Chapter: Real Numbers

Euclid's division lemma: For any two given positive integers m and n, there exist unique whole numbers q and r such that,

$$m = nq + r, \text{ where } 0 \le r < n$$

Here, m is called as dividend, n as divisor, q as quotient and r as remainder.

Using Euclid's division lemma to find the properties of numbers

Euclid's division lemma/algorithm has several applications related to finding properties of numbers. We give some examples of these applications below:

Example: Show that every positive even integer is of the form 2q, and that every positive odd integer is of the form 2q + 1, where q is some integer.

Solution: Let m be any positive integer and n = 2.

Then, by Euclid's algorithm, $m = 2q + r$, where $0 \le r < 2$.

Therefore, the values of r can be 0 and 1.

When $r = 0 \Rightarrow m = 2q$

In this case m is a multiple of 2 and therefore is an even number.

When $r = 1 \Rightarrow m = 2q+1$

In this case, whenever we add 1 to any even number then we get an odd number.

Hence, every positive even integer is of the form 2q and every positive odd integer is of the form 2q+1 for some integer q.

Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes and this expression is unique, except from the order in which the prime factors occur.

Highest Common Factor (HCF) = The highest or biggest number which can divide the given numbers without leaving any remainder. Sometimes it is also termed as Greatest Common Factor(GCF) or Greatest Common Divisor(GCD).

Example: HCF(10,15) = 5

Lowest Common Multiple (LCM) = The lowest or the first common multiple of the given numbers.

Example: LCM(10,15) = 30

When we have to calculate either LCM or HCF or both for two or more numbers, then first we have to do prime factorisation of each number separately and then work according to the following points:

For HCF = Product of the smallest power of each common prime factor in the numbers.

For LCM = Product of the greatest power of each prime factor involved in the numbers.

Note: Product of two given numbers = Product of their HCF and LCM

m × n = HCF (m,n) × LCM (m,n)

Decimal representation of Rational Numbers

Rational numbers: The numbers which can be written or expressed in the form of $\frac{p}{q}$, where p and q are integers, and q≠0, are called rational numbers.

Rational numbers in decimal form: Every rational number when expressed in decimal form, then its decimal expansion is either terminating or non-terminating repeating/recurring.

To check whether a given rational number is terminating or non-terminating repeating:

Let x is a rational number with simplest form $\frac{p}{q}$, where p and q are integers and q ≠0, then:

(i) If the prime factorisation of q is of the form ($2^m \times 5^n$), for some non-negative integers m and n, then the decimal expansion of x is terminating.

If the prime factorisation of q is **not** of the form ($2^m \times 5^n$), for some non-negative integers m and n, then the decimal expansion of x is non-terminating repeating.

Irrational numbers

Irrational numbers: The numbers whose decimal expansion is non-terminating and non-repeating are termed as irrational numbers.

Types of irrational number

Type 1: 0.3030030003…… or 1.323323332….. etc are all non-terminating as well as non-repeating decimal numbers.

Type 2: If *a* is a positive integer and is not a perfect square, then $\sqrt{a}$ is an irrational number. Example: $\sqrt{3}$, $\sqrt{5}$, $\sqrt{11}$ etc.

Type 3: If *a* is a positive integer and is not a perfect cube, then $\sqrt[3]{a}$ is an irrational number. Example: $\sqrt[3]{15}$, $\sqrt[3]{43}$, $\sqrt[3]{133}$ etc.

Important Questions with Solutions

1. Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Solution: To Prove: Any Positive odd integer is of the form 6q + 1, 6q + 3, 6q + 5

Proof: To prove the statement by Euclid's lemma we have to consider divisor as 6 and then find out the possible remainders when divided by 6

By taking,' a' as any positive integer and b = 6.

Applying Euclid's algorithm

$a = 6q + r$
As divisor is 6 the remainder can take only 6 values from 0 to 5

Here, r = remainder = 0, 1, 2, 3, 4, 5 and $q \geq 0$

So, total possible forms are $6q + 0$, $6q + 1$, $6q + 2$, $6q + 3$, $6q + 4$ and $6q + 5$

$6q + 0$, (6 is divisible by 2, its an even number)

$6q + 1$, (6 is divisible by 2 but 1 is not divisible by 2, its an odd number)

$6q + 2$, (6 and 2 both are divisible by 2, its an even number)

$6q + 3$, (6 is divisible by 2 but 3 is not divisible by 2, its an odd number)

$6q + 4$, (6 and 4 both are divisible by 2, its an even number)

$6q + 5$, (6 is divisible by 2 but 5 is not divisible by 2, its an odd number)

Therefore, odd numbers will be in the form $6q + 1$, or $6q + 3$, or $6q+5$
Hence, Proved.

2. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution: Suppose, both groups are arranged in 'n' columns, for completely filling each column,
The maximum no of columns in which they can march is the highest common factor of their number of members.
i.e. n = HCF(616, 32)

By using, Euclid's division algorithm

$616 = 32\times19+8$

Remainder $\neq 0$

So, again Applying Euclid's division algorithm

$32 = 8\times4+0$

HCF of (616, 32) is 8.

So,

They can march in 8 columns each.

3. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.

Solution:

To Prove: Square of any number is of the form 3 m or 3 m +1
Proof: to prove this statement from Euclid's division lemma, take any number as a divisor, in question we have 3m and 3m + 1 as the form
So,
By taking, ' a' as any positive integer and b = 3.

Applying Euclid's algorithm a = bq + r.

$a = 3q + r$

Here, r = remainder = 0, 1, 2 and $q \geq 0$ as the divisor is 3 there can be only 3 remainders, 0, 1 and 2.

So, putting all the possible values of the remainder in, $a = 3q + r$
$a = 3q$ or $3q+1$ or $3q+2$

And now squaring all the values,
When $a= 3q$
Squaring both sides we get,

$a^2 = (3q)^2$
$a^2 = 9q^2$
$a^2 =3 (3q^2)$
$a^2 = 3 k_1$
Where $k_1 = 3q^2$
When $a=3q+1$
Squaring both sides we get,
$a^2 = (3q + 1)^2$
$a^2 = 9q^2 + 6q + 1$
$a^2 =3(3q^2 + 2q)+ 1$
$a^2 = 3k_2 + 1$
Where $k_2 = 3q^2 + 2q$
When $a = 3q+2$
Squaring both sides we get,
$a^2 = (3q + 2)^2$
$a^2 = 9q^2 + 12q + 4$

$a^2 = 9q^2 + 12q + 3+1$
$a^2 = 3(3q^2 + 4q + 1) +1$

$a^2 = 3k_3 + 1$

Where $k_3 = 3q^2 + 4q + 1$

Where k_1, k_2 and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form 3m or 3m+1.

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Solution:

Given: HCF of (306, 657) = 9

We know that,

LCM × HCF = product of two numbers

LCM×HCF = 306×657

LCM = $\frac{306\times657}{HCF} = \frac{306\times657}{9}$

LCM = 22338

5. Check whether 6^n can end with the digit 0 for any natural number *n*.

Solution: We need to find can 6^n end with zero
If any number has last digit 0,

Then, it should be divisible by 10
Factors of 10 = 2×5

So,

Value 6^n should be divisible by 2 and 5

Prime factorisation of $6^n = (2\times3)^n$

Hence,

6^n is divisible by 2 but not by 5.

It cannot end with 0.

6. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Solution: Composite numbers are those numbers, which can be written in the form of the product of two or more integers, and at least one of them should not be 1

(i)
7 × 11 × 13 + 13

= (7 × 11 × 13) + (13 × 1)

Taking 13 as common, we get,

= 13 × (7 × 11 + 1)
= 13 × (77 + 1)
= 13 × 78
= 13 × 13 × 6

As the given no is a multiple of two or more integers, one of them being other than 1.
Hence, it is a composite number.
Therefore, it is a composite number.

(ii)

7 × 6 × 5 × 4 × 3 × 2 × 1 + 5

= (7 × 6 × 5 × 4 × 3 × 2 × 1) + (5 × 1)

Taking 5 as common , we get,

= 5×(7×6×4×3×2×1+1)

= 5× (1008+1)

= 5×1009

As the given no is a multiple of two integers, one of them being other than 1.

Hence, it is a composite number.

7. Prove that $\sqrt{5}$ is irrational.

Solution: Let's assume that $\sqrt{5}$ is a rational number.

Hence, $\sqrt{5}$ can be written in the form a/b [where a and b ($b \neq 0$) are co-prime (i.e. no common factor other than 1)]

$\therefore \sqrt{5} = a/b$

$\Rightarrow \sqrt{5}\, b = a$

Squaring both sides,

$\Rightarrow (\sqrt{5}\, b)^2 = a^2$

⇒ $5b^2 = a^2$

⇒ $a^2/5 = b^2$

Hence, 5 divides a^2

By theorem, if p is a prime number and p divides a^2, then p divides a, where a is a positive number.

So, 5 divides a too

Hence, we can say a/5 = c where, c is some integer

So, a = 5c

Now we know that,

$5b^2 = a^2$

Putting a = 5c,

⇒ $5b^2 = (5c)^2$

⇒ $5b^2 = 25c^2$

⇒ $b^2 = 5c^2$

∴ $b^2/5 = c^2$

Hence, 5 divides b^2

By theorem, if p is a prime number and p divides a^2, then p divides a, where a is a positive number

So, 5 divides b too

By earlier deductions, 5 divides both a and b

Hence, 5 is a factor of a and b

∴ a and b are not co-prime.

Hence, the assumption is wrong.

∴ By contradiction,

∴ √5 is irrational

8. Prove that $3 + 2\sqrt{5}$ is irrational.

Solution: To Prove: $3 + 2\sqrt{5}$ is irrational
Proof:

Let $3 + 2\sqrt{5}$ is rational.

A number is said to be rational if it can be expressed in the form p/q where q ≠ 0

Therefore,

We can find two integers p & q where, (q ≠ 0) such that

$$3+2\sqrt{5}=\frac{p}{q}$$

$$2\sqrt{5}=\frac{p}{q}-3$$

Since p and q are integers, $\frac{1}{2}\left(\frac{p}{q}-3\right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

We know that √5 is irrational but according to above statement it has to be rational

So, both the comments are contradictory,

Hence, the number should have been irrational to make the statement correct.

Therefore, $3+2\sqrt{5}$ is irrational

9. Without actually performing the long division, state whether $\frac{35}{50}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

Solution:

$$\frac{35}{50}=\frac{7\times5}{10\times5}=\frac{7}{10}$$

Factorize the denominator we get,

$10 = 2\times5$

The denominator is in the form $2^m\times5^n$

Hence, the decimal expansion of $\frac{35}{50}$ is terminating.

10. Without actually performing the long division, state whether $\frac{64}{455}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

Solution:

We have, $\frac{64}{455}$

Factorize the denominator we get,

455 = 5×7×13

Since, the denominator is not in the form of $2^m \times 5^n$, and it also contains 7 and 13 as its factors,

Its decimal expansion will be non-terminating repeating.

Chapter: Polynomials

A **polynomial** is an algebraic expression in which the exponent on any variable is a **whole number.**

Zero of a Polynomial

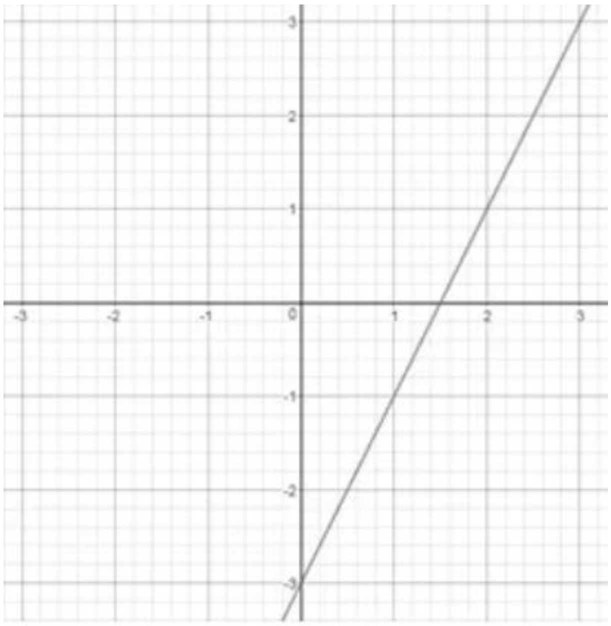
A real number k is a zero of a polynomial f(x), if f(k) = 0. Which means that on substituting the given real number in a polynomial in place of. Which means that if on substituting the given real number in place of the variable in a polynomial gives zero, then the given real number is termed as the zero of the given polynomial.

Geometrical meaning of the zeroes of a polynomial

Geometrically, zeroes of a polynomial are the x-coordinates of the points where its graph crosses or touches x-axis.

Let us see some graphs of different polynomials.

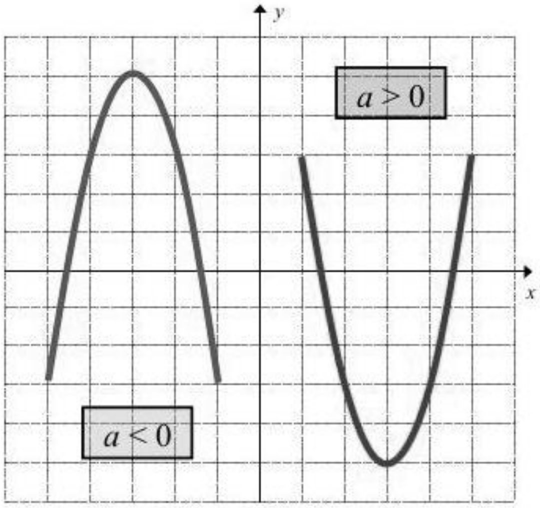
Graph of linear Polynomial:



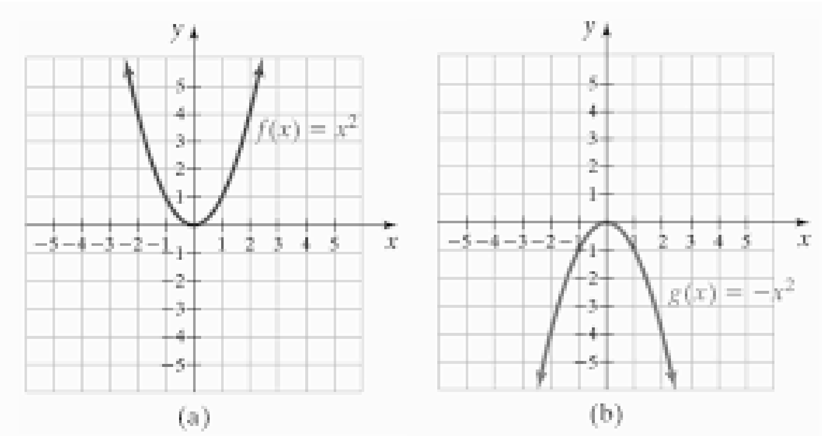
Graph of a linear polynomial is a straight line which intersects the x-axis at one point only, so a linear polynomial has 1 degree.

Graph of Quadratic Polynomial

Case 1: When the graph cuts the x-axis at the two points than these two points are the two zeroes of that quadratic polynomial.

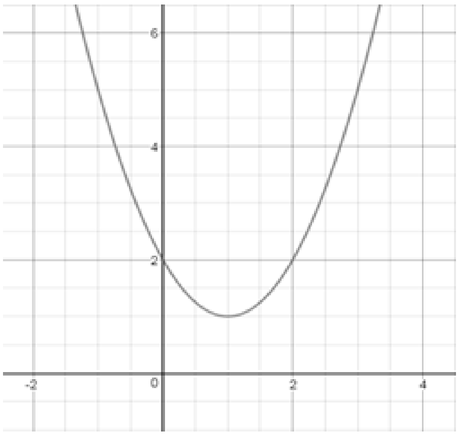


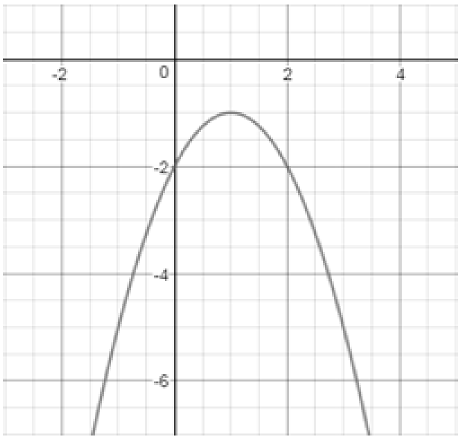
Case 2: When the graph cuts the x-axis at only one point then that particular point is the zero of that quadratic polynomial and the equation is in the form of a perfect square



(a) (b)

Case 3: When the graph does not intersect the x-axis at any point i.e. the graph is either completely above the x-axis or below the x-axis then that quadratic polynomial has no zero as it is not intersecting the x-axis at any point.





Hence the quadratic polynomial can have either two zeroes, one zero or no zero. Or you can say that it can have maximum two zero only.

Relationship between zeroes and coefficient of a quadratic polynomial

Let α and β be the two zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$, where $a \neq 0$.

Then, sum of zeros = $\alpha + \beta = -\frac{\text{Coefficient of x}}{\text{Coefficient of } x^2} = -\frac{b}{a}$

Product of zeros = $\alpha . \beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$

Remember:

If α and β are the zeros of a quadratic polynomial f(x). Then, polynomial f(x) is given by:

$f(x) = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

$f(x) = k\{x^2 - (\text{Sum of zeros})x + \text{Product of zeros}\}$

Division algorithm for polynomials

If f(x) and g(x) are any two polynomials with g(x) ≠ 0, and degree of g(x) is less than or equal to f(x), then we can always find polynomials q(x) and r(x) such that,

f(x) = q(x)g(x) + r(x), where r(x) = 0 or degree of r(x) < degree of g(x).

Important Questions with Solutions

1. Find a quadratic polynomial whose sum of the zeros is √2 and product of the zeros is $\frac{1}{3}$.

Solution: we know that for a quadratic equation in the form $ax^2 + bx + c = 0$, and its zeroes are α and β, then
sum of zeroes is

$$\alpha + \beta = \frac{-b}{a}$$

and product of zeroes is

$$\alpha\beta = \frac{c}{a}$$

Let the polynomial be $ax^2 + bx + c$, then

$$\alpha + \beta = \sqrt{2} = \frac{-b}{a}$$

and

$$\alpha\beta = -1 = \frac{1}{3} = \frac{c}{a}$$

If a = 3, then b = $-3\sqrt{2}$, and c = 1

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}\,x + 1$

2. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder where $p(x) = x^3 - 3x^2 + 5x - 3,\ g(x) = x^2 - 2$

Solution: By long division method we have,

$$\begin{array}{r|l} & x - 3 \\ \hline x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\ & x^3 \qquad\quad - 2x \\ & - \qquad\quad + \\ \hline & -3x^2 + 7x - 3 \\ & -3x^2 \qquad + 6 \\ & + \qquad\quad - \\ \hline & 7x - 9 \end{array}$$

Quotient = $x - 3$

Remainder = $7x - 9$

3. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:
$t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

Solution: We have, $t^2-3 = t^2+0t-3$

$$\begin{array}{r|l} & 2t^2 + 3t + 4 \\ \hline t^2 + 0t - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\ & 2t^4 + 0t^3 - 6t^2 \\ & - \quad - \qquad + \\ \hline & 3t^3 + 4t^2 - 9t - 12 \\ & 3t^3 + 0t^2 - 9t \\ & - \quad - \qquad + \\ \hline & 4t^2 + 0t - 12 \\ & 4t^2 + 0t - 12 \\ & - \quad - \qquad + \\ \hline & 0 \end{array}$$

Since the remainder is 0,

Hence, $t^2 - 3$ is a factor of $2t^4+3t^3-2t^2-9t-12$.

Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

$p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ *and* $-\sqrt{\frac{5}{3}}$.

$\therefore \left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$.

Therefore, we divide the given polynomial by $x^2 - \frac{5}{3}$.

$$\begin{array}{r|l}
 & \quad 3x^2+6x+3 \\
\hline
x^2+0x-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4+0x^3-5x^2 \\
 & -\quad\ -\quad\ + \\
\cline{2-2}
 & \qquad 6x^3+3x^2-10x-5 \\
 & \qquad 6x^3+0x^2-10x \\
 & \qquad -\quad\ -\quad\ + \\
\cline{2-2}
 & \qquad\qquad 3x^2+0x-5 \\
 & \qquad\qquad 3x^2+0x-5 \\
 & \qquad\qquad -\quad\ -\quad\ + \\
\cline{2-2}
 & \qquad\qquad\qquad 0 \\
\cline{2-2}
\end{array}$$

We know,
Dividend = (Divisor × quotient) + remainder

$3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2-\frac{5}{3}\right)(3x^2+6x+3)$

$3x^4 + 6x^3 - 2x^2 - 10x - 5 = 3\left(x^2-\frac{5}{3}\right)(x^2+2x+1)$

As, $(a+b)^2 = a^2 + b^2 + 2ab$

So, $x^2 + 2x + 1 = (x+1)^2$

$3x^4 + 6x^3 - 2x^2 - 10x - 5 = 3\left(x^2-\frac{5}{3}\right)(x+1)^2$

Therefore, its zero is given by $x + 1 = 0$.
$\Rightarrow x = -1,-1$

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$ and - 1 , -1.

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x) the quotient and remainder were (x - 2) and (-2x + 4), respectively. Find g(x).

Solution: Given,
Polynomial, $p(x) = x^3 - 3x^2 + x + 2$ (dividend)

Quotient = $(x - 2)$

Remainder = $(-2x + 4)$

To find : divisor = g(x)
we know,
Dividend = Divisor × Quotient + Remainder
$\Rightarrow x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

$\Rightarrow x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x - 2)$

$\Rightarrow x^3 - 3x^2 + 3x - 2 = g(x)(x - 2)$

$g(x)$ is the quotient when we divide $(x^3 - 3x^2 + 3x - 2)$ by $(x - 2)$

$$\begin{array}{r|l}
 & x^2-x+1 \\ \hline
x-2 & x^3-3x^2+3x-2 \\
 & x^3-2x^2 \\
 & - \quad + \\ \hline
 & \quad -x^2+3x-2 \\
 & \quad -x^2+2x \\
 & \quad + \quad - \\ \hline
 & \qquad\qquad x-2 \\
 & \qquad\qquad x-2 \\
 & \qquad\qquad - \quad + \\ \hline
 & \qquad\qquad 0 \\ \hline
\end{array}$$

$\therefore g(x) = (x^2 - x + 1)$

5. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are (a - b), a and (a + b). Find a and b.

Solution: Given,

p(x) = $x^3 - 3x^2 + x + 1$

zeroes are = a – b , a + b , a

Comparing the given polynomial with $mx^3 + nx^2 + px + q$, we get

= $m = 1, n = -3\,, p = 1\,, q = 1$

Sum of zeroes = $a - b + a + a + b = -\frac{n}{m}$

$3a = \ -\frac{-3}{1} = 3$

a = $\frac{3}{3} = 1$

The zeroes are = (1 - b), 1 and (1 + b)

Product of zeroes = (1 - b)(1 + b)

(1 - b)(1 + b) = -q/m

$1 - b^2 \ = -\frac{1}{1} = -1$

$b^2 = 2$

$b = \ \pm\sqrt{2}$

So,

We get, $a = 1 \ and \ b = \ \pm\sqrt{2}$

6. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Solution: 2+√3 and 2-√3 are zeroes of given equation,

Therefore,

(x - 2 + √3)(x - 2 - √3) should be a factor of given equation.

Also, (x - 2 + √3)(x - 2 - √3) = $x^2 - 2x - \sqrt{3}x - 2x + 4 + 2\sqrt{3} + \sqrt{3}x - 2\sqrt{3} - 3$

$= x^2 - 4x + 1$

To find other zeroes, we divide given equation by $x^2 - 4x + 1$

$$
\begin{array}{r l}
 & x^2 - 2x - 35 \\
x^2 - 4x + 1 & \overline{)\,x^4 - 6x^3 - 26x^2 + 138x - 35} \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

We get ,
$x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

Now factorizing $x^2 - 2x - 35$ we get,

$x^2 - 2x - 35$ *is also a factor of given polynomial and* $x^2 - 2x - 35 = (x - 7)(x + 5)$

The value of polynomial is also zero when ,

x - 7 = 0

or

x = 7

And, x + 5 = 0

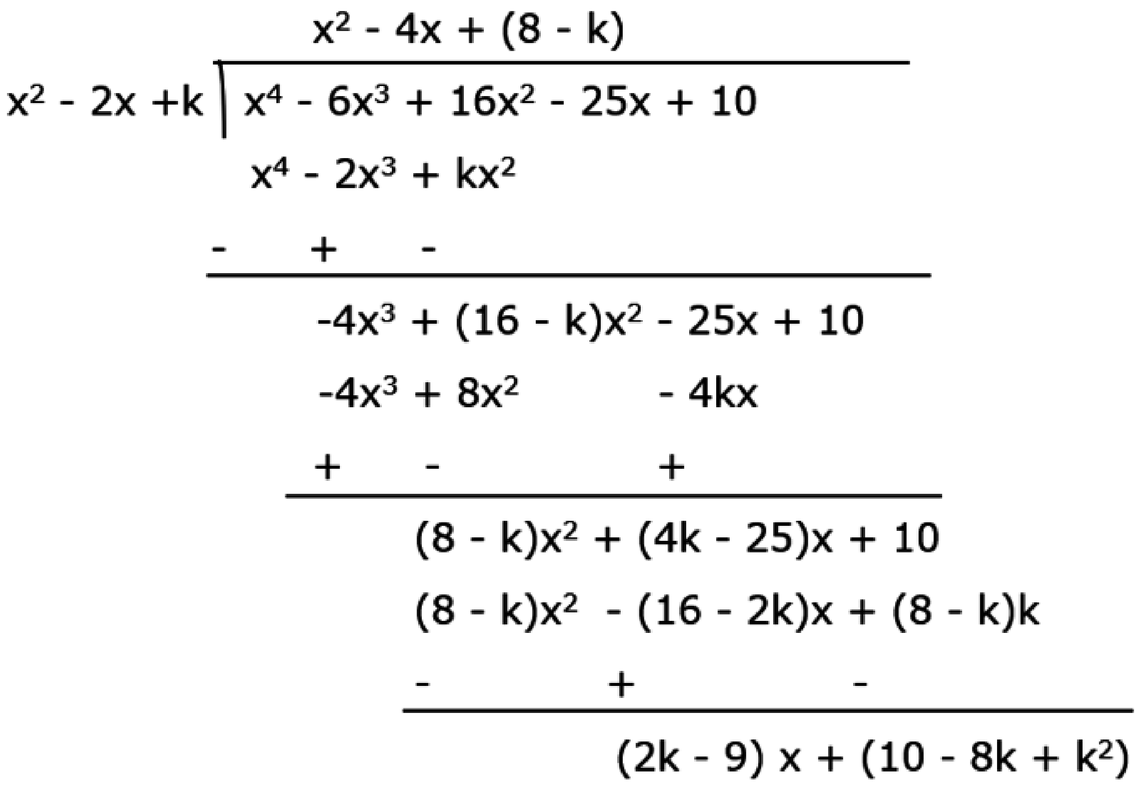
or

$x = -5$

Hence, 7 and -5 are also zeroes of this polynomial.

7. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2k + k$ the remainder comes out to be $x + a$, find k and a.

Solution: To solve this question divide $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$ by long division method
Let us divide, $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k,$

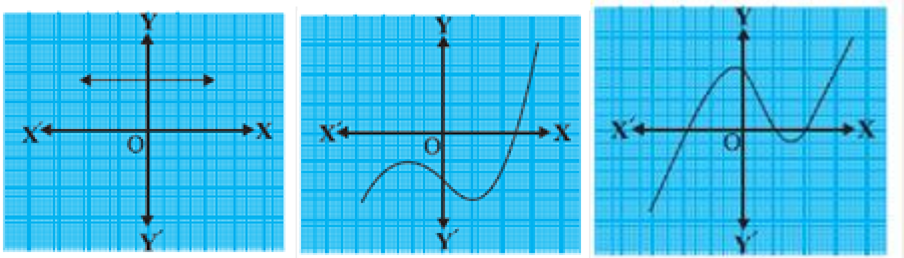


$$\begin{array}{l}
x^2 - 4x + (8 - k) \\
x^2 - 2x + k \,\big)\, x^4 - 6x^3 + 16x^2 - 25x + 10 \\
x^4 - 2x^3 + kx^2 \\
- \quad + \quad - \\
\hline
-4x^3 + (16 - k)x^2 - 25x + 10 \\
-4x^3 + 8x^2 \qquad - 4kx \\
+ \quad - \qquad + \\
\hline
(8 - k)x^2 + (4k - 25)x + 10 \\
(8 - k)x^2 - (16 - 2k)x + (8 - k)k \\
- \qquad + \qquad - \\
\hline
(2k - 9)x + (10 - 8k + k^2)
\end{array}$$

So, remainder $= (2k - 9)x + (10 - 8k + k^2)$
But given remainder $= x + a$
$\Rightarrow (2k - 9)x + (10 - 8k + k^2) = x + a$
Comparing coefficient of x, we have
$2k - 9 = 1$
$\Rightarrow 2k = 10$
$\Rightarrow k = 5$
and
Comparing constant term,
$10 - 8k + k^2 = a$
$\Rightarrow a = 10 - 8(5) + 5^2$

⇒ a = 10 - 40 + 25
⇒ a = -5
So, the value of k is 5 and a is -5.

8. The graphs of y = p(x) are given in Fig. 2.10 below, for some polynomials, p(x). Find the number of zeroes of p(x) in each case.



(i) (ii) (iii)

Solution:

The number of zeroes for any graph is the number of values of x for which y is equal to zero. And y is equal to zero at the point where a graph cuts x axis.
So, to find the number of zeroes of a polynomial, watch the number of times it cuts the x axis.
(i) The number of zeroes is 0 as the graph does not cut the x-axis at any point.

(ii) The number of zeroes is 1 as the graph intersects the x-axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

9. Find the zeroes of $p(x) = x^2 - 2x - 8$

Solution: We have, $p(x) = x^2 - 2x - 8$
So, the zeroes will be the values of x at which p(x) = 0
Therefore,

⇒ $x^2 - 4x + 2x - 8 = 0$

(We will factorize 2 such that the product of the factors is equal to 8 and difference is equal to 2)
⇒ x(x - 4) + 2(x - 4) = 0
= (x - 4)(x + 2)

The value of $x^2 - 2x - 8$ is zero when $x - 4 = 0$ or $x + 2 = 0$,

i.e, $x = 4$ or $x = -2$

Therefore, the zeroes of $x^2 - 2x - 8$ are 4 and −2.

10. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -14 respectively.

Solution: For a cubic polynomial equation, $ax^3 + bx^2 + cx + d$, and zeroes α, β and γ

we know that

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

Let the polynomial be $ax^3 + bx^2 + cx + d$, and zeroes α, β and γ.
A cubic polynomial with respect to its zeroes is given by,
x^3 - (sum of zeroes) x^2 + (Sum of the product of roots taken two at a time) x - Product of Roots = 0
$x^3 - (2)\ x^2 + (-7)\ x - (-14) = 0$
$x^3 - (2)\ x^2 + (-7)\ x + 14 = 0$
Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Chapter: Linear Equation in one Variable

Linear Equation

Equations in which the power of all the variables involved is one are called linear equations. The degree of a linear equation is always one.

General form of a Linear Equation in Two Variables

The general form of a linear equation in two variables is $ax + by + c = 0$,

where a, b and c are the real numbers ($a \neq 0$ and $b \neq 0$) and x and y are the two variables,

Here a and b are the coefficients and c is the constant of the equation.

Pair of Linear Equations

Two Linear Equations having two same variables are known as the pair of Linear Equations in two variables.

$$\mathbf{a_1x + b_1y + c_1 = 0}$$
$$\mathbf{a_2x + b_2y + c_2 = 0}$$

Nature of 2 straight lines in a plane

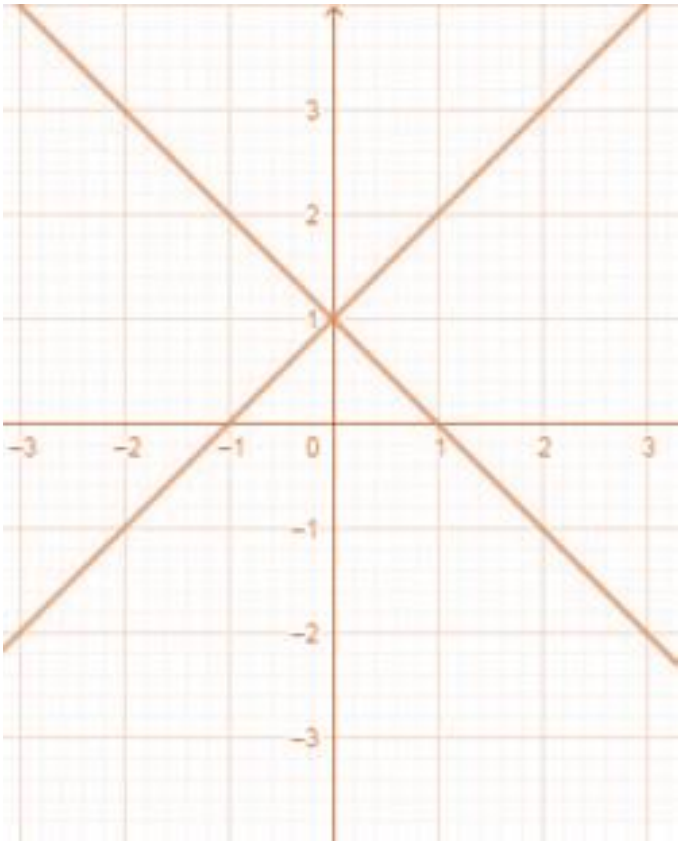
Consider a pair of linear equation in one variable as follows:

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

For a pair of straight lines on a plane, there are three possibilities

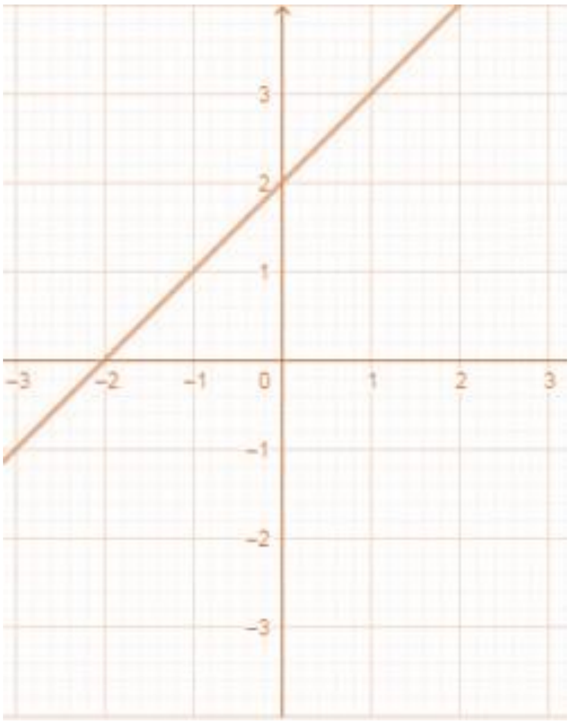
i) They intersect at exactly one point

When $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of equation is called consistent. The lines will intersect at a single point in the graph (as shown in graph below) and the pair of equation will have a unique solution.



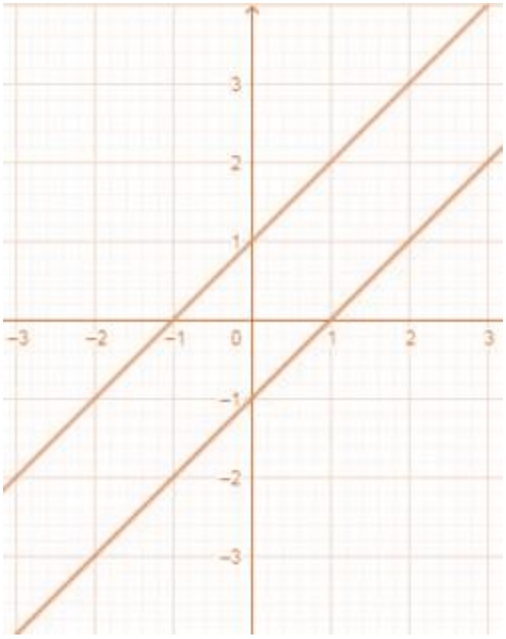
ii) They are coincident

When $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the pair of equation is again called consistent. The lines will coincide or overlap(as shown in graph below) and the pair of equation will have infinitely many solutions.



iii) They are parallel

When $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the pair of equation is called inconsistent. The lines will never intersect each other and will run parallel (as shown in graph below) and the pair of equation will have no solution.



Graphical Method:

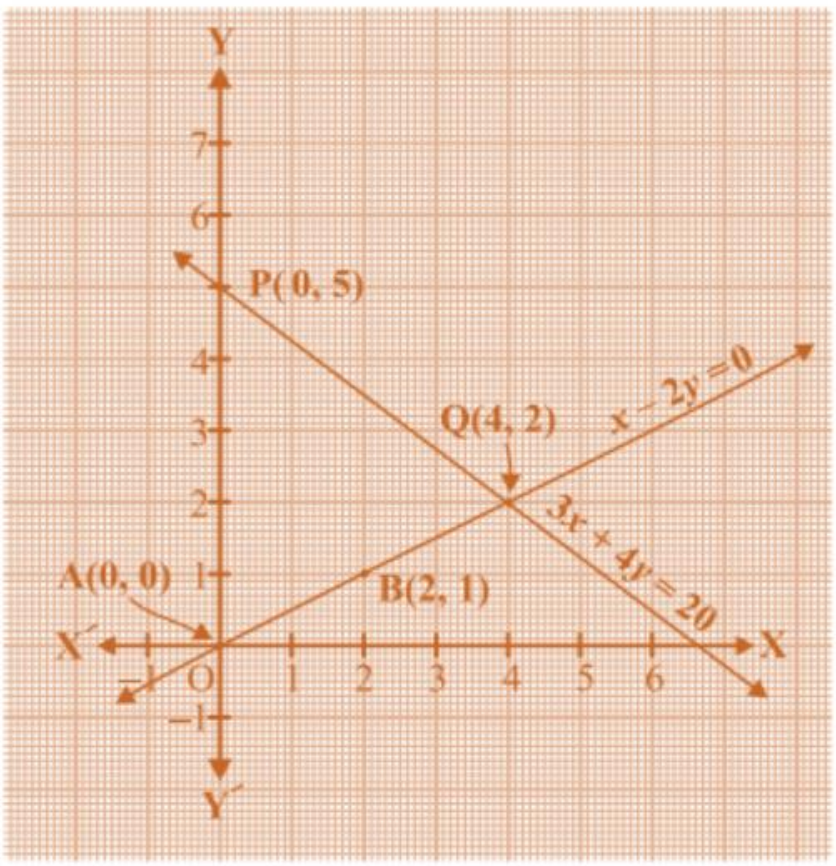
Graphical Method for finding the solution to a pair of linear equations is as follows:

- Plot both the equations (two straight lines)
- Find the point of intersection of the lines.

The point of intersection is the solution.

Let say we have to solve x – 2y = 0 and 3x + 4y = 20 graphically. Then we have to follow the given steps:

- Plot the first equation, i.e. x – 2y = 0 on the graph.
- Now plot the second equation, i.e. 3x + 4y = 20 on the same graph.
- The point where these two straight lines will intersect will be the solution of the given pair of linear equations.



Here, (4,2) is the solution of the given pair of linear equations.

Algebraic Methods of Solution for Linear Equations in Two Variables

The solution of a pair of linear equations is of the form (x,y) which satisfies both the equations simultaneously. Solution for a consistent pair of linear equations can be found out using

i) Substitution Method

ii) Elimination method

Representing Linear Equation for word problems

To represent a word problem as a linear equation

- Identify unknown quantities and denote them by variables.
- Represent the relationships between quantities in a mathematical form with the help of variables and obtain different equations depending on the number of variables. That means if there are two variables than we must have at-least two equations to find their values.
- Use any of the methods discussed previously to solve the linear equations and obtain the value of the unknowns.

The following keywords will help you in developing the linear equations:

Twice/Thrice = ×2/×3

Product/Times = Multiply

Later/Together/Hence/After/Increased/Faster/Sum = Addition

Ago/Difference/Before/Slower = Subtraction

Distribute = Division

Linear equations can be used in solving a wide range of word problems which includes geometrical problems, problems on ages, problems related to numbers etc.

Important Questions with Solutions

1. 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz. Find the solution graphically.

Solution: For representing the situation graphically and algebraically, we need to form linear equations
Let number of girls = x

Let number of boys = y

According to the question, Total no of students is equal to 10,

$$x + y = 10$$

$\Rightarrow x = 10 - y$ ....eq(i)

Now we will find different points to plot the equation. We can take any value of y and put in eq (i) to obtain the value of x at that point

Putting y = 4, 5 and 6. we get,

at x = 4

X = 10 – 4 = 6

at x = 5

X = 10 – 5 = 5

at x = 6

X = 10 – 6 = 4

x	4	5	6
y	6	5	4

Number of girls is 4 more than number of boys …………Given

So,

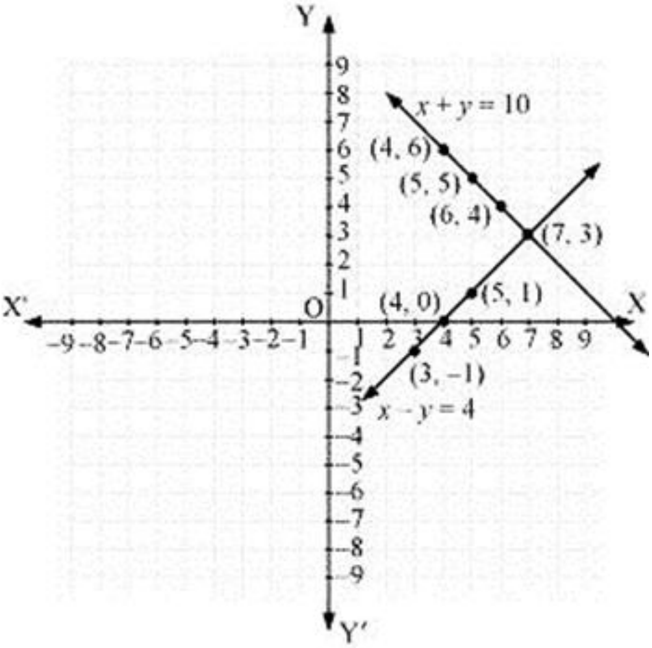
x = y + 4
⇒ y = x - 4 ....(ii)

Now for plotting the points on graph, take any values of x and put them in eq (ii) to obtain values of y

Putting x = 3 ,5 and 7 we get
at x = 3
y = 3 - 4 = -1
at x = 4
y = 5 - 4 = 1
at x = 7
y = 7 - 4 = 3

x	3	5	7
Y	-1	1	3

Graphical representation:
Plotting the points obtain on graph we get,



As, both lines intersect each other at (7, 3)
Solution of this pair of equation is (7, 3)
i.e.
No of girls, x = 7
No of boys, y = 3

2. Find out whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident:

$5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Solution: Comparing these equation with

$= a_1x + b_1y + c_1 = 0$

$= a_2x + b_2y + c_2 = 0$

We get,

$= a_1 = 5, b_1 = -4, c_1 = 8$

$= a_2 = 7, b_2 = 6, c_2 = -9$

Hence,

$= \frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = -\frac{4}{6} \text{ and } \frac{c_1}{c_2} = \frac{8}{-9}$

We find that, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, both lines intersect at one point.

3. Find out whether the following pair of linear equations are consistent, or inconsistent.

$3x + 2y = 5; \quad 2x - 3y = 7$

Solution: We get,

$= \frac{a_1}{a_2} = \frac{3}{2}$

$= \frac{b_1}{b_2} = -\frac{2}{3}$

Hence,

$= \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, these linear equations will intersect at one point only and have only one possible solution and pair of linear equations is inconsistent.

4. For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$2x + 3y = 7$

$(a - b)x + (a + b)y = 3a + b - 2$

Solution: We have,

2x + 3y - 7 =0

(a - b)x + (a + b)y - (3a + b - 2) = 0

we know, a pair of linear equations (say $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$) have infinite solution,
if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

therefore from the given equations,

$$\frac{a_1}{a_2} = \frac{2}{a-b}$$

$$\frac{b_1}{b_2} = \frac{3}{a+b}$$

$$\frac{c_1}{c_2} = \frac{-7}{-(3a+b-2)} = \frac{7}{(3a+b-2)}$$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a-b} = \frac{7}{3a+b-2}$$

6a + 2b - 4 = 7a – 7b

a – 9b = -4.............. (i)

$$\frac{2}{a-b} = \frac{3}{a+b}$$

2a + 2b = 3a – 3b

a – 5b = 0...............(ii)

Subtracting (i) from (ii), we obtain

4b = 4

b = 1

Substituting this in equation (ii), we obtain

a - 5 × 1 = 0

a = 5

Hence, a = 5 and b = 1 are the values for which the given equations give infinitely many solutions.

5. A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

Solution: Let x be the fixed charge of the food and y be the charge for food per day. According to the given information,

x + 20y = 1000 .... (1)

x + 26y = 1180 .... (2)

Subtracting equation (1) from equation (2), we obtain

6y = 180

y = 30

Substituting this value in equation (1), we obtain

x + 20 × 30 = 1000

x = 1000 - 600 = 400

x = 400

Hence, fixed charge = Rs 400 And charge per day = Rs 30

6. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Solution: Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x-1}{y}=\frac{1}{3}\rightarrow 3x-y=3 \quad ....(i)$$

$$\frac{x}{y+8}=\frac{1}{4}\rightarrow 4x-y=8 \quad ....(ii)$$

Subtracting equation (i) from equation (ii), we obtain

x = 5 .... (iii)

Putting this value in equation (i), we obtain

15 – y = 3

y = 12

Hence, the fraction is $\frac{5}{12}$.

7. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Solution: Let the speed of the train be *x* km/h and the time taken by train to travel the given distance be *t* hours and the distance to travel was *d* km.

We know that,

$$speed=\frac{distance\ travelled}{time\ taken\ to\ travel\ that\ distance}$$

$x = \frac{d}{t}$

Or, $d = xt$ (*i*)

If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time

⇒ (x + 10) = $\frac{d}{t-2}$

⇒ (x + 10)(t - 2) = d

⇒ xt + 10t - 2x - 20 = d

From (i), we have
⇒ d + 10t - 2x - 20 = d

⇒ - 2*x* + 10*t* = 20
⇒ x - 5t = -10
⇒ x = 5t - 10 (*ii*)

Also,
if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time
⇒ (x - 10) = $\frac{d}{t+3}$
⇒ (x - 10)(t + 3) = d

⇒ xt - 10t + 3x - 30 = d

By using equation (*i*),
⇒ d - 10t + 3x - 30 = d
⇒ 3*x* - 10*t* = 30 (*iii*)

Substituting the value of x from eq (ii) into eq (iii), we get
⇒ 3(5t - 10) - 10t = 30

⇒ 15t - 30 - 10t = 30
⇒ 5t = 60
⇒ *t* = 12 hours

Putting this in eq(ii)
⇒ x = 5t - 10
= 5(12) - 10
= 50 km/h

From equation (*i*), we obtain

Distance to travel = *d* = *xt*

= 50 × 12

= 600 km

Hence, the distance covered by the train is 600 km.

8. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Solution: Let the number of rows be x and number of students in a row be y.

Total students of the class = Number of rows x Number of students in a row = xy

Using the information given in the question,

Condition 1

Total number of students = $(x - 1)(y + 3)$

$= (x - 1)(y + 3)$

$= xy - y + 3x - 3$

$3x - y - 3 = 0$

$3x - y = 3$ ....(*i*)

Condition 2

Total number of students = $(x + 2)(y - 3)$

$= xy + 2y - 3x - 6$

$\Rightarrow 3x - 2y = -6$ ... (*ii*)

Subtracting equation (*ii*) from (*i*),

$(3x - y) - (3x - 2y) = 3 - (-6)$

$\Rightarrow - y + 2y = 9$

$\Rightarrow 3 + 6\ y = 9$

By using equation (*i*), we obtain $3x - 9 = 3$,

$\Rightarrow 3x = 9 + 3$

$\Rightarrow 3x = 12$

$\Rightarrow x = 4$

From (i),

⇒ $3(4) - y = 3$

⇒ $12 - y = 3$

⇒ $9 = y$

Number of rows = $x = 4$

Number of students in a row = $y = 9$

Number of total students in a class = Number of students in 1 row × Number of rows

$= xy$

$= 4 \times 9$

$= 36$

9. Solve $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$ and $\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$

Solution: Putting $\frac{1}{3x+y} = p\ and\ \frac{1}{3x-y} = q\ we\ get,$

$p + q = \frac{3}{4} \ldots\ldots\ldots..(i)$

$\frac{p}{2}-\frac{q}{2}=-\frac{1}{8}$

$p - q = -\frac{1}{4} \ldots\ldots\ldots.(ii)$

Adding (i) and (ii) we get,

$2p = \frac{3}{4}-\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$

$p = \frac{1}{4}$

Putting value of p in (ii) we get,

$\frac{1}{4}-q=-\frac{1}{4}$

$q = \frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

Now,

p = $\frac{1}{3x+y}$ $so\,, 3x + y = 4 \ldots\ldots..(iii)$

q = $\frac{1}{3x-y}$ $= \; 3x - y = 2 \ldots\ldots\ldots.(iv)$

Adding equations (iii) and (iv) we get,

6x = 6

x = 1

Putting value of x in equation (iii) we get,

3(1) + y = 4

y = 1

Hence, x = 1 and y = 1

10. Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

Solution: Let the speed of Ritu in still water and the speed of stream be *x* km/h and *y* km/h respectively.
While rowing upstream, Ritu's speed slows down and the speed will be her speed minus speed of stream and while rowing downstream her speed will increase and will be equal to sum of her speed and speed of stream. Therefore,

The speed of Ritu while rowing

Upstream = (x - y) km/h

Downstream = (x + y) km/h

According to the question:

Ritu can row downstream 20 km in 2 hours, and
distance = speed x time

⇒ 2 (x+y) = 20

⇒ x+y = 10............(1)

also,
Ritu can row upstream 4 km in 2 hours

⇒ $2(x - y) = 4$
⇒ $x-y = 2$ ....(2)

Adding equation (1) and (2),
we obtain

⇒ $x + y + x - y = 10 + 2$
⇒ $2x = 12$
⇒ $x = 6$

Putting this in equation (1),

$6 + y = 10$

we obtain $y = 4$

Hence, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

Chapter: Quadratic Equations

Quadratic Equation

When we equate the quadratic polynomial to any constant then it is called a Quadratic Equation i.e. if **p(x) = k**, where k is any constant.

Example: $2x^2 = 5$, $y^2 - 3y + 7 = 8$ etc

Standard form of Quadratic Equation

The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b, c are the real numbers and $a \neq 0$.
Example: $3x^2 + x + 4 = 0$ is a quadratic equation in the standard form.

Important points:

1. If α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, then:
Sum of roots = $\alpha + \beta = -\frac{\text{Coefficient of x}}{\text{Coefficient of x}^2} = -\frac{b}{a}$
Product of roots = $\alpha \times \beta = \frac{\text{Constant Term}}{\text{Coefficient of x}^2} = \frac{c}{a}$
2. If α and β are the roots of a quadratic equation in x, then the quadratic equation is given by:
$K\{x^2 - (\text{sum of roots}) + \text{product of roots}\}$

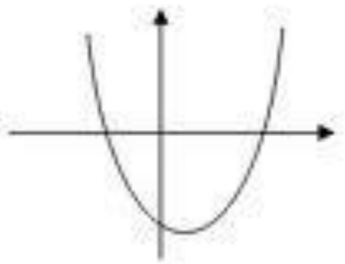
Roots of a Quadratic Equation

As the degree of a quadratic equation is 2, it means it has two roots. Now, roots are those values of the variable which satisfies the given quadratic equation.
Let $x = \alpha$ where α is a real number. If α satisfies the Quadratic Equation $ax^2 + bx + c = 0$ such that $a\alpha^2 + b\alpha + c = 0$, then α is one of the roots of the Quadratic Equation.

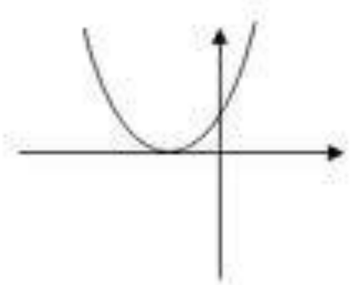
Graphical meaning of roots:

Graphically, the roots of a quadratic equation are the points where the graph of the quadratic polynomial cuts the x-axis.

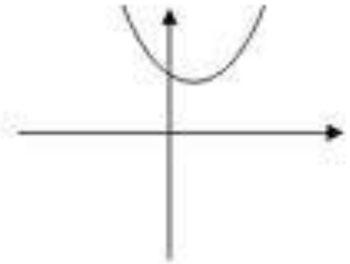
- If the graph of the quadratic polynomial cuts the x-axis at two distinct points, then it has real and distinct roots.
- If the graph of the quadratic polynomial touches the x-axis, then it has real and equal roots.
- If the graph of the quadratic polynomial does not cut or touch the x-axis then it does not have any real roots.



two roots



one root



no roots

Method of Factorisation:

In this method, we factorise the equation into two linear factors and equate each factor to zero to find the roots of the given equation.

Step 1: Given Quadratic Equation in the form of $ax^2 + bx + c = 0$.
Step 2: Split the middle term bx as mx + nx so that the sum of m and n is equal to b and the product of m and n is equal to the product of **a and c**.
Step 3: By factorization we get the two linear factors (x + p) and (x + q)
$ax^2 + bx + c = 0 = (x + p)(x + q) = 0$
Step 4: Now we have to equate each factor to zero to find the value of x. The obtained value of x are the roots of the given quadratic equation.

Quadratic Formula

One more method of finding the roots of a quadratic equation is by Quadratic formula. Let $ax^2 + bx + c = 0$ be a quadratic equation in the standard form, where $a \neq 0$, then its roots are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $b^2 - 4ac = D$ known as Discriminant.

Discriminant helps in identifying the nature of roots. We will discuss about it in detail in the next class. But remember that we will workout the roots of the quadratic equations whose discriminant is either equals to or greater than zero.

The quadratic formula can also be re-written in the form of D as:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

If $D > 0$, roots are real and unequal

If $D = 0$, roots are real and equal

If $D < 0$, roots are not real or imaginary roots

Important Questions with Solutions

1. Find the roots of $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Solution: We have,

$$\sqrt{2}x^2 + 7x + 5\sqrt{2}$$

$$= \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2}$$

$$= x\left(\sqrt{2}x + 5\right) + \sqrt{2}\left(\sqrt{2}x + 5\right)$$

$$= \left(\sqrt{2}x + 5\right)\left(x + \sqrt{2}\right)$$

Roots of this equation are the values for which $\left(\sqrt{2}x + 5\right)\left(x + \sqrt{2}\right) = 0$

$\therefore \sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$

$$x = \frac{-5}{\sqrt{2}}, \; x = -\sqrt{2}$$

2. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles

produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Solution:

To find: Number of articles produced and cost of each article.

Let the number of articles produced be x.

Therefore, cost of production of each article = Rs $(2x + 3)$

The total cost of production = Total quantity produced x cost of one article

It is given that the total cost of production is Rs 90.

$\therefore x\ (2x + 3) = 90$

$\Rightarrow 2x^2 + 3x = 90$

$\Rightarrow 2x^2 + 3x - 90 = 0$

$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow x\ (2x + 15) - 6(2x + 15) = 0$

$\Rightarrow (2x + 15)\ (x - 6) = 0$

Either $2x + 15 = 0$ or $x - 6 = 0$,

i.e., $x = \frac{-15}{2}$ or $x = 6$

As the number of articles produced can only be a positive integer,

Therefore, x can only be 6.

Hence, the number of articles produced = 6

Cost of each article = $2 \times 6 + 3$
$= 12 + 3$
= Rs 15

3. Find the roots of the following equations:

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$$

Solution: We have,

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$$

$$\frac{[(x-7)-(x+4)]}{[(x+4)(x-7)]} = \frac{11}{30}$$

$$-\frac{11}{[(x+4)(x-7)]} = \frac{11}{30}$$

$$(x+4)(x-7) = -30$$

$$x^2 - 3x - 28 = -30$$

$$x^2 - 3x + 2 = 0$$

Factorizing the quadratic equation, such that the product of two numbers is 2 and their sum is 3

x(x - 2) - 1(x - 2) = 0

(x - 2)(x - 1) = 0
i.e x - 2 = 0 or x - 1 = 0
therefore,

x = 1 and x = 2 are the roots of the given equation.

4. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Solution:
Let the marks obtained in Mathematics by Shefali be 'a'.

Given, sum of the marks obtained by Shefali in Mathematics and English is 30.

Marks obtained in english = 30 – a

Also, she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210.

⇒(a + 2)(30 – a – 3) = 210
⇒ (a + 2)(27 – a) = 210

$\Rightarrow 27a - a^2 + 54 - 2a = 210$

$\Rightarrow -a^2 + 25a + 54 = 210$
$\Rightarrow -a^2 + 25a + 54 - 210 = 0$
$\Rightarrow -a^2 + 25a - 156 = 0$

$\Rightarrow a^2 - 25a + 156 = 0$

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\Rightarrow a^2 - 13a - 12a + 156 = 0$

$\Rightarrow a(a - 13) - 12(a - 13) = 0$

$\Rightarrow (a - 12)(a - 13) = 0$

$\Rightarrow a = 12$ or 13

If marks in mathematics is 12
marks in english is 30 - a = 30 - 12 = 18

If marks in mathematics is 13
marks in english is 30 - a = 30 - 13 = 17

Hence

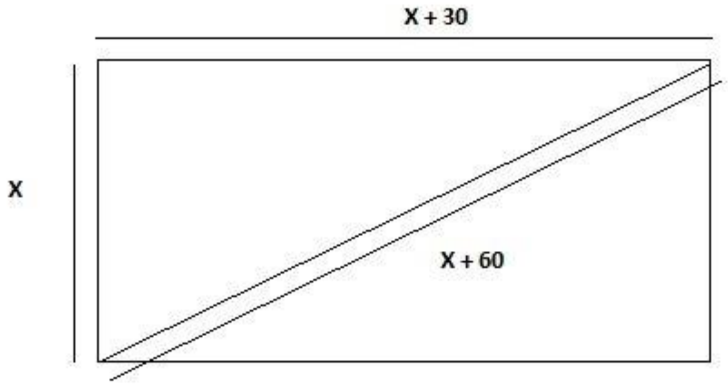
Marks in Mathematics = 12, Marks in English = 18

Or

Marks in Mathematics = 13, Marks in English = 17

5. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Solution:



Let the shorter side of the rectangle be *x* m.

Then, larger side of the rectangle = (x + 30) m

we know,
By Pythagoras theorem,
$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$
Diagonal of a rectangle is $\sqrt{[(\text{length})^2 + (\text{breadth})^2]}$

Diagonal of the rectangle = $\sqrt{x^2 + (x+30)^2}$

It is given that the diagonal of the rectangle is 60 m more than the shorter side.

$\therefore \sqrt{x^2 + (x+30)^2} = x+60$

Squaring both sides, we get,

$\Rightarrow x^2 + (x+30)^2 = (x+60)^2$

$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$

$\Rightarrow x^2 - 60x - 2700 = 0$

Now for solving this quadratic equation, we need to factorize 60 in such a way that the product is 2700 and the difference is 60

$\Rightarrow x(x - 90) + 30(x - 90) = 0$

$\Rightarrow (x - 90)(x+30) = 0$

$\Rightarrow x = 90, -30$

However, side cannot be negative. Therefore, the length of the shorter side will be 90 m.

Hence, length of the larger side will be (90 + 30) m = 120 m

6. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Solution: Assumption: Let the larger and smaller number be x and y respectively.
Given:

According to the given question difference of squares of two numbers is 180. and the square of smaller number is 8 times square of the larger number.
So,

$\Rightarrow x^2 - y^2 = 180 \qquad \text{eq(i)}$

⇒ $y^2 = 8x$ eq(ii)
putting value of eq(ii) in eq(i)

⇒ $x^2 - 8x = 180$

⇒ $x^2 - 8x - 180 = 0$

For factorizing this quadratic equation, the product of numbers should be 180 and their difference should be 8

⇒ $x^2 - 18x + 10x - 180 = 0$

⇒ $x(x - 18) + 10(x - 18) = 0$

⇒ $(x - 18)(x+10) = 0$
⇒ $x = 18, -10$

The larger number cannot be negative as it makes the square of smaller number negative, which is not possible.
Therefore, the larger number will be 18 only.

$x = 18$

∴ $y^2 = 8x$
$= 8\times18$
$= 144$

$= y = \pm\sqrt{144} = \pm12$

∴ smaller number = ±12

Therefore, the numbers are 18 and 12 or 18 and −12.

7. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution:
Let the speed of the train be *x* km/hr.

Time taken to cover 360 km = $\frac{360}{x}$ hr, As

$$time = \frac{distance}{speed}$$

Now, given that if the speed would be 5 km/hr more, the same distance would be covered in 1 hour less, i.e.
if speed = x + 5, and

$$time = (\frac{360}{x} - 1)\ hours$$

then, using distance = speed x time, we have

$$(x+5)\left(\frac{360}{x}-1\right)=360$$

Now we can form the quadratic equation from this equation

$$360-x+\frac{1800}{x}-5=360$$

$$\frac{360x-x^2+1800-5x}{x}=360$$

Now, cross multiplying we get
⇒ $360x - x^2 + 1800 - 5x = 360x$

⇒ $x^2 + 5x - 1800 = 0$
Now we have to factorize in such a way that the product of the two numbers is 1800 and the difference is 5
⇒ $x^2 + 45x - 40x - 1800 = 0$

⇒ $x(x + 45) - 40(x+45) = 0$

⇒ $(x+45)(x - 40) = 0$

⇒ $x = -45, 40$

Since, the speed of train can't be negative,
so, speed will be 40 km/hour.

8. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution: Let the time taken by the smaller pipe to fill the tank be *x* hr.

Time taken by the larger pipe = $(x - 10)$ hr

Part of the tank filled by a smaller pipe in 1 hour = $\frac{1}{x}$

Part of the tank filled by the larger pipe in 1 hour = $\frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8}=\frac{75}{8}$ hours by both the pipes together.
So 75/8 hours, multiplied by the sum of parts filled with both pipes in one hour equal to complete work i.e 1.

$$\frac{75}{8}\left(\frac{1}{x}+\frac{1}{x-10}\right)=1$$

$\Rightarrow \frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$

$\Rightarrow \frac{x-10+x}{x(x-10)}=\frac{8}{75}$

$\Rightarrow \frac{2x-10}{x(x-10)}=\frac{8}{75}$

$\Rightarrow 75(2x - 10) = 8x^2 - 80x$

$\Rightarrow 150x - 750 = 8x^2 - 80x$

$\Rightarrow 8x^2 - 230x + 750 = 0$

$\Rightarrow 8x^2 - 200x - 30x + 750 = 0$

$\Rightarrow 8x(x - 25) - 30(x - 25) = 0$
$\Rightarrow (x - 25)(8x - 30) = 0$

$\Rightarrow x = 25, \frac{30}{8}$

Time taken by the smaller pipe cannot be $\frac{30}{8}$ = 3.75 hours.

As in this case, the time taken by the larger pipe will be negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 − 10 =15 hours respectively.

9. Find the nature of the roots of $3x^2 - 4\sqrt{3}x + 4 = 0$

Solution: We have,

Comparing this equation with $ax^2 + bx + c = 0$,

we obtain,

a = 3

b = $-4\sqrt{3}$

c = 4

$b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4(3)(4)$

Discriminant = 48 − 48 = 0

As, $b^2 - 4ac = 0$,

Therefore, real roots exist for the given equation and they are equal to each other.

And the roots will be $\frac{-b}{2a}$ and $\frac{-b}{2a}$.

$$\frac{-b}{2a} = \frac{-(-4\sqrt{3})}{2\times3} = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$$

Therefore, the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$

10. Find the values of k for which $kx\ (x - 2) + 6 = 0$ will have two equal roots.

Solution: We have,

$kx\ (x - 2) + 6 = 0$

or $kx^2 - 2kx + 6 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain,

$a = k, b = -2\ k, c = 6$

Discriminant $= b^2 - 4ac = (-\ 2k)^2 - 4\ (k)\ (6) = 4k^2 - 24k$

For equal roots, $b^2 - 4ac = 0$

$= 4k^2 - 24k = 0$

$= 4k\ (k - 6) = 0$

Either $4k = 0$ or $k = 6$

$= k = 0$ or $k = 6$

However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.

Therefore, if this equation has two equal roots, k should be 6 only.

Chapter: Arithmetic Progression

Arithmetic Progression is a set of numbers in which the difference of a term and its predecessor is always constant, i.e., $a_n - a_{n-1} = d$, where d is the common difference. The successive terms of an *A.P.* are a, $a + d$, $a + 2d$, $a + 3d$, $a + 4d$ ....

Choice of terms in *A.P.*

No. of terms	Terms	Common difference
3	$a - d, a, a + d$	d
4	$a - 3d, a - d, a + d, a + 3d$	$2d$
5	$a - 2d, a - d, a, a + d, a + 2d$	d
6	$a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$	$2d$

General Term

General term or *n*th term of an *A.P.* is:

$a_n = a + (n - 1)d$

where a = first term, d = common difference

Sum of n terms of an AP

Sum of n terms of an AP is given by:

$$s_n = \frac{n}{2}[2a + (n - 1)d]$$

Or

$$s_n = \frac{n}{2}[a + l]$$

Note: $a_n = s_n - s_{n-1}$

Properties of an A.P. -

A sequence which is in A.P. follows certain properties as follows –

- The sum of terms of an A.P. equidistant from the beginning and the end of the A.P is constant and equal to the sum of the first and last term of the A.P.

- The sum of the third term from the beginning and the end is $(a + 2d) + a + (n-3)d = 2a + (n - 1)d$

- The sum of the first and last terms is $a + a + (n - 1)d = 2a + (n - 1)d$

- If each term of an A.P is increased decreased, multiplied or divided by the same non zero number, then the resulting sequence is also in an A.P.
- The sum or difference of two A.P.s is an A.P.
- If the terms are selected at a regular interval from within the A.P, the resulting sequence is also in an A.P.
- If three numbers a, b and c are in A.P. then $2b = a + c$

Important Questions and Solutions

1. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?

Solution: Given,

First term, $a = 3$,

Common difference, $d = a_2 - a_1 = 8 - 3 = 5$,

nth term, $a_n = 78$

To find: $n = ?$

We know that nth term of an A.P is given by:

$a_n = a + (n - 1)d$

$78 = 3 + (n - 1)5$

$(n - 1) 5 = 78 - 3 = 75$

$5n - 5 = 75$
$5n = 80$

$n = 80/5$

$n = 16$

Thus, 78 is the 16th term of given AP.

2. Find the number of terms in the given AP : 7, 13, 19, . . . , 205

Solution: **Given: $a = 7, d = 6, a_n = 205, n = ?$**

We know that nth term of an AP is given by, $a_n = a + (n - 1)d$

$205 = 7 + (n - 1)6$

$(n - 1)6 = 205 - 7 = 198$

$$n - 1 = \frac{198}{6}$$

$n - 1 = 33$

$n = 34$

Thus, 205 is the 34th of term of this AP.

3. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73

Solution:

We know that $a_n = a + (n - 1)d$

where, a_n = nth terms of AP
a = first term of AP
n = number of terms
d = common difference

Hence,
$a_{11} = a + (11 - 1)d$
$a_{11} = a + 10d = 38$ ....eq(i)

And,
$a_{16} = a + (16 - 1)d$
$a_{16} = a + 15d = 73$ ....eq(ii)

Subtracting eq(i) from eq(ii), we get following:

$a + 15d - (a + 10d) = 73 - 38$

$a + 15d - a - 10d = 35$

Or, $5d = 35$

Or, $d = 7$

Substituting the value of d in eq(i) we get;

$a + 10 \times 7 = 38$

Or, $a + 70 = 38$

Or, $a = 38 - 70 = -32$

Now 31st term can be calculated as follows:

$a_{31} = a + (31 - 1)d$

$a_{31} = a + 30d$

= - 32 + 30 x 7

= - 32 + 210 = 178
So, 31st term is 178

4. If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP iszero?

Solution:

Given, a_3 = 4 and a_9 = - 8

a_3 = a + 2d = 4

a_9 = a + 8d = - 8

Subtracting 3rd term from 9th term, we get;

a + 8d – a – 2d = - 8 – 4 = - 12

6d = - 12

d = - 2

Substituting the value of d in 3rd term, we get;

a + 2(-2) = 4

a – 4 = 4

a = 8

Now; 0 = a + (n – 1)d

0 = 8 + (n – 1)(- 2)

(n – 1)(- 2) = - 8

n – 1 = 4

n = 5

Thus, 5th term of this AP is zero.

5. Which term of the AP : 3, 15, 27, 39, . . . will be 132 more than its 54th term?

Solution
nth term of an AP is given by:
a_n = a + (n - 1) d
where,
a = first term of AP
n = no. of terms of AP
d = common difference of AP

54th term can be given as follows:

a_{54} = a + (54 - 1)d

= 3 + 53 x 12

= 3 + 636
= 639

In question we have to find the term which is 132 more than 639 i.e. 771
Now a_n = 771, n = ?
Again applying the formula of nth term

771 = a + (n – 1)d

771 = 3 + (n -1)12

(n – 1)12 = 771 – 3 = 768

n - 1 = 768/12

n – 1 = 64

n = 65

Thus, the required term is 65th term.

6. How many three-digit numbers are divisible by 7?

Solution:
Since, 100 is the smallest three digit number and it gives a reminder of 2 when divided by 7, therefore, 105 is the smallest three digit number which is divisible by 7

Since, 999 is greatest three digit number, and it gives a remainder of 5, thus 999 – 5 = 994 will be the greatest three digit number which is divisible by 7

Therefore, here we have,

First term (a) = 105,

The last term (a_n) = 994

The common difference = 7

We know that, $a_n = a + (n - 1)d$

Or, $994 = 105 + (n - 1)7$

Or, $(n - 1)7 = 994 - 105 = 889$

Or, $n - 1 = 127$

Or, $n = 128$

Thus, there are 128 three digit numbers which are divisible by 7.

7. Find the 20th term from the last term of the AP : 3, 8, 13, . . ., 253

Solution:

In the given AP
First term,a = 3
common difference, d = 5
we know, nth term of an AP is
$a_n = a + (n - 1)d$

Now, Let the no of terms in given AP is n
last term = 253

$\Rightarrow 253 = a + (n - 1) d$

$\Rightarrow 253 = 3 + (n -1) \times 5$

$\Rightarrow 253 = 3 + 5n - 5 = - 2$

$\Rightarrow 5n = 253 + 2 = 255$

$\Rightarrow n = 255/5 = 51$

Total number of terms in AP is 51

If there are n terms in an AP then some mth term from end will be equal to n - m + 1 term from beginning

Therefore, 20th term from the last term will be 32th term from starting (as, 51 - 20 + 1 = 32)

$a_{32} = a + 31d$

$= 3 + 31 \times 5$

$= 3 + 155 = 158$

Thus, required term is 158

8. Which term of the AP: 121, 117, 113, . . ., is its first negative term?

Solution:

Here,
first term, $a = 121$
common difference, $d = a_2 - a_1 = 117 - 121 = -4$
Let the first negative term be 'a_n'.
Also, we know that, nth term of an AP is given by
$a_n = a + (n - 1)d$
We have to find least value of n, such that
$a_n < 0$
$\Rightarrow a + (n - 1)d < 0$
$\Rightarrow 121 + (n - 1)(-4) < 0$
$\Rightarrow 121 - 4(n - 1) < 0$
$\Rightarrow 4(n - 1) > 121$
$\Rightarrow 4n - 4 > 121$
$\Rightarrow 4n > 125$
$\Rightarrow n > 31.25$
Therefore, n is 32 [least positive integer greater than 31.25 is 32]
Hence, the 32nd term of AP is first negative term.
Also,
$a_{32} = a + 31d$
$= 121 + 31(-4)$
$= 121 - 124 = -3$

9. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Solution:

nth term of an AP is given by the formula
$a_n = a + (n - 1) d$
where, a_n = nth term
n = number of term
d = common difference
So now its given that sum of third and seventh term is 6, thus we need to find 3rd and 7th term first,
$a_3 = a + 2d$

$a_7 = a + 6d$

As per question

$a_3 + a_7 = 6$

So now,

$a + 2d + a + 6d = 6$

$2a + 8d = 6$

$a + 4d = 3$

$a = 3 – 4d$ ....eq (i)

Similarly,

Product of third and seventh term is given as 8. So,

$(a + 2d)(a + 6d) = 8$

$a^2 + 6ad + 2ad + 12d^2 = 8$ ....eq(ii)

Substituting the value of a in equation (ii), we get;

$(3 – 4d)^2 + 8(3 – 4d)d + 12d^2 = 8$

$9 – 24d + 16d^2 + 24d – 32d^2 + 12d^2 = 8$

$9 – 4d^2 = 8$

$2d = 1$

$d = \pm 1/2$

Using the value of d in equation (1), we get;

$a = 3 – 4d$

$$a = 3 - 4 \times \frac{1}{2}$$

Or, $a = 3 – 2 = 1$

Sum of first 16 terms is calculated as follows:

$$S = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{16} = \frac{16}{2}[2 \times 1 + (16 - 1) \times \frac{1}{2}$$

S_{16} = 8[2 + (15/2)]

= 4 x 19

S_{16} = 76

Thus, sum of first 16 terms of this AP is 76.
Now by taking d = -1/2, we get,
a = 3 - 4 (-1/2)
a = 3 + 2 = 5

$$S = \frac{16}{2}[2 \times 5 + (16 - 1)\frac{-1}{2}]$$

S = 8[10 - 15/2]
S = 4[20 - 15]
S = 4[5] = 20
So, another possible value of sum is 20.

10. A ladder has rungs 25 cm apart.(see Fig. 5.7). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

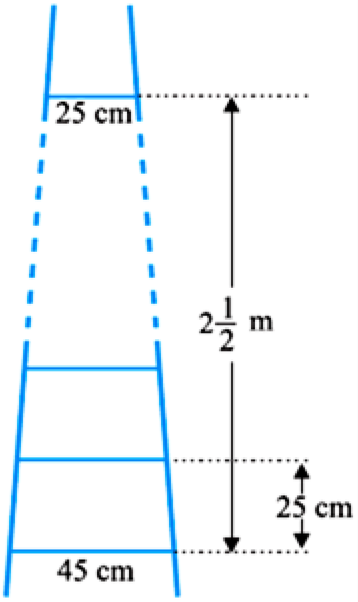


Fig. 5.7

Solution:

Total distance between top and bottom rung = 2 m 50 cm

Distance between any two rungs = 25 cm

$$Number\ of\ rungs = \frac{250}{25} + 1$$

$$Number\ of\ rungs = 11$$

And it is also given that bottom most rungs is of 45 cm length and top most is of 25 cm length.

As it is given that the length of rungs decrease uniformly, it will for an AP with a = 25, a_{11} = 45 and n = 11

nth term of an AP is given by

$a_n = a + (n - 1)d$

$a_{11} = a + 10d$

45 = 25 + 10d

10d = 45 – 25 = 20

d = 2

Total length of wood will be equal to the sum of 11 terms:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$= \frac{11}{2}[2 * 25 + 10 * 2]$$

= 11[25 + (10)]

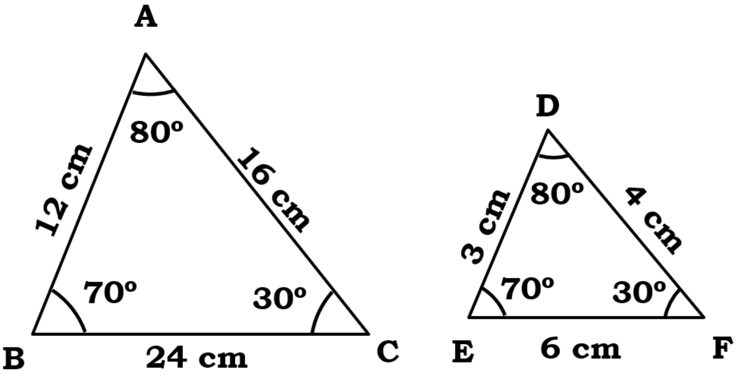
= 11 x 35

= 385 cm

Therefore, total wood required for rungs is equal to 385 m

Chapter: Triangles

Two triangles are said to be similar when they have the same shape but their sizes are proportional.



When two triangles are similar, we denote it as follows –

$$\Delta ABC \sim \Delta DEF$$

This implies that all the corresponding angles of the two triangles are equal and their corresponding sides are in the same ratio.

$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

And, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Similarity in triangles can also be checked by various criteria, like congruence. Some common criteria for similarity are as follows –

(i) **SSS similarity** – Unlike the congruency, since in case of similarity, the sides are not equal but proportional, if in two triangles, all the three corresponding sides are proportional, then the triangles are said to be similar by SSS criterion.

(ii) **AAA/AA similarity –** If in two triangles, all the three corresponding angles are equal, then they are said to be similar by AAA criterion.

NOTE: If in a triangle, 2 angles are equal to other two angles of another triangle, then the third one will automatically be equal due to Angle sum property. Thus, AAA criterion can also be confirmed by checking only for two angles and is hence also addressed as AA criterion of similarity.

(iii). SAS similarity – If in two triangles, a pair of corresponding sides is proportional and the included angle between them is equal, then the triangles are said to be similar.

Important points:

- The ratio of the areas of two similar triangles is equal to the ratio of the squares of their **corresponding sides**.
- The ratio of the areas of two similar triangles is equal to the ratio of the squares of their **corresponding altitudes**.

- The ratio of the areas of two similar triangles is equal to the ratio of the squares of their **corresponding medians**.

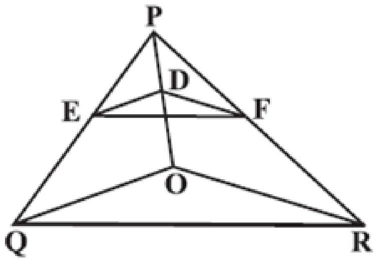
- The ratio of the areas of two similar triangles is equal to the ratio of the squares of their **corresponding angle-bisector segments**.

The Pythagoras theorem states that, "**In a right - angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides".**

Converse of Pythagoras theorem, "**In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle."**

Important Questions and Solutions

1. In the figure, DE || OQ and DF || OR. Show that EF || QR.



Solution:

To Prove: EF ll QR

Given: In triangle POQ, DE parallel to OQ

Proof:

In triangle POQ, DE parallel to OQ

Hence,

$\frac{PE}{EQ} = \frac{PD}{DO}$ (Basic proportionality theorem) (i)

Now,
$In\ triangle\ POR, DF\ parallel\ OR$
Hence,

$\frac{PF}{FR} = \frac{PD}{DO}$ (Basic proportionality theorem) (ii)

From (i) and (ii), we get

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

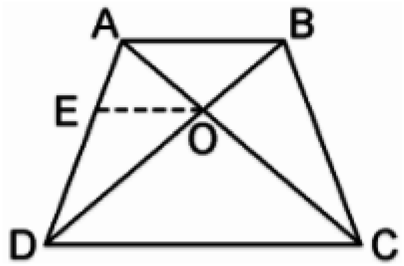
Therefore,

EF is parallel to QR (Converse of basic proportionality theorem)

Hence, Proved.

2. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

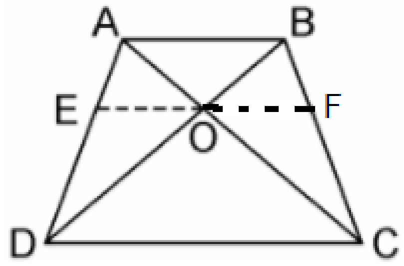
Solution:
The figure is given below:



Given: ABCD is a trapezium
AB ll CD
Diagonals intersect at O

To Prove = $\frac{AO}{BO} = \frac{OC}{OD}$

Construction: Construct a line EF through point O, such that EF is parallel to CD.



Proof:

In ΔADC, EO is parallel to CD
According to basic proportionality theorem, if a side is drawn parallel to a ny side of the triangle then the corresponding sides formed are

proportional

Now, using basic proportionality theorem in ΔABD and ΔADC, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \quad (i)$$

In ΔABD, OE is parallel to AB

So, using basic proportionality theorem in ΔEOD and ΔABD, we get

$$\frac{ED}{AE} = \frac{OD}{BO}$$

$$\frac{AE}{ED} = \frac{BO}{OD} \quad (ii)$$

From (i) and (ii), we get

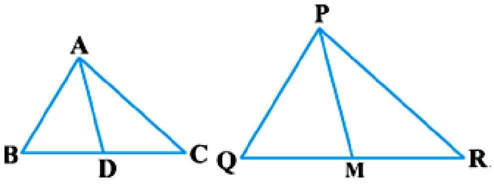
$$\frac{AO}{OC} = \frac{BO}{OD}$$

Therefore, by cross multiplying we get,

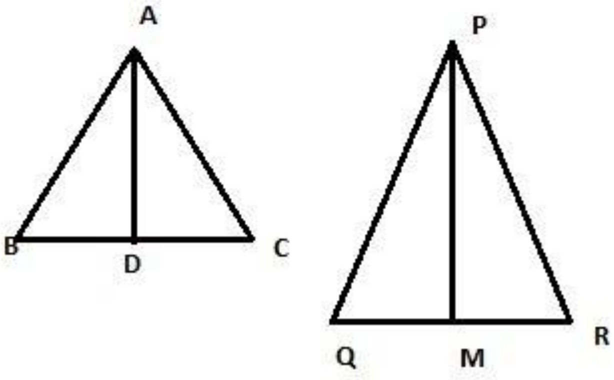
$$\frac{AO}{BO} = \frac{OC}{OD}$$

Hence, Proved

3. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of Δ PQR. Show that Δ ABC ~ Δ PQR.



Solution



To Prove: Δ ABC ~ Δ PQR

Given:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

Proof:
Median divides the opposite side

BD = $\frac{BC}{2}$ and,

QM = $\frac{QR}{2}$

Now,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

Multiplying and dividing by 2, we get,

$$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In Δ ABD and Δ PQM,

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

ΔABD $\sim$ ΔPQM (By SSS similarity)

∠ABD = ∠PQM (Corresponding angles of similar triangles)

In ΔABC and ΔPQR,

∠ABD = ∠PQM (Proved above)

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

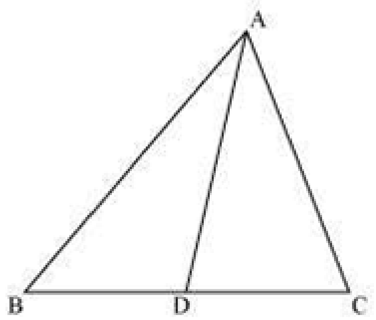
ΔABC $\sim$ ΔPQR (By SAS similarity)

Hence, Proved.

4. D is a point on the side BC of a triangle ABC such that ∠ ADC = ∠ BAC. Show that CA^2 = CB.CD

Solution

In ΔADC and ΔBAC,



To Prove : CA^2 = CB . CD

Given: ∠ADC = ∠BAC

Proof: Now In ΔADC and ΔBAC,

∠ADC = ∠BAC

∠ACD = ∠BCA (Common angle)

According to AA similarity, if two corresponding angles of two triangles are equal then the triangles are similar

ΔADC ~ ΔBAC (By AA similarity)

We know that corresponding sides of similar triangles are in proportion

Hence in ΔADC and ΔBAC,

$$\frac{CA}{CB} = \frac{CD}{CA}$$

$$CA^2 = CB \times CD$$

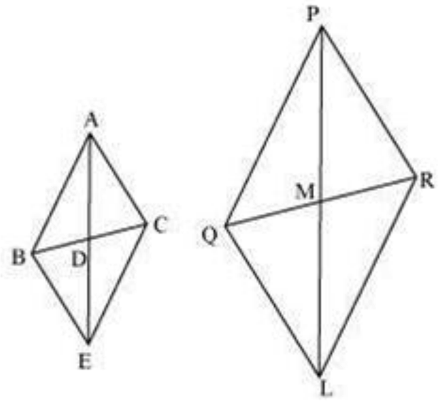
Hence Proved

5. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that Δ ABC ~ Δ PQR

Solution
To Prove: Δ ABC ~ Δ PQR
Given:

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Proof:



Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML.
Then, join B to E, C to

E, Q to L, and R to L

We know that medians divide opposite sides.

Hence, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC,

Diagonals AE and BC bisect each other at point D.

Therefore,

Quadrilateral ABEC is a parallelogram.

AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given in the question that,

$$\frac{AB}{PQ}=\frac{AC}{PR}=\frac{AD}{PM}$$

$$\frac{AB}{PQ}=\frac{BE}{QL}=\frac{2AD}{2PM}$$

$$\frac{AB}{PQ}=\frac{BE}{QL}=\frac{AE}{PL}$$

ΔABE $\sim$ ΔPQL (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

∠BAE = ∠QPL ..... (i)

Similarly, it can be proved that

ΔAEC ~ ΔPLR and

∠CAE = ∠RPL ..... (ii)

Adding equation (i) and (ii), we obtain

∠BAE + ∠CAE = ∠QPL + ∠RPL

⇒∠CAB = ∠RPQ .... (iii)

In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{AC}{PR}$ (Given)

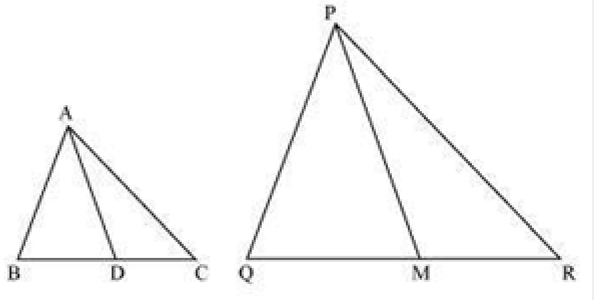
∠CAB = ∠RPQ [Using equation (iii)]

ΔABC ~ ΔPQR (By SAS similarity criterion)
Hence, Proved.

6. If AD and PM are medians of triangles ABC and PQR, respectively

where Δ ABC ~ Δ PQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Solution:
It is given that ΔABC is similar to ΔPQR



$To\ Prove: \frac{AB}{PQ} = \frac{AD}{PM}$

Given: Δ ABC ~ Δ PQR

AD and PM are medians
We know that the corresponding sides of similar triangles are in proportion

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ ....eq(i)

And also the corresponding angles are equal

∠A = ∠P

∠B = ∠Q

∠C = ∠R ....eq(ii)

Since AD and PM are medians, they divide their opposite sides in two equal parts

BD = $\frac{BC}{2}$ and,

QM = $\frac{QR}{2}$ ....eq(iii)

From (i) and (iii), we get

$\frac{AB}{PQ} = \frac{BD}{QM}$...... eq(iv)

In ΔABD and ΔPQM,

∠B = ∠Q [Using (ii)]

$\frac{AB}{PQ} = \frac{BD}{QM}$ [Using (iv)]

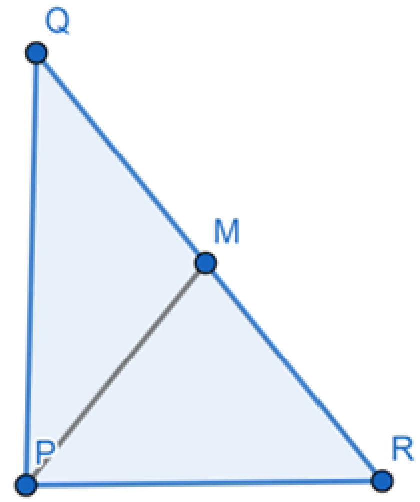
ΔABD $\sim$ ΔPQM (Since two sides are proportional and one angle is equal then by SAS similarity)

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

Hence, Proved

7. PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that PM^2 = QM×MR

Solution:



⇒ Let ∠MPR = x

⇒ In Δ MPR, ∠MRP = 180-90-x

⇒ ∠MRP = 90-x

Similarly, in Δ MPQ,

∠MPQ = 90-∠MPR = 90-x

⇒ ∠MQP = 180-90-(90-x)

⇒ ∠MQP = x

In Δ QMP and Δ PMR

⇒ ∠MPQ = ∠MRP

⇒ ∠PMQ = ∠RMP

⇒ ∠MQP = ∠MPR

⇒ Δ QMP ~ Δ PMR

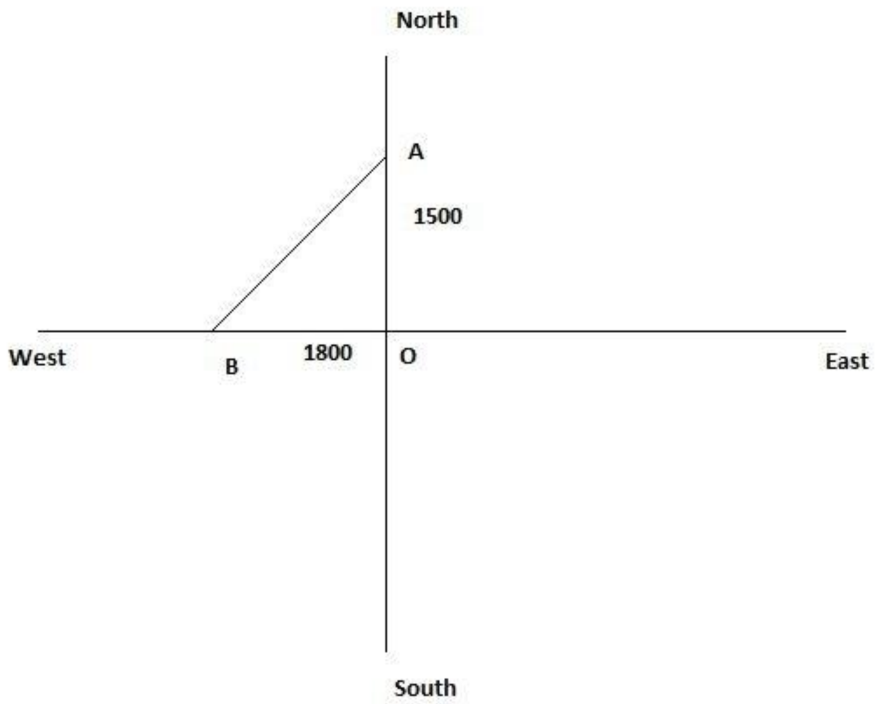
⇒ $\frac{QM}{PM} = \frac{MP}{MR}$

⇒ PM^2 = MR × QM

Hence proved

8. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Solution:



We know,

Distance = speed × time

Distance travelled by the plane flying towards north in $1\frac{1}{2}hrs$

= 1,000 x $1\frac{1}{2}$

= 1,500 km

Similarly, distance traveled by the plane flying towards west in $1\frac{1}{2}hrs$

= 1,200 x $1\frac{1}{2}$

= 1,800 km

Let these distances be represented by OA and OB respectively.

Applying Pythagoras theorem,

$$AB^2 = OA^2 + OB^2$$

$$AB = \sqrt{OA^2 + OB^2}$$

$$AB = \sqrt{(1500)^2 + (1800)^2}$$

$$AB = \sqrt{2250000 + 3240000}$$

$$AB = \sqrt{5490000}$$

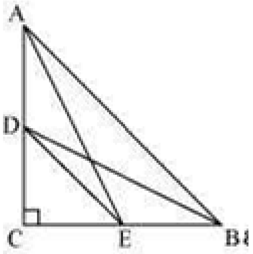
$$AB = 300\sqrt{61}$$

Distances between planes is 300√61 km

9. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$

Solution:
To Prove: $AE^2 + BD^2 = AB^2 + DE^2$

Given: D and E are midpoints of AD and CB and ABC is right angled at C
Applying Pythagoras theorem in ΔACE, we obtain



Pythagoras theorem: It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

$AC^2 + CE^2 = AE^2$ ....eqn(i)

Applying Pythagoras theorem in triangle BCD, we get

$BC^2 + CD^2 = BD^2$ ....eqn(ii)

Adding equations (i) and (ii), we get

$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2$ ....eqn (iii)

Applying Pythagoras theorem in triangle CDE, we get

$DE^2 = CD^2 + CE^2$

Applying Pythagoras in triangle ABC, we get

$AB^2 = AC^2 + CB^2$

Putting these values in eqn(iii), we get

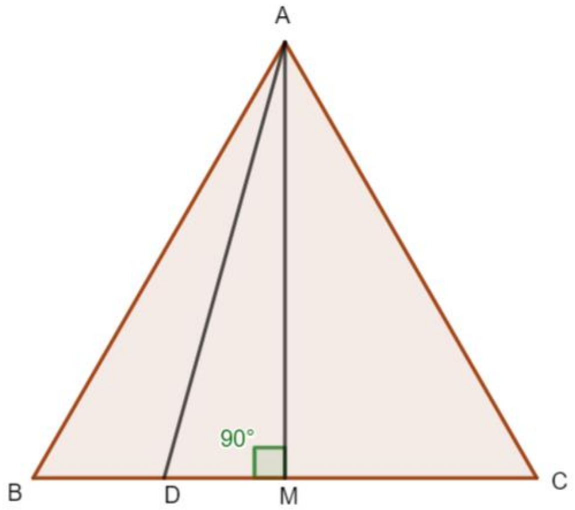
$DE^2 + AB^2 = AE^2 + BD^2$

Hence, Proved.

10. In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC.$ Prove that $9\ AD^2 = 7\ AB^2$.

Solution:

The figure is given below:



Given: BD = BC/3

To Prove: $9\ AD^2 = 7\ AB^2$

Proof:

Let the side of the equilateral triangle be *a*, and AM be the altitude of ΔABC

BM = MC = BC/2 = a/2 [Altitude of an equilateral triangle bisect the side]

And then, in ΔABM, by Pythagoras theorem we write,

$AM^2 = AB^2 - BM^2$

or $AM^2 = a^2 - a^2/4$

$$AM^2 = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4}$$

$$AM = \frac{a\sqrt{3}}{2}$$

BD = a/3 [BC = a]

DM = BM – BD

= a/2 – a/3

= a/6

Applying Pythagoras theorem in ΔADM, we obtain

$AD^2 = AM^2 + DM^2$

$$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$$

$$AD^2 = \frac{3a^2}{4} + \frac{a^2}{36}$$

$$AD^2 = \frac{27a^2 + a^2}{36}$$

$$AD^2 = \frac{28a^2}{36}$$

Now, a = AB or $a^2 = AB^2$

$$AD^2 = \frac{28AB^2}{36}$$

$36\ AD^2 = 28\ AB^2$

$9\ AD^2 = 7\ AB^2$

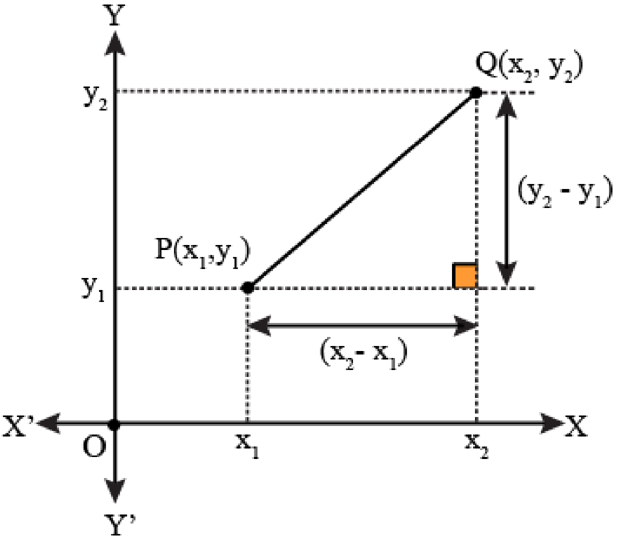
Hence, Proved

Chapter: Coordinate Geometry

Distance Formula

The distance between any two points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is calculated by:

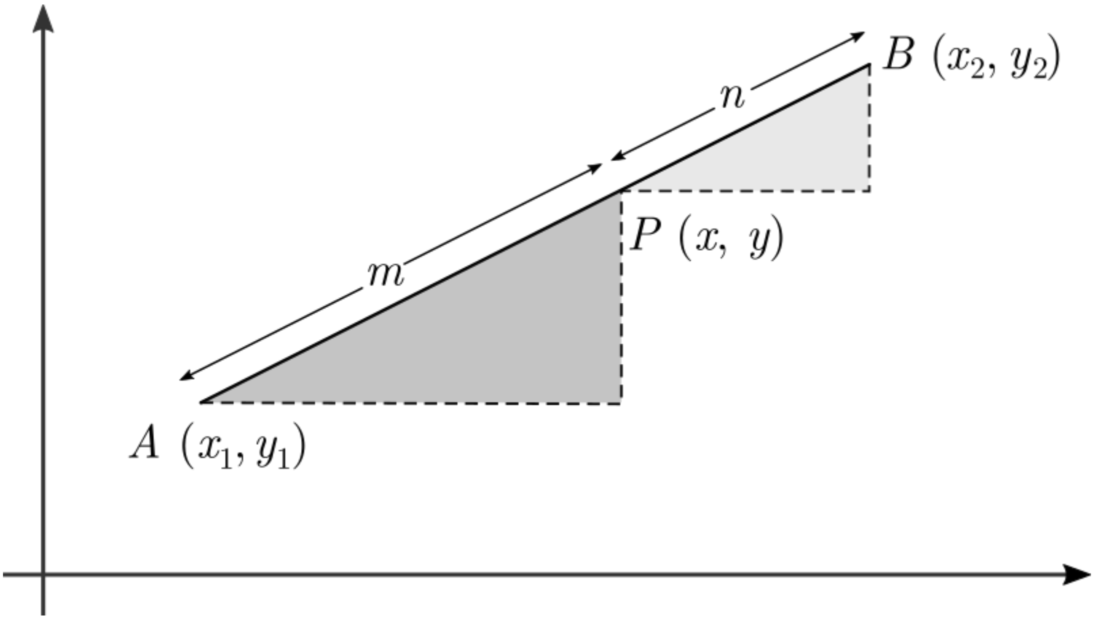
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Section Formula

If the point P(x,y) **divides** the line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$ **internally** in the **ratio m:n**, then, the coordinates of P are given by the **section formula** as

$$P(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$



Mid-Point Formula

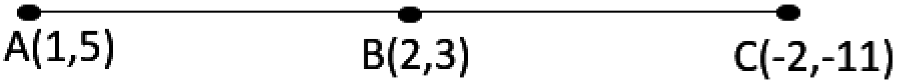
If P(x, y) is the mid-point of the line segment AB, which divides AB in the ratio of 1:1, then the coordinates of the point P(x, y) will be:

Coordinate of P = $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$, as m = 1 and n = 1

Important Questions and Solutions

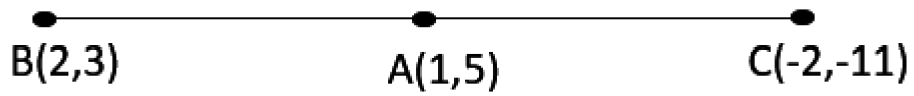
1. Determine if the points (1, 5), (2, 3) and (– 2, – 11) are collinear.

Solution: Let the points (1, 5), (2, 3), and (- 2,-11) be representing the vertices A, B, and C of the given triangle respectively.
Let A = (1, 5), B = (2, 3) and C = (- 2,-11)
Case 1)



$$\therefore AB = \sqrt{(1-2)^2+(5-3)^2} = \sqrt{5}$$
$$BC = \sqrt{(2-(-2))^2+(3-(-11))^2} = \sqrt{4^2+14^2} = \sqrt{16+196} = \sqrt{212}$$
$$CA = \sqrt{(1-(-2))^2+(5-(-11))^2} = \sqrt{3^2+16^2} = \sqrt{9+256} = \sqrt{265}$$

Since AB + BC ≠ CA
Case 2)
Now,

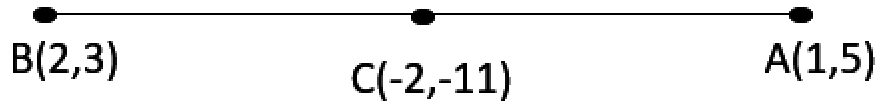


$$BA = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{5}$$
$$AC = \sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$$
$$BC = \sqrt{(2-(-2))^2 + (3-(-11))^2} = \sqrt{(4)^2 + (14)^2} = \sqrt{16+196} = \sqrt{212}$$

BA + AC ≠ BC

Case 3)

Now,



$$BC = \sqrt{(2-(-2))^2 + (3-(-11))^2} = \sqrt{(4)^2 + (14)^2} = \sqrt{16+196} = \sqrt{212}$$
$$CA = \sqrt{(1+2)^2 + (5+11)^2} = \sqrt{(3)^2 + (16)^2} = \sqrt{9+256} = \sqrt{265}$$
$$BA = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{5}$$

As BC+CA≠BA

As three of the cases are not satisfied.

Hence the points are not collinear.

2. Find the point on the x-axis which is equidistant from (2, –5) and (–2, 9).

Solution:

We have to find a point on *x*-axis.

Hence, its *y*-coordinate will be 0

Let the point on *x*-axis be (x, 0)

By distance formula, Distance between two points A(x_1, y_1) and B(x_2, y_2) is

AB = $\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

Distance between (x, 0) and (2, -5) = $\sqrt{[(x- 2)^2 + (0+ 5)^2]}$

= $\sqrt{[(x- 2)^2 + (5)^2]}$

Distance between (x, 0) and (-2, 9) = $\sqrt{[(x + 2)^2 + (0 + 9)^2]}$

= $\sqrt{[(x + 2)^2 + (9)^2]}$

By the given condition, these distances are equal in measure

$\sqrt{[(x - 2)^2 + (5)^2]} = \sqrt{[(x + 2)^2 + (9)^2]}$
Squaring both sides we get,

$(x - 2)^2 + 25 = (x + 2)^2 + 81$

$\Rightarrow x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$
$\Rightarrow (x^2 - x^2) - 4x - 4x = 81 - 25$
$\Rightarrow - 8 x = 56$

$\Rightarrow 8x = -56$

$\Rightarrow x = - 7$

Therefore, the point is (- 7, 0)

3. Find the values of y for which the distance between the points P(2, – 3) and Q (10, y) is 10 units.

Solution
By distance formula, Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$AB = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

Therefore,

$\sqrt{[(2 - 10)^2 + (-3- y)^2]} = 10$

$\sqrt{[(-8)^2 + (3+ y)^2]} = 10$

Squaring both sides to remove the square root,

$\Rightarrow 64 + (y + 3)^2 = 100$

$\Rightarrow (y + 3)^2 = 100 - 64$

$\Rightarrow (y + 3)^2 = 36$

$\Rightarrow y + 3 = \pm 6$

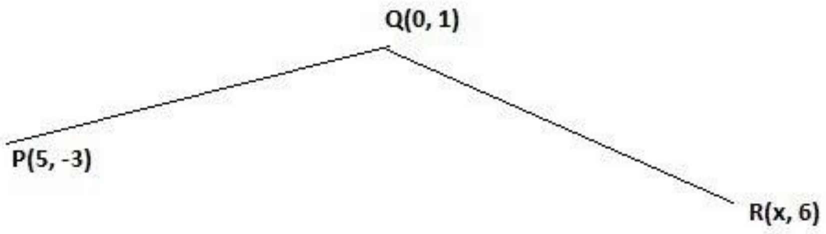
⇒ (y + 3) will give two values 6 and -6 as answer because both the value s when squared will give 36 as answer.)

$y + 3 = 6$ or $y + 3 = -6$

Therefore, y = 3 or -9

4. If Q(0, 1) is equidistant from P(5, –3) and R(x, 6), find the values of x. Also find the distances QR and PR

Solution:



We know, By distance formula distance between two coordinates A(x_1, y_1) and B(x_2, y_2) is

AB = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Now, as

PQ = QR

$\sqrt{[(5 - 0)^2 + (- 3 - 1)^2]} = \sqrt{[(0 - x)^2 + (1 - 6)^2]}$

$\sqrt{25 + 16} = \sqrt{x^2} + 25$

Squaring both sides,

$41 = x^2 + 25$

$x^2 = 16$

x = ± 4

Hence, point R is (4, 6) or (- 4, 6).

When point R is (4, 6)

PR = $[(5 - 4)^2 + (-3 - 6)^2]^{1/2}$

$= \sqrt{1 + 81}$

$= \sqrt{82}$

QR = $[(0 - 4)^2 + (1 - 6)^2]^{1/2}$

$= \sqrt{16 + 25}$

= $\sqrt{41}$

When point R is (- 4, 6)

PR = $[(5 + 4)^2 + (-3 - 6)^2]^{1/2}$

= $\sqrt{81 + 81}$

= 9√2

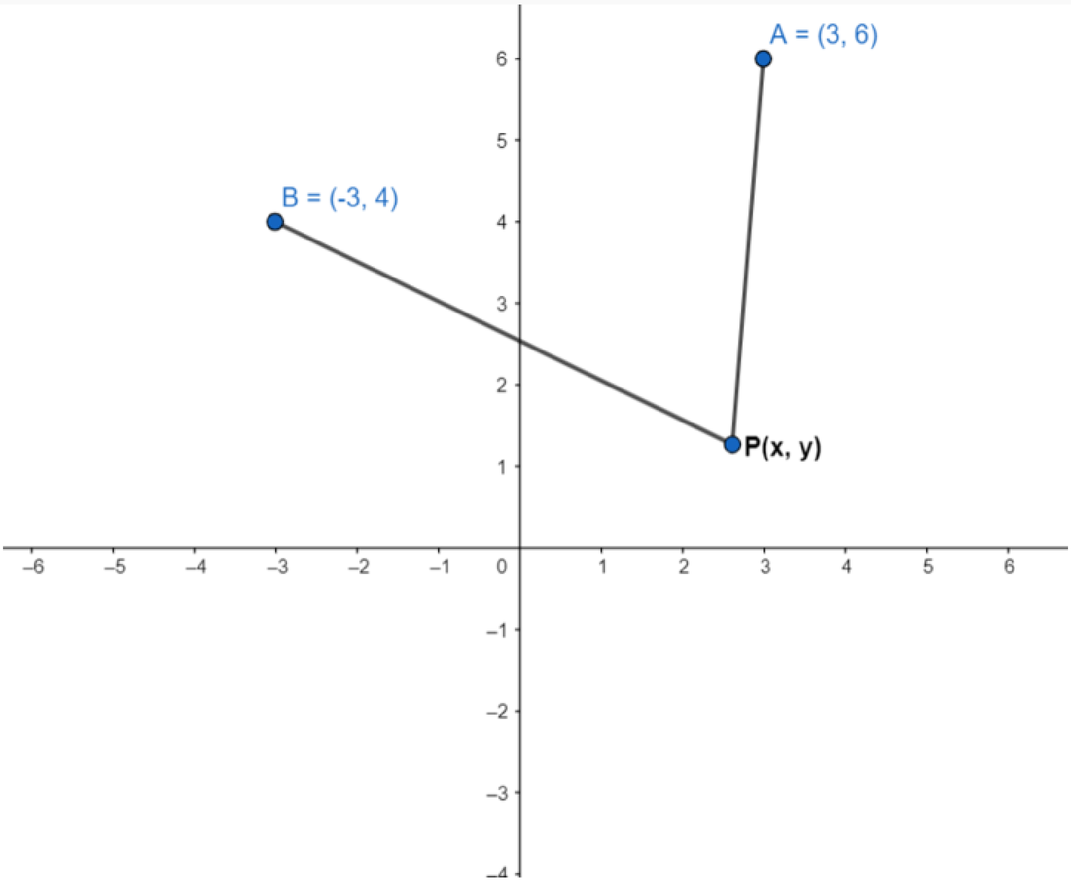
QR = $[(0 + 4)^2 + (1 - 6)^2]^{1/2}$

= $\sqrt{16 + 25}$

= $\sqrt{41}$

5. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (– 3, 4).

Solution:



From the figure it can be seen that Point (*x*, *y*) is equidistant from (3, 6) and (- 3, 4)
This means that the distance of (x, y) from (3, 6) will be equal to distance of (x, y) from (- 3, 4)

We know by distance formula that, distance between two points A(x_1, y_1), B(x_2, y_2) is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Therefore,

$\sqrt{[(x- 3)^2 + (y - 6)^2]} = \sqrt{[(x + 3)^2 + (y- 4)^2]}$

Squaring both sides, we get

$(x - 3)^2 + (y - 6)^2 = (x + 3)^2 + (y - 4)^2$

$x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$

$36 - 16 = 6x + 6x + 12y - 8y$

$20 = 12x + 4y$

$3x + y = 5$

$3x + y - 5 = 0$ is the relation between x and y

6. Find the coordinates of the points of trisection of the line segment joining (4,−1) and (−2, −3).

Solution:

The points of trisection means that the points which divide the line in three equal parts. From the figure C, and D are these two points.
Let C (x_1, y_1) and D (x_2, y_2) are the points of trisection of the line segment joining the given points i.e., BC = CD = DA
Let BC = CD = DA = k
Point C divides the BC and CA as:
BC = k
CA = CD + DA
= k + k
= 2k
Hence ratio between BC and CA is:

$$\frac{BC}{CA} = \frac{k}{2k} = \frac{1}{2}$$

Therefore, point C divides BA internally in the ratio 1:2

then by section formula we have that if a point P(x, y) divide two points P (x_1, y_1) and Q (x_2, y_2) in the ratio m:n then,
the point (x, y) is given by

$$(x,\ y)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$$

Therefore C(x, y) divides B(−2, −3) and A(4,−1) in the ratio 1:2, then

$$C(x,y)=\frac{(1\times 4)+(2\times -2)}{1+2},\frac{(1\times -1)+(2\times -3)}{1+2}$$

$$C(x,y)=\frac{4-4}{1+2},\frac{-1-6}{1+2}$$

$$C(x,y)=0,\frac{-7}{3}$$

Point D divides the BD and DA as:
DA = k
BD = BC + CD
= k + k
= 2k
Hence ratio between BD and DA is:

$$\frac{BD}{DA}=\frac{2k}{k}=\frac{2}{1}$$

The point D divides the line BA in the ratio 2:1
So now applying section formula again we get,

$$D(x,y)=\frac{(2\times 4)+(1\times -2)}{2+1},\frac{(2\times -1)+(1\times -3)}{2+1}$$

$$D(x,y)=\frac{8-2}{3},\frac{-2-3}{3}$$

$$D(x,y)=\frac{6}{3},\frac{-5}{3}$$

$$D(x,y)=2,\frac{-5}{3}$$

7. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

Solution:
Let the ratio in which the line segment joining (- 3, 10) and (6, - 8) is divided by point (- 1, 6) be *k* : 1

Using section formula

i.e. the coordinates of the points P(x, y) which divides the line segment joining the points A(x_1, y_1) and B(x_2,y_2), internally in the ratio m : n are

$$\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

$$(-1,6) = \left(\frac{k(6)+1(-3)}{k+1}, \frac{k(-8)+1(10)}{k+1}\right)$$

Therefore,

$-1 = \frac{6k-3}{k+1}$

⇒ -k – 1 = 6k – 3

⇒ - k -6k = -3 + 1

⇒ - 7k = -2

⇒ 7k = 2

⇒ k = $\frac{2}{7}$

Therefore, the required ratio is 2: 7.

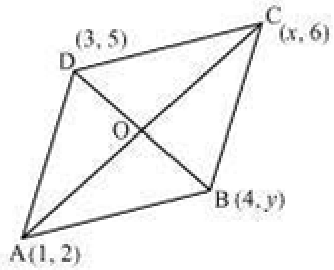
8. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Solution:

Let (1, 2), (4, *y*), (*x*, 6), and (3, 5) are the coordinates of A, B, C, D vertices of a parallelogram ABCD.

Intersection point O of diagonal AC and BD also divides these diagonals.

Therefore, O is the mid-point of AC and BD.



By mid point formula, If (x, y) is the midpoint of the line joining points A(x_1, y_1) and B(x_2, y_2) Then,

$$(x,\ y)\ =\ (\frac{x_1\ +\ x_2}{2},\ \frac{y_1\ +\ y_2}{2})$$

If O is the mid-point of AC, then the coordinates of O are:

$(\frac{1+x}{2}, \frac{2+6}{2}) = (\frac{x+1}{2}, 4)$

If O is the mid-point of BD, then the coordinates of O are:

$(\frac{4+3}{2}, \frac{5+y}{2}) = (\frac{7}{2}, \frac{5+y}{2})$

Since both the coordinates are of the same point O

Comparing the x coordinates we get,

$$\frac{x+1}{2} = \frac{7}{2}$$

x + 1 = 7

x = 6

And,

Comparing the y coordinates we get,

$\frac{5+y}{2} = 4$

5 + y = 8

y = 3

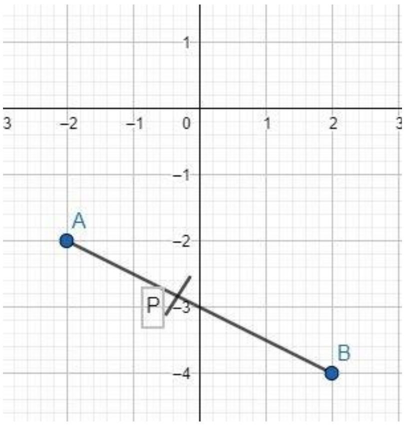
Hence, x = 6 and y = 3.

9. If A and B are (– 2, – 2) and (2, – 4), respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB

Solution:

The coordinates of point A and B are (- 2, - 2) and (2, - 4) respectively

$AP = \frac{3}{7}AB$



Therefore, AP: PB = 3:4

Point P divides the line segment AB in the ratio 3:4
By section formula,

If a point divides the point (x_1, y_1) and (x_2, y_2) in the ratio m:n

then,

$$(X,\ Y) = (\frac{mx_2 + nx_1}{m + n},\ \frac{my_2 + ny_1}{m + n})$$

$$(X,\ Y) = (\frac{3 \times 2 + 4 \times (-2)}{3 + 4},\ \frac{3 \times (-4) + 4 \times (-2)}{3 + 4})$$

$$(X,\ Y) = (\frac{-2}{7},\ \frac{-20}{7})$$

(-2/7, -20/7) is the point which divides line in the ratio of 3:4.

10. Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A (2, – 2) and B (3, 7)

Solution:
Let the given line divide the line segment joining the points A(2, - 2) and B(3, 7) in a ratio *k* : 1

Coordinates of the point of division = $(\frac{3k+2}{k+1}, \frac{7k-2}{k+1})$

This point also lies on 2*x* + *y* - 4 = 0

Therefore,

$$2\ (\frac{3k+2}{k+1}) + (\frac{7k-2}{k+1}) - 4 = 0$$

$\frac{6k+4+7k-2-4k-4}{k+1} = 0$

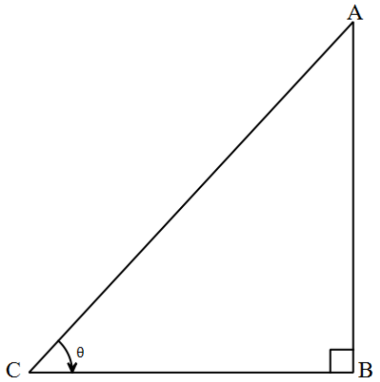
$9k - 2 = 0$

$k = \frac{2}{9}$

Hence, the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, - 2) and B(3, 7) is **2:9**.

Chapter: Introduction to Trigonometry

Let us consider a right-angled triangle ABC, right angled at B (as shown below) and ∠C = θ, where 0° < θ < 90°. The three sides of the ΔABC are:



AB = Perpendicular (side opposite to θ)

BC = Base (side on which both θ and 90° angles lie)

AC = Hypotenuse (side opposite to 90°)

Now, the ratio of these sides are termed as trigonometric ratios. As, there are three sides, so maximum there can be six ratios. All these six ratios are given different names. The various trigonometric ratios are as follows:

$\text{Sine } \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$\text{Cosine } \theta = \frac{\text{Base}}{\text{Hypotenuse}}$

$\text{tangent } \theta = \frac{\text{Perpendicular}}{\text{Base}}$

$\text{cosecant } \theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

$\text{secant } \theta = \frac{\text{Hypotenuse}}{\text{Base}}$

cotangent θ = $\frac{\text{Base}}{\text{Perpendicular}}$

The ratios defined above are abbreviated as sin θ, cos θ, tan θ, cosec θ, sec θ and cot θ respectively. Note that the ratios cosec θ, sec θ and cot θ are respectively, the reciprocals of the ratios sin θ, cos θ and tan θ.

sin θ = $\frac{1}{\text{cosec } \theta}$

cos θ = $\frac{1}{\sec \theta}$

tan θ = $\frac{1}{\cot \theta}$

Also, tan θ = $\frac{\sin \theta}{\cos \theta}$

The following table shows the value of different trigonometric ratios for different angles.

∠A	0°	30°	45°	60°	90°
sin A	0	1/2	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	1/2	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot a	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Clearly for the angles between 0° to 90°, the value of sine increases from 0 to 1 and the value of cosine decreases from 1 to 0.

The basic trigonometric identities which are very useful in proving other identities are:

$$\sin^2 \theta + \cos^2 \theta = 1$$

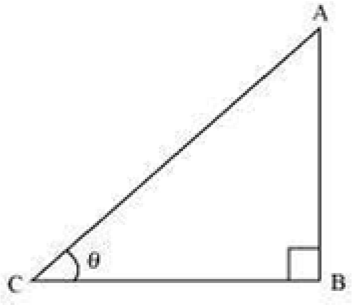
$$\tan^2\theta + 1 = \sec^2\theta$$

$1 + \cot^2\theta = \mathrm{cosec}^2\theta$

Important Questions and Solutions

1. If $\cot\,\theta = \frac{7}{8}$, evaluate: $\frac{(1+\sin\,\theta)(1-\sin\,\theta)}{(1+\cos\,\theta)(1-\cos\,\theta)}$,

Solution: Let us consider a right triangle ABC, right-angled at point B



Given:

$$\cot\,\theta = \frac{Base}{Perpendicular}$$

$$\cot\,\theta = \frac{7}{8}$$

Let
Base = 7
Perpendicular = 8
According to Pythagoras theorem,
$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8)^2 + (7)^2$

$= 64^2 + 49^2$

$= 113^2$

$AC = \sqrt{113}$

Now we have all the three sides of the triangle,
Base = 7
Perpendicular = 8
Hypotenuse = $\sqrt{113}$

Now applying other trigonometric angle formulas

$$\sin\ \theta\ =\ \frac{Perpendicular}{Hypotenuse}$$

Sin θ = $\frac{AB}{AC}$

= $\frac{8}{\sqrt{113}}$

$$\cos\ \theta\ =\ \frac{Base}{Hypotenuse}$$

Cos θ = $\frac{BC}{AC}$

= $\frac{7}{\sqrt{113}}$

Putting the obtained trigonometric ratios into the expression we get,

$\frac{(1+sin\theta)(1-\sin\theta)}{(1+\cos\theta)\ (1-\cos\theta)}$

= $(1 - \sin^2 \theta)/(1 - \cos^2 \theta)$

$$\frac{(1\ +\ \sin\ \theta)(1\ -\ \sin\ \theta)}{(1\ +\ \cos\ \theta)(1\ -\ \cos\ \theta)} =\ \cot^2\ \theta \qquad [\ 1\ -\ \sin^2\ \theta\ =\ \cos\ ^2\ \theta,\ 1\ -\ \cos^2\ \theta\ =\ \sin^2\ \theta]$$

= $\frac{49}{64}$

2. If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$; $0° < A+B \le 90°; A > B,$ find A and B

Solution:
Now, we know that

$$\tan\ 30^{\circ}\ =\ \frac{1}{\sqrt{3}}$$

$$\tan\ 60^{\circ}\ =\ \sqrt{3}$$

So, from the question we can tell,

tan (A + B) = $\sqrt{3}$

tan (A + B) = tan 60^{o}

A + B = 60° ....eq (i)

And

tan (A – B) = $\frac{1}{\sqrt{3}}$

tan (A - B) = tan 30°

A - B = 30 ....eq(ii)

On adding both equations, we obtain

2A = 90°

⇒ A = 45

From equation (i), we obtain

45 + B = 60

B = 15°

Therefore, ∠A = 45° and ∠B = 15°

3. Prove $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta$

Solution: $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta$

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$$

$$=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{sin^3\theta - cos^3\theta}{cos\theta\ \sin\theta(sin\theta - cos\theta)}$$

Use the formula $a^3 - b^3 = (a^2 + b^2 + ab)(a - b)$

$$= \frac{sin\theta - cos\theta(sin^2\theta + cos^2\theta + \sin\theta\ \cos\theta)}{cos\theta\ sin\theta(sin\theta - cos\theta)}$$

Cancelling (sin θ - cos θ) from numerator and denominator

$$= \frac{1 + sin\theta\ cos\theta}{cos\theta\ sin\theta}$$

$$= \frac{1}{cos\theta\ sin\theta} + \frac{sin\theta\ cos\theta}{cos\theta\ sin\theta}$$

As cosθ = 1/secθ and sinθ = 1/cosecθ

= 1 + sec θ cosec θ

= RHS

Hence, proved.

4. Prove $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

Solution: $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

LHS

$$\frac{1+\sec A}{\sec A}$$

use the formula secA = 1/cosA

$$= \frac{1+\frac{1}{cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\cos A+1}{cos A}}{\frac{1}{cos A}}$$

= cos A + 1

RHS

$$\frac{\sin^2 A}{1-\cos A}$$

Use the identity $\sin^2\theta + \cos^2\theta = 1$

$$=\frac{1-\cos^2 A}{1-cos A}$$

Use the formula $a^2 - b^2 = (a - b)(a + b)$

$$=\frac{(1-cos A)(1+\cos A)}{1-cos A}$$

= cos A + 1

LHS = RHS

Hence, proved.

5. Prove $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \cos ec\, A + \cot A,$

Solution:

$$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \cos ec\, A + \cot A,$$

LHS

$$\frac{\cos A - \sin A + 1}{cos A + sin A - 1}$$

Dividing Numerator and Denominator by sin A

$$=\frac{\frac{cos A - sin A + 1}{\sin A}}{\frac{cos A + sin A - 1}{\sin A}}$$

$$=\frac{\frac{\cos A}{\sin A}-\frac{\sin A}{\sin A}+\frac{1}{\sin A}}{\frac{cos A}{sin A}+\frac{sin A}{sin A}-\frac{1}{sin A}}$$

Use the formula $\cot\theta = \cos\theta / \sin\theta$

$$=\frac{\cot A-1+\operatorname{cosec} A}{\cot A+1-\operatorname{cosec} A}$$

Using the identity $\cos ec^2 A = 1 + \cot^2 A.$

$$=\frac{\cot A-\left(\operatorname{cosec}^2 A-\cot^2 A\right)+\operatorname{cosec} A}{\cot A+1-\operatorname{cosec} A}$$

$$=\frac{(cot A+cosec A)-\left(cosec^2 A-cot^2 A\right)}{cot A+1-cosec A}$$

Use the formula $a^2 - b^2 = (a - b)(a + b)$

$$=\frac{(cot A+cosec A)-\left[(cosec A-cot A)(\operatorname{cosec} A+\cot A)\right]}{cot A+1-cosec A}$$

$$=\frac{(cot A+cosec A)\left[1-(cosec A-cot A)\right]}{cot A+1-cosec A}$$

$$=\frac{(cot A+cosec A)(1-cosec A+cot A)}{cot A+1-cosec A}$$

= cot A + cosec A

= RHS

6. Prove $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Solution: $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Dividing numerator and denominator of LHS by cos A

$$\sqrt{\frac{\frac{1}{\cos A}+\frac{\sin A}{\cos A}}{\frac{1}{cos A}+\frac{sin A}{cos A}}}$$

As cosθ = 1/secθ and tanθ = sinθ /cosθ

$$=\sqrt{\frac{\sec A+\tan A}{\sec A-\tan A}}$$

Rationalize the square root to get,

$$=\sqrt{\frac{(secA+tanA)\times(secA+tanA)}{(secA-tanA)\times(secA+tanA)}}$$

Use the formula $a^2 - b^2 = (a - b)(a + b)$ to get,

$$=\sqrt{\frac{(secA+tanA)^2}{\left(sec^2A-tan^2A\right)}}$$

$$=\frac{\sqrt{(secA+tanA)^2}}{\sqrt{sec^2A-tan^2A}}$$

Use the identity $sec^2\theta = 1 + tan^2\theta$ to get,

$$=\frac{secA+tanA}{1}$$

= sec A + tan A

= RHS

Hence, proved.

7. Prove $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

Solution:

LHS

$$\frac{\sin\theta-2\ \sin^3\theta}{2\ \cos^3\theta-\cos\theta}$$

$$=\frac{sin\theta\left(1-2\,sin^2\theta\right)}{\cos\theta\left(2\,cos^2\theta-cos\theta\right)}$$

Since $\sin^2\theta = 1 - \cos^2\theta$

$$=\frac{sin\theta\left(1-2\,(1-cos^2\theta\right)}{cos\theta\left(2\,cos^2\theta-1\right)}$$

$$=\frac{sin\theta\left(1-2\ +2cos^2\theta\right)}{cos\theta\left(2\ cos^2\theta-1\right)}$$

$$=\frac{sin\theta(2cos^2\theta-1)}{cos\theta\left(2\ cos^2\theta-1\right)}$$

As tanθ = sinθ/cosθ
= tanθ
= R.H.S
Hence, Proved.

8. Prove $(\sin A+\operatorname{cos ec} A)^2\ (\cos A+\sec A)^2\ =7+\tan^2 A+\cot^2 A$

Solution:
LHS = (sin A + cosec A)^2 + (cos A + sec A)^2

Use the formula (a+b)^2 = a^2 + b^2 + 2ab to get,

= (sin^{2}A + cosec^{2}A + 2sin A cosec A) + (cos^{2}A + sec^{2}A + 2 cos A sec A)
Since sinθ=1/cosecθ and cosθ = 1/secθ

$$=\left(sin^2A+cosec^2A+2\ sinA\ \frac{1}{sinA}\right)+\left(cos^2A+sec^2A+2\ cosA\ \frac{1}{cosA}\right)$$

= sin^{2}A + cosec^{2}A + 2 + cos^{2}A + sec^{2}A + 2
=(sin^{2}A + cos^{2}A)+cosec^{2}A+sec^{2}A+2+2
Use the identities sin^{2}A + cos^{2}A = 1, sec^{2}A = 1 + tan^{2}A and cosec^{2}A = 1 + cot^{2}A to get

= 1+ 1 + tan^{2}A + 1 + cot^{2}A + 2 + 2

= 1 + 2 + 2 + 2 + tan^{2}A + cot^{2}A

= 7 + tan^{2}A + cot^{2}A

= RHS

Hence, proved.

9. Prove $(\operatorname{cos ec} A-\sin A)(\sec A-\cos A))\ =\frac{1}{\tan A+\cot A}$

Solution:

To Prove:

$$(\operatorname{cos ec} A-\sin A)(\sec A-\cos A)=\frac{1}{\tan A+\cot A}$$

[Hint: Simplify LHS and RHS separately]

Proof:
LHS = (cosec A – sin A) (sec A – cos A)
Use the formula sinθ = 1/cosecθ and cosθ = 1/secθ

$$= (\frac{1}{\sin\ A} - \sin\ A)\ (\frac{1}{\cos\ A} - \cos\ A)$$

$$= \frac{(1 - sin^2\ A)}{\sin\ A} \times \frac{(1 - \cos^2\ A)}{\cos\ A}$$

$$= \frac{\cos^2\ A}{\sin\ A} \times \frac{\sin^2\ A}{\cos\ A} \qquad [\ (1 - \sin^2\ A) = \cos^2\ A]$$

$$[\ (1 - \cos^2\ A) = \sin^2\ A]$$

= cos A sin A

RHS

$$\frac{1}{\tan\theta + \cot\theta}$$

use the formula tanθ=sinθ/cosθ and cotθ=cosθ/sinθ

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{sin^2 A + \cos^2 A}{cos A\ \sin A}}$$

$$= \frac{\cos A\ \sin A}{\sin^2 A + \cos^2 A}$$

Use the identity $\sin^2 A + \cos^2 A = 1$

$$= \frac{cos A\ sin A}{1}$$

= cos A sin A

LHS = RHS

Hence, proved.

10. Prove $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Solution:

To Prove:

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$$

Proof:
Taking left most term
Since, cot A is the reciprocal of tan A, we have

$$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}}$$

$$=\frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}}$$

$$=\frac{1}{\frac{1}{\tan^2 A}}$$

$$=\tan^2 A$$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2=\left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$=\left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2$$

$$=\left(\frac{1}{\frac{-1}{\tan A}}\right)^2$$

$$=(-\tan A)^2$$

$$=\tan^2 A$$

Hence, Proved.