gradeup

gradeup SUPER

Rajasthan RVUNL

Civil Engineering

Strength of Materials

Important Formula Notes

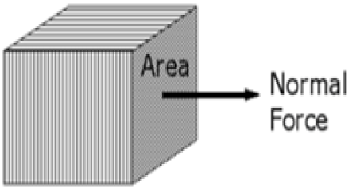
IMPORTANT FORMULAS ON IRRIGATION ENGINEERING

Stress: When a material is subjected to an external force, a resisting force is set up in the component. The internal resistance force per unit area acting on a material is called the stress at a point. It is a tensor quantity having unit of N/m^2 or Pascal.

$$Stress = \frac{Force}{Area}$$

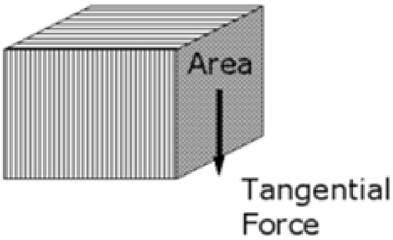
Types of Stresses

- **Normal stress**



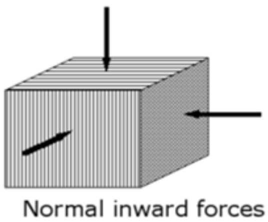
$$Normal\ stress = \frac{Normal\ force}{area} \qquad \sigma_n = \frac{F_n}{A}$$

- **Shear Stress**



$$\text{Shear stress} = \frac{\text{tangential force}}{\text{area}} \qquad \sigma_t = \frac{F_t}{A}$$

- **Bulk Stress**

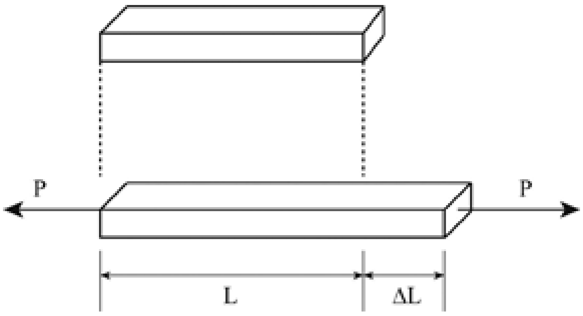


$$\text{Bulk stress} = \frac{\text{normal inward force}}{\text{area}} \qquad \sigma_B = P$$

Strain: It is the deformation produced in the material due to simple stress. It usually represents the displacement between particles in the body relative to a reference length.

Types of Strains

- **Normal Strain:** The normal strain of a body is generally expressed as the ratio of total displacement to the original length.



$$\text{Normal Strain} = \frac{\text{change in normal length}}{\text{original normal length}}$$

$$\varepsilon_n = \frac{\delta l}{l}$$

Strain is dimensionless quantity.

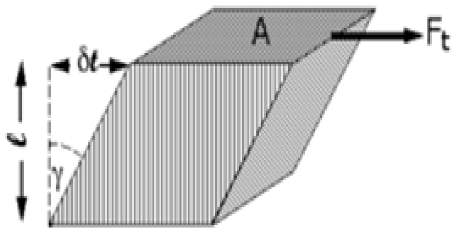
It is of two types: Longitudinal strain and Lateral Strain

Longitudinal strain is the defined as the ratio of the change in length of the body due to the deformation to its original length in the direction of the force.

Lateral Strain is defined as the ratio of the change in length (breadth of a rectangular bar or diameter of a circular bar) of the body due to the deformation to its original length (breadth of a rectangular bar or diameter of a circular bar) in the direction perpendicular to the force.

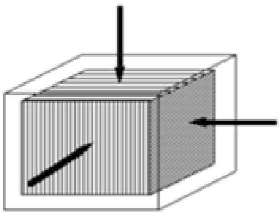
$$\text{Poisson's ratio}(\mu) = -\frac{\textit{Lateral Strain}}{\textit{Longitudinal Strain}}$$

- **Shear strain**



$$\text{Shear Strain} = \frac{\text{tangential dislacement}}{\text{original normal length}}$$

- **Bulk Strain or Volumetric Strain**



$$\text{Bulk Strain} = \frac{-(\text{change in volume})}{\text{original volume}}$$

$$\varepsilon_B = \frac{-\delta V}{V}$$

Stress and Strain both are tensor quantity i.e. it has both change in magnitude as well as direction.

True Stress and True Strain

- The **true stress** is defined as the ratio of the load to the cross section area at any instant.

$$(\sigma_T) = \frac{\text{load}}{\text{Instantaneous area}} = \sigma(1+\varepsilon)$$

Where σ and ε is the engineering stress and engineering strain respectively.

- The **true strain** is defined as

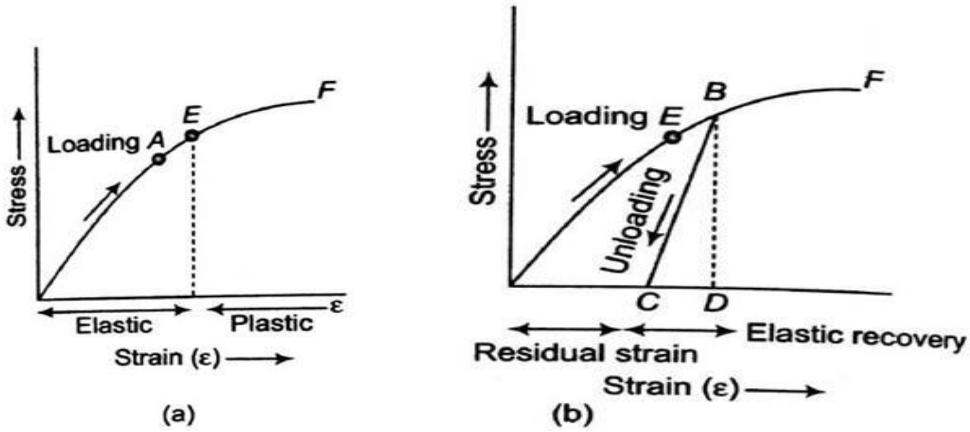
$$(\varepsilon_T) = \int_{L_o}^{L} \frac{dl}{l} = \ln\left(\frac{L}{L_o}\right) = \ln(1+\varepsilon) = \ln\left(\frac{A_o}{A}\right) = 2\ln\left(\frac{d_o}{d}\right)$$

Lo- original length, L-successive values of the length as it changes

- The volume of the specimen is assumed to be constant during plastic deformation.

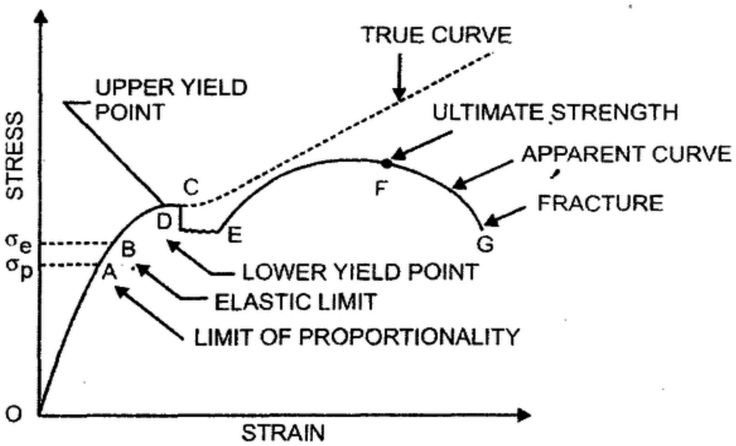
Stress-Strain Relationship

- The stress-strain diagram is shown in the figure. In brittle materials, there is no appreciable change in the rate of strain. There is no yield point and no necking takes place.



- In figure (a), the specimen is loaded only upto point A, when load is gradually removed the curve follows the same path AO and strain completely disappears. Such a behaviour is known as the elastic behaviour.
- In figure (b), the specimen is loaded upto point B beyond the elastic limit E. When the specimen is gradually loaded the curve follows path BC, resulting in a residual strain OC or permanent strain.

Comparison of engineering stress and the true stress-strain curves shown below:



- True stress-strain curve gives a true indication of deformation characteristics because it is based on the instantaneous dimension of the specimen.
- In engineering stress-strain curve, stress drops down after necking since it is based on the original area.
- In true stress-strain curve, the stress however increases after necking since the cross sectional area of the specimen decreases rapidly after necking.

Hooke's Law:

According to Hooke's law the stress is directly proportional to strain i.e. normal stress (σ) $\propto$ normal strain (ε)

and shearing stress (ζ) $\propto$ shearing strain (γ).

$\sigma = E\varepsilon$ and $\zeta = \gamma G$

The co-efficient E is called the modulus of elasticity i.e. its resistance to elastic strain. The coefficient G is called the shear modulus of elasticity or modulus of rigidity.

Properties of Materials

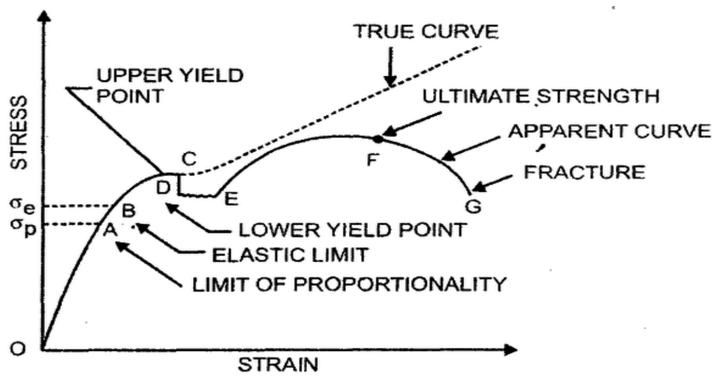
Some properties of materials which judge the strength of materials are given below:

- **Elasticity:** Elasticity is the property by virtue of which a material is deformed under the load and is enabled to return to its original dimension when the load is removed.
- **Plasticity:** Plasticity is the converse of elasticity. A material in the plastic state is permanently deformed by the application of load and it has no tendency to recover. The characteristic of the material by which it undergoes inelastic strains beyond those at the elastic limit is known as plasticity.

- **Ductility:** Ductility is the characteristic which permits a material to be drawn out longitudinally to a reduced section, under the action of a tensile force (large deformation).
- **Brittleness:** Brittleness implies the lack of ductility. A material is said to be brittle when it cannot be drawn out by tension to the smaller section.
- **Malleability:** Malleability is a property of a material which permits the material to be extended in all directions without rapture. A malleable material possesses a high degree of plasticity, but not necessarily great strength.Malleability is a physical property of metals that defines their ability to be hammered, pressed, or rolled into thin sheets without breaking
- **Toughness:** Toughness is the property of a material which enables it to absorb energy without fracture
- **Hardness:**Hardness is the ability of a material to resist indentation or surface abrasion. Brinell hardness test is used to check hardness.
- **Strength:** The strength of a material enables it to resist fracture under load.

Engineering Stress-Strain Curve

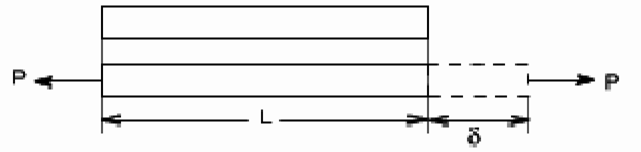
- The stress-strain diagram is shown in the figure. The curve starts from an origin. Showing thereby that there is no initial stress of strain in the specimen.
- The stress-strain curve diagram for a ductile material like mild steel is shown in the figure below.
- Upto point A, Hooke's Law is obeyed and stress is proportional to strain. Point A is called limit of proportionality.



- Point B is called the elastic limit point.
- At point B the cross-sectional area of the material starts decreasing and the stress decreases to a lower value to point D, called the lower yield point.
- The apparent stress decreases but the actual or true stress goes on increasing until the specimen breaks at point C, called the upper yield point
- From point E onwards, the strain hardening phenomena become predominant and the strength of the material increases thereby requiring more stress for deformation, until point F is reached. Point F is called the ultimate point.

Elongation

A prismatic bar loaded in tension by an axial force



For a prismatic bar loaded in tension by an axial force P. The elongation of the bar can be determined as

δ=PL/AE

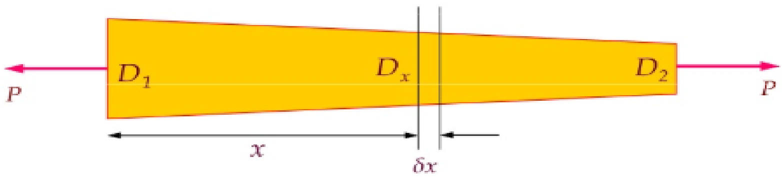
Elongation of composite body

Elongation of a bar of varying cross section A_1 ,A_2 ,----------,A_n of lengths l_1 , l_2,--------l_n respectively

$$\delta = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} - - - - - - - - + \frac{l_n}{A_n}\right]$$

Elongation of a tapered body

Uniformly tapering circular bar



$$D_x = D_1 - \left(\frac{D_1 - D_2}{L}\right)x$$

$$\Delta = \frac{P\delta x}{\left(\frac{\pi D_x^2}{4}\right)E}$$

$$\therefore \text{Total elongation, } \Delta L = \int_0^L \frac{P\delta x}{\left(\frac{\pi D_x^2}{4}\right)E} = \frac{4PL}{\pi E D_1 D_2}$$

$$\text{When } D_1 = D_2 = D, \ \Delta L = \frac{4PL}{\pi E D^2}$$

Elongation of a body due to self weight

(i) Elongation of a uniform rod of length 'L' due to its own weight 'W'

$$\delta = \frac{WL}{2AE}$$

The deformation of a bar under its own weight as compared to that when subjected to a direct axial load equal to its own weight will be half.

ii) Total extension produced in rod of length 'L' due to its own weight ' ω ' per with length.

$$\delta = \frac{\omega L^2}{2EA}$$

(iii) Elongation of a conical bar due to its self weight

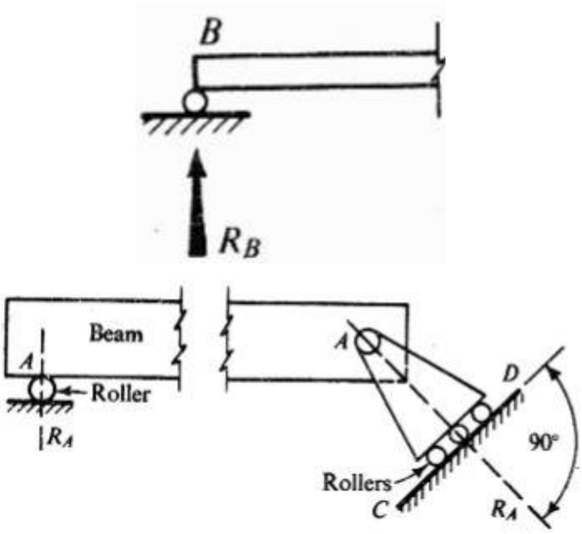
$$\delta = \frac{\rho g L^2}{6E} = \frac{WL}{2A_{max}E}$$

Shear Force and Bending Moment Diagrams

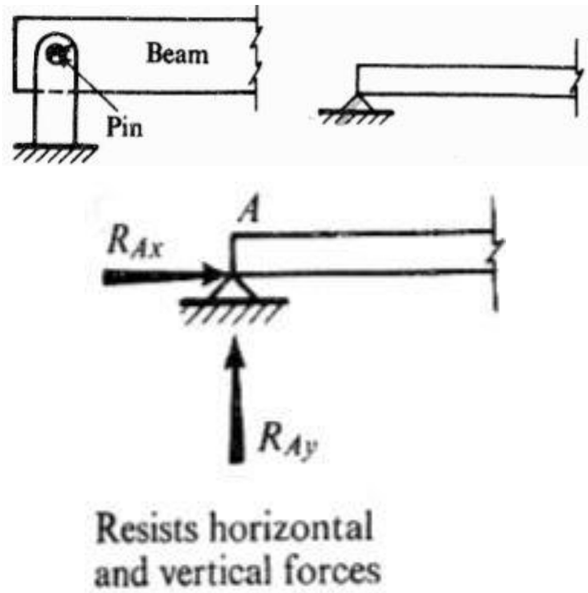
Beam is one of the most important structural components.Beams are usually long, straight, prismatic members and always subjected forces perpendicular to the axis of the beam

- A Shear Force Diagram (SFD) indicates how a force applied perpendicular to the axis (i.e., parallel to cross-section) of a beam is transmitted along the length of that beam.
- A Bending Moment Diagram (BMD) will show how the applied loads to a beam create a moment variation along the length of the beam.

Types of Supports (a) Roller Support – resists vertical forces only

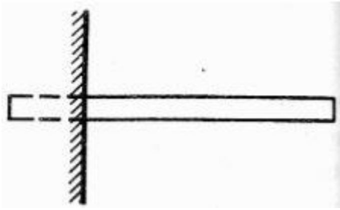


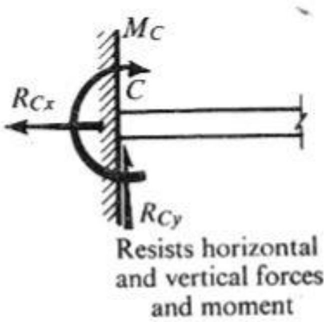
(b) Hinge support or pin connection – resists horizontal and vertical forces



- Hinge and roller supports are called as simple supports

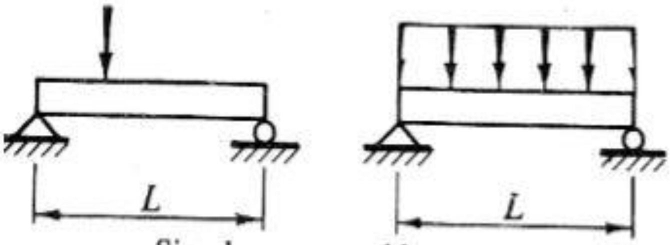
(c) Fixed support or built-in end





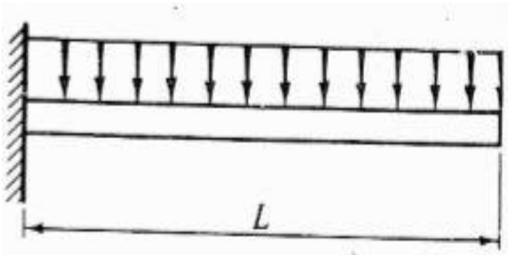
- **The distance between two supports is known as "span".**

Types of beams : Beams are classified based on the type of supports: (1) Simply supported beam: A beam with two simple supports

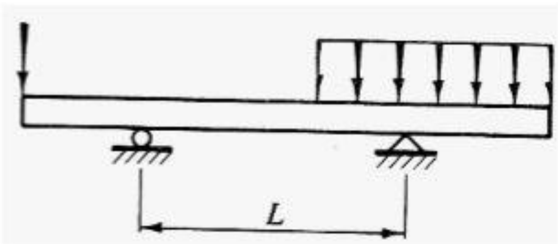


Simply supported beams

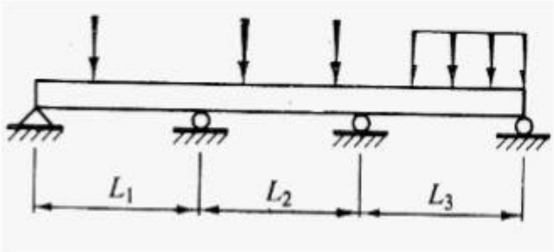
(2) Cantilever beam: Beam fixed at one end and free at other



(3) Overhanging beam



(4) Continuous beam: More than two supports



Shear Force

Shear force has a tendency to slide the surface, it acts parallel to surface.

$\sum F_{vert} = 0$

$V - q\,dx - (V + dV) = 0$

$$\frac{dV}{dx} = -q \Rightarrow \int_A^B dV = -\int_A^B q\ dx\ V_B - V_A = -\int_A^B q\ dx$$

Only for distributed load not for point load.

Bending Moment

Any moment produced by forces acting on the beam must be balance by an equal opposite moment produced by internal forces acting in beam at the section. This moment is called bending moment.

$\sum M = 0$

$$-M - q dx\left(\frac{dx}{z}\right) - (V + dV)dx + M + dm = 0$$

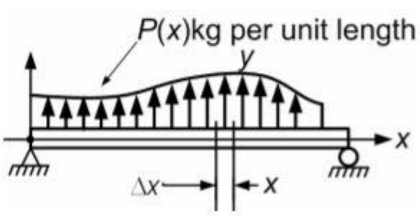
$$\frac{dM}{dx} = V \Rightarrow M_a - M_A = \int V\ dx$$

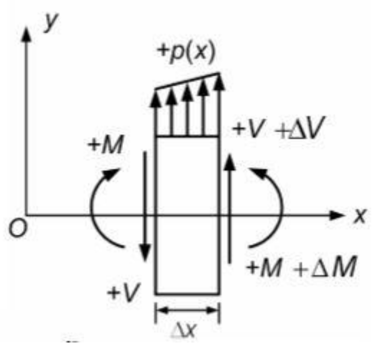
Only for distributed and concentrated load not for couple.

- The necessary internal forces to keep the segment of the beam in equilibrium are

$$\Sigma F_x = 0 \Rightarrow P$$
$$\Sigma F_y = 0 \Rightarrow V$$
$$\Sigma F_z = 0 \Rightarrow M$$

Differential equations of equilibrium





Sign Conventions :

$$[\Sigma F_x = 0 \rightarrow +]$$

$$\left[\Sigma F_y = 0 \uparrow +\right]$$

So the differential equations would be:

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \frac{dV}{dx} = -P$$

$$\lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = \frac{dM}{dx} = -V$$

From equation $\frac{dV}{dx} = -P$,we can write

$$V_D - V_C = -\int_{X_C}^{X_D} P dx$$

From equation $\frac{dy}{dx} = -M$,we can write

$$M_D - M_C = -\int V dx$$

Bending and Shear Stress

Bending Moments and Shear Stress Distribution

Bending stress and shear stress distribution are classified in the following groups

Bending Moment in Beam:

Transverse loads or lateral loads: Forces or moments having their vectors perpendicular to the axis of the bar.

Classification of structural members.

- Axially loaded bars :-Loaded in perpendicular dimension along the axis of bar. Supports forces having their vectors directed along the axis of the bar.
- Bar in tension:- Supports torques having their moment vectors directed along the axis to generate a force in the outward direction
- Beams :- Subjected to lateral loads.Beams undergo bending (flexure) because of lateral loads.

When beam is subjected to a bending moment it induces longitudinal or bending stress in cross-section.

Equation of Pure Bending:

$$\frac{\sigma_x}{y}=\frac{M}{I}=\frac{E}{R}$$

Assumptions:

1. The material of the beam is homogeneous and isotropic.
2. The value of Young's Modulus of Elasticity is same in tension and compression.
3. The transverse sections which were plane before bending, remain plane after bending also.
4. The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.
5. The radius of curvature is large as compared to the dimensions of the cross-section.
6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

At the neutral axis, there is no stress of any kind. At one side of the neutral axis, there are compressive stresses and on the other side , there are tensile stresses.

Section Modulus : Section modulus is a geometric property for a given cross-section used in the design of beams or flexural members.

$$z=\frac{I}{y_{max}} \Rightarrow \frac{M}{I}=\frac{\sigma}{y}$$

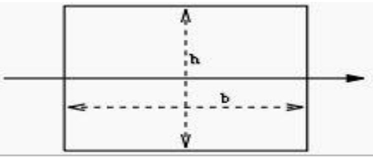
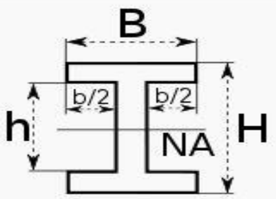
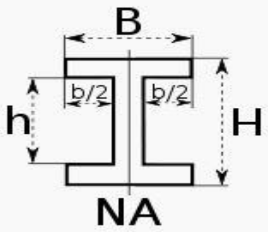
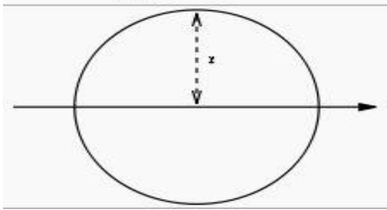
$$M=\sigma_{max}\frac{I}{y_{max}} \Rightarrow M=\sigma_{max} x z$$

Rectangular section : $I=\frac{bd^3}{12}$

Modulus of section :$z=bd^2/6$

Circular section : $I=\frac{\pi}{64}d^4$

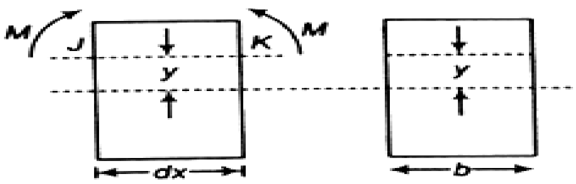
Modulus of section : $z=\frac{\pi}{32}d^3$

	$S = \frac{bh^2}{6}$
	$Sx = \frac{BH^2}{6} - \frac{bh^3}{6H}$ $Sx = \frac{Ix}{y}$, with $y = \frac{H}{2}$
	$Sy = \frac{B^2(H-h)}{6} + \frac{(B-b)^3 h}{6B}$
	$S = \frac{\pi r^3}{4} = \frac{\pi d^3}{32}$ [4]

Shearing Stress

Shearing stress on a layer JK of beam at distance y from neutral axis.

$$\tau = \frac{VA\overline{y}}{Ib}$$



Shearing stress on a beam

Where,

V = Shearing force

$A\overline{y} =$ First moment of area

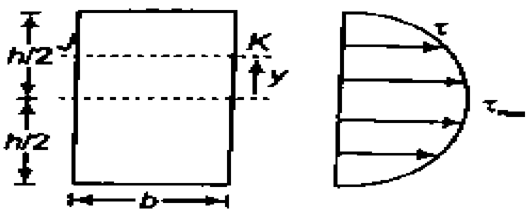
$$\tau = \frac{VQ}{Ib}$$

Shear stress in Rectangular Beam

$\tau = \frac{V}{2I}(\frac{h^2}{4} - y^2)$

τ_{max} = 3/2 V/A

$\tau_{max} = 1.5\tau_{avg}$



Rectangular beam

Circular Beam

Centre of gravity of semi-circle lies at distance from centre or base line. As it is symmetrical above neutral axis, hence at neutral axis shear stress will be maximum.

$$\tau = V\frac{\left(r^2 - y^2\right)}{3l}$$

$$Q = A\bar{y} = \frac{\pi r^2}{2}\left(\frac{4r}{3\pi}\right) = \frac{2r^3}{3}$$

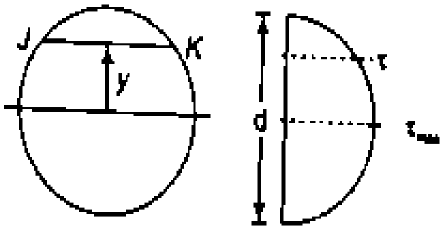
$$b = 2r$$

$$\tau_{\max} = \frac{V\frac{2r^3}{3}}{\frac{\pi r^4}{4}(2r)} = \frac{4}{3}\frac{V}{A}$$

For $\tau_{\max}$ subsitituting $y = 0$

$$l = \frac{\pi d^4}{64}$$

$$\tau_{\max} = \frac{4V}{3A} = \frac{4}{3}\tau_{av}$$



Circular beam

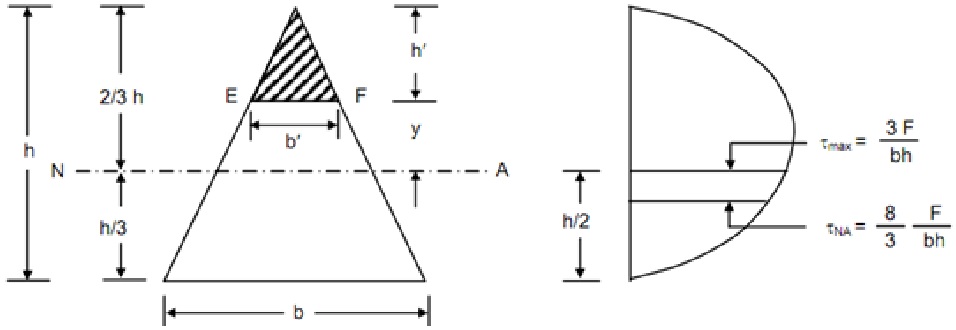
Shears Stress in Hollow Circular Cross-Section

In hollow circular cross-section, if we have to calculate τ at neutral axis by the formula

$$\tau_{\max} = \frac{4V}{3A}\left(\frac{r_2^2 + r_1 r_2 + r_1^2}{r_2^4 - r_1^4}\right)$$

Shear Stress in Triangular Section

In a triangular cross-section, if we have to calculate τ at neutral axis, then in formula

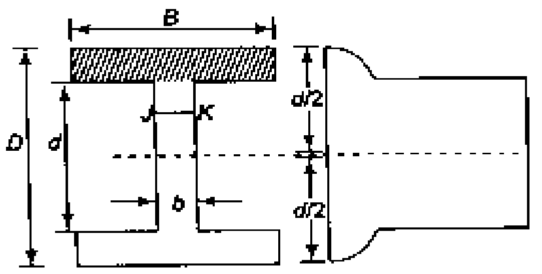


$$\tau = \frac{V}{3I}\left(hx - x^2\right) \Rightarrow \tau_{max} = \frac{3}{2}\tau_{av}$$

Shear Stress in I-section

$$\tau = \frac{VX}{Ib}\left[\frac{B(D^2-d^2)}{8} + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

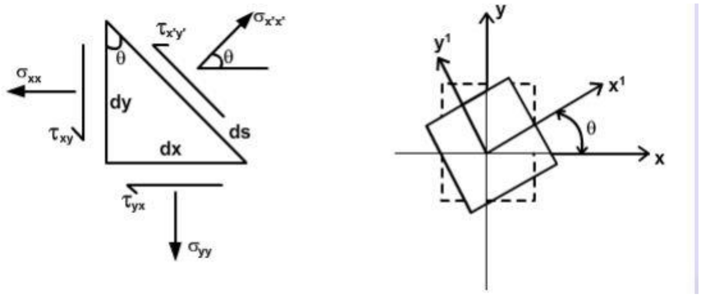
$$\tau_{max} = \frac{V}{Ib}\left[\frac{B}{8}(D^2-d^2) + \frac{bd^2}{8}\right]$$



I-Section diagram

Principal Stress and Principal Strain

Transformation of stress



$$\sigma_{x'x'} = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = \frac{-\left(\sigma_{xx}-\sigma_{yy}\right)}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Replacing θ by θ + 90°, in expression of equation, we get the normal stress along y' direction.

$$\sigma_{y'y'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

Above Equations are the transformation equations for plane stress using which the stress components on any plane passing through the point can be determined. Notice here that,

$$\sigma_{xx} + \sigma_{yy} = \sigma_{x'x'} + \sigma_{y'y'}$$

Invariably, the sum of the normal stresses on any two mutually perpendicular planes at a point has the same value.

Principal stresses and maximum shear stress

Among those varying stresses, finding the maximum and minimum values and the corresponding planes are important from the design considerations. By taking the derivative of σ_{xx} in equation with respect to θ and equating it to zero, we get

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}$$

Here, θp has two values θp1, and θp2 that differ by 90° with one value between 0° and 90° and the other between 90° and 180°. These two values define the principal planes that contain maximum and minimum stresses.

These two values define the principal planes that contain maximum and minimum stresses. Substituting these two θp values in first equations, the maximum and minimum stresses, also called as principal stresses, are obtained.

$$\sigma_{max,min} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau^2_{xy}}$$

The plus and minus signs in the second term of the above equation, indicate the algebraically larger and smaller principal stresses, i.e. maximum and minimum principal stresses.

In the equation, if $\tau_{x'y}$ is taken as zero, then the resulting equation is same as equation of $\tan 2\theta_P$.

Thus, the following important observation pertained to principal planes is made.

The shear stresses are zero on the principal planes

To get the maximum value of the shear stress, the derivative of $\tau_{x'y'}$ in equation with respect to θ is equated to zero and by solving we get

$$\tan 2\theta_s = \frac{-(\sigma_{xx} - \sigma_{yy})}{2\tau_{xy}}$$

Hence, θs has two values, θs1 and θs2 that differ by 90° with one value between 0° and 90° and the other between 90° and 180°.

Hence, the maximum shear stresses that occur on those two mutually perpendicular planes are equal in algebraic value and are different only in sign due to its complementary property.

Comparing above equations,

$$\tan 2\theta_p = -\frac{1}{\tan 2\theta_s}$$

It is understood from above equation that the tangent of the angles 2θp and 2θs are negative reciprocals of each other and hence, they are separated by 90° . Hence, we can conclude that θp and θs differ by 45° , i.e., the maximum shear stress planes can be obtained by rotating the principal plane by 45° in either direction.

The principal planes do not contain any shear stress on them, but the maximum shear stress planes may or may not contain normal stresses as the case may be. Maximum shear stress value is found out by substituting θs values in the first equation

$$\tau_{max} = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau^2_{xy}}$$

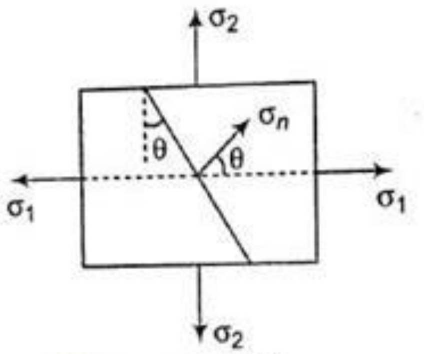
Another expression for τ_{max} is obtained from the principal stresses,

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

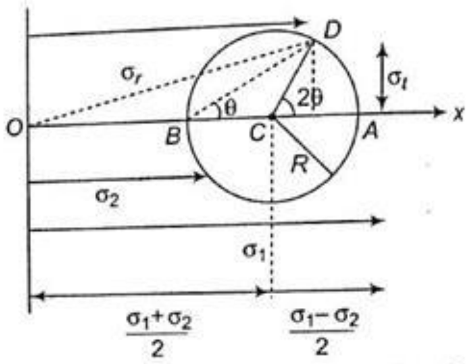
Mohr's Circle for Plane Stress

Mohr's circle is the locus of points representing magnitude of ***normal*** and ***shear stress*** at various plane in a given stress element. Graphically, variation of normal stress and shear stress are studied with the help of Mohr's circle.

σ_1, σ_2 are Principal Stress then normal and shear stress on lane which is inclined at angle 'θ' from major principal plane, then



Different stress diagram



Mohr's circle for plane stress and strain

Normal stress:

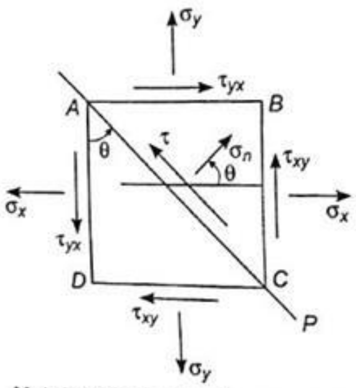
$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

Shear stress:

$$\tau = -\frac{\sigma_1 + \sigma_2}{2}\sin 2\theta$$

General State of Stress at an Element:

If σ_x, σ_y are normal stress on vertical and horizontal plane respectively and this plane is accompanied by shear stress then normal stress and shear stress on plane, which is inclined at an angle θ from plane of



Mohr's circle approach as state of stress

then,

$$\sigma_\theta = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right).\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_\theta = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta - \tau_{xy}\cos 2\theta$$

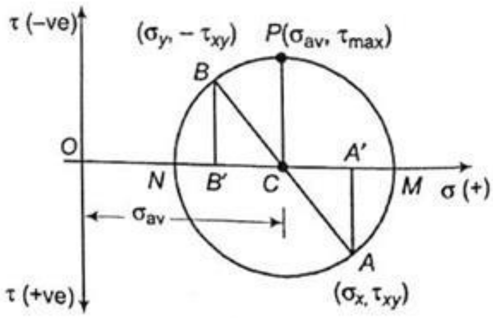
Let be two normal stresses(both tensile) and be shear stress then,

- Maximum and Minimum Principal Stresses are:

$$\sigma_1/\sigma_2 = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

- Radius of Mohr’s circle:

$$r=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2_{xy}}=\frac{\sigma_1-\sigma_2}{2}$$



Mohr's circle for plane stressed

Observations from Mohr's Circle

The following are the observations of Mohr's circle as

* At point M on circle σ_n is maximum and shear stress is zero.

∴ Maximum principal stress ≡ coordinate of M

* At point N on circle σ_n is minimum and shear stress τ is zero.

∴ minimum principal stress ≡ coordinate of N

* At point P on Circle τ is maximum.

Maximum shear stress ≡ ordinate of P(i.e. radius of circle)

Also, normal stress on plane of maximum shear stress

$$\equiv abscissa\ of\ P\left(i.e., \sigma_n=\sigma_{av}=\frac{\sigma_x+\sigma_y}{2}\right)$$

Where, σ_n ≡ Average stress

* Mohr's circle becomes zero at a point if radius of circle has the following consideration.

Radius of circle

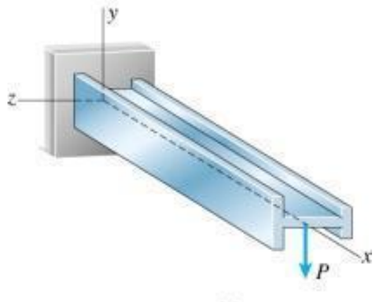
$$=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2 xy}$$

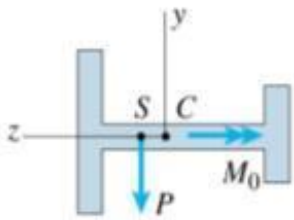
* If $\sigma_x = \sigma_y$, then radius of Mohr's circle is zero and $\tau_{xy} = 0$

Shear Centre and Pressure Vessels

Shear Centre

The shear centre is defined as the point about which the external load has to be applied so that it produces no twisting moment.

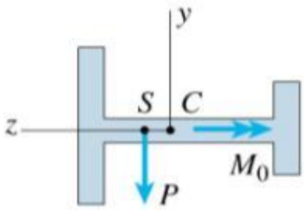


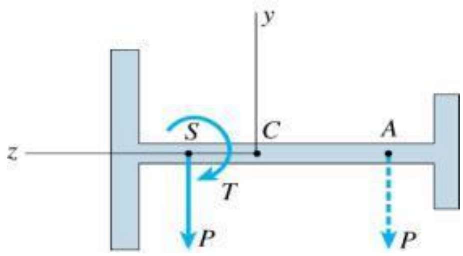


Shear Center

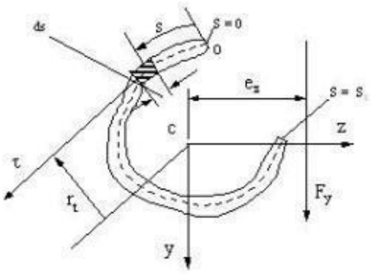
- A lateral load acting on a beam will produce bending without twisting only if it acts through the shear center

  For a doubly symmetric section Shear centre and Centroid coincide





Locating Shear Centre

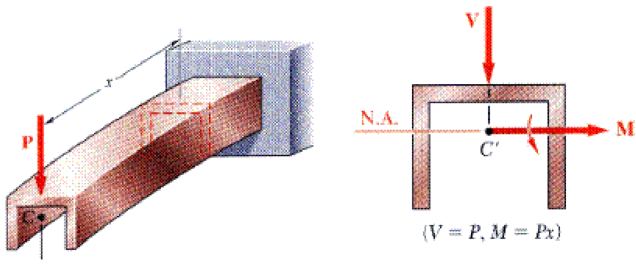


- Moment due to applied force $= F_y . e_z$
- Force on shaded element $= \tau . t . ds = Q . ds$
- Moment due to that force $= Q . ds . r_t$
- Total resisting moment $= \int_0^{s_1} Q . r_t . ds$
- Equating applied moment and resisting moment $\therefore \int_0^{s_1} Q . r_t . ds = F_y . e_z$

$$Q = -\frac{F_y}{I_{zz}} . \int_0^{s} y . t . ds \qquad \therefore -\frac{F_y}{I_{zz}} . \int_0^{s_1} \left(r_t . \int_0^{s} y . t . ds \right) . ds = F_y . e_z$$

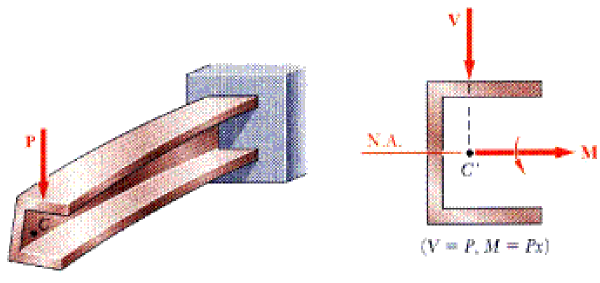
$$\boxed{\therefore e_z = -\frac{1}{I_{zz}} . \int_0^{s_1} \left(r_t . \int_0^{s} y . t . ds \right) . ds}$$

Unsymmetric Loading of Thin-Walled Members



Beam loaded in a vertical plane of symmetry deforms in the symmetry plane without twisting.

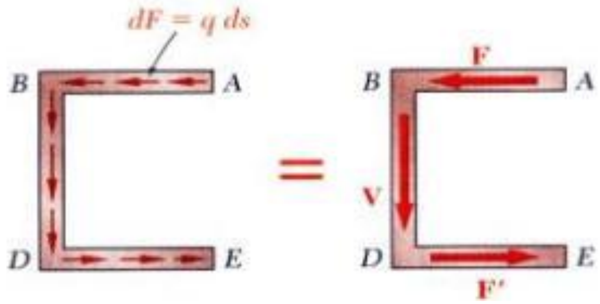
$$\sigma_x = -\frac{My}{I} \qquad \tau_{ave} = \frac{VQ_z}{It}$$



Beam without a vertical plane of symmetry bends and twists under loading.

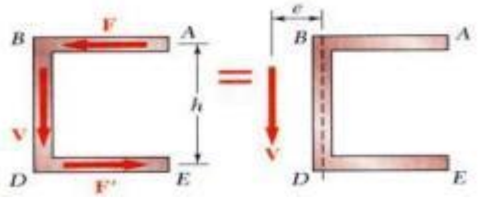
$$\sigma_x = -\frac{My}{I} \qquad \tau_{ave} \neq \frac{VQ_z}{It}$$

Unsymmetric Loading of Thin-Walled Members



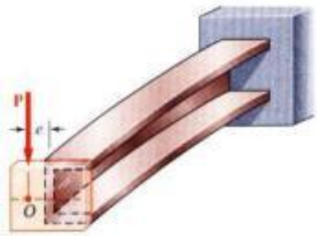
If the shear load is applied such that the beam does not twist, then the shear stress distribution satisfies

$$\tau_{ave} = \frac{VQ_z}{It} \quad V = \int_B^D Q\, ds \quad F = \int_A^B Q\, ds = -\int_D^E Q\, ds = -F'$$



F and *F′* indicate a couple *Fh* and the need for the application of a torque as well as the shear load.

$$Fh = Ve$$



When the force P is applied at a distance ***e*** to the left of the web centerline, the member bends in a vertical plane without twisting.

For symmetric sections subject to bending about one axis

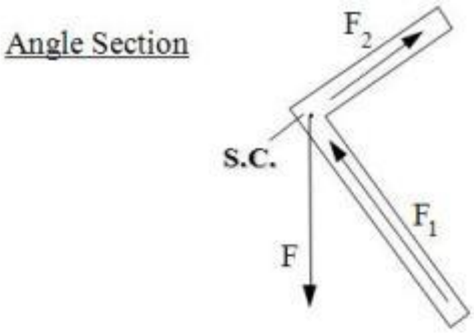
Elements parallel to bending axis-Linear distribution

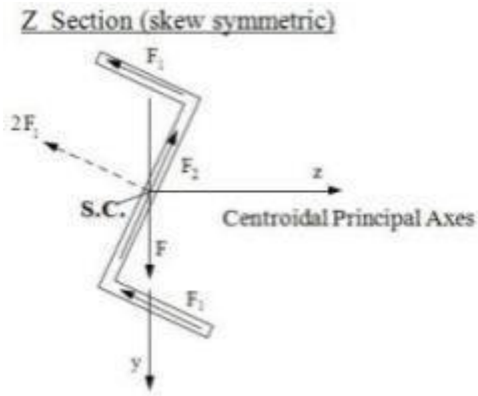
Elements normal to bending axis-Parabolic distribution

For unsymmetric sections shear flow in all elements is parabolic

When moving from one element to another the end value of shear in one element equals the initial value for the subsequent element (from equilibrium)

Shear Centres for Some Other Sections





Pressure Vessels

Types of Pressure Vessels Pressure vessels are mainly of two type:

- **Thin shells**

If the thickness of the wall of the shell is less than 1/10 to 1/15 of its diameter, then shell is called **Thin** shells.

$$t < \frac{D_i}{10} \text{ to } \frac{D_i}{15}$$

- **Thick shells**

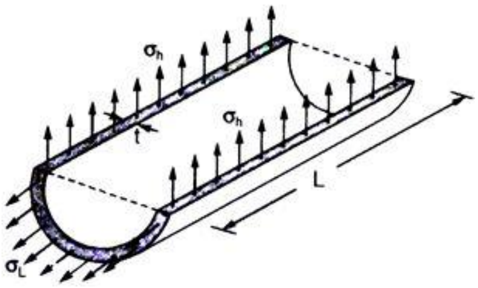
If the thickness of the wall of the shell is greater than 1/10 to 1/15 of its diameter, then shell is called **Thick** shells.

$$t > \frac{D_i}{10} \text{ to } \frac{D_i}{15}$$

where Nature of stress in thin cylindrical shell subjected to internal pressure

1. Hoop stress/circumferential stress will be tensile in nature.
2. Longitudinal stress/axial stress will be tensile in nature
3. Radial stress will be compressive in nature.

Stresses in Thin Cylindrical Shell



- **Circumferential Stress /Hoop Stress**

$\sigma_h = \frac{pd}{2t} \Rightarrow \sigma_h = \frac{pd}{2t\eta}$

Where, p = Intensity of internal pressure

d = Diameter of the shell

t = Thickness of shell

η = Efficiency of joint

- Longitudinal Stress

$\sigma_l = \frac{pd}{4t} \Rightarrow \sigma_l = \frac{pd}{4t\eta}$

- **Hoop Strain**

$\varepsilon_h = \frac{pd}{4tE}(2-\mu)$

- **Longitudinal Strain**

$$\varepsilon_L = \frac{pd}{4tE}(1-2\mu)$$

- **Ratio of Hoop Strain to Longitudinal Strain**

$$\frac{\varepsilon_h}{\varepsilon_L} = \frac{2-\mu}{1-2\mu}$$

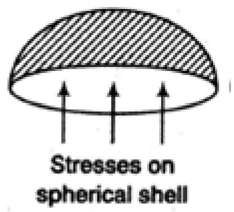
- **Volumetric Strain of Cylinder**

$\varepsilon_v = \frac{pd}{4tE}(5-4\mu)$

Stresses in Thin Spherical Shell

- **Hoop stress/longitudinal stress**

$\sigma_L = \sigma_h = \frac{pd}{4t}$



- **Hoop stress/longitudinal strain**

$$\varepsilon_L = \varepsilon_h = \frac{pd}{4tE}(1-\mu)$$

- **Volumetric strain of sphere**

$$\varepsilon_L = \frac{3pd}{4tE}(1-\ \mu)$$

Lame's Theory/Analysis of Thick Cylinders Lame's theory is based on the following assumptions

Assumptions

1. Homogeneous, isotropic and linearly elastic material.
2. Plane section of cylinder, perpendicular to longitudinal axis remains plane.

- Hoop stress at any section $\sigma_r = \dfrac{b}{r^2} + a$
- Radial pressure $p_r = \dfrac{b}{r^2} - a$

Subjected to Internal Pressure (*p*)

- At $r = r_i,\ \sigma_{ri} = p\left(\dfrac{r_0^2 + r_i^2}{r_0^2 - r_i^2}\right)$
- At $r = r_0,\ \sigma_{r0} = \dfrac{2\,pr_i^2}{r_0^2 - r_i^2}$

Subjected to External Pressure (*p*)

- At $r = r_i,\ \sigma_{ri} = \dfrac{-2\,pr_0^2}{r_0^2 - r_i^2}$
- At $r = r_0,\ \sigma_{r0} = p\left(\dfrac{r_0^2 + r_i^2}{r_0^2 - r_i^2}\right)$

Note: Radial and hoop stresses vary hyperbolically.

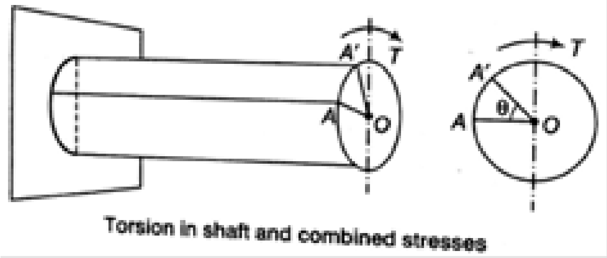
Uniform Torsion

Torsion of Shaft and Combined Stresses

Torsion means twisting a structural Member when it is loaded by a **couple** that Produces rotation about the **longitudinal** axis.

If τ be the intensity of shear stress, on any layer at a distance *r* from the centre of shaft, then

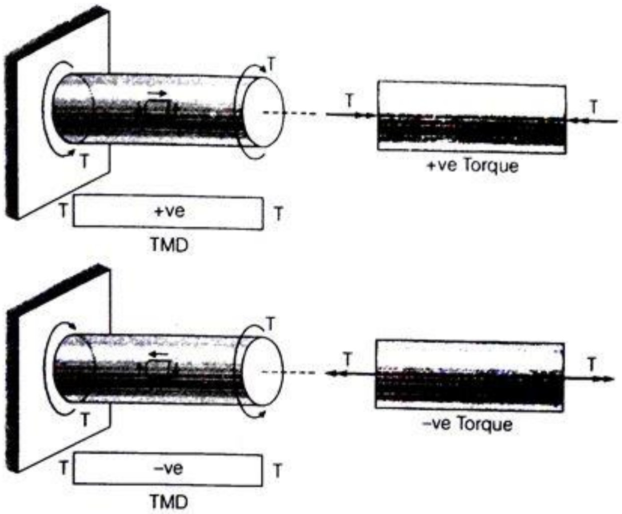
$$\frac{\tau_l}{r} = \frac{T}{J} = \frac{G\theta}{l}$$



Torsion in shaft and combined stresses

Sign Convention

- Sign convention of torque can be explained by **right hand thumb rule**.
- A positive torque is that in which there is **tightening effect of nut** on the bolt. From either side of the cross-section. If torque is applied in the direction of right hand fingers than right hand thumbs direction represents movement of the nut.



TMD = Torsion moment diagram

T = Torque

Total angle of twist :

$$\theta = \frac{Tl}{GJ}$$

Where, T = Torque,

J = Polar moment of inertia

G = Modulus of rigidity,

θ = Angle of twist

L = Length of shaft,

GJ = Torsional rigidity

$\frac{GJ}{l} \to$ Torsional stiffness;

$\frac{l}{GJ} \to$ Torsional flexibility

$\frac{EA}{l} \rightarrow$ Axial stiffness

$\frac{l}{EA} \rightarrow$ Axial flexibility

Moment of Inertia About polar Axis:

- For solid circular shaft,:

$J = \frac{\pi d^4}{32}$, $\tau_{max} = \frac{16T}{\pi d^3}$

- For hollow circular shaft:

$J = \frac{\pi}{32}(d_0^4 - d_i^4)$

Power Transmitted in the Shaft

- Power transmitted by shaft:

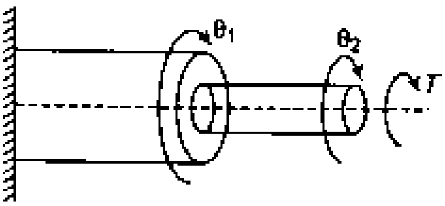
$$P = \frac{2\pi NT}{60000} kW$$

Where, N = Rotation per minute.

Compound Shaft

An improved type of compound coupling for connecting in series and parallel are given below

1. **Series connection:** Series connection of compound shaft as shown in figure. Due to series connection the torque on shaft 1 will be equal to shaft 2 and the total angular deformation will be equal to the sum of deformation of 1st shaft and 2nd shaft.



Series connection

$$\theta = \theta_1 + \theta_2$$
$$T = T_1 = T_2$$

Therefore,

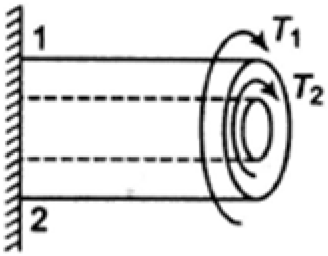
$$\theta = \frac{TL_1}{G_1J_1} + \frac{TL_2}{G_2J_2}$$

Where,

θ_1 = Angular deformation of 1st shaft

θ_2 = Angular deformation of 2nd shaft

1. **Parallel connection:** Parallel connection of compound shaft as shown in figure. Due to parallel connection of compound shaft the total torque will be equal to the sum of torque of shaft 1 and torque of shaft 2 and the deflection will be same in both the shafts.



Parallel connection

$$\theta_1 = \theta_2$$
$$T = T_1 + T_2$$

Therefore,

$$\frac{T_1L}{G_1J_1} = \frac{T_2L}{G_2J_2}$$

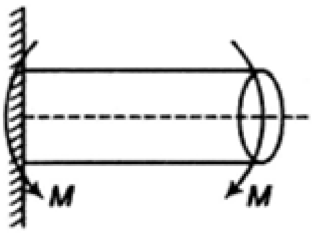
Strain energy (U) stored in shaft due to torsion:

$$U = \frac{1}{2}T.\theta = \frac{1}{2}\frac{T^2L}{G.J} = \frac{\tau_{max}^2}{4G}.Volume\ of\ shaft$$

- G = Shear modulus
- T = Torque
- J = Moment of inertia about polar axis

Effect of Pure Bending on Shaft

The effect of pure bending on shaft can be defined by the relation for the shaft,



Pure bending on shaft

$$\sigma = \frac{32M}{\pi D^3}$$

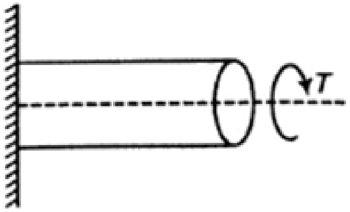
Where, σ = Principal stress

D = Diameter of shaft

M = Bending moment

Effect of Pure Torsion on Shaft

It can be calculated by the formula, which are given below



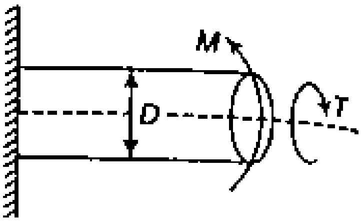
Pure torsion on shaft

$$\tau_{max} = \frac{16T}{\pi D^3}$$

Where, τ = Torsion

D = Diameter of shaft

Combined effect of bending and torsion



Bending and torsion effect

- Principal stress

$$= \frac{16}{\pi D^3}[M \pm \sqrt{M^2 + T^2}]$$

- Maximum shear stress

$$= \frac{16}{\pi D^3}\sqrt{M^2 + T^2}$$

- Equivalent bending moment

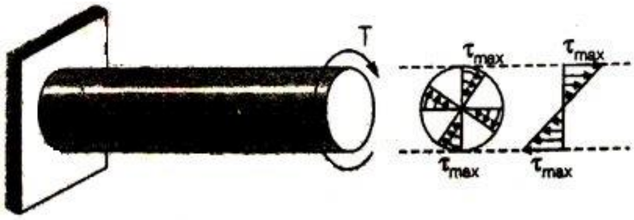
$$M_{eq} = \frac{1}{2}[M \pm \sqrt{M^2 + T^2}]$$

- Equivalent torque

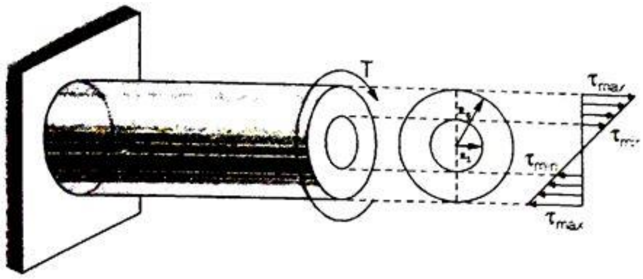
$$T_{eq} = \sqrt{T^2 + M^2}]$$

Shear Stress Distribution:

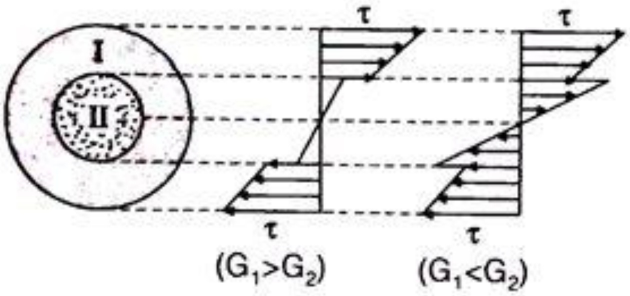
- Solid Circulation Section:



- Hollow Circulation Section



- Composite Circular Section



- Thin Tubular section: In view of small thickness-shear stress is assumed to be uniform

Deflections of Beam

Slope of a Beam: Slope of a beam is the angle between deflected beam to the actual beam at the same point.

Deflection of Beam: Deflection is defined as the vertical displacement of a point on a loaded beam. There are many methods to find out the slope and deflection at a section in a loaded beam.

- The maximum deflection occurs where the slope is zero.

Methods of Determining Beam Deflections

Numerous methods are available for the determination of beam deflections. These methods include:

Double Integration Method

- This is most suitable when concentrated or udl over entire length is acting on the beam.
- Deflection of beams is so small, such that the slope of the elastic curve dy/dx is very small, and squaring this expression the value becomes practically negligible, hence
- If EI is constant, the equation may be written as:

$$EIy''=M$$

where x and y are the coordinates shown in the figure of the elastic curve of the beam under load,

- y is the deflection of the beam at any distance x.
- E is the modulus of elasticity of the beam,
- I represent the moment of inertia about the neutral axis, and
- M represents the bending moment at a distance x from the end of the beam.

The product **EI is called the flexural rigidity** of the beam.

$$EI\,\frac{d^2y}{dx^2} = -M$$

Integrating one time

$$EI\,\frac{dy}{dx} = -\int M$$

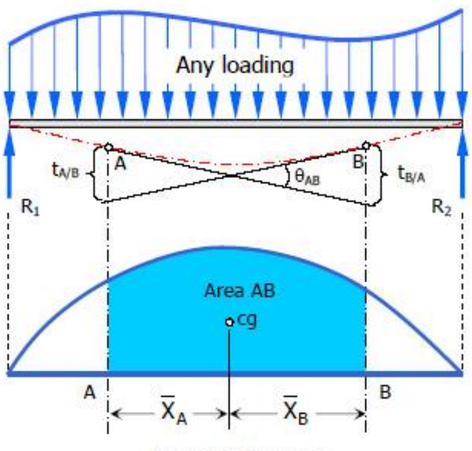
The first integration y'(dy**/dx) yields the Slope of the Elastic Curve**

Second Integration $$EI\,y = -\int\int M$$

The second integration y gives the **Deflection of the Beam at any distance x**.

Area Moment Method

- Another method of determining the slopes and deflections in beams is the area-moment method, which involves the area of the moment diagram.
- The moment-area method is a semi graphical procedure that utilizes the properties of the area under the bending moment diagram. It is the quickest way to compute the deflection at a specific location if the bending moment diagram has a simple shape.



Moment Diagram

Theorems of Area-Moment Method

- **Theorem 1**
  - The change in slope between the tangents drawn to the elastic curve at any two points A and B is equal to the product of 1/EI multiplied by the area of the moment diagram between these two points

θ_{AB}=(1/EI)(Area between A and B)

- **Theorem 2**
    - The deviation of any point B relative to the tangent drawn to the elastic curve at any other point A, in a direction perpendicular to the original position of the beam, is equal to the product of 1/EI multiplied by the moment of an area about B of that part of the moment diagram between points A and B.

$t_{B/A}$=(1/EI)(Area between A and B)·X‾$_B$

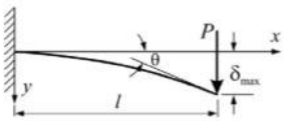
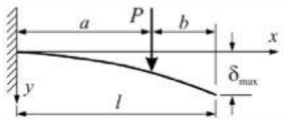
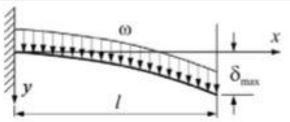
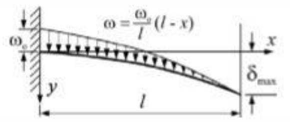
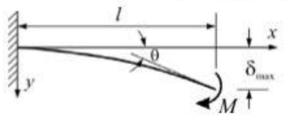
and

$t_{A/B}$=(1/EI)(Area between A and B)·X‾A

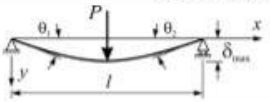
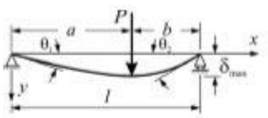
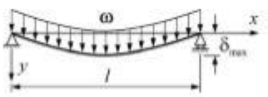
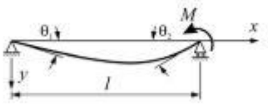
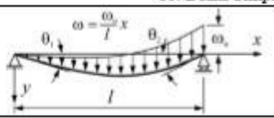
Method of Superposition: The method of superposition, in which the applied loading is represented as a series of simple loads for which deflection formulas are available. Then the desired deflection is computed by adding the contributions of the component loads(principle of superposition)

Beam Deflection Formula (Standard Results)

- **Cantilever Beams:**

BEAM TYPE	SLOPE AT FREE END	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM DEFLECTION
1. Cantilever Beam – Concentrated load P at the free end			
	$\theta = \frac{Pl^2}{2EI}$	$y = \frac{Px^2}{6EI}(3l - x)$	$\delta_{max} = \frac{Pl^3}{3EI}$
2. Cantilever Beam – Concentrated load P at any point			
	$\theta = \frac{Pa^2}{2EI}$	$y = \frac{Px^2}{6EI}(3a - x)$ for $0 < x < a$ $y = \frac{Pa^2}{6EI}(3x - a)$ for $a < x < l$	$\delta_{max} = \frac{Pa^2}{6EI}(3l - a)$
3. Cantilever Beam – Uniformly distributed load ω (N/m)			
	$\theta = \frac{\omega l^3}{6EI}$	$y = \frac{\omega x^2}{24EI}(x^2 + 6l^2 - 4lx)$	$\delta_{max} = \frac{\omega l^4}{8EI}$
4. Cantilever Beam – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta = \frac{\omega_o l^3}{24EI}$	$y = \frac{\omega_o x^2}{120lEI}(10l^3 - 10l^2x + 5lx^2 - x^3)$	$\delta_{max} = \frac{\omega_o l^4}{30EI}$
5. Cantilever Beam – Couple moment M at the free end			
	$\theta = \frac{Ml}{EI}$	$y = \frac{Mx^2}{2EI}$	$\delta_{max} = \frac{Ml^2}{2EI}$

- **Simply supported Beams:**

BEAM TYPE	SLOPE AT ENDS	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM AND CENTER DEFLECTION
6. Beam Simply Supported at Ends – Concentrated load P at the center			
	$\theta_1 = \theta_2 = \frac{Pl^2}{16EI}$	$y = \frac{Px}{12EI}\left(\frac{3l^2}{4} - x^2\right)$ for $0 < x < \frac{l}{2}$	$\delta_{max} = \frac{Pl^3}{48EI}$
7. Beam Simply Supported at Ends – Concentrated load P at any point			
	$\theta_1 = \frac{Pb(l^2 - b^2)}{6lEI}$ $\theta_2 = \frac{Pab(2l - b)}{6lEI}$	$y = \frac{Pbx}{6lEI}\left(l^2 - x^2 - b^2\right)$ for $0 < x < a$ $y = \frac{Pb}{6lEI}\left[\frac{l}{b}(x-a)^3 + \left(l^2 - b^2\right)x - x^3\right]$ for $a < x < l$	$\delta_{max} = \frac{Pb\left(l^2 - b^2\right)^{3/2}}{9\sqrt{3}\,lEI}$ at $x = \sqrt{\left(l^2 - b^2\right)/3}$ $\delta = \frac{Pb}{48EI}\left(3l^2 - 4b^2\right)$ at the center, if $a > b$
8. Beam Simply Supported at Ends – Uniformly distributed load ω (N/m)			
	$\theta_1 = \theta_2 = \frac{\omega l^3}{24EI}$	$y = \frac{\omega x}{24EI}\left(l^3 - 2lx^2 + x^3\right)$	$\delta_{max} = \frac{5\omega l^4}{384EI}$
9. Beam Simply Supported at Ends – Couple moment M at the right end			
	$\theta_1 = \frac{Ml}{6EI}$ $\theta_2 = \frac{Ml}{3EI}$	$y = \frac{Mlx}{6EI}\left(1 - \frac{x^2}{l^2}\right)$	$\delta_{max} = \frac{Ml^2}{9\sqrt{3}\,EI}$ at $x = \frac{l}{\sqrt{3}}$ $\delta = \frac{Ml^2}{16EI}$ at the center
10. Beam Simply Supported at Ends – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta_1 = \frac{7\omega_o l^3}{360EI}$ $\theta_2 = \frac{\omega_o l^3}{45EI}$	$y = \frac{\omega_o x}{360lEI}\left(7l^4 - 10l^2x^2 + 3x^4\right)$	$\delta_{max} = 0.00652\frac{\omega_o l^4}{EI}$ at $x = 0.519l$ $\delta = 0.00651\frac{\omega_o l^4}{EI}$ at the center

****