NDA II 2021 Previous Year's Question Paper: Mathematics

1. Ans. A.

Let $x=\sin^{-1}\frac{3}{5}$, then

$$\sin x=\frac{3}{5}, \cos x=\sqrt{1-\left(\frac{3}{5}\right)^2}=\frac{4}{5}$$
$$\cot x=\frac{4}{3}$$

By using the formula, $\cot^{-1}x+\cot^{-1}y=\cot^{-1}\left(\frac{xy-1}{y+x}\right)$

$$\cot\left[\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right]$$
$$=\cot\left[\cot^{-1}\frac{4}{3}+\cot^{-1}\frac{3}{2}\right]$$
$$=\cot\left[\cot^{-1}\left(\frac{\frac{4}{3}.\frac{3}{2}-1}{\frac{3}{2}+\frac{4}{3}}\right)\right]$$
$$=\frac{\frac{4}{3}.\frac{3}{2}-1}{\frac{3}{2}+\frac{4}{3}}$$
$$=\frac{12-6}{9+8}$$
$$=\frac{6}{17}$$

2. Ans. C.

Given that $4\sin^2 x = 3$, where $0 \le x \le \pi$

Now,

$$\sin^2 x = \frac{3}{4}$$
$$1 - \cos^2 x = \frac{3}{4}$$
$$\cos^2 x = \frac{1}{4}$$
$$\tan^2 x = 3, \tan x = \sqrt{3}$$

As, we know that

$$\tan 3x = \frac{\tan x\left(3 - \tan^2 x\right)}{1 - 3\tan^2 x}$$
$$= \frac{\sqrt{3}(3-3)}{1 - 3\times 3}$$
$$= 0$$

3. Ans. C.

Given that the first term is p and the common difference is d, then

$$T_1 = p$$
$$T_3 = p + 2d = q$$
$$T_5 = p + 4d = 3$$

Find the value of p and q in the form of d.

$$T_5 - T_3 = 2d = 3 - q$$
$$q = 3 - 2d$$
$$p = 3 - 4d$$

Let, $X = pq = (3-2d)(3-4d) = 9 - 18d + 8d^2$

For minimum value of X=pq, when $X' = 0$

$$X' = -18 + 16d = 0$$
$$d = \frac{18}{16} = \frac{9}{8}$$

4. Ans. A.

Roots of the equation $x^3 - 8 = 0$

$$x^3 = 2^3$$
$$x = 2, 2\omega, 2\omega^2$$

These are the points lie on a circle of radius 2 unit which will 120° angle apart.

And they are not collinear.

5. Ans. C.

Given that

$$\text{secx.cosecx} = p$$
$$\frac{1}{\sin x.\cos x} = p$$
$$\frac{2}{\sin 2x} = p$$
$$p|_{\min} = \frac{2}{\sin 2x|_{\max}} = \frac{2}{1} = 2$$

6. Ans. B.

Let Y = sinθ + sinθ.cosθ = sinθ + 0.5 sin2θ

For maximum value of Y, Y' = 0

$$Y' = \cos\theta + \cos 2\theta = 0$$
$$2\cos^2\theta + \cos\theta - 1 = 0$$
$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$
$$2\cos\theta - 1 = 0 \quad or \quad \cos\theta + 1 = 0$$
$$\cos\theta = \frac{1}{2} \qquad or \quad \cos\theta = -1$$
$$\theta = \frac{\pi}{3} \qquad or \qquad \theta = \pi$$

But in the given range, $\theta = \frac{\pi}{3}$

$$Y = \sin\theta(1+\cos\theta)$$
$$Y = 2\sin\theta.\cos^2\frac{\theta}{2}$$

7. Ans. A.

1. Let three sets are A, B and C.

The union over intersection of sets is $A\cup(B\cap C)$.

We can apply the distributive law of sets.

$$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$$

So, we can say The union over intersection of sets is distributive.

2. Let A and B are two sets, $(A\cup B)'=A'\cap B'$

The complement of union of two sets is equal to intersection of their complements.

3. let the two sets are A = {2,3} and B = {1,2,3,4}

Here, A – B = {} = empty set, but $A\neq B$.

So, only statement-1 and statement-2 are correct.

8. Ans. C.

Given that three sets X, Y and Z having 6, 5 and 4 elements respectively and all are distinct. Its mean they all are disjoint sets.

Let P = X – Y,

n(P) = n(X – Y) = n(X) – n(X$\cap$Y) = 6 – 0 = 6 (X and Y are disjoint)

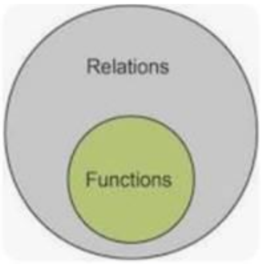
And S = (X – Y) $\cup$ Z = P $\cup$ Z

n(S) = n(P$\cup$Z) = n(P) + n(Z) = 6 + 4 =10 (they all are disjoint sets)

so, number of proper subsets of S = $2^{10}-1=1023$

9. Ans. B.

1. All functions are relations, but not all relations are functions.



2. If A and B are two sets and their cartesian product are A X B.

3. Then every subset of A X B is a relation from A to B.

If A X A is cartesian product A to A, then the relation on A is a subset of A X A.

10. Ans. B.

Given that, $\log_{10} 2\log_2 10 + \log_{10}(10^x) = 2$

$$\frac{\ln 2}{\ln 10}\cdot\frac{\ln 10}{\ln 2} + x\ln_{10} 10 = 2 \qquad \left(\log_a b = \frac{\ln b}{\ln a}, \ln_a a = 1\right)$$

$$1 + x = 2$$

$$x = 1$$

11. Ans. D.

ABC be a triangle then $\angle A + \angle B + \angle C = \pi$

Cos2A + cos2B + cos2C = − 1

2cos(A+B).cos(A-B)=1+cos2C

-2cosC.cos(A-B)=2cos^{2}C

cosC[cosC+cos(A-B)]=0

cosC[cos(A-B)-cos(A+B)]=0

cosC[2sinA.sinB]=0

sinA.sinB.cosC=0

12. Ans. B.

$$\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$$
$$= \cos C(0+\tan A.\sin B) - \tan A(\sin B.\cos C - 0) + 0$$
$$= \tan A.\sin B.\cos C - \tan A.\sin B.\cos C$$
$$= 0$$

13. Ans. C.

Let A = set of first 250 natural numbers that are multiples of 3

A = { 3,6,9,……750}, n(A) = 750

And B = set of first 200 even natural numbers

B = {1,2,3,…..200}, n(B) = 200

$$A \cap B = \{3, 6, 9, \ldots 198\}$$

Let there are n term in the series 3,6,9,…198

Tn = 3+(n-1).3=198 or n = 66, $n(A \cap B) = 66$

Now,

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 250 + 200 - 66 \\ &= 384 \end{aligned}$$

14. Ans. C.

If S_κ denote the sum of first κ terms of an AP, then $S_k = \frac{k}{2}(2a+(k-1)d)$

$$\frac{S_{30}}{S_{20} - S_{10}} = \frac{15(2a+29d)}{10(2a+19d) - 5(2a+9d)} = \frac{3(2a+29d)}{2a+29d} = 3$$

15. Ans. C.

Let the roots are r and r+1, then

Sum of roots, $2r+1=\frac{5k+1}{4}\,or\,r=\frac{5k-3}{8}$

Product of roots, $r\left(r+1\right)=\frac{5k}{4}$

$$\left(\frac{5k-3}{8}\right)\left(\frac{5k-3}{8}+1\right)=\frac{5k}{4}$$

$$\begin{aligned}\frac{5\left(5k-3\right)\left(k+1\right)}{64}&=\frac{5k}{4}\\5k^2+2k-3&=16k\\5k^2-14k-3&=0\\\left(5k+1\right)\left(k-3\right)&=0\\k&=\frac{-1}{5}\,or\,3\end{aligned}$$

16. Ans. B.

If $x^2 + x + 1 = 0$, then roots are $x=\omega,\omega^2$

Now,

$$\begin{aligned}x^{199}+x^{200}+x^{201}&=x^{199}\left(1+x+x^2\right)\\&=\omega^{199}\left(1+\omega+\omega^2\right)\\&=0\qquad\qquad\left(1+\omega+\omega^2=0\right)\end{aligned}$$

17. Ans. A.

x, y, z are in GP,

Then $y^2=xz$

Taking log, 2lny = lnx + lnz,

ln(x), ln(y), ln(z) are in A.P

adding xyz, xyx + ln(x), xyz + ln(y), xyz + ln(z) are in A.P

again $y^2 = xz$ or $(3y)^2 = (3x)(3z)$

Taking log, 2ln3y=ln3x+ln3z, So, ln(3x), ln(3y), ln(3z) are in AP.

Only statement i is correct.

18. Ans. C.

Given that $\log_{10}2$, $\log_{10}(2^x - 1)$, log10$(2^x + 3)$ are in AP, then

$$2\log_{10}\left(2^x-1\right)=\log_{10}2+\log_{10}\left(2^x+3\right)$$
$$\log_{10}\left(2^x-1\right)^2=\log_{10}2.\left(2^x+3\right)$$
$$\left(2^x-1\right)^2=2.\left(2^x+3\right)$$
$$2^{2x}-4.2^x-5=0$$
$$\left(2^x-5\right)\left(2^x+1\right)=0$$
$$2^x=5 \quad or \quad 2^x\neq-1$$
$$x=\log_2 5$$

19. Ans. C.

For a number less than 500, we can fill 2, 3 and 4 by 3 ways rest of the position can be filled by 6 and 5 ways. So, Total number of ways = 3 x 6 x 5 = 90.

20. Ans. A.

Given that p = 1111 … up to n digits

9p = 9999… up n digit

9p = 10000….up to (n+1) digit – 1

9p+1 = 10^n

Now, $9p^2+p=p(9p+1)=10^n.p$

21. Ans. D.

The quadratic equation $3x^2 – (k^2 + 5k)x + 3k^2 – 5k = 0$, has real roots of equal magnitude and opposite sign.

Then, sum of roots = 0 and $D^2 > 0$

$$\frac{k^2+5k}{3}=0$$
$$k(k+5)=0$$
$$k=0,-5 \qquad ...i$$

$$\left(k^2+5k\right)^2-4.3.\left(3k^2-5k\right)>0$$
$$0-12k(3k-5)>0$$
$$k(3k-5)<0$$
$$0<k<\frac{5}{3} \qquad ...ii$$

From the above two condition no such value of k is possible.

22. Ans. D.

Given that If $a_n = n(n!)$, now

$$a_1+a_2+a_3+...+a_{10}$$
$$1(1!)+2(2!)+3(3!)+...+10(10!)$$
$$=(2-1)(1!)+2(2!)+3(3!)+...+10(10!)$$
$$=2(1!)+2(2!)+3(3!)+...+10(10!)-1(1!)$$
$$=2!+2(2!)+3(3!)+...+10(10!)-1$$
$$=2!(1+2)+3(3!)+4(4!)...+10(10!)-1$$
$$=3!+3(3!)+4(4!)+...+10(10!)-1$$
$$...$$
$$=10!(1+10)-1$$
$$=11!-1$$

23. Ans. B.

p and q are the non-zero roots of the equation $x^2 + px + q = 0$, then

Sum of roots, $p + q = -p$ or $q = -2p$

Product of roots, pq = q , q = 0 or p =1

For p = 1 , q = -2

So, possible non-zero roots are 1 and -2.

24. Ans. D.

$$\begin{vmatrix} 3d+5g & 4a+7g & 6g \\ 3e+5h & 4b+7h & 6h \\ 3f+5i & 4c+7i & 6i \end{vmatrix}$$

$$= \begin{vmatrix} 3d & 4a+7g & 6g \\ 3e & 4b+7h & 6h \\ 3f & 4c+7i & 6i \end{vmatrix} + \begin{vmatrix} 5g & 4a+7g & 6g \\ 5h & 4b+7h & 6h \\ 5i & 4c+7i & 6i \end{vmatrix}$$

$$= 3\times 6 \begin{vmatrix} d & 4a+7g & g \\ e & 4b+7h & h \\ f & 4c+7i & i \end{vmatrix} + 5\times 6 \begin{vmatrix} g & 4a+7g & g \\ h & 4b+7h & h \\ i & 4c+7i & i \end{vmatrix}$$

$$= 18 \begin{vmatrix} d & 4a & g \\ e & 4b & h \\ f & 4c & i \end{vmatrix} + 18 \begin{vmatrix} d & 7g & g \\ e & 7h & h \\ f & 7i & i \end{vmatrix} + 0 \qquad [C_1 = C_2]$$

$$= 18\times 4 \begin{vmatrix} d & a & g \\ e & b & h \\ f & c & i \end{vmatrix} + 18\times 7 \begin{vmatrix} d & g & g \\ e & h & h \\ f & i & i \end{vmatrix}$$

$$= 72 \begin{vmatrix} d & a & g \\ e & b & h \\ f & c & i \end{vmatrix} + 0 \qquad [C_2 = C_3]$$

$$= -72 \begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} \qquad [C_1 \to C_2]$$

$$= -72 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= -72\Delta$$

25. Ans. A.

Given that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in HP

⇒ b + c, c + a, a + b are in AP

⇒ -(b + c), -(a + b), -(a +b) are in AP

⇒ (a + b + c) − (b + c), (a + b + c) - (a + b), (a + b + c) - (a + b) are in AP

⇒ a, b, c are in AP

If $(b + c)^2$, $(c + a)^2$, $(a + b)^2$ are in GP

⇒ (b + c), (c + a), (a + b) are in GP

But given that it is in AP.

So, only statement 1 is correct.

26. Ans. A.

Given that $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

Now, $A^2 = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}$

And $A^2 = A^2 . A = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3a \\ 0 & 1 \end{bmatrix}$

So, $A^{25} = \begin{bmatrix} 1 & 25a \\ 0 & 1 \end{bmatrix}, A^{50} = \begin{bmatrix} 1 & 50a \\ 0 & 1 \end{bmatrix}, A^{100} = \begin{bmatrix} 1 & 100a \\ 0 & 1 \end{bmatrix}$

Now,

$$A^{100} - A^{50} - 2A^{25}$$
$$= \begin{bmatrix} 1 & 100a \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 50a \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 25a \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 50a \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 50a \\ 0 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$
$$= -2I$$

27. Ans. A.

$$\begin{vmatrix} a & -b & a-b-c \\ -a & b & -a+b-c \\ -a & -b & -a-b+c \end{vmatrix} - kabc = 0$$
$$a\left(-ab-b^2+bc-ab+b^2-bc\right)+a\left(ab+b^2-bc-ab+b^2-bc\right)-a\left(ab-b^2+bc-ab+b^2+bc\right)=kabc$$
$$-2a^2b+2ab^2-2abc-2abc=kabc$$
$$kabc=-4abc$$
$$k=-4$$

$$\begin{vmatrix} a & -b & a-b-c \\ -a & b & -a+b-c \\ -a & -b & -a-b+c \end{vmatrix} - kabc = 0$$
$$\begin{vmatrix} a & -b & a \\ -a & b & -a \\ -a & -b & -a \end{vmatrix} + \begin{vmatrix} a & -b & -b \\ -a & b & b \\ -a & -b & -b \end{vmatrix} + \begin{vmatrix} a & -b & -c \\ -a & b & -c \\ -a & -b & c \end{vmatrix} = kabc$$
$$0+0+a(0)+a(-2bc)-a(2bc)=kabc$$
$$-4abc=kabc$$
$$k=-4$$

28. Ans. A.

$$\sum_{n=1}^{8n+7} i^n = i + i^2 + i^3 + i^4 + i^5 + \ldots + i^{8n+7}$$
$$= (i+i^2+i^3+i^4) + \ldots (i^{8n+5}+i^{8n+6}+i^{8n+7}+i^{8n+8}) - i^{8n+8}$$
$$= 0 + 0 + \ldots + 0 - (i^4)^{2n+2}$$
$$= -1^{2n+2}$$
$$= -1$$

29. Ans. C.

If z = x + iy, then $z\bar{z}+|z|^2+4(z+\bar{z})-48=0$

$$(x+iy)(x-iy)+\left(x^2+y^2\right)+4(x+iy+x-iy)-48=0$$
$$x^2+y^2+x^2+y^2+8x-48=0$$
$$x^2+4x+y^2-24=0$$
$$(x+2)^2+y^2=\left(\sqrt{28}\right)^2$$

It is an equation of circle.

30. Ans. A.

From the option add and subtract the ib

$$2a+2\sqrt{a^2+b^2}$$
$$=2a+2\sqrt{(a+ib)(a-ib)}-ib+ib$$
$$=(a+ib)+2\sqrt{(a+ib)}.\sqrt{(a-ib)}+(a-ib)$$
$$=\left(\sqrt{a+ib}\right)^2+2\sqrt{(a+ib)}.\sqrt{(a-ib)}+\left(\sqrt{a-ib}\right)^2$$
$$=\left(\sqrt{a+ib}+\sqrt{a-ib}\right)^2$$

So, square root of $2a+2\sqrt{a^2+b^2}$ is $\pm\left(\sqrt{a+ib}+\sqrt{a-ib}\right)$

31. Ans. A.

There are 4 cases where a digit will repeat

Let the set of number = (3,3,5,7,9)

Total number = $\frac{5!}{2!}=60$

Since there are total 4 cases so total number = 60 x 4 = 240

32. Ans. D.

With 4 entries we can make three matrix such that

$$[a\ b\ c\ d],\begin{bmatrix} a & b \\ c & d \end{bmatrix},\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

For the 4 entries we have two choices 1 or 2, so total number of ways = 2 x 2 x 2 x 2 = 16

So, total number of matrices = 16 x 3 = 48.

33. Ans. B.

Given that $i^{-2n} = \left(i^2\right)^{-n} = (-1)^{-n} = (-1)^n$

For n = even, then i^{-2n} = 1

For n = odd, then i^{-2n} =-1

i^{-2n} have only 2 values.

34. Ans. A.

$$x = \frac{a}{b-c}, y = \frac{b}{c-a}, z = \frac{c}{a-b}$$
$$\frac{1}{x} = \frac{b-c}{a}, \frac{1}{y} = \frac{c-a}{b}, \frac{1}{z} = \frac{a-b}{c}$$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \frac{b-c}{a}+\frac{c-a}{b}+\frac{a-b}{c}$$
$$= \frac{b-c}{a}+\frac{c^2-ac+ab-b^2}{bc}$$
$$= \frac{b-c}{a}+\frac{(b-c)(a-b-c)}{bc}$$
$$= (b-c)\left[\frac{bc+a-b-c}{abc}\right]$$

$$\begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -x & x \\ 0 & 1+x & -y-x \\ 0 & z+x & 1-x \end{vmatrix}$$

$$= (1+x)(1-x) + (x+y)(z+x)$$

$$= 1 - x^2 + xz + x^2 + yz + xy$$

$$= 1 + xz + yz + xy$$

$$=$$

$$\begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix}$$

$$= 1(1+yz) + x(1+y) + x(z-1)$$

$$= 1 + yz + x + xy + xz - x$$

$$= 1 + yz + xy + xz$$

$$= 1 + \left(\frac{1}{x} + \frac{1}{z} + \frac{1}{y}\right) xyz$$

$$=$$

35. Ans. C.

Determinant of matrix A

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

, |A| =0, So, inverse of A does not exist.

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 3A$$

$$A^3 = A^2.A = 3A.A = 3A^2 = 3.3A = 9A$$

Only 1 and 3 are correct.

36. Ans. D.

Let the equation of circle $x^2+y^2+2gx+2hy+r=0$

The circle is passing through the points (5, –8), (–2, 9) and (2, 1), So these points will satisfy the equation of circle.

For (5,-8) $25+64+10g-16h+r=0 \Rightarrow 10g-16h+r+89=0$ ...i

For (-2,9) $4+81-4g+18h+r=0 \Rightarrow -4g+18h+r+85=0$ ...ii

For (2,1) $4+1+4g+2h+r=0 \Rightarrow 4g+2h+r+5=0$ ...iii

By (i)-(ii) and (i)-(iii)

$14g-34h+4=0 \Rightarrow 7g-17h+2=0$

$6g-18h+84=0 \Rightarrow g-3h+14=0$

From above two equation, g = 58, h = 24 and r = 285

Centre, $(-g,-h)=(-58,-24)$

37. Ans. D.

Let the equation of circle $x^2+y^2+2gx+2hy+r=0$

The circle is passing through the points (5, –8), (–2, 9) and (2, 1), So these points will satisfy the equation of circle.

For (5,-8) $25+64+10g-16h+r=0 \Rightarrow 10g-16h+r+89=0$ ...i

For (-2,9) $4+81-4g+18h+r=0 \Rightarrow -4g+18h+r+85=0$ ...ii

For (2,1) $4+1+4g+2h+r=0 \Rightarrow 4g+2h+r+5=0$ ...iii

By (i)-(ii) and (i)-(iii)

$14g-34h+4=0 \Rightarrow 7g-17h+2=0$

$6g-18h+84=0 \Rightarrow g-3h+14=0$

From above two equation, g = 58, h = 24 and r = -285

Radius, $=\sqrt{g^2+h^2-r}=\sqrt{58^2+24^2-285}=\sqrt{4225}=65$

38. Ans. C.

Let the third vertices is C(h,k) and given two vertices are A(0,0), B(2,2)

Then $AC^2=BC^2, \qquad AC^2=AB^2$

$$h^2+k^2=(h-2)^2+(k-2)^2, \qquad h^2+k^2=2^2+2^2$$
$$h^2+k^2=h^2+k^2-4h-4k+8, \quad h^2+k^2=8$$
$$h+k=2, \qquad ...(i) \qquad h^2+k^2=8 \qquad ...(ii)$$

$$h^2+k^2+2hk=4$$
$$8+2hk=4$$
$$hk=-2 \qquad ...(iii)$$

$h-k=\sqrt{(h-k)^2}=\sqrt{h^2+k^2-2hk}=\sqrt{8+4}=\sqrt{12}=2\sqrt{3}$ ...(iv)

Now, $h=1+\sqrt{3}, \quad k=1-\sqrt{3}$

Length of the vertices AB = $2\sqrt{2}$

Area of triangle $=\frac{\sqrt{3}}{4}AB^2=2\sqrt{3}$

Both the statements are correct.

39. Ans. D.

Let the third vertices is C(h,k) and given two vertices are A(0,0), B(2,2)

Then $AC^2=BC^2, \qquad AC^2=AB^2$

$$h^2+k^2=(h-2)^2+(k-2)^2, \qquad h^2+k^2=2^2+2^2$$
$$h^2+k^2=h^2+k^2-4h-4k+8, \quad h^2+k^2=8$$
$$h+k=2, \qquad ...(i) \qquad h^2+k^2=8 \qquad ...(ii)$$

$$h^2+k^2+2hk=4$$
$$8+2hk=4$$
$$hk=-2 \qquad ...(iii)$$

$$h-k=\sqrt{(h-k)^2}=\sqrt{h^2+k^2-2hk}=\sqrt{8+4}=\sqrt{12}=2\sqrt{3}$$ ...(iv)

Now, $h=1+\sqrt{3}, \quad k=1-\sqrt{3}$

The coordinate of the third vertex is $\left(1+\sqrt{3},1-\sqrt{3}\right)$

And the difference = $\left|\left(1+\sqrt{3}\right)-\left(1-\sqrt{3}\right)\right|=2\sqrt{3}$

40. Ans. C.

Let the coordinate of D is (g, h), then in a parallelogram

The slope of AB = slope of DC and slope of AD = slope of BC

$$\frac{3-2}{1+1}=\frac{h-5}{g-3} \qquad and \qquad \frac{3-h}{1-g}=\frac{2-5}{-1-3}$$
$$g-3=2h-10 \qquad and \qquad -12+4h=-3+3g$$
$$g-2h+7=0 \qquad and \qquad 3g-4h+9=0$$

After solving we get the coordinate of D is (5,6).

Now equation of the diagonal BD,

$$y-6=\frac{6-2}{5+1}(x-5)$$
$$3y-18=2x-10$$
$$2x-3y+8=0$$

41. Ans. C.

Let the vector of $\overrightarrow{BA} = -\hat{i} - 2\hat{k}$ and $\overrightarrow{BC} = -4\hat{i} - 3\hat{j}$

Area of diagonal = $\frac{1}{2}\left|\overrightarrow{BA} \times \overrightarrow{BC}\right| = \frac{1}{2}\left|\left(-1\hat{i} - 2\hat{j}\right) \times \left(-4\hat{i} - 3\hat{j}\right)\right| = \frac{1}{2}\left|-5\hat{k}\right| = \frac{5}{2}$ square unit

42. Ans. D.

The intersection point of side AB and BC is point B(2,-1).

The altitude through B on AC is perpendicular to AC,

If equation of CA is x + 2y – 4 = 0, then the perpendicular line on CA, y - 2x = c

And it passes through B(2,-1), So, $-1 - 4 = c \Rightarrow c = -5$

So, the equation of line, y – 2x = -5 or 2x – y – 5 = 0.

43. Ans. A.

The intersection point of side AB and BC is point B(2,-1).

The intersection point of side BC and CA is point C(6,-1).

The intersection point of side CA and AB is point A(2,1).

Let the midpoint of AB, D = (2,0) and equation of AB, x – 2 = 0

The equation of perpendicular of AB, *y=c*

And it passes through D(2,0), so, y = 0

Let the midpoint of BC, E = (4,-1) and equation of BC, y + 1 = 0

The equation of perpendicular of AB, *x=c*

And it passes through E(4,-1), so, x = 4

The circumcentre of triangle is (4,0).

44. Ans. B.

We have the end points of the latus rectum.

Focus lies on the midpoint of the segment joining the end points of the latus rectum.

So, the focus of the parabola is fixed.

We have the length of the latus rectum.

So, the distance between the focus and the directrix is fixed.

Directrix is parallel to the latus rectum.

So, the slope of directrix is fixed.

So, we can have two lines which are parallel to latus rectum and are at a fixed distance from the focus.

So, we have one focus and two possible directrices.

So, a maximum of two parabolas can be drawn.

45. Ans. C.

Length of the latus rectum, 4a = 8

Let the equation of the parabola is $(y-k)^2 = 4a(x-h)$

Or $(y-k)^2 = \pm 8(x-h)$

And it passes through the points (−2, 4) and (−2, −4).

So, $(4-k)^2 = \pm 8(-2-h)$ and $(-4-k)^2 = \pm 8(-2-h)$

$$(4-k)^2 = (-4-k)^2$$
$$k^2 - 8k + 16 = k^2 + 8k + 16$$
$$k = 0$$ and $h = -4.0$

So, equation of parabola is $y^2 = 8(x-4), y^2 = 8x$

One of the parabola pass through the origin.

The focus will be the midpoint of the end point of latus rectum.

So, focus of parabola = F(-2,0)

46. Ans. C.

Sum of the roots, $\sin\theta+\cos\theta=\frac{-b}{a}$ ...i

And product of the roots, $\sin\theta.\cos\theta=\frac{c}{a}$ ...ii

$$(\sin\theta+\cos\theta)^2-2\sin\theta.\cos\theta=\left(\frac{-b}{a}\right)^2-\frac{2c}{a}$$

$$\sin^2\theta+\cos^2\theta=\frac{b^2-2ac}{a^2}$$

$$a^2=b^2-2ac$$

$$a^2-b^2+2ac=0$$

47. Ans. A.

C(n, 4), C(n, 5) and C(n, 6) are in AP, then

$$2C(n,5)=C(n,4)+C(n,6)$$

$$\frac{2n!}{(n-5)!.5!}=\frac{n!}{4!.(n-4)!}+\frac{n!}{6!(n-6!)}$$

$$\frac{2}{5(n-5)}=\frac{1}{(n-4)(n-5)}+\frac{1}{6\times 5}$$

$$\frac{2}{5(n-5)}=\frac{30+n^2-9n+20}{5\times 6\times(n-4)(n-5)}$$

$$12n-48=n^2-9n+50$$

$$n^2-21n+98=0$$

$$(n-14)(n-7)=0$$

$$n=7\,or\,14$$

48. Ans. A.

In word 'LUCKNOW', there are 2 vowel and 5 consonants.

So, total 4-letter words containing two vowels = ${}^5C_2.{}^2C_2.4!=240$

49. Ans. B.

By joining two points we can draw a line, since all the points are on the circle its mean they are not linear. So, total number of lines $={}^{20}C_2=190$

By joining three points we can draw a triangle, since all the points are on the circle its mean they are not linear. So, total number of triangles = ${}^{20}C_3=\frac{20\times19\times18}{1\times2\times3}=1140$

Only statement (ii) is correct.

50. Ans. D.

We can simplify this as

$$\left(\frac{a^2}{b^2}+\frac{b^2}{a^2}+2\right)^{21}=\left[\left(\frac{a}{b}+\frac{b}{a}\right)^2\right]^{21}=\left(\frac{a}{b}+\frac{b}{a}\right)^{42}$$

So, total numbers of terms = 42 + 1 = 43

51. Ans. B.

A system of equations is said to be consistent if it has at least one solution.

$$6kx+y+1=0\Rightarrow y=-1-6kx$$

$$7x-2y+3=0\Rightarrow 7x+2+12kx+3=0\Rightarrow x=\frac{-5}{7+12k}\text{ and }y=-1+\frac{30k}{7+12k}=\frac{18k-7}{7+12k}$$

Now put value of x and y in equation 1

$$2k^2x + 3y - 1 = 0$$

$$2k^2\left(\frac{-5}{7+12k}\right) + 3\left(\frac{18k-7}{7+12k}\right) - 1 = 0$$

$$-10k^2 + 54k - 21 - 7 - 12k = 0$$

$$10k^2 - 42k + 28 = 0$$

$$5k^2 - 21k + 14 = 0$$

$$k = \frac{21 \pm \sqrt{21^2 - 4\times 5\times 14}}{2\times 5}$$

$$k = \frac{21 \pm \sqrt{161}}{10}$$

52. Ans. A.

Let $B = A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix}$ and we know that $A = \left(A^{-1}\right)^{-1} = B^{-1}$

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ a matrix, then inverse of this matrix will be $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Now, $A = \frac{2}{-1}\begin{bmatrix} -\frac{1}{2} & -1 \\ -\frac{3}{2} & -2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

53. Ans. B.

Period of $\ln(2+\sin^2 x)$ = period of $2+\sin^2 x$ = period of $\sin^2 x$ = π

54. Ans. D.

sin(A + B) = 1 $\Rightarrow$ sinA.cosB + cosA.sinB = 1

And $\sin(A-B) = \frac{1}{2} \Rightarrow \sin A.\cos B - \cos A.\sin B = \frac{1}{2}$

Dividing by cosA.cosB

$$\tan A + \tan B = \sec A.\sec B \quad ...i$$

$$\tan A - \tan B = \frac{\sec A.\sec B}{2} \quad ...ii$$

Dividing (i) by (ii)

$$\frac{\tan A + \tan B}{\tan A - \tan B} = 2$$
$$\frac{2\tan A}{2\tan B} = \frac{2+1}{2-1} \qquad [componendo\ \&\ divindo]$$
$$\tan A : \tan B = 3:1$$

55. Ans. B.

Total number of triangles = ${}^{10}C_3 = 120$

Number of triangles that have one common side = n(n – 4) = 10(10 – 4) = 60

Number of triangles that have two common sides = n = 10

So, number of triangles that have no common side with any of the sides of the polygon = Total number of triangles - Number of triangles that have one common side - Number of triangles that have two common sides = 120 – 60 – 10 = 50

56. Ans. C.

$$x^4 - 10x^2 + 9 = 0$$
$$(x^2 - 9)(x^2 - 1) = 0$$
$$x = \pm 1, \pm 3$$

Sum of the absolute value of the roots = $|1| + |-1| + |3| + |-3| = 8$

57. Ans. C.

$$(1+x)^n = 1 + nx + ... + nx^{n-1} + x^n$$

From above p = 1, q = n, r = n and s = 1

ps + qr = 1 + n^2

58. Ans. C.

$$\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$$

x = y = z = 1 can satisfy this equation

$$LHS=\sin^{-1}1+\sin^{-1}1+\sin^{-1}1=\frac{\pi}{2}+\frac{\pi}{2}+\frac{\pi}{2}=\frac{3\pi}{2}=RHS$$

Now, $x^{1000}+y^{1001}+z^{1002}=1^{1000}+1^{1001}+1^{1002}=1+1+1=3$

59. Ans. A.

sin x + sin y = cos x + cos y

$$2\sin\frac{x+y}{2}.\cos\frac{x-y}{2}=2\cos\frac{x+y}{2}.\cos\frac{x-y}{2}$$

$$\tan\left(\frac{x+y}{2}\right)=1$$

$$\tan\left(\frac{x}{2}+\frac{y}{2}\right)=1$$

60. Ans. D.

If $A=\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}$, then

$$mI+nA=\begin{bmatrix}m & 0\\0 & m\end{bmatrix}+\begin{bmatrix}0 & 2n\\-2n & 0\end{bmatrix}=\begin{bmatrix}m & 2n\\-2n & m\end{bmatrix}$$

$$(mI+nA)^2=\begin{bmatrix}m & 2n\\-2n & m\end{bmatrix}\begin{bmatrix}m & 2n\\-2n & m\end{bmatrix}=\begin{bmatrix}m^2-4n^2 & 4mn\\-4mn & m^2-4n^2\end{bmatrix}$$

$$(mI + nA)^2 = A$$

$$\begin{bmatrix} m^2 - 4n^2 & 4mn \\ -4mn & m^2 - 4n^2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

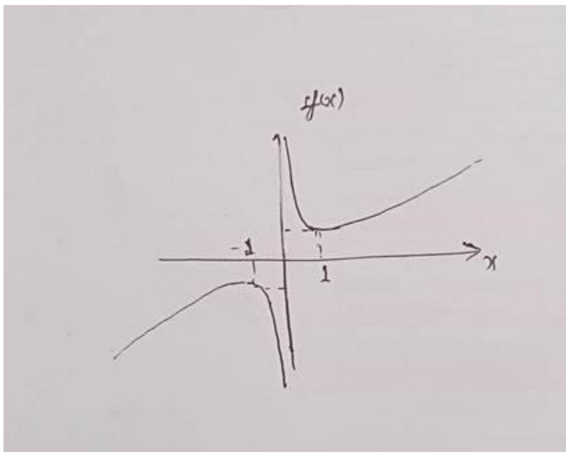
$$m^2 - 4n^2 = 0 \Rightarrow m^2 = 4n^2 \Rightarrow m = \pm 2n$$

$$\Rightarrow 4mn = 2 \Rightarrow mn = \frac{1}{2} \Rightarrow 2n^2 = \frac{1}{2} \Rightarrow n = \pm\frac{1}{2}$$

$$\Rightarrow m = \pm 1$$

M and n should be of same sign, so $m + n = \pm\frac{3}{2}$

61. Ans. A.



$$f(x) = x + \frac{1}{x}$$
$$f'(x) = 1 - \frac{1}{x^2} \leq 0$$
$$(x^2 - 1) \leq 0$$
$$(x - 1)(x + 1) \leq 0$$

$$f''(x) = \frac{1}{x^3}$$

$f"(1) = 1 > 0$ is a minimum value

$f(1)|_{\min}$

$f"(-1) = -1 < 0$ is a maximum value

$f(-1)|_{max}$

f(x) is decreasing in (-1,0) and (0,1)

Critical points are -1 and 1.

Local maxima at point x=-1 and local minima at x=1.

The local maximum value of f(x) is less than its local minimum value.

The local maximum value of f(x) occurs at x = -1

62. Ans. C.

We know that,

The maximum area of a rectangle that can be inscribed in a circle is square,

whose diagonal is diameter of the circle.

So,

Side of square = $\frac{\text{diameter}}{\sqrt{2}} = 2\sqrt{2}$

So, area is $= (2\sqrt{2})^2 = 8$

63. Ans. A.

$$\int \frac{dx}{x(x^2+1)}$$

$$\Rightarrow \int \frac{dx}{x^3\left(1+\frac{1}{x^2}\right)}$$

$$\text{let}\left(1+\frac{1}{x^2}\right) = \text{tthen},$$

$$-\frac{2}{x^3}dx = dt$$

so,

$$\Rightarrow \int \frac{-1}{2}\frac{dt}{t}$$

$\Rightarrow \dfrac{-1}{2}\ln t + c$

$\Rightarrow \dfrac{-1}{2}\ln\left(1+\dfrac{1}{x^2}\right) + c \Rightarrow \frac{1}{2}\ln\left(\frac{x^2}{x^2+1}\right) + c$

64. Ans. A.

$= \dfrac{de^{e^x}}{de^x}$

Let $e^x = t$ then,

$e^{e^x} = e^t$

Now,

$= \dfrac{de^t}{dt}$

$= e^t \text{put} t = e^x = e^{e^x}$

65. Ans. B.

$f(x) = x^3 + x^2 + kx$

For no local extremum f(x) always increasing.

So,

$\Rightarrow f'(x) > 0 \Rightarrow 3x^2 + 2x + k > 0$

$D < 0$

$\Rightarrow 2^2 - 4 \times 3k < 0$

$\Rightarrow 1 - 3k < 0 \Rightarrow 3k > 1$

66. Ans. D.

$F(x) = 2^x$ then $f'(x) = 2^x.\ln 2$

$\Rightarrow \int_2^{10} \frac{f'(x)}{f(x)} dx \Rightarrow \int_2^{10} \frac{2^x \ln 2}{2^x} dx$

$\Rightarrow \int_2^{10} \ln 2 . dx$

$\Rightarrow \ln 2 . (x)_2^{10}$

$\Rightarrow l\ln 2 . (10 - 2)$

$\Rightarrow 8 \ln 2$

67. Ans. D.

We know that the modulus of any function is always greater than or equal to that function $|f(x)| \geq f(x)$

Hence, $\int_{-2}^{0} |f(x)| dx \geq \int_{-2}^{0} f(x) dx = k$

68. Ans. A.

function f(x) = x^2 – kx is monotonically increasing

$$\begin{aligned} &f'(x) > 0 \\ &2x - k > 0 \\ &\quad k < 2x \qquad (i) \end{aligned}$$

Given that it is increasing in the interval (1, ∞).

$1 < x < \infty$

$2 < 2x < \infty (ii)$

Now, $k < 2$

69. Ans. C.

$y = [x]$

In between x = −1.5 and x = −1.8, value of y is same i.e. -2.

Then,

Area $= -2 \times (-1.8 + 1.5) = 0.6$

70. Ans. D.

We know that,

slope $= \tan\theta = \frac{dy}{dx}$,

Then,

$$\Rightarrow \tan\theta = \frac{dy}{dx}$$
$$\Rightarrow \tan\theta = \frac{dx^2}{dx}$$
$$\Rightarrow \tan\theta = 2xatx = 1$$
$$\Rightarrow \tan\theta = 2$$
$$\Rightarrow \theta = \tan^{-1}(2) = 63.47 \Rightarrow \frac{\pi}{3} < \theta < \frac{\pi}{2}$$

71. Ans. C.

We know that, $P(E \cup F) = P(E) + P(F) - P(E \cap F) \leq 1$

It is always true.

2. $P(E \cup F) = P(E) + P(F) + P(E \cap F)$wrong$P(E \cup F) = P(E) + P(F) - P(E \cap F)$right

3. $P(E \cup F) \leq P(E) + P(F)$ it is correct, inequality is valid when there is no intersection part.

72. Ans. A.

$$P(A|B) < P(A) \frac{P(B \cap A)}{P(B)} < P(A)$$
$$\frac{P(B \cap A)}{P(A)} < P(B) \, P(B|A) < P(B)$$

73. Ans. C.

Standard deviation is used over other statistical point as it is shows the spread from mean unlike sharp and quartile deviation.

It is also used for other infinancial statistics like correlation, regression and unlike mean deviation.

74. Ans. D.

Given that,

The solving probabilities of A, B and C, of a problem independently are $\frac{1}{2}, \frac{3}{4}$ and p respectively, and

probability that the problem can be solved is $\frac{29}{32}$.Then,

$$\Rightarrow p(A \cup B \cup C) = p(A) + p(B) + p(C) - p(A \cap B) - p(B \cap C) - p(A \cap C) + p(A \cap B \cap C)$$

$$\Rightarrow \frac{29}{32} = \frac{1}{2} + \frac{3}{4} + p - \frac{1}{2} \times \frac{3}{4} - \frac{3}{4} \times p - \frac{1}{2} \times p + \left(\frac{1}{2} \times \frac{3}{4} \times p\right)$$

$$\Rightarrow \frac{29}{32} = \frac{1}{2} + \frac{3}{4} + p - \frac{1}{8} - \frac{3p}{4} - \frac{p}{2} + \left(\frac{3}{8}p\right)$$

$$\Rightarrow \frac{29}{32} - \frac{7}{8} = \frac{p}{8} \Rightarrow p = \frac{1}{4}$$

Or

$$P(A \cup B \cup C) = 1 - P(A \cap B \cap C)' = 1 - P(A)'P(B)'P(C)'$$

$$\frac{29}{32} = 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{4}\right)(1 - p)$$

$$\frac{29}{32} = 1 - \frac{1}{2} \times \frac{1}{4} \times (1 - p)$$

$$\frac{1 - p}{8} = \frac{3}{32} p = \frac{1}{4}$$

75. Ans. D.

Given, a batsman hits a six 8 times out of 60 balls he plays,

Total no. of outcome = 60

Total no. of favourable outcome = 60-8 = 52

Probability that not hits six is $\frac{52}{60} = \frac{13}{15}$.

76. Ans. D.

It forms a plane at z=7 which is parallel to xy plane.

77. Ans. C.

A line in space can have infinitely many direction ratios. Such as $(1,2,0.5), (2,4,1), (4,8,2)$ are same.

It is possible for certain line that the sum of the squares of direction cosines can be equal to sum of its direction cosines.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \cos\alpha + \cos\beta + \cos\gamma$$

If we put $\alpha = \beta = \gamma = 0 \Rightarrow \text{LHL} = \text{RHL} = 1$
$\alpha = \beta = \gamma = \frac{\pi}{2} \Rightarrow \text{LHL} = \text{RHL} = 0$

78. Ans. C.

Given that,

The xy-plane divides the line segment joining the points (−1, 3, 4) and (2, −5, 6),

xy-plane it means z- coordinate equal to zero.

So, using line dividing formula

$\Rightarrow \frac{r}{1} = \frac{rz_2 + z_1}{r+1}$
$\Rightarrow 0 = \frac{r6 + 4}{r+1}$
$\Rightarrow -4 = 6r \Rightarrow r = \frac{-2}{3}$

Negative sign denote it divided externally.

79. Ans. C.

In the 3-d, there are 8 octants, so number of spheres of radius r touching the coordinate axes is 8.

80. Ans. B.

Side AB, AG, and AC are 12i, 12i+6j+4k and 12i+6j. If α is the angle between AB and AG.

$$\cos\alpha = \frac{(12i).(12i+6j+4k)}{|12|.\sqrt{|12^2+6^2+4^2|}} = \frac{144}{12.14} = \frac{6}{7}$$

$$\cos\beta = \frac{(12i+6j).(12i+6j+4k)}{\left|\sqrt{12^2+6^2}\right|.\left|\sqrt{12^2+6^2+4^2}\right|} = \frac{144+36}{(\sqrt{180}).(\sqrt{196})} = \frac{180}{\sqrt{180}.14} = \frac{6\sqrt{5}}{14} = \frac{3\sqrt{5}}{7}$$

So, $\cos 2\alpha + \cos 2\beta = 2(\cos^2\alpha + \cos^2\beta - 1) = 2\left(\frac{36}{49} + \frac{45}{49} - 1\right) = \frac{64}{49}$

81. Ans. A.

$1. \vec{a} \times \vec{b} = |\vec{a}|\left|\vec{b}\right| \sin\theta\, \vec{n} = 1 \times 1 \sin\theta\, \vec{c} = \sin\theta\, \vec{c}$ where n and c are normal vector.

$2. \vec{a}.\left(\vec{b} \times \vec{c}\right) = \vec{c}.\left(\vec{a} \times \vec{b}\right)$ so, $\vec{c}\text{and}\left(\vec{a} \times \vec{b}\right)$ are parallel to each other so it cannot be zero.

82. Ans. D.

Solve both equations $\vec{a} + 3\vec{b} = 3\hat{\imath} - \hat{\jmath}$ and $2\vec{a} + \vec{b} = \hat{\imath} - 2\hat{\jmath}$, and we get

$\vec{a} = -\hat{\jmath}\text{and}\vec{b} = \hat{\imath}$ then,

$\Rightarrow \vec{a}.\vec{b} = |\vec{a}|\left|\vec{b}\right| \cos\theta$
$\Rightarrow (-\hat{\jmath}).(\hat{\imath}) = |-\hat{\jmath}||\hat{\imath}| \cos\theta$
$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

83. Ans. A.

Given that,

$\left(\vec{a} + \vec{b}\right)$is per perpendicular to $\vec{a}$ and magnitude of $\vec{b}$ is twice that of $\vec{a}$, Then,

$\vec{b} = 2\vec{a}\ldots\ldots(1)$

$\Rightarrow (\vec{a}+\vec{b}).\vec{a} = 0$
$\Rightarrow |\vec{a}|^2 + \vec{a}.\vec{b} = 0 \Rightarrow \vec{a}.\vec{b} = -|\vec{a}|^2\ldots\ldots.(2)$

$\Rightarrow (4\vec{a}+\vec{b}).\vec{b}$
$\Rightarrow 4\vec{a}.\vec{b} + |\vec{b}|^2$
$\Rightarrow -4|\vec{a}|^2 + 4|\vec{a}|^2 \text{fromeqn. } 1\&2 \Rightarrow 0$

84. Ans. C.

We know that,

$(\vec{a}\times\vec{b})$ is perpendicular to $\vec{a}, \vec{b} \text{and} \vec{c}$, after that cross product with vector $\vec{c}$ then,

$(\vec{a}\times\vec{b})\times\vec{c}$ will be coplanar with $\vec{a}$ and $\vec{b}$ and perpendicular to $\vec{a}$ x $\vec{b}$

85. Ans. C.

Given that,

Two position vectors of A and B are $(\sqrt{2}-1)\hat{i}-\hat{j}$ and $\hat{i}+(\sqrt{2}+1)\hat{j}$ respectively,

$\overrightarrow{AB} = \hat{i}+(\sqrt{2}+1)\hat{j}-(\sqrt{2}-1)\hat{i}-\hat{j}$
$= (2-\sqrt{2})i + (2+\sqrt{2})j$
$|\overrightarrow{AB}| = \sqrt{(2-\sqrt{2})^2 + (2+\sqrt{2})^2}$
$= \sqrt{4+2+4+2} = 2\sqrt{3}$

86. Ans. B.

$y = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$ then,

$$\left.\frac{dy}{dx}\right|_{x=0} = 1(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$$
$$+ (1+x)(2\underline{x})(1+x^4)(1+x^8)(1+x^{16})$$
$$+(1+x)(1+x^2)(4\underline{x})(1+x^8)(1+x^{16})+\ldots..$$
$$= 1+0+0+0\ldots..= 1$$

87. Ans. A.

$$\Rightarrow y = \cos x.\cos 4x.\cos 8x$$
$$\Rightarrow \frac{dy}{dx} = -[\sin x\cos 4x.\cos 8x + 4\sin 4x.\cos x\ \cos 8x + 8\cos x.\cos 4x.\sin 8x]$$
$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -[\tan x + 4\tan 4x + 8\tan 8x]$$
$$\Rightarrow \left.\frac{1}{y}\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = -\left[\tan\frac{\pi}{4} + 4\tan 4\frac{\pi}{4} + 8\tan 8\frac{\pi}{4}\right] = -1$$

88. Ans. C.

F(x) is an polynomial then and $fof(x) = x^4$ then,

It is clearly seen $f(x) = x^2$

Now,

$$\Rightarrow f'(x) = 2x \Rightarrow f'(1) = 2$$

89. Ans. C.

Given that a > b > 1 then $\frac{b}{a} < 1$

Now, $\lim_{n\to\infty}\frac{a^n+b^n}{a^n-b^n}$ $\Rightarrow \lim_{n\to\infty}\frac{a^n\left[1+\left(\frac{b}{a}\right)^n\right]}{a^n\left[1-\left(\frac{b}{a}\right)^n\right]}$

$$\Rightarrow \lim_{n\to\infty}\frac{1+0}{1-0} \Rightarrow 1$$

90. Ans. D.

$\lim\limits_{x\to 2} f(x)$ exists, then

$$\Rightarrow 1+\frac{x}{2k}=kx at x=2$$
$$\Rightarrow 1+\frac{2}{2k}=k2 at x=2$$
$$\Rightarrow 2k^2-k-1=0$$
$$\Rightarrow (2k+1)(k-1)=0 \Rightarrow k=1,\frac{-1}{2}$$

91. Ans. C.

There are two cases

Case:I

The regression line of y on x $\Rightarrow y=\frac{3}{4}x+2$

The regression line of x on y is $x=\frac{3}{4}y+\frac{1}{4}$.

We know that, $r^2=m_1m_2$ and $0\le r^2\le 1$

Where, m_1 = slope of line 1, m_2 = slope of line 2, y = coefficient of correlation

Here $r^2=\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}<1$

This is true.

Case:II

The regression line of y on x $\Rightarrow x=\frac{4}{3}y-\frac{8}{3}$

The regression line of x on y is $\Rightarrow y=\frac{4}{3}x-\frac{1}{3}$

Here $r^2=\frac{4}{3}\times\frac{4}{3}=\frac{16}{9}>1$

This is not valid.

The intercepts of line on axes are $x = 4, y = 5$

Here y-intercepts is positive and it is true only for case-1.

Thus both statements are true.

92. Ans. A.

The regression line of y on x $\Rightarrow y = \frac{3}{4}x + 2$

The regression line of x on y is $x = \frac{3}{4}y + \frac{1}{4}$.

We know that, $r^2 = m_1 m_2$ and $0 \le r^2 \le 1$

Where, m_1 = slope of line 1, m_2 = slope of line 2, y = coefficient of correlation

Here $$r^2 = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16} < 1$$

$$r = \pm\frac{3}{4}$$

But regression coefficients m_1, m_2 and r have same value.

Thus $r = \frac{3}{4}$

The intercepts of line on axes are means of x and y

And $x = 4, y = 5$

So, only statement 1 is correct.

93. Ans. C.

We know that,

Marks	Number of students	Cumulative frequency	Mid
15 – 20	4	4	17.5
20 – 25	5	9	22.5
25 – 30	11	20	27.5
30 – 35	6	26	32.5
35 – 40	5	31	37.5
40 – 45	8	39	42.5
45 – 50	9	48	47.5
50 – 55	6	54	52.5
55 – 60	4	58	57.5
60 – 65	2	60	62.5

$\Rightarrow \frac{n}{2} = \frac{60}{2} = 30$

Coloured data is median class, so

$$\text{median} = l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times c$$
$$= 35 + \frac{30 - 26}{5} \times 5 = 39$$

94. Ans. B.

In a given interval, $\text{mod} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

where l is the lower limit of modal class,

h is the size of class interval, f_1 is the frequency of the modal class,

f_0 is the frequency of the class preceding the modal class, and

f_2 is the frequency of the class succeeding the modal class.

We see the first highest frequency i.e. 11.

Now,

$$\text{mod} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$
$$= 25 + \left(\frac{11 - 5}{2 \times 11 - 5 - 6}\right) \times 5$$
$$= 25 + \left(\frac{30}{11}\right)$$
$$= 25 + 2.72$$
$$= 27.72$$

95. Ans. C.

$$\text{mean} = \frac{\sum_1^{64} n - \sum_1^{14} n}{64 - 14}$$
$$= \frac{\frac{1}{2}(64 \times 65 - 14 \times 15)}{50}$$
$$= \frac{4160 - 210}{100} = 39.5$$

96. Ans. A.

total number of data = 5, mid-term = $\frac{5+1}{2} = 3$

Mean = $\frac{x+x+x+2+x+3+x+10}{5} = \frac{5x+15}{5} = x + 3$

Median = $\text{mid} - \text{term} = x + 2$

Mode = x

We can see, Mean > Mode and Median < Mean

97. Ans. B.

Given that,

The mean of 10 observation is 5.5,

Let the sum of observations is x, then

$$\Rightarrow \frac{x}{10} = 5.5 \Rightarrow x = 55$$

Now,

$$\text{newmean} = \frac{44 \times 10 - 4x}{10}$$
$$= \frac{440 - 4 \times 55}{10} = \frac{440-220}{10} = 22$$

98. Ans. C.

Total no. of terms in the GP is 10.

We know that,

Geometric mean is always lies between two mid terms for the even no. of terms.

So, geometric mean lies between 5^{th} and 6^{th} term i.e. $32 < g < 64$.

99. Ans. C.

$$\Rightarrow \frac{\frac{1}{x} + \frac{1}{60}}{2} = \frac{1}{48}$$
$$\Rightarrow \frac{x + 60}{120x} = \frac{1}{48}$$
$$\Rightarrow 2x + 120 = 5x$$
$$\Rightarrow 3x = 120 \Rightarrow x = 40$$

100. Ans. A.

$$\text{mean}, \bar{x} = \frac{2 + 4 + 6 + \ldots\ldots + 20}{10}$$
$$= \frac{2(1 + 2 + 3 + \ldots\ldots + 10)}{10} = 2 \times \frac{10 \times 11}{2 \times 10} = 11$$

$$\text{meandeviation} = \frac{1}{n}\sum |(x_i - \bar{x})|$$
$$= \frac{1}{10}(9 + 7 + 5 + 3 + 1) \times 2 = 5$$

101. Ans. B.

Variance = $\frac{1}{n}\sqrt{\sum x^2 - \frac{(\sum x)^2}{n}}$

$= \frac{1}{10} \times \left(1540 - \frac{(110)^2}{10}\right) = \frac{1}{10} \times (1540 - 1210)$

$= \frac{1}{10} \times 330 = 33$

102. Ans. A.

Given that,

3-digit numbers are formed using the digits 1, 3, 7 without repetition of digits.

Divisibility rule for 3 is sum of all digit numbers must be divisible by 3.

1 + 3 + 7 = 11 which is not divisible by 3, so favourable outcome is zero.

So, probability also be zero.

103. Ans. A.

Discriminant, $D \geq 0$

$\Rightarrow \frac{b^2-4ac}{2a} \geq 0 \Rightarrow \frac{1-4n}{2} \geq 0$

$\Rightarrow n \leq \frac{1}{4}$

Since n ϵ N then, there is no any natural number so probability is zero.

104. Ans. D.

Given that,

A and B are two events such that P(A') = $\frac{7}{10}$, P(B') = $\frac{3}{10}$ and P(A/B) = $\frac{3}{14}$,

So,

P(A) = $\frac{3}{10}$, P(B) = $\frac{7}{10}$ and P(A/B) = $\frac{3}{14}$

Now,

$$\Rightarrow p(A|B) = \frac{p(A \cup B)}{p(B)}$$
$$\Rightarrow \frac{3}{14} = \frac{p(A \cup B)}{\frac{7}{10}}$$
$$\Rightarrow p(A \cup B) = \frac{3}{20}$$
then,
$$\Rightarrow p(B|A) = \frac{p(A \cup B)}{p(A)}$$
$$\Rightarrow p(B|A) = \frac{\frac{3}{20}}{\frac{3}{10}} \Rightarrow p(B|A) = \frac{1}{2}$$

105. Ans. A.

Given that,

Seven white balls and three black balls are randomly placed in a row,

No. of total outcomes is $\frac{10!}{7!3!}$,

For the favourable outcome, we have to placed the balls as placed below

_W_W_W_W_W_W_

There 8 places at around White balls, we will place 3 black ball there.

No. of favourable outcome is $^{8}C_{3} = \frac{8!}{3!5!}$

Probability is

$$= \frac{\text{favourable}}{\text{totaloutcome}}$$

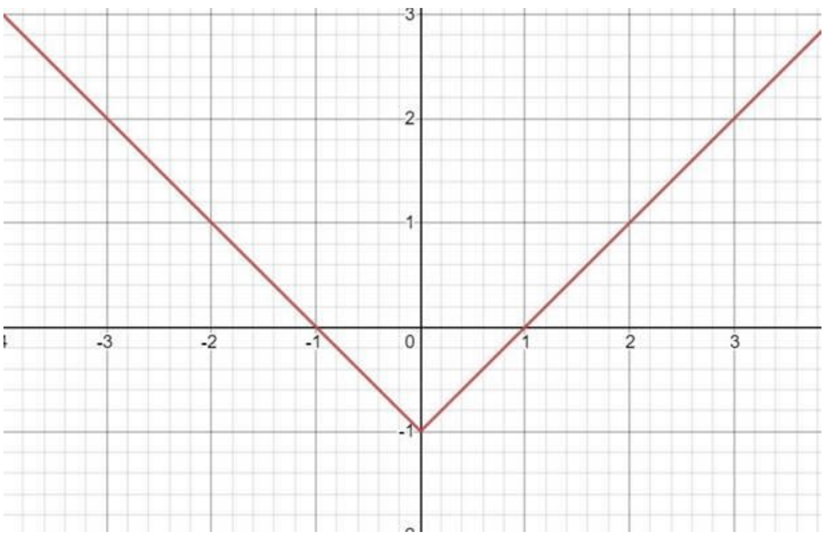
$$= \frac{\frac{8!}{5!\,3!}}{\frac{10!}{7!\,3!}}$$

$$= \frac{8!}{5!\,3!} \times \frac{7!\,3!}{10!}$$

$$= \frac{7 \times 6}{10 \times 9} = \frac{7}{15}$$

106. Ans. A.

As from the graph of y = |x| - 1



As we can see from above graph the function is continuous at x= 0 but not differentiable as slope changes. And at x = 1 the function is continuous and differentiable.

107. Ans. C.

For $x \to 1^- \Rightarrow [x] = 0 \Rightarrow \text{LHL} = \lim_{x \to 1^-} \frac{[x]}{|x|} = \lim_{x \to 1^-} \frac{0}{-x} = 0$

For $x \to 1^+ \Rightarrow [x] = 1 \Rightarrow \text{RHL} = \lim_{x \to 1^+} \frac{[x]}{|x|} = \lim_{x \to 1^+} \frac{1}{x} = 1$

For limit to be exist $\text{RHL} = \text{LHL} = 1$

108. Ans. B.

The value of function cannot be predictable at x=0 because function make sin(∞) = (-1,1) at x=0.

But at $x = \frac{2}{\sqrt{\pi}}$

$$\mathrm{LHL} = \lim_{x \to \frac{2}{\sqrt{\pi}}^{-}} f(x) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\mathrm{RHL} = \lim_{x \to \frac{2}{\sqrt{\pi}}^{+}} f(x) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

.

As $\mathrm{LHL} = \mathrm{RHL}$ So, function is continuous.

109. Ans. B.

We know that,

Range of f(x) = range of [1-sinx] = 1 – range(sinx) = 1-[-1,1] = [0,2].

110. Ans. A.

$$y = \cos^{-1}(\cos x)$$
$$\cos y = \cos x$$
$$-\sin y \,.\, y' = -\sin x$$
$$y' = \frac{\sin x}{\cos y} = \frac{\sin x}{\cos x} = \tan x \; y'\left(-\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right) = -1$$

111. Ans. D.

f(x) = 1 + x^2 + x^4

$$\Rightarrow \int (1 + x^2 + x^4)\, dx^2$$

$$\Rightarrow \int (1 + x^2 + x^4)\, 2x dx$$

$$\Rightarrow \int 2(x + x^3 + x^5)\,dx$$
$$\Rightarrow 2\left(\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6}\right) + c \Rightarrow \left(x^2 + \frac{x^4}{2} + \frac{x^6}{3}\right) + c$$

112. Ans. D.

As domain of the function

Maximum value of the function, $f(x)|2_{\max}$

And minimum value of the function, $f(x)|2_{\min}$

113. Ans. B.

Given that,

f(x) satisfies f(1) = f(4),

$$\Rightarrow \int_1^4 f'(x)\,dx$$
$$\Rightarrow [f(x)]_1^4$$
$$\Rightarrow f(4) - f(1) \Rightarrow 0 \because f(4) = f(1)$$

114. Ans. C.

$$\int_0^{\frac{\pi}{2}} e^{\ln(\cos x)}\,dx$$

We know that,

$$\Rightarrow \int_0^{\frac{\pi}{2}} \cos x\,dx \qquad \left(\because a^{\log_a b} = b\right)$$
$$\Rightarrow [\sin x]_0^{\frac{\pi}{2}}$$
$$\Rightarrow 1$$

115. Ans. B.

$$\int \sqrt{1-\sin 2x}\ dx = A\sin x + B\cos x + C$$
$$= \int \sqrt{(\cos x - \sin x)^2}\ dx$$
$$= \int (\cos x - \sin x)\ dx$$
$$A\sin x + B\cos x + C = \sin x + \cos x + C$$

By comparison on both sides, A=B=1

And A+B-2=0

116. Ans. B.

Family of all ellipses whose axes are along the coordinate axes,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Now,

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

diff.

$$\Rightarrow \frac{2x}{a^2} + \frac{2yy'}{b^2} = 0$$

$$\Rightarrow \frac{y}{x}y' = -\frac{b^2}{a^2}$$

$$\text{againdiff.} \Rightarrow \left(\frac{y'x-y}{x^2}\right)y' + \frac{y}{x}y'' = 0$$

Order of differential eqn. is 2.

Short cut: Order of an differential equation is equal to number of constant.

117. Ans. B.

Family of circles that touching both the coordinate axes in the first quadrant,

$$\Rightarrow (x-a)^2 + (y-a)^2 = a^2 \Rightarrow x^2 + y^2 - 2ax - 2ay + a^2 = 0.....(1)$$

Now differentiate

$$\Rightarrow x^2 + y^2 - 2ax - 2ay + a^2 = 0$$
$$\Rightarrow 2x + 2yy' - 2a - 2ay' = 0$$
$$\Rightarrow a = \frac{x + yy'}{1 + y'} \ldots . (2)$$

Now put the value of a in eqn. (1) form eqn. (2)

$$\Rightarrow x^2 + y^2 - 2\left(\frac{x+yy'}{1+y'}\right)x - 2\left(\frac{x+yy'}{1+y'}\right)y + \left(\frac{x+yy'}{1+y'}\right)^2 = 0$$

After simplifying we get degree of highest order is 2, then degree is 2.

118. Ans. B.

$$y = A - \frac{B}{x}$$
$$y' = \frac{B}{x^2} \ldots .. (1) y" = -2\frac{B}{x^3} \ldots . (2)$$

By eqn. (1) and (2)

$$\Rightarrow \frac{y'}{y"} = -\frac{x}{2} \Rightarrow xy" + 2y' = 0$$

119. Ans. A.

We know that,

$$\Rightarrow I = \int_0^{\pi} \ln\left(\tan\frac{x}{2}\right) dx \ldots \ldots (1)$$
$$\Rightarrow I = \int_0^{\pi} \ln\left(\tan\frac{\pi - x}{2}\right) dx$$
$$\Rightarrow I = \int_0^{\pi} \ln\left(\cot\frac{x}{2}\right) dx \ldots \ldots (2)$$

addingeqn. 1&2

$$\Rightarrow 2I = \int_0^{\pi} \ln\left(\cot\frac{x}{2}\tan\frac{x}{2}\right) dx$$

$$\Rightarrow 2I = \int_0^{\pi} \ln 1 \, dx$$
$$\Rightarrow I = 0$$

120. Ans. B.

The tangent to the curve $y = e^x$ at the point (0, 1)

$$\text{slope} = \frac{de^x}{dx} = e^x = 1$$

Then,

Tangent at point (0,1) is $y = x+1$.

Point at which tangent meet the x-axis it means $y=0$,

So,

$$\Rightarrow y = x + 1$$
$$\Rightarrow 0 = x + 1 \Rightarrow x = -1$$

Point is (-1,0)