MEGA MOCK CHALLENGE

gradeup

GATE 2022

Mechanical Engineering

Subject-Wise Mini Mock Challenge

(April 21st - April 22nd 2021)

Engineering Mathematics & General Aptitude

Questions & Solutions

1. log 75/16 - 2 log 5/9 + log 32/243 = ?

A. 4 log 2 | B. 2 log 2
C. log 2 | D. none of these

Ans. C

Sol. Tip –

(i) log ab = log a + log b

(ii) log a/b = log a – log b

(iii) log a^n = n log a

log 75/16 - 2 log 5/9 + log 32/243

= log 75/16 – log $(5/9)^2$ + log (32/243)

$$= \log\left(\frac{75 \times 81 \times 32}{16 \times 25 \times 243}\right)$$

$$= \log\ 2$$

Hence, (C) is the correct answer

2. If A, B and C can respectively complete a piece of work in 20, 24 and 36 days respectively, how many days will they take to complete the work, if they work together?

A. $8\frac{16}{43}$ | B. $6\frac{1}{4}$
C. $9\frac{1}{4}$ | D. $7\frac{19}{20}$

Ans. A

Sol. Let total work = LCM (20, 24, 36) = 360

Efficiency of A = $\frac{360}{20} = 18$

Efficiency of B = $\frac{360}{24} = 15$

Efficiency of C = $\frac{360}{36} = 10$

Total efficiency (A + B + C) = 18 + 15 + 10 = 43

Time taken = $\frac{360}{43} = 8\frac{16}{43}$.

3. Anand & Ravi start together to cover a certain distance. The speed of Anand is 7 km/h and speed of Ravi is 6 km/h. If Anand reaches the destination 1 1/2hrs before Ravi reaches the destination, find the distance.

A. 63 Km B. 64 Km

C. 65 Km D. 66 Km

Ans. A

Sol. LCM of Anand's & Ravi's speed = 42

So, if distance is 42 km then

Anand covers that distance in 6 hrs.

Ravi cover that distance in 7 hrs.

Which does not satisfy the condition given in question

So, if we assume the desired distance is 63 km (63 also a LCM of 6 & 7)

Then, Anand cover the distance in 9 hrs. (speed = $\frac{Distance}{Time} \Rightarrow \frac{63}{7} = 9hrs.$)

Ravi cover the distance in 10.5 hrs. (speed = $\frac{Distance}{Time} \Rightarrow \frac{63}{6} = 10$.5hrs.)

Now, the distance = 63 km.

4. The product of the ages of Ankita and Sunita is 240. If twice the age of Sunita is more than Ankita's age by 4 years. What is Sunita's age?

A. 12 years B. 20 years

C. 24 years D. 16 years

Ans. A

Sol. Let the Ankita's age be x years, then Sunita's age will be 240/x years.

According to question,

2*(240/x) = x + 4

$\Rightarrow x^2 + 4x - 480 = 0$

$\Rightarrow x^2 + 24x - 20x - 480 = 0$

$\Rightarrow (x + 24)(x - 20) = 0$

$\Rightarrow x = 20$ years

And Sunita's age = 240/20 = 12 years

5. The difference between squares of two consecutive odd integer is always divisible by.

A. 3 B. 6

C. 7 D. 8

Ans. D

Sol. Can be easily verified as

$3^2 - 1^2 = 8$

$5^2 - 3^2 = 16$, both are divisible by 8

6. A train leaves station P at 5 AM and reaches station Q at 8 AM on the same day. Another train leaves station Q at 7 AM and reaches station P at 9 AM on the same day. The time at which the two trains cross each other is :

A. 7:12 AM B. 7:36 AM

C. 7:44 AM D. 7:24 AM

Ans. D

Sol. Let A be the train leaving station P towards Q and

B be the train leaving station Q towards P.

Time taken by A = 3 hrs

Time taken by B = 2 hrs

Ratio of time taken by A and B = 3:2

Speed ∝ (1/ Time)

$$\frac{Speed\ of\ A}{Speed\ o\ B} = \frac{2}{3}$$

Now, let the speed of A and B be 10 kmph and 15 kmph respectively.

So, the distance between P and Q will be 10 x 3 = 30kms.

Now distance travelled by A in 2 hrs (till B has not moved) = 10 x 2 = 20

Distance left to be travelled = 30 - 20 = 10 km

Trains moving in opposite direction will cross each other in = (10 / 25) x 60 mins

= 24 mins. ie.they'll cross each other at 7:24 AM

7. Sheela is Ravi's sister-in-law. Rani is Ravi's brother. Ram's wife is Sheela. Deepa is Ravi's sister. Deepa's mother is Shanthi. How is Sheela related to Shanthi?

   A. Mother-in-law B. Daughter-in-law

   C. Granddaughter D. Daughter

Ans. B

Sol.

Shanthi is mother of Ravi, Ram and Deepa.

Sheela is the wife of Ram.

Therefore, Sheela is the daughter-in-law of Shanthi.

Hence the correct response is (B).

8. Find the pair of letters which will come in blank spaces marked as '?' ___________.

   V W X Y E D C B R S T ? ?

   A. GU B. UI

   C. IU D. IG

Ans. B

Sol.

A careful look at the alphabets in the given series shows a pattern:

VWXY – EDCB RST? etc

$V \leftrightarrow E;\ W \leftrightarrow D;\ X \leftrightarrow C; Y \leftrightarrow B;$ i.e. 5^{th} from end corresponds to a* from beginning etc. It is a cluster of 4 consecutive letters which is taken at a time, "Therefore, the next 4 letters will be RSTU-IHGF etc.

Hence the last 2 letters will be U and I i.e. option (b).

9. A group of boys and girls know either French or Spanish. The number of boys and girls are in the ratio 1 : 4. 30% of the girls know Spanish and the rest of them know French. On the other hand, 50% of the boys know Spanish and the rest of them know French. A student is chosen at random from the group of students who knows Spanish. What is the probability that the chosen student is a girl?

A. $\frac{2}{7}$ B. $\frac{12}{17}$

C. $\frac{20}{41}$ D. $\frac{8}{13}$

Ans. B

Sol. Let the number of students be 100. Since ratio of number of boys : girls is 1 : 4; it means number of boys is 20 and number of girls is 80. 30% of girls know Spanish ⇒ number of girls who know Spanish = 30% of 80 = 24 50% of the boys know Spanish ⇒ number of boys who know Spanish = 50% of 20 = 10

Total number of students who know Spanish = 24 + 10 = 34

Hence probability that a gill is chosen from the group of students who know English

$=\frac{24}{34}=\frac{12}{17}$ i.e. (b).

10. A man had 7 children. When their average age was 12, a child aged 6 years died. The average age of the remaining six children is:

A. 13 years B. 10 years

C. 11 years D. 14 years

Ans. A

Sol. Total age of 7 children = 12 × 7 = 84 years

Since a child aged 6 years died.

Therefore,

The average age of the remaining 6 children = $\frac{84-6}{6}=\frac{78}{6}=13$

Hence, option A is correct.

11. Train A having length of 420 m crosses a platform whose length is 50% more than that of train A in 35 second. In how much time it can cross another train B, whose length is 380 m and running in the opposite direction with the speed of 180 km/hr?

A. 20 seconds B. 30 seconds

C. 15 seconds D. 10 seconds

E. 50 seconds

Ans. D

Sol. Let the speed of train A be ‘D m/s’

Length of the platform = 420 × 1.50

= 630 meters

A.T.Q,

$\frac{(420+630)}{D}$ = 35 seconds

1050 = 35 × D

D = 30m/s

Speed of train A = 30 m/s

Speed of train B = 180 km/h

= 180 × 5 ÷ 18 m/s

= 50 m/s

Time taken by train B to cross train A = $\frac{380+420}{30+50}$

= 10 seconds

12. The product of two numbers is 2028 and their H.C.F is 13. The numbers of such pair is:

A. 1

B. 2

C. 3

D. 4

Ans. B

Sol. Let the numbers be 13*a* and 13*b*.

Then, 13*a* × 13*b* = 2028

ab = 2028/169 = 12

Now, the co-primes with product 12 are (1, 12) and (3, 4).

So, the required numbers are (13 × 1, 13 × 12) and (13 × 3, 13 × 4).

Clearly, there are 2 such pairs.

Hence, option B is correct.

13. We are given a square of side 22 cm. A circle of maximum possible diameter is inscribed in this square. If a point is chosen at random inside the square, then the probability that it will lie inside the circle is _____________.

Ans.

Sol.

$$\text{Required probability} = \frac{\text{Area of the circle}}{\text{Area of the square}}$$

Biggest possible circle that can be inscribed in the given square would, be touching all the four sides of the square internally implying that:

The diameter of this circle = side of the square = 22 cm

$$\text{Required probability} = \frac{\text{Area of the circle}}{\text{Area of the square}} = \frac{\pi(11 \times 11)}{(22 \times 22)} = \frac{\pi}{4} = 0.785$$

14. Which of the following means ‘Cast Iron Stomach’?

A. The ability to eat foods that others may find unusual, unpalatable, or too spicy

B. In thorough and exact detail

C. To refuse to do something due to fear (real or perceived).

D. To eat, often quickly or enthusiastically.

E. None of these

Ans. A

Sol. ‘Cast iron stomach’ refers to the ability to eat food that others may find unusual, unpalatable, or too spicy.

15. Two solutions of alcohol A and B were mixed to obtain 20 litres of new solution C. Before they were mixed, the first solution A contained 1.6 litres of alcohol while the second solution B contained 1.2 litres of alcohol. Before mixing if the percentage of alcohol in the first solution A was twice that in the second B, what was the volume of the first solution A before mixing?

A. 6 litres

B. 6.4 litres

C. 7.2 litres

D. 8 litreS

Ans. D

Sol.

Let the volume of the first A in the mixture be “x” litres, the volume of the second B in the mixture must be (20 - x).

$$\%\text{ of alcohal in A} = \left(\frac{1.6}{x}\right) \times 100$$

$$\% \text{ of alcohal in } B = \left(\frac{1.2}{20-x}\right) \times 100$$

$$\left(\frac{1.6}{x}\right) \times 100 = 2 \times \left(\frac{1.2}{20-x}\right) \times 100$$

16 – 0.8x = 1.2x

x = 8 litres

16. G is in north of H at distance of 13m who is in the west of I at distance of 7m. M is in the west of G at distance of 8m. Now, M is in which direction with respect to I?

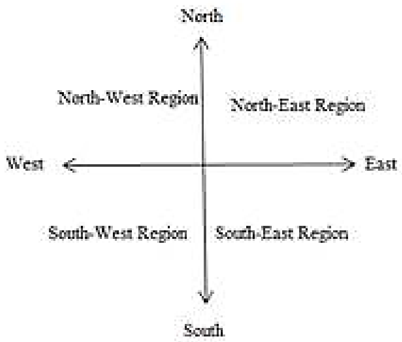
A. South-East B. North-East

C. North-West D. South-West

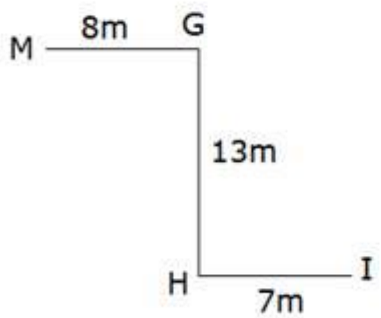
Ans. C

Sol.

We know that:



We can show the given data in the following figure:



From the above figure, its clearly can be seen that M is in north-west direction with respect to I.

So, the correct answer is option C.

17. A series is given with one term missing. Select the correct alternative from the given ones that will complete the series. 480, ?, 24, 8, 4

A. 98　　B. 96

C. 104　　D. 88

Ans. B

Sol. The series will be,

$480 \div 5 = 96$

$96 \div 4 = 24$

$24 \div 3 = 8$

$8 \div 2 = 4$

Hence, option B is the correct response.

18. |||Common||| In the following question, out of the four alternatives, select the word similar in meaning to the given word. |||End|||

Vindicate A. Disprove

B. Validate

C. Rebut

D. Refute

Ans. B

Sol. Vindicate means to prove that what someone said or did was right or true, after other people thought it was wrong.

Disprove means to prove that something is false.

Validate means to check or prove the validity or accuracy of something.

Rebut means to claim or prove that evidence or an accusation is false.

Refute means to prove that something is wrong.

Thus, the word "validate" is the correct synonym of the given word.

19. A rectangular swimming pool of length 30 m and breadth 15 m has a uniformly sloping floor. It is 1 m in depth on one end of its length and 2 m at the other. How much water is needed to completely fill this pool?

A. 1350 cubic mts　　B. 2025 cubic mts

C. 675 cubic mts　　D. 2700 cubic mts

Ans. C

Sol. Length of pool= 30m

Breadth of pool= 15 m

Height of pool at one end=1m

And at other end=2m

In this way we have a trapezium whose height is 30 m and other two sides are 1m and 2 m

So area of trapezium = $\frac{1}{2}(30)(1+2) = 45 sqm$

Volume of water required =45× 15=675 cubic mts

20. If a landlord sell a portion of land for Rs. 40000, one-fourth of the outlay was gained. What should be the selling price in order to have loss of 25%?

A. Rs. 38750 B. Rs. 38000

C. Rs. 24000 D. Rs. 22000

Ans. C

Sol. Given that, if a landlord sell a portion of land for Rs. 40000, one-fourth of the outlay was gained.

We know that,

$$CP = \frac{100 \times SP}{100 + gain\%}$$

$$SP = \left[1 - \frac{loss\%}{100}\right] \times CP$$

Gain = CP/4

$$Gain\% = \frac{gain}{CP} \times 100$$

⇒Gain% = $\frac{\frac{1}{4} \times CP}{CP} \times 100 = 25\%$

Thus, $CP = \frac{100 \times 40000}{100 + 25}$ = Rs.32000

Now, SP in order to have loss of 25% = $\left[1 - \frac{25}{100}\right]$ × 32000

= 3/4 × 32000

= Rs. 24000

21. Laplace transform for $\frac{1}{4}e^{2t}\cos 4t$ is

A. $\frac{s-4}{4s^2-16s+80}$ B. $\frac{s-4}{4s^2-16s-80}$

C. $\frac{s-2}{4s^2-16s+80}$ D. $\frac{s-2}{4s^2-16s-80}$

Ans. C

Sol.

$$L(e^{at}\cos bt) = \frac{(s-a)}{(s-a)^2+b^2}$$

$$L\left(\frac{1}{4}e^{2t}\cos 4t\right) = \frac{1}{4}\frac{(s-2)}{(s-2)^2+16} = \frac{s-2}{4(s^2-4s+20)} = \frac{s-2}{4s^2-16s+80}$$

22. Let $P = f(z) = z^2 + 2z = u + iv$, Where, z is a complex variable such that z = x + iy, then value of u at z = 1+ i is _____.

Ans.

Sol.

P = z^2 + 2z

P = $(x + iy)^2$ + 2(x + iy)

P = $x^2 - y^2$ + 2ixy + 2x + 2iy

P = $(x^2 - y^2 + 2x) + i(2xy + 2y)$

P = u + iv

∴ u = $x^2 - y^2 + 2x$

at $z(1+i)$

∴ (x, y) = (1, 1)

∴ u = 1 – 1 + 2 = 2

23.The particular integral of $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 2\log x$ is ___________.

A. $e^x + 2$ B. $2e^x + 4$

C. $\log x + 2$ D. $2\log x + 4$

Ans. D

Sol. Let x = e^z

$\log x = \ln e^z \Rightarrow x = z$

$$x\frac{dy}{dx} = \theta y,\ x^2\frac{d^2x}{dy^2} = \theta(\theta-1)$$

$$\therefore \left(\theta(\theta-1)-\theta+1\right)y = 2z$$

$$(\theta-1)^2 y = 2z$$

$$\therefore\ y_p = \frac{1}{(\theta-1)^2}2z = 2(1-\theta)^{-2}z = 2\left(1+2\theta+3\theta^2+....\right)z$$

$$y_p = 2z+4$$

$$\therefore\ y_p = \log x + 4$$

y_p = 2logx+4

24. The value of $\oint_C (3x-2y)\,dx + (x-4y)dy$, where C is a circle $x^2 + y^2 = 4$

A. 4п

B. 8п

C. 12п

D. 16п

Ans. C

Sol. Using Green’s theorem:

$$\oint_C Pdx + Qdy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy$$

$P = 3x - 2y$

$Q = x - 4y$

$\frac{\partial P}{\partial y}$ = -2 , $\frac{\partial Q}{\partial x} = 1$

$$\Rightarrow \iint_R (1-(-2))dxdy = \int_R 3\ dR = 3\cdot R = 3\cdot \pi r^2 = 12\pi$$

So, the correct option is (c).

25. By Newton-Raphson method, the root of $x^4 - x - 10 = 0$ which is near to x = 2 will be -------------- (round of to 2 decimal places)

Ans.

Sol. Here $f(x) = x^4 - x - 10$, so that $f'(x) = 4x^3 - 1$

Therefore

$X_{n+1} = X_n - f(X_n)/f'(X_n) = X_n - (x_n^4 - x_n - 10)/(4x_n^3 - 1)$

i.e, $X_{n+1} = (3x_n^4 + 10)/(4x_n^3 - 1)$ --------------------(1)

For n = 1 & put $x_0 = 2$ in (1) we get $X_1 = 1.871$

On further two iterations i.e, $X_2 = 1.856$ and $X_3 = 1.856$

since $X_2 = X_3$, the required root will be 1.856.

26. Find the value of integral $\int_1^3 \frac{|x-2|}{x} dx$ is __________.

Ans.

Sol.

$$|x-2| = \begin{cases} -(x-2); x < 2 \\ (x-2); x > 2 \end{cases}$$

$$\int_1^3 \frac{|x-2|}{x} dx = \int_1^2 \frac{-(x-2)}{x} dx + \int_2^3 \frac{(x-2)}{x} dx$$

$$= \int_1^2 \left(-1 + \frac{2}{x}\right) dx + \int_2^3 \left(1 - \frac{2}{x}\right) dx$$

$$= -(2-1) + (2\,ln\,x)_1^2 + (x)_2^3 - 2(ln\,x)_2^3$$

$$= 2\,ln\,2 - 2\,ln\left(\frac{3}{2}\right)$$

$$= 2\,ln\left(\frac{2}{\frac{3}{2}}\right) = 2\,ln\left(\frac{4}{3}\right) = 0.575$$

27. The value of $\int_0^{\pi/4} \log(1 + tan\theta)\, d\theta$ will be ----

A. $\frac{\pi}{4} log2$

B. $\frac{\pi}{8} log2$

C. $\frac{\pi}{2} log2$

D. 1

Ans. B

Sol.

Let

$I = \int_0^{\pi/4} \log(1 + tan\theta)\, d\theta = \int_0^{\pi/4} \log\left(1 + \tan\left(\frac{\pi}{4} - \theta\right)\right) d\theta$

Since, $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

So,

$$I = \int_0^{\pi/4} \log\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right) d\theta = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right) d\theta$$

$$I = \int_0^{\pi/4} \log 2\, d\theta - \int_0^{\pi/4} \log(1+\tan\theta)\, d\theta$$

$$I = \int_0^{\pi/4} \log 2\, d\theta - I$$

$$2I = \frac{\pi}{4} log 2$$

$$I = \frac{\pi}{8} log 2$$

28. Two persons A and B take turns in throwing a pair of dice. The first person to throw 9 from both dice well be awarded the prize. If A throws first, then the probability that B wins the game is

A. $\frac{9}{17}$

B. $\frac{8}{17}$

C. $\frac{8}{9}$

D. $\frac{1}{9}$

Ans. B

Sol. The probability of throwing 9 with two dice is $p = \frac{4}{36} = \frac{1}{9}$

Therefore, the probability of not throwing with two dice is

$$q = (1-p) = 1 - \frac{1}{9} = \frac{8}{9}$$

Case 1: If A has to win, he should throw 9 in 1st or 3rd or 5th attempt.

Case 2: If B has to win, he should throw the dice 9 in 2nd or 4th chance.

Therefore, B's chance of winning the game is

$$= \left(\frac{8}{9}\right)\times\left(\frac{1}{9}\right)+\left(\frac{8}{9}\right)^3\times\frac{1}{9}+\ldots\ldots\ldots\ldots\ldots = \frac{\frac{8}{9}\times\frac{1}{9}}{1-\left(\frac{8}{9}\right)^2} = \frac{8}{17}$$

29. There are 3 fair coins and 1 false coin with tails on both sides. A coin is chosen at random and tossed 4 times. If 'tails' occurs in all 4 times, then the probability that the false coin has been chosen for tossing is ____.

Ans.

Sol. $Required\ Probablity\ = \frac{Favorable\ Outcomes}{Total\ possible\ outcomes}$

Favorable outcomes = A false coin is chosen and flipped every time

Probability of selecting a false coin $= \frac{1}{4}$

Probability of getting a tail on every flip of false coin =1.

$\therefore$ Favorable outcome $= \frac{1}{4} \times 1 = \frac{1}{4}$

Total possible outcomes = Favourable outcomes + Unfavorable outcomes

Unfavorable outcomes = A fair coin is chosen and flipped every time to get tail

Probability of selecting a fair coin $= \frac{3}{4}$

Probability of flipping a fair coin 4 times and getting

tails every time $= \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$\therefore$ Unfavourable outcomes $= \frac{3}{4} \times \frac{1}{16} = \frac{3}{64}$

Total possible outcomes $= \frac{1}{4} + \frac{3}{64} = \frac{19}{64}$

$\therefore$ Required probability $= \frac{\frac{1}{4}}{\frac{19}{64}} = \frac{16}{19} = 0.84$

30. From the solution to the initial value problem:

$$t^2 \frac{dy}{dt} + ty = 1$$

if y(1) =1 then pick the closest estimate of y(2).

A. 0.55

B. 0.75

C. 0.846

D. 1.25

Ans. C

Sol. Given,

$$t^2\frac{dy}{dt}+ty=1$$

$$\frac{dy}{dt}+\frac{y}{t}=\frac{1}{t^2}$$

$$\text{Integrating factor(I.F.)} = e^{\int \frac{1}{t}dt} = t$$

the solution is

$$y(\text{I.F.}) = \int \frac{1}{t^2}(\text{I.F.})\,dt$$

$$yt = \int \frac{1}{t^2}\times t\,dt$$

$$yt = \ln(t) + C$$

given, $y(1) = 1$

$C = 1$

$$yt = \ln(t) + 1$$

$$y = \frac{\ln(t)+1}{t}$$

put $t = 2$

$$y = \frac{\ln(2)+1}{2} = 0.846$$

31. Given that: $u = x^2$ and $v = y^2$

The value of $\frac{\partial(u,v)}{\partial(x,y)}$ at the point where $x = 1$ and $y = 3$ is _____.

A. 10

B. 11

C. 12

D. 15

Ans. C

Sol. Given that: $u = x^2$ and $v = y^2$

$u_x = 2x$; $u_y = 0$; $v_x = 0$; $v_y = 2y$;

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = \begin{vmatrix} 2x & 0 \\ 0 & 2y \end{vmatrix} = 4xy$$

At x = 1, y = 3:

= 4 × 1 × 3 = 12

32. The value of the integral $\oint \frac{3z+1}{z^2-4z+3}dz$ evaluated around the closed curve $|z-3|$ =1 is_____?

A. −4πi

B. 6πi

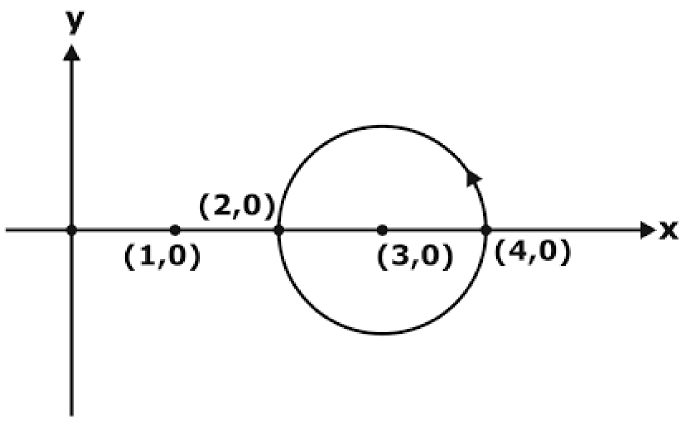
C. 0

D. 10πi

Ans. D

Sol. Given,

Closed curve $|z-3|$ =1



The poles are determined as

$z^2 - 4z + 3 = 0$

(z−3)(z−1) = 0

z = 3, 1

There is only one pole z =3 lies inside the given circle $|z-3|$ =1

So, the value of integral =

$$Res.\ f(z)_{z=3} = \lim_{z \to 3}(z-3)f(z) = \lim_{z \to 3}\frac{(z-3)(3z+1)}{(z-3)(z-1)} = 5$$

$$\oint f(z)dz = 2\pi i(Sum\ of\ residue\ of\ f(z)\ at\ all\ its\ poles\ inside\ c)$$

$$= 2\pi i(\ 5)$$

$$= 10\pi i$$

33. Given the following three vectors:

$\vec{a} = \alpha\,\hat{i} - 9\hat{j} - \hat{k}, \vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}\,, \vec{c} = 4\hat{i} + 2\hat{j} + \hat{k}$

The value of α such that the vector $\vec{a}\ is\ perpendicular\ to\ \vec{b}x\vec{c}$ is :

Ans.

Sol.

For $\vec{a}\ to\ be\ perpendicular\ to\ \vec{b}x\vec{c}$, we must have : $\vec{a}.\vec{b}x\vec{c} = 0$

Which is the scalar triple product

Now,

$$\vec{a}.\vec{b}x\vec{c} = \begin{vmatrix} \alpha & -9 & -1 \\ 3 & 3 & 1 \\ 4 & 2 & 1 \end{vmatrix}$$

$\alpha(3-2) + 9(3-4) - 1(6-12) = 0$

$\alpha - 9 + 6 = 0\ giving\ \alpha = 3$

34. $\lim\limits_{n\to\infty}\left[\frac{1}{n^3+1} + \frac{4}{n^3+1} + \frac{9}{n^3+1} + \ldots\ldots + \frac{n^2}{n^3+1}\right]$

A. 1

B. $\frac{2}{3}$

C. $\frac{1}{3}$

D. 0

Ans. C

Sol. Given,

$= \lim\limits_{n\to\infty}\left[\frac{1}{n^3+1} + \frac{4}{n^3+1} + \frac{9}{n^3+1} + \ldots\ldots + \frac{n^2}{n^3+1}\right]$

$$= \lim_{n\to\infty} \frac{1}{(n^3+1)}\left[1+4+9+\ldots.+n^2\right]$$

$$= \lim_{n\to\infty} \frac{1}{(n^3+1)}\left[\frac{n\,(n+1)\,(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} \frac{(n^2+n)\,(2n+1)}{6\,(n^3+1)}$$

$$= \lim_{n\to\infty} \frac{(2n^3+n^2+2n^2+n)}{6\,(n^3+1)}$$

$$= \lim_{n\to\infty} \frac{(2n^3+3n^2+n)}{6\,(n^3+1)}$$

$$= \lim_{n\to\infty} \frac{n^3\left[2+\frac{3n^2}{n^3}+\frac{n}{n^3}\right]}{n^3\,6\left(1+\frac{1}{n^3}\right)}$$

$$= \frac{2}{6} = \frac{1}{3}$$

35. Consider system of linear equations $\begin{bmatrix} 4 & 1 & 3 \\ 2 & 3 & 5 \\ 8 & 5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 10 \end{bmatrix}$ it has

A. unique solution

B. Infinitely many solution

C. No solution

D. Exactly 3 solutions

Ans. A

Sol. Given system of equations

$$\begin{bmatrix} 4 & 1 & 3 \\ 2 & 3 & 5 \\ 8 & 5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 10 \end{bmatrix}$$

Augmented matrix is $\left[\begin{array}{ccc:c} 4 & 1 & 3 & 4 \\ 2 & 3 & 5 & -3 \\ 8 & 5 & 4 & 10 \end{array}\right]$

Since all minors of matrix A is non zero

rank of (A) = 3

so that rank of (A|B) : 3 : No. of variables

So, given set of equations has unique solution

36. The line integral $\oint$ [(y + sin x) dx + (x + z cosy) dy + siny dz] calculated over the boundary of the rectangle $0 \le x \le 5\pi$; $0 \le y \le 4$ and $z = 7$ is_______.

A. 1 B. 2

C. 3 D. 0

Ans. D

Sol. Given,

line integral = ∮ [(y + sin x) dx + (x + z cosy) dy + siny dz]

boundary of the rectangle 0 ≤ x ≤ 5π ;0 ≤ y ≤ 4 and z = 7,

Using the Stokes theorem:

$$\oint \bar{F}.dr = \iint_s (\nabla \times \bar{F}).\hat{\eta}ds$$

$$\nabla \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin x + y & z\cos y + x & \sin y \end{vmatrix}$$

$$= i(\cos y - \cos y) - j(0) + \hat{k}(1 - 1) = 0$$

37. Which one of the following is an eigen vector of the matrix:-

A = $\begin{bmatrix} -4 & 2 \\ 4 & 3 \end{bmatrix}$

A. $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ B. $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

C. $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ D. $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Ans. C

Sol.

Firstly we will find its eigen values by solving its characteristics equation

|A - λI| = 0

Where A is given matrix and I is identity matrix

$\begin{bmatrix} -4-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix} = 0$

$\lambda^2 + \lambda - 20 = 0$

λ = -5, 4

now we will find the eigen vector corresponding to λ = -5

$\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$x_1 + 2x_2 = 0$

Put $x_2 = k$

$x_1 = -2k$

$$\text{eigen vector} = \begin{bmatrix} -2k \\ k \end{bmatrix} = k\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\text{So eigen vector} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

38. The value of $\int_0^1 \int_x^1 x\sqrt{1+y^3}\,dydx$ is________.

A. $2\sqrt{2}$

B. $\frac{2\sqrt{2}-1}{2}$

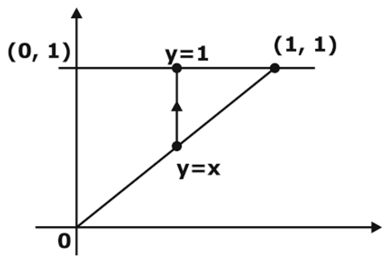
C. $\frac{2\sqrt{2}-1}{8}$

D. $\frac{2\sqrt{2}-1}{9}$

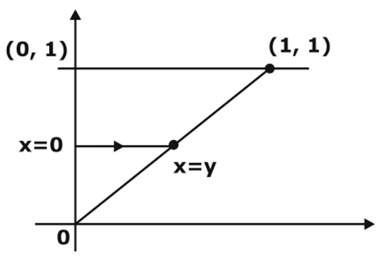
Ans. D

Sol. Given,

$$\int_0^1 \int_x^1 x\sqrt{1+y^3}\,dydx$$



By changing the order of integration,



$$= \int_0^1 \int_0^y x\sqrt{1+y^3}\,dxdy$$

$$= \int_0^1 \left(\frac{x^2}{2}\right)_0^y \sqrt{1+y^3}\ dy$$

$$= \int_0^1 \left(\frac{y^2}{2}\right) \sqrt{1+y^3}\ dy$$

let $1 + y^3 = t$

$3y^2 dy = dt$

$y^2 dy = \frac{dt}{3}$

at $y = 0$, $t = 1$

at $y = 1$, $t = 2$

$$\text{Integral} = \int_1^2 \sqrt{t}\,\frac{dt}{6}$$

$$= \left(\frac{1}{6} \times \frac{2t^{\frac{3}{2}}}{3}\right)_1^2$$

$$= \frac{\left(2\sqrt{2} - 1\right)}{9}$$

39. The value of integral $\oint_c \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz,$ where C is the circle $|z| = 1$, will be__________.

A. $2\pi i$

B. $-2\pi i$

C. $-\pi i$

D. πi

Ans. D

Sol. Given,

$z = \frac{\pi}{6} = 0.523$ is a pole of order 3 lies inside the circle $|z| = 1$

So, $$\oint_c \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz = \frac{2\pi i}{2!}\left(\frac{d^2}{dz^2}(\sin^2 z)\right)\Bigg|_{z=\frac{\pi}{6}}$$

$= \pi i(2\cos 2z)\big|_{z=\frac{\pi}{6}} = 2\pi i \cos(\pi/3) = \pi i$

40. The given eigen values of the following matrix are represented by λ_1 and λ_2

$$A = \begin{bmatrix} 1 & -1 \\ \frac{4}{9} & \frac{-1}{3} \end{bmatrix}$$

The value of $\lambda_1^{\,2} + \lambda_2^{\,2} + \lambda_1\lambda_2$ is ____.

A. 0.27

B. 0.30

C. 0.33

D. 0.37

Ans. C

Sol. Given,

$$\lambda_1^{\,2} + \lambda_2^{\,2} + \lambda_1\lambda_2 = \lambda_1^{\,2} + \lambda_2^{\,2} + 2\lambda_1\lambda_2 - \lambda_1\lambda_2$$

$$= \left(\lambda_1 + \lambda_2\right)^2 - \lambda_1\lambda_2$$

Sum of eigen values,

$= \lambda_1 + \lambda_2 =$ trace of matrix

= sum of diagonal elements

$= 1 - \frac{1}{3} = \frac{2}{3}$

Products of eigen values,

$\lambda_1\lambda_2 =$ determinant of matrix

$$= 1 \times \left(\frac{-1}{3}\right) - (-1)\left(\frac{4}{9}\right) = \frac{1}{9}$$

$$\left(\lambda_1 + \lambda_2\right)^2 - \lambda_1\lambda_2 = \left(\frac{2}{3}\right)^2 - \frac{1}{9} = 0.333$$

****