Real Numbers

Euclid's division lemma: For any two given positive integers m and n, there exist unique whole numbers q and r such that,

$$m = nq + r, \text{ where } 0 \leq r < n$$

Here, m is called as dividend, n as divisor, q as quotient and r as remainder.

Therefore,

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

Fundamental theorem of arithmetic

Every composite number can be expressed as a product of primes and this expression is unique, except from the order in which the prime factors occur.

Product of two given numbers = Product of their HCF and LCM

$m \times n = \text{HCF}(m,n) \times \text{LCM}(m,n)$

Polynomials

Zero of a Polynomial

A real number *k* is a zero of a polynomial f(x), if f(k) = 0. Which means that on substituting the given real number in a polynomial in place of. Which means that if on substituting the given real number in place of the variable in a polynomial gives zero, then the given real number is termed as the zero of the given polynomial.

Zero of a linear polynomial ax + b $= -\frac{\text{Constant term}}{\text{Coefficient of x}} = -\frac{b}{a}$

Relationship between zeroes and coefficient of a quadratic polynomial

Let α and β be the two zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$, where $a \neq 0$.

Then, sum of zeros = $\alpha + \beta = -\frac{\text{Coefficient of x}}{\text{Coefficient of } x^2} = -\frac{b}{a}$

Product of zeros = $\alpha.\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$

Remember:
If α and β are the zeros of a quadratic polynomial f(x). Then, polynomial f(x) is given by:
$f(x) = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$
$f(x) = k\{x^2 - (\text{Sum of zeros})x + \text{Product of zeros}\}$

Relationship between zeroes and coefficient of a cubic polynomial

Let α, β and γ be the zeros of a cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

Then, sum of zeros $= \alpha + \beta + \gamma = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{b}{a}$

Sum of product of zeros $= \alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a}$

Product of zeros $= \alpha\beta\gamma = -\frac{\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{d}{a}$

Remember:
If α, β and γ are the zeros of a quadratic polynomial f(x). Then, polynomial f(x) is given by:
$f(x) = k\{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$
$f(x) = k\{x^3$ - (sum of zeroes)x^2 + (Sum of the product of roots taken two at a time)x - Product of Roots}

Division algorithm for polynomials

If f(x) and g(x) are any two polynomials with $g(x) \neq 0$, and degree of g(x) is less than or equal to f(x), then we can always find polynomials q(x) and r(x) such that,

$f(x) = q(x)g(x) + r(x)$, where $r(x) = 0$ or degree of r(x) < degree of g(x).

Pair of Linear Equation in two variables

Two Linear Equations having two same variables are known as the pair of Linear Equations in two variables.

$$\mathbf{a_1x + b_1y + c_1 = 0}$$
$$\mathbf{a_2x + b_2y + c_2 = 0}$$

Nature of 2 straight lines in a plane

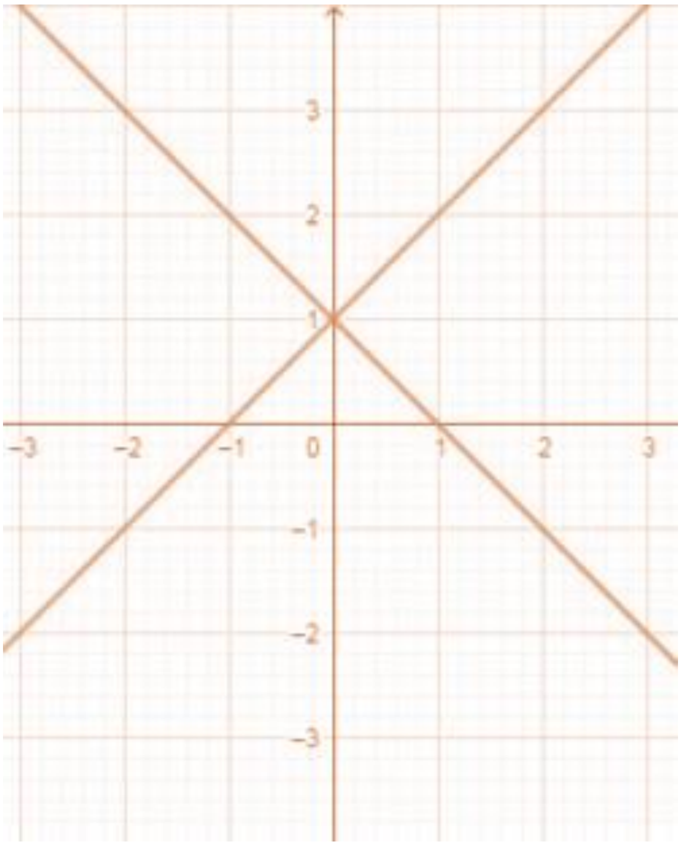
Consider a pair of linear equation in one variable as follows:

$$\mathbf{a_1x + b_1y + c_1 = 0}$$
$$\mathbf{a_2x + b_2y + c_2 = 0}$$

For a pair of straight lines on a plane, there are three possibilities

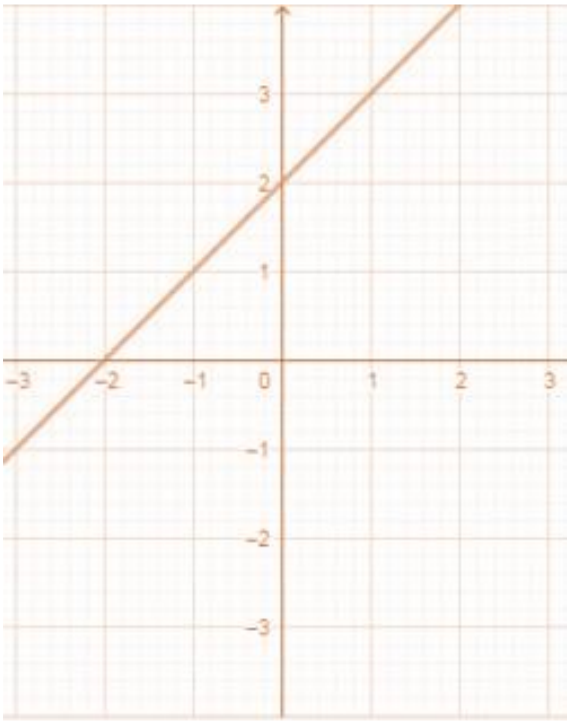
i) They intersect at exactly one point

When $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of equation is called consistent. The lines will intersect at a single point in the graph (as shown in graph below) and the pair of equation will have a unique solution.



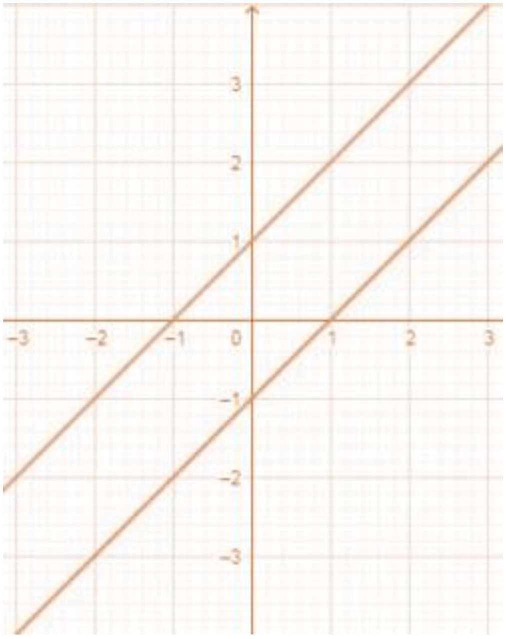
ii) They are coincident

When $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the pair of equation is again called consistent. The lines will coincide or overlap(as shown in graph below) and the pair of equation will have infinitely many solutions.



iii) They are parallel

When $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the pair of equation is called inconsistent. The lines will never intersect each other and will run parallel (as shown in graph below) and the pair of equation will have no solution.



Quadratic Equations

Quadratic Polynomial

A polynomial, whose degree is 2, is called a quadratic polynomial. It is in the form of **$p(x) = ax^2 + bx + c$,** where $a \neq 0$

Quadratic Equation

When we equate the quadratic polynomial to any constant then it is called a Quadratic Equation i.e. if **p(x) = k**, where k is any constant.

Example: $2x^2 = 5$, $y^2 - 3y + 7 = 8$ etc

Standard form of Quadratic Equation

The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b, c are the real numbers and $a \neq 0$.

Roots of a Quadratic Equation

As the degree of a quadratic equation is 2, it means it has two roots. Now, roots are those values of the variable which satisfies the given quadratic equation.

Let $x = \alpha$ where α is a real number. If α satisfies the Quadratic Equation $ax^2 + bx + c = 0$ such that $a\alpha^2 + b\alpha + c = 0$, then α is one of the roots of the Quadratic Equation.

Remember:

1. If α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, then:

Sum of roots $= \alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{b}{a}$

Product of roots $= \alpha \times \beta = \frac{\text{Constant Term}}{\text{Coefficient of } x^2} = \frac{c}{a}$

2. If α and β are the roots of a quadratic equation in x, then the quadratic equation is given by:

K{x^2 –(sum of roots) + product of roots}

One more method of finding the roots of a quadratic equation is by Quadratic formula. Let $ax^2 + bx + c = 0$ be a quadratic equation in the standard form, where $a \neq 0$, then its roots are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $b^2 - 4ac = D$ known as Discriminant.

Discriminant helps in identifying the nature of roots. We will discuss about it in detail in the next class. But remember that we will workout the roots of the quadratic equations whose discriminant is either equals to or greater than zero.

The quadratic formula can also be re-written in the form of D as:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

If D < 0, roots are not real or imaginary roots

Arithmetic Progression (A.P.)

Arithmetic Progression is a set of numbers in which the difference of a term and its predecessor is always constant, i.e., $a_n - a_{n-1} = d$, where d is the common difference.

The successive terms of an *A.P.* are a, $a + d$, $a + 2d$, $a + 3d$, $a + 4d$ ……….

Choice of terms in A.P.

No. of terms	Terms	Common difference
3	$a - d, a, a + d$	d
4	$a - 3d, a - d, a + d, a + 3d$	$2d$
5	$a - 2d, a - d, a, a + d, a + 2d$	d
6	$a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$	$2d$

A sequence which is in A.P. follows certain properties as follows –

1. The sum of terms of an A.P. equidistant from the beginning and the end of the A.P is constant and equal to the sum of the first and last term of the A.P.

   a, a+d, a+2d,…………a+(n-3)d, a+(n-2)d, a+(n-1)d

2. The sum of the third term from the beginning and the end is

   $(a + 2d) + a + (n-3)d = 2a + (n - 1)d$

3. The sum of the first and last terms is: $a + a + (n - 1)d = 2a + (n - 1)d$

4. If each term of an A.P is increased decreased, multiplied or divided by the same non zero number, then the resulting sequence is also in an A.P.

5. The sum or difference of two A.P.s is an A.P.

6. If the terms are selected at a regular interval from within the A.P, the resulting sequence is also in an A.P.

7. If three numbers a, b and c are in A.P. then $b - a = c - b$, $2b = a + c$.

Sum of n terms of an A.P

$s_n = \frac{n}{2}[2a + (n-1)d]$

Or

$s_n = \frac{n}{2}[a + l]$

Where,

a = First term,

d = common difference

n = number of terms

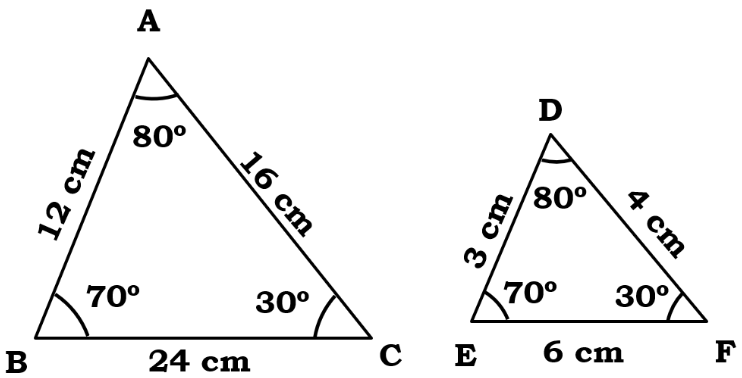
l = last term

S_n = Sum of n terms

Also, $a_n = s_n - s_{n-1}$

Triangles

Two triangles are said to be similar when they have the same shape but their sizes are proportional.



When two triangles are similar, we denote it as follows –

$$\Delta\ ABC \sim \Delta\ DEF$$

This implies that all the corresponding angles of the two triangles are equal and their corresponding sides are in the same ratio.

$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

And, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Similarity in triangles can also be checked by various criteria, like congruence. Some common criteria for similarity are as follows –

(i) **SSS similarity** – Unlike the congruency, since in case of similarity, the sides are not equal but proportional, if in two triangles, all the three corresponding sides are proportional, then the triangles are said to be similar by SSS criterion.

(ii) **AAA/AA similarity –** If in two triangles, all the three corresponding angles are equal, then they are said to be similar by AAA criterion.

(iii) **SAS similarity –** If in two triangles, a pair of corresponding sides is proportional and the included angle between them is equal, then the triangles are said to be similar.

Ratio of area of two similar triangles

The theorem states that, **"The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides".**

i.e., If Δ ABC ~ Δ DEF, then

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

The Pythagoras theorem states that, **"In a right - angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides".**

$\text{Hypotenuse}^2 = \text{Perpendicular}^2 + \text{Base}^2$

Coordinate Geometry

Distance Formula

The distance between any two points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is calculated by:

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Section Formula

If the point P(x,y) **divides** the line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$ **internally** in the **ratio m:n**, then, the coordinates of P are given by the **section formula** as

$$P(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

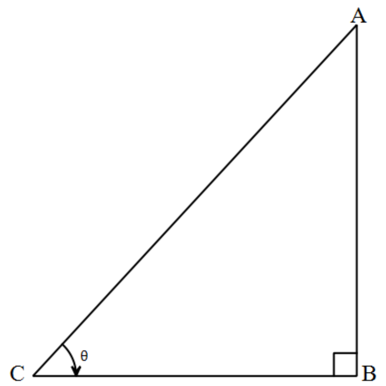
Mid-Point Formula

If P(x, y) is the mid-point of the line segment AB, which divides AB in the ratio of 1:1, then the coordinates of the point P(x, y) will be:

Coordinate of P = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, as m = 1 and n = 1

Introduction to Trigonometry

Let us consider a right-angled triangle ABC, right angled at B (as shown below) and ∠C = θ, where 0° < θ < 90°. The three sides of the ΔABC are:



AB = Perpendicular (side opposite to θ)

BC = Base (side on which both θ and 90° angles lie)

AC = Hypotenuse (side opposite to 90°)

Sine θ = $\frac{\text{Perpendicular}}{\text{Hypotenuse}}$

Cosine θ = $\frac{\text{Base}}{\text{Hypotenuse}}$

tangent θ = $\frac{\text{Perpendicular}}{\text{Base}}$

cosecant θ = $\frac{\text{Hypotenuse}}{\text{Perpendicular}}$

secant θ = $\frac{\text{Hypotenuse}}{\text{Base}}$

cotangent θ = $\frac{\text{Base}}{\text{Perpendicular}}$

Relation of trigonometric ratios

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

tan θ = $\frac{1}{\cot\theta}$

Also, tan θ = $\frac{\sin\theta}{\cos\theta}$

The following table shows the value of different trigonometric ratios for different angles.

∠ A	0°	30°	45°	60°	90°
sin A	0	1/2	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	1/2	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot a	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Basic Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

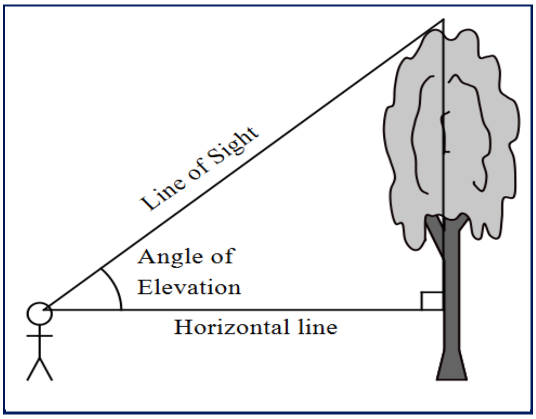
$1 + \cot^2\theta = \text{cosec}^2\theta$

Some Applications of Trigonometry

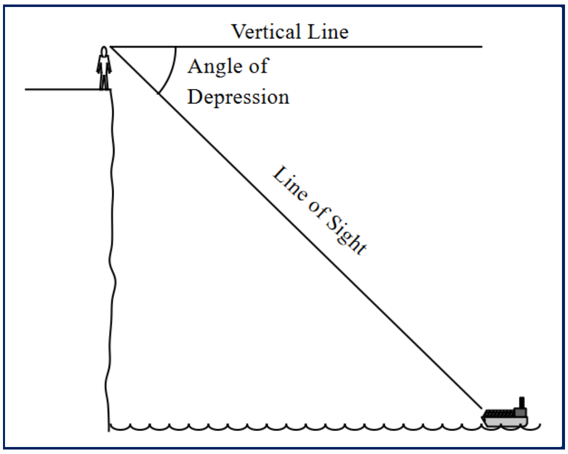
Line of Sight: It is an imaginary straight line from the eye of an observer to the point of the object under observation. It can be inclined upwards as well as downwards.

Horizontal line: It is a line parallel to the ground between the observer and the object.

Angle of elevation: The angle between the line of sight and the horizontal line is termed as angle of elevation. Basically, it is formed when the observer is observing something in the diagonally upward direction (as shown in the diagram below).



Angle of depression: When the observer is looking any object from a height in downward direction, then the angle made between the line of sight and the vertical line is termed as angle of depression (as shown in the diagram below).



Calculating Heights and Distances

To, calculate heights and distances, we can make use of trigonometric ratios.

Step 1: Draw a **line diagram** corresponding to the problem.

Step 2: Mark all known heights, distances and angles and denote unknown lengths by variables.

Step 3: Use the values of various **trigonometric ratios** of the angles to obtain the unknown lengths from the known lengths.

CIRCLES

Theorem 1: "The tangent at any point of a circle is perpendicular to the radius through the point of contact".

Theorem 2: The converse of the above theorem, "A line drawn through the end of a radius and perpendicular to it is a tangent to the circle".

Theorem 3: "The length of tangents drawn from an external point to a circle are equal".

AREA RELATED TO CIRCLES

If r is the radius of a circle, then

(i) Circumference of circle = $2\pi r$ or πd, where $d = 2r$ is the diameter of the circle.

(ii) Area of circle = πr^2 or $\pi d^2/4$

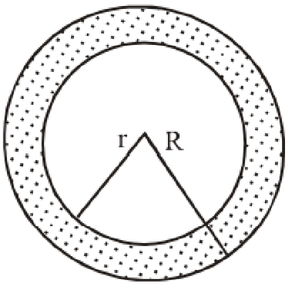
(iii) Area of semicircle = $\frac{1}{2}$ Area of the circle

(iv) Area of the quarter circle = $\frac{1}{4}$ Area of the circle

Area enclosed in a concentric circles

If R and r are the radii of two concentric circles such that $R>r$ **then,**

Area enclosed by the two circles = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$



Some useful results

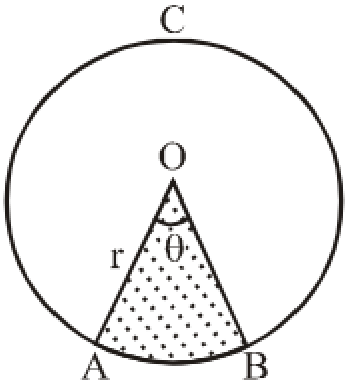
(i) If two circles touch internally, then the distance between their centres is equal to the difference of their radii.

(ii) If two circles touch externally, then the distance between their centres is equal to the sum of their radii.

(iii) Distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.

(iv) The number of revolutions completed by a rotating wheel in one minute = $\frac{\textit{Distance moved in one minute}}{\textit{circumference}}$

Sectors of a circle



Minor sector: A sector of a circle is called a minor sector if the minor arc of the circle is a part of its boundary In Fig. sector OAB is the minor sector.

Major sector: A sector of a circle is called a major sector if the major arc of the circle is a part of its boundary. In Fig. sector OACB is the major sector.

Following are some important points to remember:

(i) A minor sector has an angle θ, subtended at the centre of the circle, whereas a major sector has no angle.

(ii) The sum of the arcs of major and minor sectors of a circle is equal to the circumference of the circle.

(iii) The sum of the areas of major and minor sectors of a circle is equal to the area of the circle.

(iv) The boundary of a sector consists of an arc of the circle and the two radii.

Area of a sector of a circle

If a sector of a circle of radius **r** contains an angle of **θ** Then,

(i) Length of the arc of the sector = $\frac{\theta}{360} \times 2\pi r$

(ii) Area of the sector containing angle θ = $\frac{\theta}{360} \times \pi r^2$

Some useful results to remember

(i) Angle described by minute hand in 60 minutes = 360°

$\therefore$ Angle described by minute hand in one minute = 6°

(ii) Angle described by hour hand in 12 hours = 360°

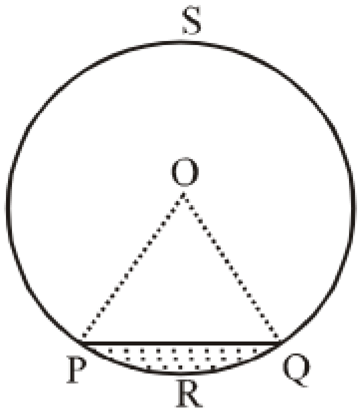
$\therefore$ Angle described by hour hand in one hour = 30°

Segment of a circle

The region enclosed by an arc and a chord is called the segment of the circle.

Minor segment: If the boundary of a segment is a minor arc of a circle, then the corresponding segment is called a minor segment.

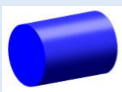
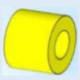
Major segment: A segment corresponding a major arc of a circle is known as the major segment.

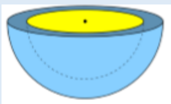
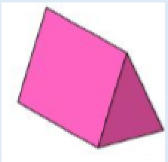
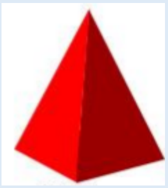


Area of the sector OPRQ = Area of the segment PRQ + Area of ΔOPQ

⇒ Area of segment PRQ = Area of sector OPRQ – Area of ΔOPQ

SURFACE AREA AND VOLUME

Name	Figure	Lateral or Curved Surface Area	Total Surface Area	Volume	Length of diagonal and nomenclature
Cube		$4l^2$	$6l^2$	l^3	√3 l = edge of the cube
Cuboid		2h(l +b)	2(lb + bh + hl)	lbh	$l^2+b^2+h^2$ l = length b = breadth h = height
Cylinder		2πrh	$2\pi r^2 + 2\pi h = 2\pi r(r + h)$	$\pi r^2 h$	r = radius h = height
Hollow cylinder		2πh (R + r)	$2\pi h\,(R + r) + 2\pi h\,(R^2 - r^2)$	$\pi h(R^2 - r^2)$	R = outer radius

					r = inner radius
Cone		$\pi r l = \pi\sqrt{h^2 + r^2}$	$\pi r^2 + \pi r l = \pi r(r + l)$	$1/3\ \pi r^2 h$	r = radius h = height l = slant height $l = \sqrt{h^2 + r^2}$
Sphere		$4\pi r^2$	$4\pi r^2$	$4/3\ \pi r^3$	r = radius
Hemisphere		$2\pi r^2$	$3\pi r^2$	$2/3\ \pi r^3$	r = radius
Spherical shell		$4\pi R^2$ (Surface area of outer)	$4\pi r^2$ (Surface area of outer)	$4/3\ \pi(R^3 - r^3)$	R = outer radius r = inner radius
Prism		Perimeter of base × height	Lateteral surface area + 2(Area of the end surface)	Area of base × height	-
pyramid		1/2 (Perimeter of base) × slant height	Lateral surface area + Area of the base	1/3 area of base × height	-

Whenever a solid is melted and re-casted into a new shape, its volume remains unchanged. This is the only concept that is required for dealing all the questions related to conversion from one form to another.

Some important results:

1. When an object is thrown in water, the amount of water displaced is equal to the volume of the object.
2. If a bigger object is melted and re-casted into smaller object, then the volume of bigger object is equal to the the total volume of the smaller objects.

STATISTICS

Mean of ungrouped data is given by = $\frac{f_1x_1 + f_2x_2 + f_3x_3 + \cdots + f_nx_n}{f_1 + f_2 + f_3 + \cdots + f_n} = \frac{\sum_{i=1}^{i=n} f_i x_i}{\sum_{i=1}^{i=n} f_i}$

Class mark for each class is calculated as: $\frac{\text{Upper class limit + lower class limit}}{2}$

Direct Method

Follow these steps to work out mean using the direct method.

1. Write the class intervals and corresponding frequencies in two separate columns.
2. Calculate the class mark for each class and write it in another column corresponding to the classes.
3. Calculate the product of frequencies and respective class mark and put them in separate column corresponding to the respective class.
4. Now use the following formula to calculate mean using the direct method.

$$\bar{x} = \frac{\sum_{i=1}^{i=n} f_i x_i}{\sum_{i=1}^{i=n} f_i}$$

Where, f_i = frequencies of different classes

x_i = class mark of different classes

Assumed Mean Method:

Sometimes when the numerical values of x_i and f_i are large, finding the product of x_i and f_i becomes tedious and time consuming. So, for such situations, let us think of a method of reducing these calculations.

1. The first step is to choose one among the x_i's as the assumed mean, and denote it by 'a'. Also, to further reduce our calculation work, we may take 'a' to be that x_i which lies in the centre of $x_1, x_2, \ldots, x_n$
2. The next step is to find the difference d_i between a and each of the x_i's, that is, the deviation of 'a' from each of the x_i's, which is given by: $d_i = x_i - a$

Mode of Grouped Data

In the ungrouped data the most frequently occurring no. is the mode of the sequence, but in the grouped data we can find the class interval only which has the maximum frequency number i.e**. the modal class.**

The value of mode in that modal class is calculated by

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

l = lower class limit of the modal class

h = class interval size

f_1 =frequency of the modal class
f_0 =frequency of the preceding class
f_2 = frequency of the succeeding class

3. The third step is to find the product of di with the corresponding f_i , and take the sum of all the $f_i d_i$'s.

4. Now use the following formula to calculate the mean.

$$\bar{x} = a + \frac{\sum_{i=1}^{i=n} f_i d_i}{\sum_{i=1}^{i=n} f_i}$$

Where, a is the assumed mean

And $d_i = x_i - a$

Median of Grouped Data

To find the median of a grouped data, we need to find the cumulative frequency and n/2

Then we have to find the median class, which is the class of the cumulative frequency near or greater than the value of n/2.

Cumulative Frequency is calculated by adding the frequencies of all the classes preceding the given class.
Then substitute the values in the formula

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

where l = lower limit of median class

n = no. of observations

cf = cumulative frequency of the class preceding to the median class

f = frequency of the median class

h = size of class

Empirical Formula

The empirical relation between the three measures of central tendency is

3 Median = Mode + 2 Mean

PROBABILITY

Probability is a measure of how likely something is to happen.

Many events can't be predicted with total certainty. The best we can say is how likely they are to happen, using idea of probability.

$$\textbf{Probability of an event} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes (sample space)}}$$

Important points

1. Probability of an event and its compliment is always equal to 1, i.e. $P(E) + P(\bar{E}) = 1$
2. The probability of any event A lies in the range $0 \le P(A) \le 1$