of resistances R_2 and R_1 is:

$$\frac{R_2}{R_1} = \frac{(\theta_2 - \theta_0)}{(\theta_1 - \theta_0)}$$

- The average temperature rise Δθ of a winding under load may be estimated from measured values of the cold winding resistance R_1 at temperature θ_1 (usually ambient temperature) and the hot winding resistance R_2 at temperature θ_2, using:

$$\Delta\theta = \theta_2 - \theta_1 = \frac{(\theta_1 - \theta_0)(R_2 - R_1)}{R_1}$$

- Rearranging for per-unit change in resistance ΔR_{pu} relative to R_1:

$$\Delta R_{pu} = \frac{(R_2 - R_1)}{R_1} = \frac{(\theta_2 - \theta_1)}{(\theta_1 - \theta_0)} = \frac{\Delta\theta}{(\theta_1 - \theta_0)}$$

Copper Windings:

- The value of θ_0 for copper is −234.5°C, so that:

$$\Delta\theta = \theta_2 - \theta_1 = (\theta_1 + 234.5)\frac{(R_2 - R_1)}{R_1}$$

- If θ_1 is 20°C and $\Delta\theta$ is 1 degC:

$$\Delta R_{pu} = \frac{(R_2 - R_1)}{R_1}$$

$$= \frac{\Delta\theta}{(\theta_1 - \theta_0)} = \frac{1}{254.5} = 0.00393$$

- The temperature coefficient of resistance of copper at 20°C is 0.00393 per degC.

Aluminium Windings:

- The value of θ_0 for aluminium is - 228°C, so that:

$$\Delta\theta = \theta_2 - \theta_1 = (\theta_1 + 228)\frac{(R_2 - R_1)}{R_1}$$

- If θ_1 is 20°C and $\Delta\theta$ is 1 deg C:

$$\Delta R_{pu} = \frac{(R_2 - R_1)}{R_1} = \frac{\Delta\theta}{(\theta_1 - \theta_0)} = \frac{1}{248} = 0.00403$$

- The temperature coefficient of resistance of aluminium at 20°C is 0.00403 per deg C.

Dielectric Dissipation Factor:

- If an alternating voltage V of frequency f is applied across an insulation system comprising capacitance C and equivalent series loss resistance R_S, then the voltage V_R across R_S and the voltage V_C across C due to the resulting current I are:

  $V_R = IR_S$

  $V_C = IX_C$

  $V = (V_R^2 + V_C^2)^{1/2}$

- The dielectric dissipation factor of the insulation system is the tangent of the dielectric loss angle δ between V_C and V:

$$\tan\delta = \frac{V_R}{V_C} = \frac{R_S}{X_C} = 2\pi fCR_S$$

$$R_S = X_C \tan\delta = \frac{\tan\delta}{2\pi fC}$$

- The dielectric power loss P is related to the capacitive reactive power Q_C by:

$P = I^2R_S = I^2X_C\tan\delta = Q_C\tan\delta$

- The power factor of the insulation system is the cosine of the phase angle ϕ between V_R and V:

$$\cos\phi = \frac{V_R}{V}$$

So that δ and ϕ are related by:

$\delta + \phi = 90°$

- tanδ and $\cos\phi$ are related by:

$$\tan\delta = \frac{1}{\tan\phi} = \frac{\cos\phi}{\sin\phi} = \frac{\cos\phi}{(1-\cos^2\phi)^{1/2}}$$

So that when $\cos\phi$ is close to zero,

$\tan\delta \approx \cos\phi$

3. Synchronous Machines:

- **Principle of operation:**
  Whenever a conductor cuts the magnetic flux, an emf is induced in that conductor"
  -Faraday's law of electromagnetic induction.
- **Coil span (β):** It is the distance between two sides of the coil. It is expressed in terms of degrees, pole pitch, no. of slots / pole etc.
- **Pole pitch:** It is the distance between two identical points on two adjacent poles.
  Pole pitch is always 180° e = slots / pole.
- $\theta_{elec} = \frac{P}{2}\theta_{mech}$
- **Slot pitch or slot angle:** (T) Slot angle is the angle for each slot.
- For a machine with 'P' poles and 's' no. of slots, the slot angle = $Y = \frac{P(180°)}{d}$

$$Y = \frac{(180°)}{(s/p)}$$

- Pitch factor or coil span factor or chording factor :- (K_P)

$$K_p = \frac{\text{The emf induced | Coil in short pitched winding}}{\text{The emf induced | Coil in full pitched winding}}$$

$$= \frac{\text{The vector sum of indeed emf | Coil}}{\text{Arithmetic sum of induced emf | Coil}}$$

$$K_p = \frac{2E\cos\propto/2}{2E} = \cos\frac{\propto}{2}$$

- Pitch factor for n^{th} harmonic i.e., $K_{p_n} = \frac{\cos n\propto}{2}$
- Chording angle to eliminate n^{th} harmonics

$(\alpha) = \frac{180°}{n}$.

- Coil spam to eliminate n[th] harmonics, $(\beta) = 180\left(\frac{n-1}{n}\right)$.

- Distribution factor | Spread factor | belt factor | breadth factor (kd):

$$K_d = \frac{\text{The emf induced when the winding is distributed}}{\text{The emf induced when the winding is concentrated}}$$

$$= \frac{\text{Vector sum of emf induced}}{\text{Arithmetic sum of emf induced}}$$

$$K_d = \frac{\sin\frac{mY}{2}}{m\sin\frac{Y}{2}}$$

- The distribution factor for uniformly distributed winding is

$$kd_4 = \frac{\sin\frac{mr}{2}}{\frac{mr}{2}\times\frac{\pi}{180}}$$

For n[th] harmonic, $kd_n = \frac{\sin\frac{mnr}{2}}{m\sin\frac{ny}{2}}$

- To eliminate n[th] harmonics phase spread (my) = $\frac{360°}{n}$

- Generally, KVA rating, power output $\propto$ kd and E_{ph} (induced emf) $\propto k_d \times T_{ph}$

$$\therefore \frac{KVA_{60}(3-\phi)}{KVA_{120}(3-\phi)} = \frac{Pout_{60}(3\phi)}{Pout_{120}(3\phi)} = \frac{kd_{60}}{kd_{120}}$$

$$= \frac{\sin\frac{60}{2}}{\sin\frac{120}{2}} \times \frac{m_{120}}{m_{60}} = \frac{\sin 30°}{\sin 60°} \times \frac{120}{60} = 1.15$$

$$\frac{KVA_{60}(3\phi)}{KVA_{120}(2\phi)} = \frac{Pout_{60}(3\phi)}{Pout_{90}(2\phi)} = \frac{kd_{60}}{kd_{90}} = \frac{\sin\frac{60}{2}}{\sin\frac{90}{2}} \times \frac{90}{60} = 1.06$$

$$\frac{KVA_{60}(3\phi)}{KVA_{180}(1\phi)} = \frac{Pout_{60}(3\phi)}{Pout_{180}(1-\phi)} = \frac{kd_{60}}{kd_{180}} = \frac{\sin\frac{60}{2}}{\sin\frac{180}{2}} \times \frac{180}{60} = 1.5$$

$$\frac{KVA_{90}(2-\phi)}{KVA_{180}(1-\phi)} = \frac{Pout_{90}}{Pout_{180}} = \frac{kd_{90}}{kd_{180}} = \frac{\sin\frac{90}{2}}{\sin\frac{180}{2}} \times \frac{180}{90} = 1.414$$

- Speed of space harmonics of order $(6k \pm 1)$ is $\frac{1}{(6k \pm 1)}N_s$

  where N_s = Synchronous speed = $\frac{120f}{p}$

  The order of slot harmonics is $\left(\frac{2s}{p} \pm 1\right)$

  where S = No. of slots, P = No. of poles
- Slot harmonics can be eliminated by skewing the armature slots and fractional slot winding. The angle of skew = θ_s = γ (slot angle) = 2 harmonic pole pitches = 1 slot pitch.
- Distribution factor for slot harmonics,

  $k_d\left(\frac{2s}{p} \pm 1\right)$

  Is $kd_1 = \frac{\sin\frac{m\gamma}{2}}{m\sin\frac{\gamma}{2}}$ i.e., same that of fundamental
- Pitch factor for slot harmonics, $k_p\left(\frac{2s}{p} \pm 1\right) = k_{p_1} = \cos\frac{\propto}{2}$
- The synchronous speed N_s and synchronous angular speed of a machine with p pole pairs running on a supply of frequency f_s are:

  $\omega_s = \frac{2\pi f_s}{p}$
- Slip S = $\frac{N_S - N}{N_S}$

  Where $N_S = \frac{120f}{p}$ = synchronous speed
- The magnitude of voltage induced in a given stator phase is $E_a = \sqrt{2}\pi N_c \varnothing f = K\varnothing\omega$ Where K = constant
- The output power P_m for a load torque T_m is:

  $P_m = \omega_s T_m$
- The rated load torque T_M for a rated output power P_M is:

  $T_M = \frac{P_M}{\omega_s} = \frac{P_M p}{2\pi f_s} = \frac{120P_M}{2\pi N_s}$

Synchronous Generator:

- For a synchronous generator with stator induced voltage E_s, stator current I_s and synchronous impedance Z_s, the terminal voltage V is:

  $V = E - I_s Z_s = E_s - I_s(R_s + jX_s)$

  where R_s is the stator resistance and X_s is the synchronous reactance

$$E = \sqrt{(V\cos\phi + I_aR_a)^2 + (V\sin\phi \pm I_aX_s)^2}$$

$+ \Rightarrow$ lag p.f.

$- \Rightarrow$ leading p.f.

Synchronous Motor:

- For a synchronous motor with stator induced voltage E_s, stator current I_s and synchronous impedance Z_s, the terminal voltage V is:

  $V = E_s + I_sZ_s = E_s + I_s(R_s + jX_s)$

  where R_s is the stator resistance and X_s is the synchronous reactance

Voltage regulation:

- % regulation = $\frac{|E| - |V|}{|V|} \times 100$

  $E - V = I_aZ_s$

  $\therefore$ % regulation = $\frac{E - V}{V} = \frac{I_aZ_s}{V} \times 100$

  $\therefore$ regulation $\propto Z_s$

  $\therefore$ As Z_s increases, voltages regulation increases.

- Condition for zero | min. voltage regulation is, $\cos(\theta + \varphi) = -\frac{I_aZ_s}{2V}$

- Condition for max. Voltage regulation is, $\varphi = \theta$

- Short circuit ratio (SCR)

  $$\frac{I_{f_m}}{I_{f_a}} = \frac{1}{Z_s\,(\text{adjusted})|_{\text{unit}}} = \frac{1}{X_s\,(\text{adjusted})|_{\text{unit}}}$$

  $$\text{SCR} \propto \frac{1}{X_a} \propto \frac{1}{\text{Armature reaction}}$$

  Voltage regulation $\propto$ Armature reaction

  $$\therefore\ \text{SCR} \propto \frac{1}{\text{Voltage reaction}}$$

  $\therefore$ Small value of SCR represents poor regulation.

  $$\varphi_a = \frac{\text{Armature mmf}}{\text{Reluctance}}$$

  But reluctance $\propto$ Air gap

  $$\therefore \varphi_a = \frac{\text{Armature mmf}}{\text{Airgap}}$$

  $$\varphi_a \propto \frac{1}{\text{Airgap length}}$$

  $$\text{Armature reaction} \propto \varphi_a \propto \frac{1}{\text{Airgap length}}$$

$\therefore \text{SCR} \propto \frac{1}{\text{Armature reaction}} \propto \text{Airgap length}$

Air gap length $\propto$ SCR

$\therefore$ Machine size $\propto$ SCR Cost $\propto$ SCR

$$\text{Power} = \frac{EV}{X_a}\sin\delta \Rightarrow P \propto \frac{1}{X_s} \propto \text{SCR}$$

Power $\propto$ SCR

$\therefore$ Large value of SCR represents more power output.

- Synchronizing power coefficient or stability factor P_{sy} is given as

$$P_{sy} = \frac{dp}{d\delta} = \frac{d}{d\delta}\left(\frac{EV}{X_s}\sin\delta\right) = \frac{EV}{X_s}\cos\delta$$

P_{sy} is a measure of stability

$\therefore$ Stability $\propto P_{sy}$

But, $P_{sy} \propto \frac{1}{X_s} \propto \text{SCR}$

$\therefore$ Stability $\propto$ SCR

Stability $\propto$ SCR $\propto$ air gap length

$\therefore$ Stability $\propto$ air gap length

- When the stator mmf is aligned with the d – axis of field poles then flux φ_d per pole is set up and effective reactance offered by the alternator is X_d.

$$X_d = \frac{\text{Maximum voltage}}{\text{Minimum current}} = \frac{(V_t)\text{line}(\text{at min}, I_a)}{\sqrt{3}I_a(\text{min})} = \text{Direct axis reactance}$$

- When the stator mmf is aligned with the q – axis of field poles then flux φ_q per pole is set up and the effective reactance offered by the alternator is X_q.

$$X_q = \frac{\text{Minimum voltage}}{\text{Maximum current}} = \frac{(V_t)\text{line}(\text{at min}, I_a)}{\sqrt{3}I_a(\text{max})} = \text{Quadrature axis reactance}$$

- Cylindrical rotor Synchronous machine,

The per phase power delivered to the infinite bus is given by $P = \frac{E_f V_t}{X_s}\sin\delta$

- Salient pole synchronous machine,

The per phase power delivered to the infinite bus is given by

$$P = \frac{E_f V_t}{X_d}\sin\delta + \frac{V_t^2}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right]\sin 2\delta$$

Condition for max. power:

- For cylindrical rotor machine:

At constant V_t and E_f, the condition for max. power is obtained by putting $\frac{dp}{d\delta} = 0$

$$\therefore \frac{dp}{d\delta} = \frac{E_f V_t}{X_s} \text{cod}\delta = 0$$

$\cos\delta = 0$

$\delta = 90°$

Hence maximum power occurs at $\delta = 90°$

- For salient – pole synchronous machine:

$$\frac{dp}{d\delta} = 0$$

$$\Rightarrow \frac{V_t E_f}{X_d} \text{cod}\delta + V_t^2 \left(\frac{1}{x_q} - \frac{1}{x_d}\right) \cos 2\delta = 0$$

$\cos\delta = -\frac{E_f X_q}{4V_t (X_d - X_q)} \pm \sqrt{\frac{1}{2} + \left[\frac{E_f X_q}{4V_t (X_d - X_q)}\right]^2}$ The value of load angle is seed to be less than 90°.

$\therefore$ Maximum power occurs at $\delta < 90°$

- Synchronizing power = $P_{sy}.\Delta\delta.$ = $\frac{EV}{X_s} \text{cod}\delta\delta\Delta\delta$
- Synchronizing torque = $\frac{\text{Synchronizing power}}{\omega}$

Power flow in Alternator:

- Complex power = S = P + jQ = $VI_a{}^*$

  Where Active power flow (P) = $\frac{EV}{Z_s}\cos(\theta - \delta) - \frac{V^2}{Z_s}\cos\theta;$

  Reactive power flow (Q) = $\frac{EV}{Z_2}\sin(\theta - \delta) - \frac{V^2}{Z_s}\sin\theta;$

- Condition for max. power output:

$$P = \frac{EV}{Z_s}\cos(\theta - \delta) - \frac{V^2}{Z_2}\cos\theta$$

$\frac{dp}{d\delta} = 0$ for max power condition

i.e. $\theta - \delta = 0$

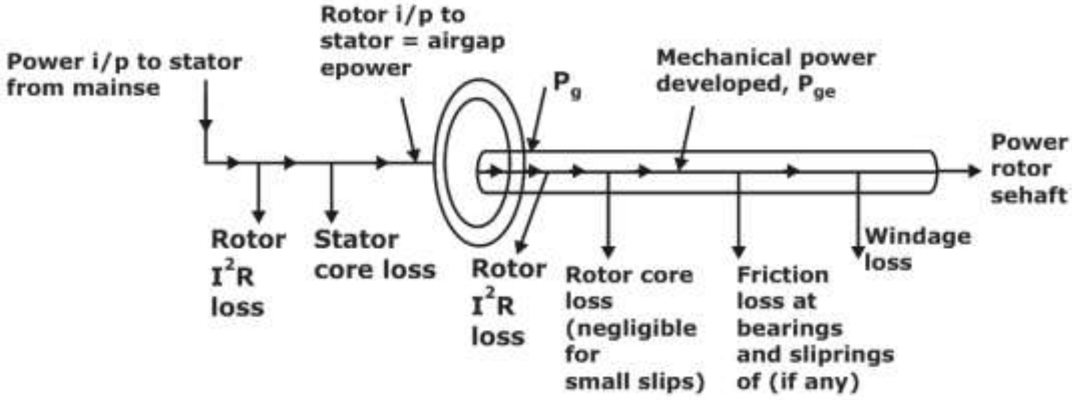
$\theta = \delta$

If $R_a = 0$; $\theta = \delta = 90°$; then max power is given by

$$P_{max} = \frac{EV}{Z_s} - \frac{V^2}{Z_s}\cos\theta$$

4. Induction Machines:

- The power flow diagram of $3-\phi$ induction motor is



The slip of induction machine is

$$(S) = \frac{n_s - n_r}{n_s} = \frac{N_s - N_r}{N_s}$$

Where N_s is synchronous speed in rpm n_s is synchronous speed in rps

$\Rightarrow N_r = N_s(1-s)$

$\Rightarrow N_s - N_r = sN_s$

∴ Rotor frequency,

$$f_2 = \frac{P \times SN_s}{120} = S\frac{PN_s}{120} = Sf_1$$

For an induction machine with rotor resistance R_r and locked rotor leakage reactance X_r, the rotor impedance Z_r at slip s is:

$Z_r = R_r + jsX_r$

The stator circuit equivalent impedance Z_{rf} for a rotor/stator frequency ratio s is:

$$Z_{rf} = \frac{R_{rs}}{s + jX_{rs}}$$

For an induction motor with synchronous angular speed ω_s running at angular speed ω_m and slip s, the airgap transfer power P_t, rotor copper loss P_r and gross output power P_m for a gross output torque T_m are related by:

$$P_t = \omega_s T_m = \frac{P_r}{s} = \frac{P_m}{(1-s)}$$

$$P_r = sP_t = \frac{sP_m}{(1-s)}$$

$P_m = \omega_m T_m = (1 - s)P_t$

The power ratios are:

$P_t : P_r : P_m = 1 : s : (1 - s)$

The gross motor efficiency η_m (neglecting stator and mechanical losses) is:

$\eta_m = \frac{P_m}{P_t} = 1 - s$

Rotor emf, Current Power:

At stand still, the relative speed between rotating magnetic field and rotor conductors is synchronous speed Ns; under this condition let the per phase generated emf in rotor circuit be E_2.

$\therefore \frac{E_2}{ph} = 4.44\ N_{phr}\ \varphi_1\ f_1\ K_{dr}\ K_{pr}$

$\frac{E_2}{ph} = 4.44\ N_{phr}\ \varphi_1\ f_1\ K_{wr}$

K_{wr} = Rotor winding factor

- But during running conditions the frequency of the rotor becomes, running with speed N_r

  $\frac{P(N_s - N_r)}{120} = \frac{PsN_s}{120} = sf_1$

  $\therefore f_r = sf_1$

  $\therefore$ Emf under running conditions is

  $E = \sqrt{2}\pi f_r Kw_2 N_{phr}\phi_1 = sE_2$

- Rotor leakage reactance = 2π (Rotor frequency) (Rotor leakage Inductance)

  $\therefore$ Rotor leakage reactance at stand still = $2\pi f_1 l_2 = x_2\Omega$

- Rotor leakage reactance at any slips = $2\pi f_2 l_2 = s\ x_2\Omega$
- Rotor leakage impedance at stand still = $\sqrt{r_2^2 + x_2^2}$
- At any slip s, rotor = $\sqrt{r_2^2 + x_2^2}$
- Per phase rotor current at any slip s is given by

  $I_2 = \frac{sE_2}{\sqrt{r_2^2 + (sx_2)^2}} = \frac{E_2}{\sqrt{(r_1 / s)^2 + x_2^2}}$

- The rotor current I_2 lags the rotor voltage E_2 by rotor power factor angle θ_2 given by

  $\theta_2 = \tan^{-1}\frac{sx_2}{r_2}$

- Per phase power input to rotor is

  $P_g = E_2 I_2 \cos\theta_2$

  $\cos\theta_2 = \frac{\text{Per phase rotor resistance}}{\text{Per phase rotor impedance}} = \frac{r_2 / s}{\sqrt{(r_2 / s)^2 + (x_2)^2}}$

  $\therefore P_g = E_2 I_2 \times \frac{r_2 / s}{\sqrt{(r_2 / s)^2 + (x_2)^2}} = \frac{E_2}{\sqrt{(r_2 / s)^2 + (x_2)^2}} \times I_s \frac{r_2}{s} = I_2^2 \frac{r_2}{s}$

- P_g is the power transferred from stator to rotor across the air gap. Therefore, P_g is called air gap power

  $$P_g = I_2^2\frac{r_2}{s} = I_2^2 r_2 + I_2^2 r_2\left[\frac{1-s}{s}\right]$$

  P_g = (Rotor ohmic loss) + Internal mechanical power developed in rotor (P_m) = sP_g +(1 − s)P_g

  $$\therefore P_m = (1-s)P_g = I_2^2 r_2\left[\frac{1-s}{s}\right]$$

  $$\text{Rotor ohmic loss} = \left[\frac{s}{1-s}\right]P_m = sP_g$$

- Internal (or gross) torque developed per phase is given by

  $$T_e = \frac{\text{Internal mechanical power developed in rotor}}{\text{Rotor speed in mechanical radian per sec}}$$

  $$T_e = \frac{P_m}{\omega_r} = \frac{(1-S)P_g}{(1-S)\,\omega_s} = \frac{P_g}{\omega_s}$$

- Electromagnetic torque T_e can also be expressed as

  $$T_e = \frac{P_g}{\omega_s} = \frac{1}{\omega_s}\times\frac{I_2^2 r_2}{s} = \frac{\text{Rotor ohmic loss}}{(\omega_s)\,\text{slip}}$$

  $$\therefore T_e = \frac{\text{Rotor ohmic loss}}{(\omega_s)\,\text{slip}}$$

- Power available at the shaft can be obtained from P_g as follows. Output or shaft power, P_{sh} = P_m – Mechanical losses
- Mechanical losses imply frication and windage losses.

  P_{sh} = P_g – Rotor ohmic loss – Friction and windage losses = Net mechanical power output or net power output output or shaft torque

  $$T_{sh} = \frac{P_{sh}}{\text{Rotor speed}} = \frac{P_{sh}}{(1-s)\,\omega_s}$$

- If the stator input is known. Then air gap power P_g is given by P_g = stator power input – stator I^2R loss – stator core loss.
- Ratio of Rotor input power, rotor copper losses and gross mechanical output is

  $$\frac{I_r^2 R_2}{s} : I_r^2 R_2 : I_r^2 R_2\left[\frac{1}{s}-1\right]$$

  $$\Rightarrow 1 : s : (1-s)$$

  ∴ Rotor copper losses = s × Rotor input

  Gross Mechanical output = (1−s) × Rotor input.

  Rotor copper losses = (Gross mechanical output) × $\frac{s}{1-s}$

  Efficiency of the rotor is approximately equal to

$$\eta_{rotor} = \frac{\text{Gross mechanical power output}}{\text{Rotor input}}$$

$$\frac{(1-s)\text{Rotor input}}{\text{Rotor input}} = 1 - s = 1 - \frac{N_S - N}{N_S} = \frac{N}{N_s}$$

$$\eta_{rotor} \simeq \frac{N}{N_s}$$

Total torque is

$$T_e = \frac{m}{\omega_s} \times \frac{V_e^2}{\left[R_e + \frac{r_2}{s}\right]^2 + (x_2 + X_e)^2} \times \frac{r_2}{s} \text{ Nm}$$

m is the number of stator phases. Torque equation can be written as

$$T_e = \frac{m}{\omega_s} \times I_2^2 \times \frac{r_2}{s} = \frac{m}{\omega_s} \times \text{ rotor input per phase.}$$

Thus, the slip s_{mT} at which maximum torque occurs is given by

$$S_{mT} = \frac{r_2}{\sqrt{R_e^2 + x^2}}$$

Substituting the value of maximum slip in the torque equation, gives maximum torque

$$T_{em} = \frac{m}{\omega_s} \times \frac{V_e^2}{2X_2}$$

If stator parameters are neglected then applying maximum transfer theorem to $\frac{r_2}{s}$ then $\frac{r_2}{s} = x_2$

Slip corresponding to maximum torque is

$$S_m = \frac{r_2}{x_2} \text{ (Breakdown slip)}$$

$N_m = N_s(1 - S_m)$

$$\Rightarrow N_m = N_s\left(1 - \frac{R_2}{x_2}\right)$$

N_m is the stalling speed at the maximum torque.

Starting Torque:

At starting, slip S = 1.00, starting torque is given by

$$\text{Test} = \frac{mV_e^2}{\omega_s} \times \frac{r_2}{(R_e + r_2)^2 + x_2}$$

Motor Torque in terms of T_{em}:

- The torque expression of an induction motor can also be expressed in terms of maximum torque T_{em} and dimension less ratio $\frac{S}{S_{mT}}$. In order to get a simple and approximate expression, stator resistance r_1, or the stator equivalent resistance R_e, is neglected.

$$\therefore \frac{T_e}{T_{em}} = \frac{2X}{\left(\frac{r_2}{s}\right)^2 + X^2} \times \frac{r_2}{s}$$

- Since r_1 or R_e is neglected

$$\frac{T_e}{T_{em}} = \frac{2X}{\left(\frac{r_2}{s}\right)^2 + X^2} \times \frac{r_2}{s}$$

- The slip at which maximum torque occurs is

$$S_{mT} = \frac{r_2}{X} \qquad \therefore r_2 = S_{mT}X$$

$$\therefore \frac{T_e}{T_{em}} = \frac{2X}{\left[\frac{S_{mT}X}{s}\right]^2 + X^2} \times \frac{S_{mT}X}{s}$$

$$\Rightarrow \frac{T_e}{T_{em}} = \frac{2}{\frac{S_{mT}}{s} + \frac{S}{S_{mT}}}$$

$$T_e = \frac{2T_{em}}{\frac{S_{mT}}{s} + \frac{S}{S_{mT}}}$$

Losses and Efficiency:

There are three cases in iron losses.

Case (i): If the ratio of voltage to frequency is constant and flux is also constant then:

Iron loss = Hysteresis loss + eddy current loss

$P_h = K_h + B_m^{1.6}$, $P_e = K_e f^2 B_m^2$

Given: $\frac{V}{f}$ is constant. As $B_m \propto \frac{V}{f}$

$\Rightarrow$ B_m is constant.

$\therefore \quad P_h \propto f$ and $P_e \propto f^2$

Case (ii): If the ratio of voltage to frequency is not constant and flux is also not constant

$$\Rightarrow \frac{V}{f} \neq \text{const} \quad \phi \neq \text{const}$$

$P_h = K_h f B_m^{1.6}$ $P_e = K_e f^2 B_m^2$

$\therefore P_h \propto V^{1.6} f^{-0.6}$ $\qquad P_e \propto V^2$

Case (iii): If frequency is constant and voltage is variable then:

$P_h = K_h f B_m^{1.6}$ $P_e = K_h f^2 B_m^2$

$$= K_h f \left(\frac{V}{f}\right)^{1.6}$$

$P_h \propto V^{1.6}$ $\qquad$ $P_e \propto V_1^2$

- Short circuit current with normal voltage applied to stator is

$$I = I_{br} \times \frac{V}{V_{br}}$$

I = Short circuit current with normal voltage

I_{br} = Short circuit current with voltage V_b,

- Power factor on short circuit is found from

$$P_{br} = \sqrt{3}V_{br}I_{br}\cos\phi_{br}$$

$$\Rightarrow \cos\phi_{br} = \frac{P_{br}}{\sqrt{3}V_{br}I_{br}}$$

- As P_{br} is approximately equal to full load cooper losses

$$R_{br} = \frac{P_{br}}{I_{br}}$$

The blocked rotor impedance is

$$Z_{br} = \frac{V_{br}}{I_{br}}$$

$\therefore$ Blocked rotor reactance = $X_{br} = \sqrt{Z_{br}^2 - R_{br}^2}$

Efficiency of induction machines:

Generally, efficiency = $\frac{\text{Output power}}{\text{Input power}}$

$\therefore$ Efficiency of induction motor = $\frac{\text{Net mechanical output}}{\text{Electrical power input}}$

$\therefore$ Efficiency of induction generator = $\frac{\text{Net electrical output}}{\text{Mechanical power input}}$

$$\therefore \frac{GD}{GF} = \frac{P_{sc} - 3I_{sc}^2 r_1}{3I_{sc}^2 r_1}$$

Wound rotor: $\frac{GD}{GF} = \frac{I_2^2 r_2}{I_1^2 r_1} = \frac{r_2}{r_1}\left(\frac{I_2}{I_1}\right)^2$

Direct-on line (across the line) starting:

- The relation between starting torque and full load torque is

$$T_e = \frac{1}{\omega_s} \times I_2^2 \frac{r_2}{s}$$

$$\therefore \frac{T_{e,st}}{T_{e,f1}} = \frac{I_{2,st}^2 \frac{r_2}{r_1}}{I_{2,ft}^2 \frac{r_2}{s_{r_1}}} = \left[\frac{I_{2,st}}{I_{2,f1}}\right]^2 \times S_{f1}$$

The above equation valid for rotor resistance remains constant.

$$\frac{T_{e,st}}{T_{e,f1}} = \left[\frac{I_{st}}{I_{fl}}\right]^2 \times S_{f1}$$

Where,

$$\frac{I_{st}}{I_{f1}} = \frac{(\text{Effective rotor to stator turns ratio})I_{2,st}}{(\text{Effective rotor to stator turns ratio})I_{2,s1}}$$

- Per phase short – circuit current at stand still (or at starting) is,

$$I_{sc} = \frac{V_1}{Z_{sc}}$$

Where, $Z_{sc} = (r_1 + r_2) + j(x_1 + x_2)$

Here shunt branch parameters of equivalent circuit are neglected.

- Therefore, for direct switching.

$$I_{st} = I_{sc} = \frac{V_1}{Z_{sc}}$$

$$\therefore \frac{T_{est}}{T_{esf}} = \left[\frac{I_{sc}}{I_{f1}}\right]^2 S_{f1}$$

Stator resistor (or reactor) starting:

Since per phase voltage is reduced to xv, the per phase starting current I_{st} is given by

$$I_{st} = \frac{xV_1}{Z_{sc}} = xI_{sc}$$

As be fore $\therefore \frac{T_{est}}{T_{esf}} = \left[\frac{I_{sc}}{I_{f1}}\right]^2 S_{f1} = \left[\frac{xI_{sc}}{I_{f1}}\right]^2 S_{f1}$

- In an induction motor, torque $\propto (\text{Voltage})^2$

$$\therefore \frac{\text{Starting torque with reactor starting}}{\text{Starting torque with direct switching}} = \left[\frac{xV_1}{V_1}\right]^2 = x^2$$

Auto transformer starting:

- Per phase starting current from the supply mains is $I_{st} = x^2 I_{sc}$

$$\frac{T_{e,st}}{T_{e,f1}} = \frac{\text{Per phase starrting current in motor winding}}{\text{Per phase motor full load current}} \times S_{f1}$$

$$\frac{T_{e,st}}{T_{e,f1}} = \frac{I_{st} I_{sc}}{I_{f1}^2} \times S_{f1}$$

$$\frac{T_{e,st}\text{with an auto transformer}}{T_{e,st}\text{with direct switching}} = \left[\frac{xV_1}{V_1}\right]^2 = x^2$$

Star – Delta method of Starting:

- $$\frac{\text{Starting torque with star delta starter}}{\text{Starting torque with direct switching in delta}} = \frac{\left[\frac{V_L}{\sqrt{3}}\right]^2}{\left[V_L\right]^2} = \frac{1}{3}$$

  $\therefore$ Star delta starter also reduces the starting torque to one-third of that produce by direct switching in delta.

- With star – delta starter, a motor behaves as if it were started by an auto transformer starter with $x = \frac{1}{\sqrt{3}} = 0.58$ i.e. with 58% tapping.

- $$\frac{\text{Starting torque with star delta starter, } T_{e,st}}{\text{Starting torque with direct switching in delta, } T_{ef1d}} = \frac{\frac{1}{w_s}\left(I_{st,y}\right)^2 \frac{r_2}{1}}{\frac{1}{w_s}\left(I_{st,d}\right)^2 \frac{r_2}{S_{f1}}}$$

$$= \frac{\left[\frac{1}{\sqrt{3}} I_{st,d}\right]^2}{\left(I_{f1,d}\right)^2} \times S_{f1} = \frac{1}{3}\left[\frac{I_{sc}d}{I_{f1}d}\right]^2 \times S_{f1}$$

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