BYJU'S
EXAM PREP

AE/JE Foundation

Electrical Engineering

Electrical Machines

Important Formula Notes

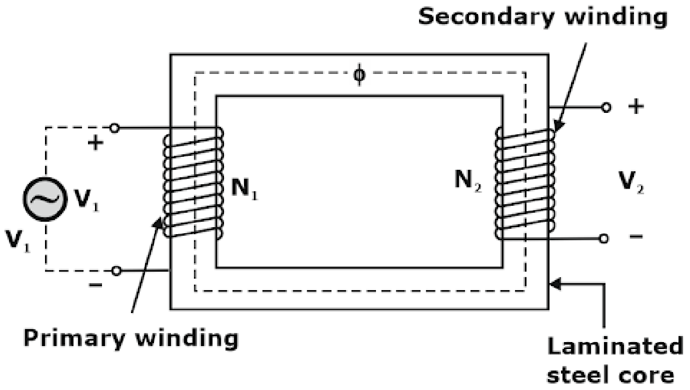
ELECTRICAL MACHINES (FORMULA NOTES)

Transformer

Features of Single-phase Transformers

Transformer is a static device which performs:

- It works on the principle of electromagnetic induction.
- Transfers electrical energy from one electrical circuit to another without changing of frequency.
- The energy transfer usually takes place with change in voltage, although this is not always necessary.



Basic Diagram of Transformer

- When the transformer raises the voltage, i.e. when output voltage of a transformer is higher than its input voltage, it is called step-up transformer and when output voltage is lowered, it is called step-down transformer.
- Transformer doesn't change the frequency of the system. Hence, it can be treated as **constant frequency device**, it transfers almost same amount of power from one circuit to another so it can be treated as **constant power device** and the amount of flux in its core remains constant, so it is also known as **constant flux device**.

Ideal Transformer

- Ideal transformer does not have any loss, so resistances are also assumed to be zero.
- Magnetic leakage flux in transformer is completely zero. (K=1, coefficient of coupling)
- Magnetic permeability of transformer core $\mu = \infty$. So, the flux can be established without any excitation current.
- Magnetization curve of transformer core is assumed to be linear.
- For an ideal two-winding transformer with primary voltage V_1 applied across N_1 primary turns and secondary voltage V_2 appearing across N_2 secondary turns:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

- The primary current I_1 and secondary current I_2 are related by:

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{V_2}{V_1}$$

- For a single-phase transformer with rated primary voltage V_1, rated primary current I_1, rated secondary voltage V_2 and rated secondary current I_2, the volt ampere rating S is:

$$S = V_1 I_1 = V_2 I_2$$

- During operation of transformer:

$$B_m \propto \frac{E_1}{f} \propto \frac{V_1}{f}$$

- B_{max} = Constant $\Rightarrow \frac{V_1}{f} = \text{Constant}$

Note:

- Gross cross-sectional area = Area occupied by magnetic material + Insulation material.
- Net cross-sectional area = Area occupied by only magnetic material excluding area of insulation material.
- Hence for all calculations, net cross-sectional area is taken since majority $\phi(\text{flux})$ flows in magnetic material.

  $\phi = BA_n$

E.M.F. EQUATION OF TRANSFORMER

Let, Flux $\Phi = \Phi_m \sin(\omega t)$

As the flux is assumed to be sinusoidal,

In primary winding side:

According to faraday's law,

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_m \sin \omega t)$$

Instantaneous value of emf in primary

$$e_1 = N_1 \phi_m \omega \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\boxed{E_{1rms} = \sqrt{2}\pi f \phi_m N_1}$$

In secondary winding side

Similarly, $\boxed{E_{2rms} = \sqrt{2}\pi f \phi_m N_2}$

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

Also, $\frac{E_1}{N_1} = \frac{E_2}{N_2} = \sqrt{2}\,\pi f \phi_m$

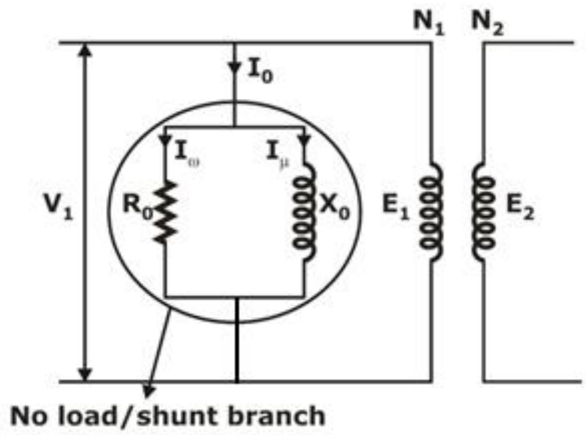
From the above equation,

EMF per turn in primary = EMF per turn in secondary

Note:

- Transformation ratio = $K = \frac{E_2}{E_1} = \frac{N_2}{N_1}$
- Turn ratio = $\frac{1}{K} = N_1 : N_2$

Equivalent circuit of Transformer under No-Load condition:



- No load current/ exciting current = $I_0 = \bar{I}_\mu + \bar{I}_w = I_0\angle -\phi_0$

  $I_w = I_0 \cos\phi_0$
  $I_\mu = I_0 \sin\phi_0$

  I_w = core loss component

  I_μ = magnetizing component

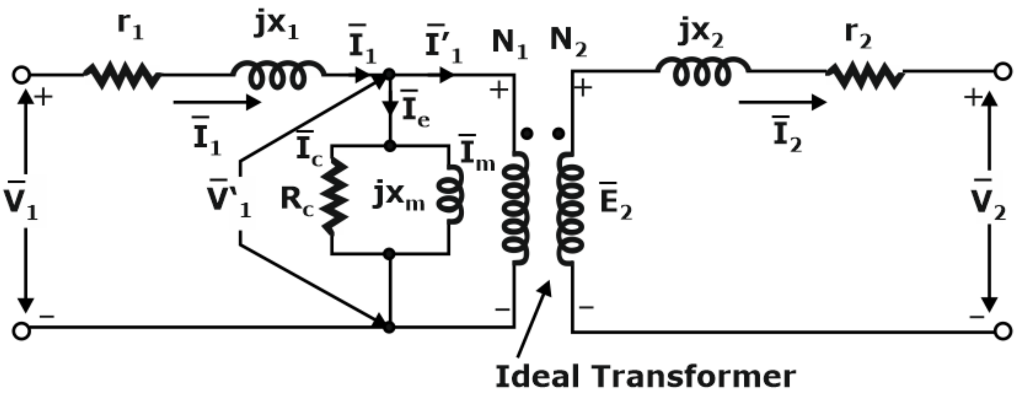
- No load power = $V_1 I_0 \cos\phi_0 = V_1 I_w$ = Iron losses

  $R_0 = \frac{V_1}{I_w}; X_0 = \frac{V_1}{I_\mu} \Rightarrow \quad I_w = \frac{\text{No load power}}{V_1}$

EQUIVALENT CIRCUIT OF A TRANSFORMER

Equivalent circuit

The equivalent circuit of a transformer having voltages transformation ratio, $k = \frac{E_2}{E_1}$ is shown in figure below:



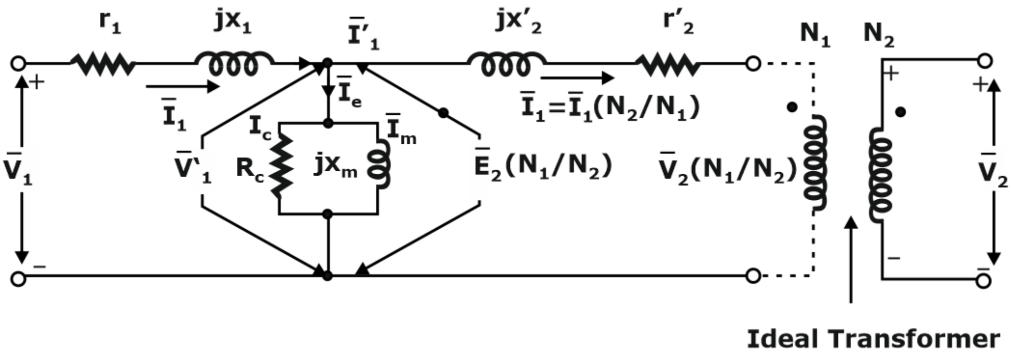
Exact Equivalent Circuit

Practical representation of simple two-winding transformer:

$$\boxed{\vec{E}_2 = \vec{V}_2 + \vec{I}_2 R_2 + j\vec{I}_2 X_2}$$

$$\boxed{\vec{V}_1 = \vec{E}_1 + \vec{I}_1 R_1 + j\vec{I}_1 X_1}$$

When all the parameters are referred to primary side

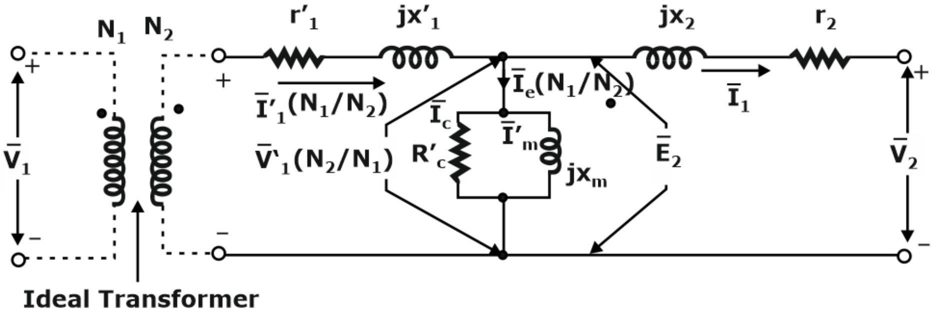


Referred to Primary side

r_{e_1} = Equivalent resistance referred to primary $= \left(r_1 + \frac{r_2}{k^2}\right)$

x_{e_1} = Equivalent reactance referred to primary $= \left(x_1 + \frac{x_2}{k^2}\right)$

When all the parameters are referred to secondary side



Referred to Secondary side

r_{e_2} = Equivalent resistance to secondary side $= \left(r_2 + k^2 r_1\right)$

x_{e_2} = Equivalent reactance referred to secondary side $= \left(x_2 + k^2 x_1\right)$

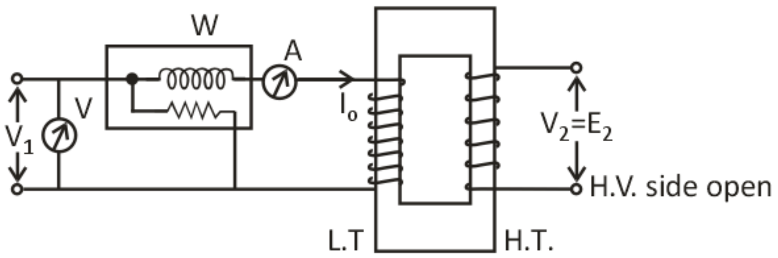
R_c = Core loss resistance

X_m = Magnetising reactance

Where, k = Transformation Ratio $= \dfrac{N_2}{N_1}$

TESTS ON TRANSFORMER:

1. **Open Circuit Test (O.C.) or No-load Test:**
   - This test is performed to determine core or iron loss P_i and no-load current I_o.
   - This test is helpful in determining the magnetizing current I_μ and the core loss components I_w and hence no-load shunt branch parameters R_o and X_o.



Open Circuit configuration

Iron loss (P_i) = Input no-load Power = W_o (wattmeter reading)

No-load current = I_o

Hence no load p.f., $\cos\phi_o = \dfrac{W_o}{V_1 I_o}$

Now from no-load phasor diagram

$I_w = I_o \cos\phi_o$

$I_\mu = I_o \sin\phi_o$

Now,

$$R_o = \frac{V_1}{I_w} = \frac{V_1}{I_o \cos\phi_o}$$

$$X_o = \frac{V_1}{I_\mu} = \frac{V_1}{I_o \sin\phi_o}$$

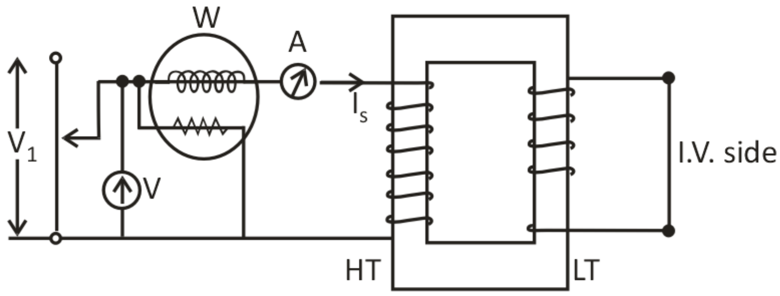
At no load efficiency of the transformer is zero because output power is zero.

No load test conducted on rated voltage and rated frequency on LV side of transformer.

2. **Short Circuit Test (S.C.):**

   S.C. test is carried out at rated current to determine the Cu loss at full load condition.

   Full load Cu-loss, $P_C = I^2{}_s r_{e_1} = W$



Short circuit of transformer

Equivalent resistance referred to HV side, $r_{e_1} = \left(\frac{W}{I_s^2}\right)$

Equivalent impedance referred to HV side, $Z_{e_1} = \left(\frac{V_s}{I_s}\right)$

Thus, equivalent reactance, $X_{e_1} = \sqrt{Z_{e_1}{}^2 - r_{e_1}{}^2}$

Note: As the applied voltage required to circulate full load current at short-circuit would be limited to 5% to 10% of the rated voltage, the core loss at such reduced voltage is ignored. Also, the exciting current at such low voltage may be neglected.

VOLTAGE REGULATION OF A TRANSFORMER

It is defined as the percentage change in secondary terminal voltage from no load to full load, expressed as a percentage (or pu) of the secondary rated voltage.

If ,

V_2 = secondary terminal voltage at rated load, and

E_2 = secondary terminal voltage at no load,

Then,

Voltage regulation $= \frac{V_{NL} - V_{FL}}{V_{FL}}$ (in pu)

Voltage regulation $= \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$ (in %)

The magnitude of change in secondary voltage depends on the load power factor, load current, total resistances and total leakage reactance of the transformer.

Therefore, voltage regulation equation can be given as:

Voltage drop in the secondary terminal voltage,

$E_2 - V_2 = I_2 r_{e_2} \cos\theta_2 \pm I_2 x_{e_2} \sin\theta_2$

Hence for any load current I_2, Per unit voltage regulation is given by:

$$V.R = \frac{V_{NL} - V_{FL}}{V_{FL}}$$

$$V.R = \frac{E_2 - V_2}{V_2}$$

$$V.R = \frac{I_2 r_{e_2}}{V_2}\cos\theta_2 - \frac{I_2 x_{e_2}}{V_2}\sin\theta_2$$

where,

$\frac{I_2 r_{e_2}}{V_2}$ = PU Equivalent resistance or PU resistance drop = R_{pu}

$\frac{I_2 X_{e_2}}{V_2}$ = PU Equivalent reactance or PU reactance drops = X_{pu}

At rated load voltage regulation in PU = $\left(R_{pu}\cos\theta_2 \pm X_{pu}\sin\theta_2\right)$

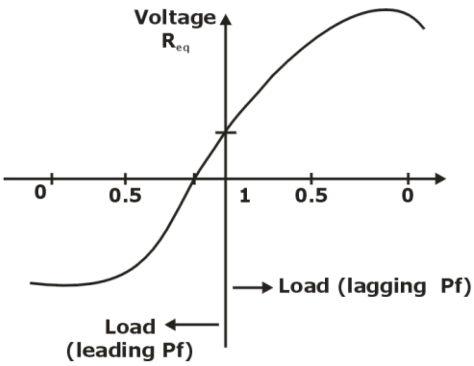
Percentage voltage drop $= \left(R_{pu}\cos\theta_2 \pm X_{pu}\sin\theta_2\right) \times 100$

Condition for Zero Voltage Regulation:

Zero voltage regulation takes place at leading pf and when load pf is $\left(\frac{X_{pu}}{Z_{pu}}\right)$.

Condition for Maximum Voltage Regulation:

Maximum voltage regulation occurs at lagging power factor and when pf is $\left(\frac{R_{pu}}{Z_{pu}}\right)$



Plot between voltage regulation and power factor

- Voltage regulation is a figure of merit of a transformer and its low value is always desired.
- The voltage regulation may be reduced by reducing the per unit impedance of the transformer.
- Per unit impedance, Z_{pu} can be reduced for reducing R_{pu} or X_{pu}.
- Per unit resistance, R_{pu} is directly kept at an optimally low value.
- X_{pu} can be reduced by reducing leakage flux.

Per Unit System

A per-unit system is the expression of system quantities as fractions of a defined base unit quantity. Calculations are simplified because quantities expressed as per-unit do not change when they are referred from one side of a transformer to the other.

Per Unit Value = $\frac{\text{Actual value in any unit}}{\text{Base value in same unit}}$

It is usual to assume the base values as:-

- Base voltage = rated voltage of the machine
- Base current = rated current of the machine
- Base impedance = base voltage /base current
- Base power = base voltage x base current

Hence it can be concluded that out of 4 parameters, any two can be selected as base value and according to that the remaining values are determined.

Values which we will choose: 1) Base Apparent Power = S_B; 2) Base Voltage = V_B

Values which we calculate: 1) Base Current $I_B = \frac{S_B}{V_B}$; 2) Base Impedance $Z_B = \frac{V_B^2}{S_B}$

For an ideal Transformer $E_{1PU} = E_{2PU}$.

Also, $Z_{1PU} = Z_{2PU}$

Direct relation for changing the base of an impedance:

$$Z_{PU\_NEW} = Z_{PU\_OLD} \times \frac{S_{B\_NEW}}{S_{B\_OLD}} \times \frac{V_{B\_OLD}^2}{V_{B\_NEW}^2}$$

Losses present in transformer

1. Copper losses: Transformer windings major losses
2. Iron losses: Transformer core
3. Stray load losses: Cu parts and Iron parts minor losses
4. Dielectric losses: Insulating materials.

1. Cu losses in transformer:

Total Cu loss = $I_1^2R_1 + I_2^2R_2 = I_1^2R_{01} = I_2^2R_{02}$

- Rated current on primary side = $\frac{\text{VA rating of transformer}}{E_1}$
- Similarly, current on secondary side = $\frac{\text{VA rating of transformer}}{E_2}$

- Cu losses $\propto I_1^2$ or I_2^2. Hence these are called as variable losses.
- P.U. full load Cu loss = $\frac{\text{FL Cu loss in watts}}{\text{VA rating of transformer}} = \frac{I_1^2 R_{01}}{E_1 I_1}$
- If VA rating of transformer is taken as base then P.U Cu loss $\propto I_1^2$ as remaining terms are constant.
- P.U. Cu loss at x (Loading factor) of full load = x^2 × PU full load Cu loss
- $\text{P.U. resistance drop ref to primary} = \frac{I_1 R_{01}}{E_1} \times \frac{I_1}{I_1} = \frac{I_1^2 R_{01}}{E_1 I_1}$

  ∴ P.U. resistance drop = P.U. Full Load Cu loss

2. Iron (or) Core losses in Transformer:

2.1 Hysteresis loss:

Steinmetz formula:

$W_h = \eta B_{max}^x f V$

Area under one hysteresis loop.

where,

η = Stienmetz coefficient

B_{max} = Maximum flux density in transformer core.

f = Frequency of magnetic reversal = supply frequency.

V = Volume of core material

x = Hysteresis coefficient (or) stienmetz exponent = 1.6 (Si or CRGO steel)

2.2 Eddy current loss:

Eddy current loss, $W_e \propto R_{ce} \times I_e^2$

As area decreases in laminated core resistance as a result conductivity decrease.

$W_e = K(\text{Constant})B_{max}^2 f^2(\text{Supply freq.}) \times t^2(\text{Thickness of lamination})$

(It is a function of σ)

During operation of transformer:

$B_m \propto \frac{V_1}{f}$

Case(i): $\frac{V_1}{f}$ = Constant, B_{max} = Constant

$$\begin{aligned} W_h &\propto f \\ W_h &= Af \end{aligned} \quad , \quad \begin{aligned} W_e &\propto f^2 \\ W_e &= Bf^2 \end{aligned}$$

$$\therefore \left.\begin{aligned} W_i &= W_h + W_e \\ W_i &= Af + Bf^2 \end{aligned}\right\} \text{When } B_{max} = \text{Constant}$$

Case(ii): $\frac{V_1}{f} \neq$ Constant, $B_{max} \neq$ Constant

$$W_h \alpha \left(\frac{V_1}{f}\right)^x f \quad , \quad W_e \propto \left(\frac{V_1}{f}\right)^2 f^2$$

$$W_h \alpha \frac{V_1^x}{f^{x-1}} \qquad W_e \propto V_1^2$$

$W_i = W_h + W_e$

$$W_i = A\frac{V_1^x}{f^{x-1}} + BV_1^2$$

P.U. iron loss:

- P.U. iron loss = $\dfrac{\text{Iron loss in watts}}{\text{VA rating of transformer}}$
- As VA rating is chosen as base then the P.U. iron loss are also constant at all load conditions.

To find out constant losses:

- W_0 = Losses in transformer under no-load condition = Iron losses + Dielectric loss + no-load primary loss ($I_0^2R_1$)
- Constant losses = $W_0 - I_{02}^2 R_1$

  Where, R_1 = LV winding resistance.

To find out variable losses:

- W_{sc} = Loss in transformer under SC condition = Full Load Cu loss + stray load losses (Cu and Iron) + Iron losses in both windings.
- Variable losses = W_{sc} – Iron losses corresponding to V_{SC}.

TRANSFORMER EFFICIENCY

Efficiency Calculation:

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \left(\frac{V_2I_2\cos\theta_2}{V_2I_2\cos\theta_2 + P_c + I_2^2 r_{e2}}\right)$$

where, P_i = core losses

$I_2^2 r_{e_2}$ = ohmic losses

$V_2 I_2$ = output kVA

$\cos\theta_2$ = load p.f.

$$\therefore \qquad \eta = \left(1 - \frac{\text{Losses}}{\text{Input power}}\right)$$

Note: Maximum efficiency takes place when variable ohmic losses are equal to the fixed core loss P_i.

$$\therefore \quad I_2^2 r_{e_2} = P_i$$

$$\Rightarrow \quad I_2 = \sqrt{\frac{P_i}{r_{e_2}}}$$

$$X = \sqrt{\frac{P_i}{full\ load\ ohmic\ loss}}$$

$$\therefore \quad (kVA)_{\max\ \eta} = (kVA)_{fl}\sqrt{\frac{P_i}{I_{fl}^2 r_{e_2}}}$$

$$S\eta_{max} = V \times I_j\sqrt{\frac{P_I}{I_j^2 R_{eq}}}$$

$$S\eta_{max} = S_j \times \sqrt{\frac{P_I}{P_{cu(j)}}}$$

Here, $S\eta_{max}$ represents the KVA at the maximum efficiency.

I_j – any current at which copper loss is known.

S_j – any kVA at which the Cu-loss is known.

All Day Efficiency:

$$\text{All Day efficiency} = \eta_{\text{all-day}} = \frac{\text{Output in KWh}}{\text{Input in KWh}}(\text{for 24 hours})$$

$$\boxed{\eta_{\text{all day}} = \frac{\text{output kWh in 24hrs}}{\text{input kWh in 24hrs}}}$$

$$\boxed{\frac{\text{output KWh in 24hr}}{\text{output KWh in 24hr} + [P_{cu}(\text{KWh}) + P_i(\text{KWh})]\text{in 24hr}}}$$

8. PARALLEL OPERATION OF TRANSFORMERS

Conditions for parallel operation of Single-Phase transformers:

Necessary conditions:

- The transformers must have the same polarities.
- The transformers should have equal turn ratios.

Desirable conditions:

- The voltages at full load across transformers internal impedance should be equal.
- The ratios of their winding resistances to reactance should be equal for both transformers. This condition ensures that both transformers operate at the same power factor, thus sharing active power and reactive volt amperes according to their ratings.

Conditions for parallel operation of Three-Phase transformers:

The condition for the parallel operation of single phase and three phase transformers is the same but with the following additions:

- The phase sequence of the transformers must be identical.
- The primary and secondary voltages of all the transformers connected in parallel must have the same phase shift.

i.e. Leakage impedance $\propto \dfrac{1}{\text{kVA rating}}$

Let two transformers A and B having Z_{ea} and Z_{eb} equivalent impedances are in parallel and S_{ar} and S_{br} are the kVA rating respectively.

$$Z_{ea} \propto \frac{1}{S_{ar}}$$

$$Z_{eb} \propto \frac{1}{S_{br}}$$

$$\Rightarrow \qquad \frac{Z_{ea}}{Z_{eb}} = \left(\frac{S_{br}}{S_{ar}}\right)$$

$$\therefore \qquad \frac{I_a}{I_b} = \left(\frac{S_{ar}}{S_{br}}\right)$$

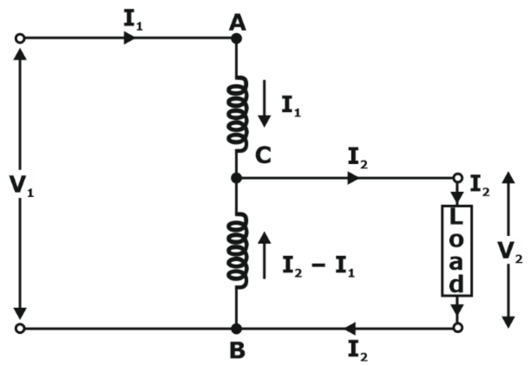
Hence, kVA shared by $A = \left(\dfrac{Z_{eb}}{Z_{ea} + Z_{eb}}\right)S$

and kVA shared by $B = \left(\dfrac{Z_{ea}}{Z_{eb} + Z_{ea}}\right)S$

where, S = Total kVA

AUTO – TRANSFORMER

A transformer whose winding is common to both primary and secondary circuits is called autotransformer. The primary and secondary are connected electrically as well as coupled magnetically.



Auto-transformer construction

- K of auto-transformer = $\frac{LV}{HV}$

  $(kVA)_{induction} = (V_1 - V_2)I_1$

  Input kVA = V_1I_1

  $$\frac{(kVA)_{induction}}{\text{input KVA}} = \frac{(V_1 - V_2)I_1}{V_1I_1} = 1 - \frac{LV}{HV} = 1 - K$$

  $\therefore$ $(kVA)_{induction} = (1 - K)$ input kVA

  $(kVA)_{conduction}$ = Input kVA − $(kVA)_{ind}$

  $(kVA)_{conduction}$ = K × input kVA

- Weight of conductor in section AB of auto $t/f \propto (N_1 - N_2)I_1$
- Weight of conductor in section BC of auto $t/f \propto (I_1 - I_2)N_1$

  $\therefore$ Total Weight of conductor in auto t/f is

  $\propto I_1(N_1 - N_2)N_1 + (N_2 - N_1)N_2$

  $\propto 2(N_1 - N_2)I_1$

- Total Weight of conductor in two winding transformer

  $\propto I_1N_1 + I_2N_2 \propto 2I_1N_1$

- $\frac{\text{Wt. of conductor in an auto } t/f}{\text{Wt. of conductor in 2 wdg } t/f} = \frac{2(N_1 - N_2)I_1}{2N_1I_1} = 1 - \frac{N_2}{N_1} = 1 - K$

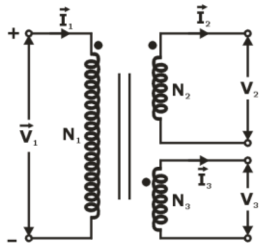
  Weight of conductor in auto-transformer = (1 − K) (Weight of conductor in in two winding transformer)

- Thus, saving of conductor material if auto-transformer is used = K × (Conductor weight in two winding transformer)

Some Important Relations:

- $\frac{\text{Weight of conductor in auto transformer}}{\text{Weight of conductor in 2-winding transformer}} = (1 - k)$
- $\frac{\text{kVA rating as an auto transformer}}{\text{kVA rating as 2-winding transformer}} = \left(\frac{1}{1 - k}\right)$
- $\frac{\text{p.u. full load losses as an auto transformer}}{\text{p.u. full load losses as a 2-winding transformer}} = (1 - k)$
- $\frac{\text{p.u. impedance drop in an auto transformer}}{\text{p.u. impedance drop in 2-winding transformer}} = (1 - k)$
- $\frac{\text{Voltage regulation as in auto transformer}}{\text{Voltage regulation in 2-winding transformer}} = (1 - k)$

TERTIARY WINDING



Tertiary winding of transformer

$$\frac{\vec{V}_1}{\vec{N}_1} = \frac{\vec{V}_2}{\vec{N}_2} = \frac{\vec{V}_3}{\vec{N}_3}$$

MMF balance:

$$N_1\vec{I}_1 - N_2\vec{I}_2 - N_3\vec{I}_3 = N_1\vec{I}_0$$

$$\vec{I}_1^* = \vec{I}_0^* + \frac{N_2}{N_1} \times \vec{I}_2^* + \frac{N_3}{N_1} \times \vec{I}_3^*$$

$$\vec{V}_1\vec{I}_1 * = \vec{V}_1\vec{I}_0 * + \left(\vec{V}_1 \times \frac{N_2}{N_1}\right) \times \vec{I}_2 * + \left(\vec{V}_1 \times \frac{N_3}{N_1}\right) \times \vec{I}_3 *$$

$$\vec{V}_1 I_1 * = \vec{V}_1\vec{I}_0 * + \vec{V}_2\vec{I}_2 * + \vec{V}_3\vec{I}_3 *$$

$$\boxed{\vec{S}_1 = \vec{S}_2 + \vec{S}_0 + \vec{S}_3}$$

Application of Different 3-phase Connections:

1. Δ/Δ → This connection is used where we have simple 3-φ load, specially at low voltage level. No mix loading possible.
2. Y/Δ → This connection is used for step down application.
3. Δ/Y → This connection is used for step up application except in distribution system where, Δ/Y is used for step down application for mix loading.
4. Y/Y → This connection is quite attractive for HV applications. It is not generally used without a tertiary Δ.

Impact of dimensions on varius parameters :

Voltage rating of transformer depends on the level of insulation of the conductor.

Voltage rating is directly proportional to the area of the transformer.

Voltage rating is directly proportional to square of the dimensions of the transformer.

Current rating of the transformer is depends on the cross sectional area of the conductor.

Current rating is directly proportional to the area of the transformer.

Current rating is directly proportional to square of the dimensions of the transformer.

KVA rating of the transformer is quartic of the dimensions of the transformer.

DC Machines

EMF EQUATION AND DEVELOPED TORQUE

Notations:

$\phi \to$ Flux/pole (Wb)

P $\to$ No. of poles

Z $\to$ Total no of conductors

N $\to$ Rotor speed (rpm)

A $\to$ Number of parallel paths

A = 2, for wave windings

A = P, for lap winding

$$E = \frac{P\phi NZ}{60A} \text{ Volts}$$

As we know that,

$$\omega_m = \frac{2\pi N}{60} \text{rad / sec}$$

$$N = \frac{60\omega}{2\pi}$$

Put this value in the above formula-

$$E = \frac{P\phi Z}{60A} \times \left(\frac{60\omega_m}{2\pi}\right)$$

$$E = \left(\frac{PZ}{2\pi A}\right)\phi\,\omega_m$$

$\boxed{E_a = k\phi\omega_m}$ Where $k = \frac{PZ}{2\pi A}$ = Machine constant

Developed Torque

Developed power $P_a = E_a I_a$ and $P_a = T.\omega_m$

$\therefore P_a = T.\omega_m$

$T = \frac{E_a I_a}{\omega_m} = k\phi I_a \quad \therefore E = k\varphi\omega_m$

$\boxed{T = k\phi I_a}$ Induced emf $\Rightarrow \boxed{E_a = k\phi\omega_m}$

Developed Torque $\Rightarrow \boxed{T_d = k\phi I_a}$

COMMUTATION PROCESS

The reversal of current in the armature coil by means of brush and commutator bars, is called commutation process. Good commutation means no sparking at the brushes and with commutator surface remaining unaffected during continuous operation of the DC machine.

- Under commutation: $T_{commutation} > T_C$
- Over commutation: $T_{commutation} < T_C$

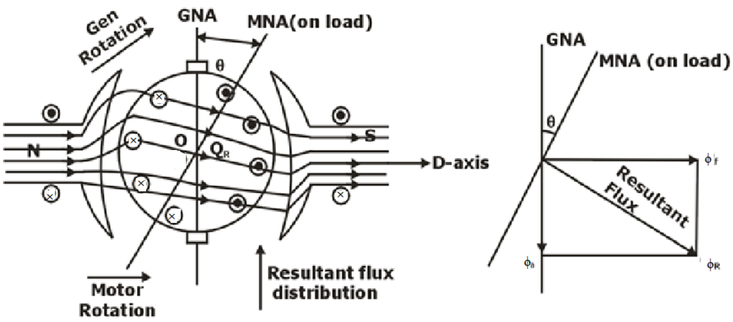
The time required by the coil current to change from $+I_C$ to $-I_C$ is called the commutation period T_C.

$$\boxed{T_C = \frac{\text{Brush width}}{\text{Commutator peripheral speed}}}$$

Where, T_C is commutation period.

ARMATURE REACTION

The effect of armature flux due to armature current over main field flux is known as Armature reaction effect.



Armature Reaction Effect

Effects of Armature Reaction

(i) If strengthening effect = weakening effect, the average flux under the pole remains same and therefore no demagnetizing effect of AR.

But due to saturation, strengthening effect is less than the weakening effect so average flux under each pole reduces and hence Armature reaction is “Demagnetizing”.

(ii) The MNA shifts in the direction of rotation for generator action and in a direction opposite to rotation for motor.

(iii) Armature flux is perpendicular to field flux. So, the nature of armature reaction is cross magnetizing.

INTERPOLES

- In large machine, interpoles also called commutating poles are used to overcome commutation problems.
- The interpolar winding is designed to neutralize armature MMF in interpolar region.
- The interpoles winding carries the armature current as it is connected in series with the armature winding. The presence of interpoles ensures sparkless linear commutation.

COMPENSATING WINDING

compensating winding consists of conductors embedded in pole faces and carry armature current in a direction opposite to armature conductor current under one pole arc. The compensating winding may be designed to completely neutralize the armature MMF of the conductors that lie under the pole are resulting into restoration of main field flux.

DC machine compensating winding mmf per pole is:

- $F_{comp}(MMF) = \left[\frac{Z/2}{P} \times \left(\frac{\text{Pole arc}}{\text{Pole Pitch}}\right) \times \frac{I_a}{A}\right] AT/pole$
- $F_{comp}(MMF)_{Interpole} = \left[\frac{Z/2}{P} \times \left(1 - \frac{\text{Pole arc}}{\text{Pole Pitch}}\right) \times \frac{I_a}{A} + \frac{B_{interpole}}{\mu_0} \times L_{interpolar}\right] AT/pole$

CLASSIFICATION OF DC MACHINES:

DC Machines have been classified into multiple categories based on the connection of field winding and the armature winding.

DC Machines

- **Separately Excited (Isolation b/w field winding and armature winding)**
- **Self-Excited**
  - **(a) Series Excited**
  - **(b) Shunt excited**
  - **(c) Compound Excited**
    - **(i) Cumulative compound**
    - **(ii) Differential compound**

Shunt Generator:

- For a shunt generator with armature induced voltage E_a, armature current I_a and armature resistance R_a, the terminal voltage V is:

$V = E_a - I_aR_a$

- The field current I_f for a field resistance R_f is: $I_f = \frac{V}{R_f}$
- The armature induced voltage E_a and torque T with magnetic flux Φ at angular speed ω are:

$E_a = k_f\Phi\omega_m = k_m\omega_m \qquad T = k_f\Phi I_a = k_m I_a$

Where, k_f and k_m are design coefficients of the machine.

Note that for a shunt generator:

- Induced voltage is proportional to speed.
- Torque is proportional to armature current.

- The airgap power P_e for shunt generator is:

$$P_e = \omega_m T = E_a I_a = k_m \omega_m I_a$$

Series Generator:

- For a series generator with armature induced voltage E_a, armature current I_a, armature resistance R_a and field resistance R_f, the terminal voltage V is:

  $V = E_a - (I_aR_a + I_aR_f) = E_a - I_a(R_a + R_f)$

  The field current is equal to the armature current.

- The armature induced voltage E_a and torque T with magnetic flux Φ at angular speed ω_m are:

$$E_a = k_f \Phi \omega_m I_a = k_m \omega_m I_a$$

$$T = k_f \Phi I_a^2 = k_m I_a^2$$

Where k_f and k_m are design coefficient of the machine.

Note that for a series generator:

- Induced voltage is proportional to both speed and armature current,
- Torque is proportional to the square of armature current.
- Armature current is inversely proportional to speed for a constant E_a

- The airgaps power P_e for a series generator is:

$$P_e = \omega_m T = E_a I_a = k_m \omega_m I_a^2$$

Cumulatively compounded DC generator: (long shunt):

(a) $I_a = I_f + I_L$

(b) $V_t = E_a - I_a (R_a + R_s)$

(c) $I_{sf} = \dfrac{V_x}{R_f} =$ Shunt field current

(d) The equivalent effective shunt field current for this machine is given by

$$I_{sf} = I_{sf} + \frac{N_{se}}{N_f} I_a - \left(\frac{\text{Armature reaction MMF}}{N_f}\right)$$

Where, N_{se} = No. of series field turns

N_f = No of shunt field turns

Differentially compounded DC generator: (long shunt):

(a) $I_a = I_f + I_L$

(b) $V_t = E_a - I_a (R_a + R_s)$

(c) $I_{sf} = \frac{V_x}{R_f} =$ Shunt field current

(d) The equivalent effective shunt field current for this machine is given by

$$I_{sf} = I_{sf} - \frac{N_{se}}{N_f} I_a - \left(\frac{\text{Armature reaction MMF}}{N_f}\right)$$

Where, N_{se} = No. of series field turns

N_f = No of shunt field turns

Shunt Motor:

- For a shunt generator with armature induced voltage E_a, armature current I_a and armature resistance R_a, the terminal voltage V is:

  $V = E_a + I_aR_a$

  The field current I_f for a field resistance R_f is: $I_f = \frac{V}{R_f}$

- The armature induced voltage E_a and torque T with magnetic flux Φ at angular speed ω are:

  $$E_a = k_f\Phi\omega = k_m\omega$$
  $$T = k_f\Phi I_a = k_m I_a$$

  Where, k_f and k_m are design coefficients of the machine.

  Note that for a shunt motor:

  - Induced voltage is proportional to speed.
  - Torque is proportional to armature current.

- The airgap power P_e for shunt generator is:

  $$P_e = \omega T = E_a I_a = k_m \omega I_a$$

- The speed of the shunt motor, $\omega = \frac{V}{k\phi} - \frac{TR_a}{(K\phi)^2}$ where, $K = \frac{PZ}{2\pi A}$

Series Motor:

- For a series motor with armature induced voltage E_a, armature current I_a, armature resistance R_a and field resistance R_f, the terminal voltage V is:

  $V = E_a + (I_aR_a + I_aR_f) = E_a + I_a(R_a + R_f)$

  The field current is equal to the armature current.

- The armature induced voltage E_a and torque T with magnetic flux Φ at angular speed ω are:

  $$E_a = k_f\Phi\omega I_a = k_m\omega I_a \qquad T = k_f\Phi I_a^2 = k_m I_a^2$$

  Where k_f and k_m are design coefficient of the machine.

  Note that for a series motor:

  - Induced voltage is proportional to both speed and armature current.
  - Torque is proportional to the square of armature current.

- Armature current is inversely proportional to speed for a constant E_a

- The airgaps power P_e for a series motor is:

  $P_e = \omega T = E_a I_a = k_m \omega I_a^2$

Losses:

- Constant losses (P_k) = P_{w-f} + P_{io}

  where, P_{io} = No-load core loss

  P_{w-f} = Windage and frictional losses

- Variable losses (P_v) = P_c + P_{st} + P_b

  Where, P_c = Copper losses = $I_a^2 R_a$

  P_{st} = Stray load loss

  P_b = Brush contact drop = V_b I_a, where, V_b = Brush voltage drop

- The total machine losses, P_L = P_k + P_v

Efficiency:

- The per unit efficiency η of an electrical machine with input power P_{in}, output power P_{out} and power loss P_{loss} is:

  $$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{(P_{out} + P_{loss})} = \frac{(P_{in} - P_{loss})}{P_{in}}$$

- Rearranging the efficiency equations:

  $$P_{in} = P_{out} + P_{loss} = \frac{P_{out}}{\eta} = \frac{P_{loss}}{(1-\eta)}$$

  $$P_{out} = P_{in} - P_{loss} = \eta P_{in} = \frac{\eta P_{loss}}{(1-\eta)}$$

  $$P_{loss} = P_{in} - P_{out} = (1-\eta) P_{in} = \frac{(1-\eta) P_{out}}{\eta}$$

Note :

- The terminal voltage of the DC generator is given by $V_t = E_a - I_a R_a$
- The terminal voltage of the DC motor is given by $V_t = E_a + I_a R_a$
- Speed regulation of DC machine is given by, $SR = \frac{\omega_{nl} - \omega_{fl}}{\omega_{fl}} \times 100\% = \frac{N_{nl} - N_{fl}}{N_{fl}} \times 100\%$
- Voltage regulation, $VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\%$

OPERATING CHARACTERISTICS OF DC GENERATOR

1. No-load characteristics:

   $E_a = f(I_f)$, with constant speed.

   This is also called **open-circuit characteristics or magnetization curve**.

2. Load Characteristics:

   $V_t = f(I_f)$, with both I_a & N constant.

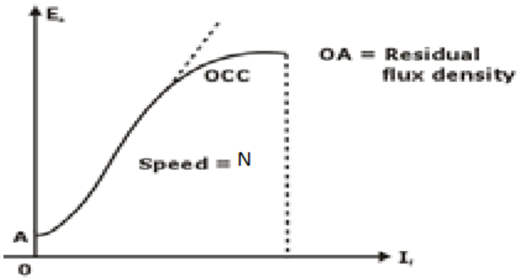
This is called as load magnetisation curve.

3. External characteristics:

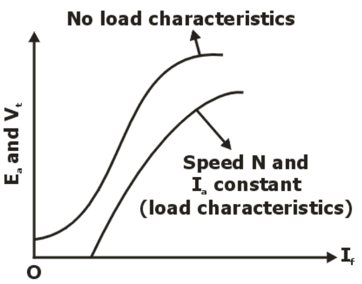
$V_t = f(I_L)$, with both I_f & N constant.

A. **Separately excited generators**

1. No-load characteristics:

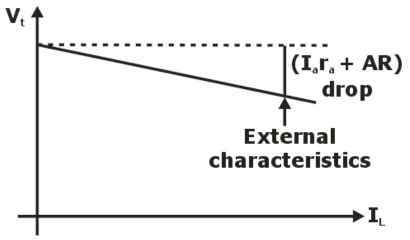


2. Load characteristics



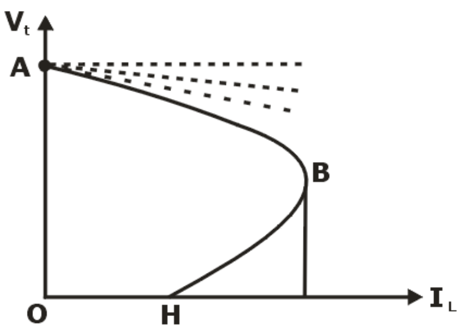
3. External characteristics

$V_t = E_a - I_aR_a$



B. Shunt Generator:

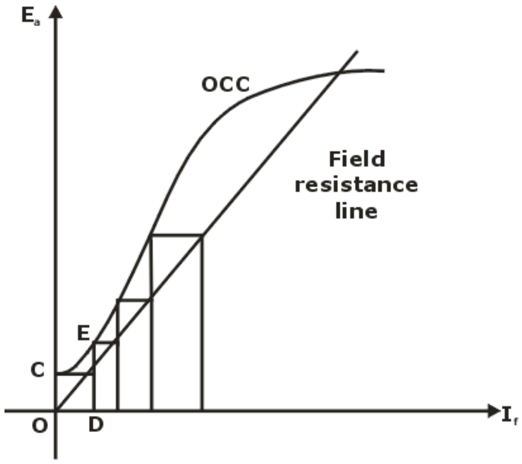
External characteristics



Terminal voltage falls due to AR drop & $I_a r_a$ drop due to which I_f decreases which leads to further decrease in V_t.

Voltage build up in Shunt generator:

The voltage build up in shunt generator requires some residual flux and emf for voltage build up.



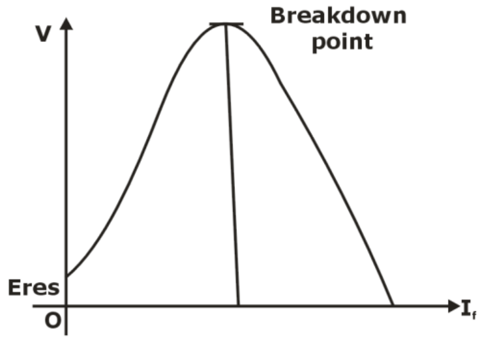
- Due to residual flux, emf OC is induced in armature.
- Due to this induced emf field current OD flows in the circuit.
- This causes an emf DE to be induced as per OCC.
- This further causes a current to flow in field winding.
- This effect goes on increasing till the field resistance line cuts OCC.

Necessary Conditions for Voltage Build up:

1. There must be a residual flux.
2. Correct polarity of field winding with respect to armature winding so that field flux aids residual flux.
3. Field resistance must be less than critical value.
4. Speed of rotation should be more than critical value for given R_f.

C. Series generator:

External characteristics:



As load resistance is reduced, armature current or field current increases which leads to increase in flux & hence induced voltage also increases upto breakdown point. After that ∅ becomes constant, at saturation, So E_a becomes constant and due to increase in armature current terminal voltage reduces.

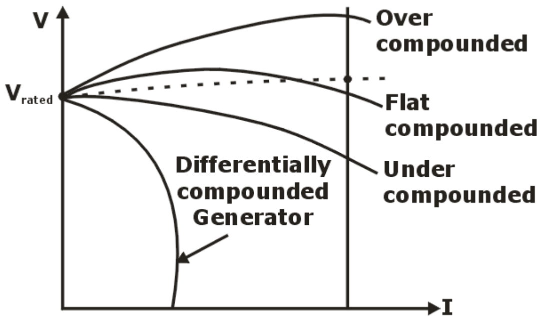
D. Compound Generator:

Cumulative compound generator are more common as compared to differentially compound generator as they have almost constant voltage from no load to full load.

In cumulative compound generator series flux aids shunt flux and depending upon number of series field turns,

- Under-compounded
- Flat- compounded
- Over-compounded

In differentially compounded generator, Series flux opposes shunt flux and thus V_t falls more rapidly with load.



Different characteristics of different generator keeping the no load voltage same in all the cases.

OPERATING CHARACTERISTICS OF DC MOTOR:

The following characteristics are-

- Speed-Armature current characteristics
- Torque-Armature current characteristics
- Speed-Torque characteristics

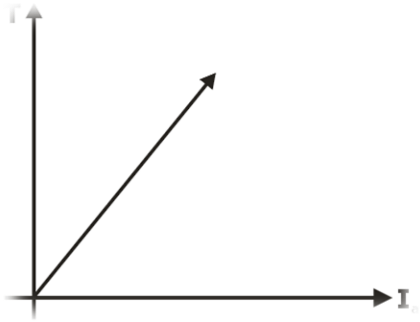
A. Separately excited DC motor

1. Torque-Armature current characteristics

Developed torque in a DC machine depends on Armature current as shown below-

$T = K\varnothing I_a$

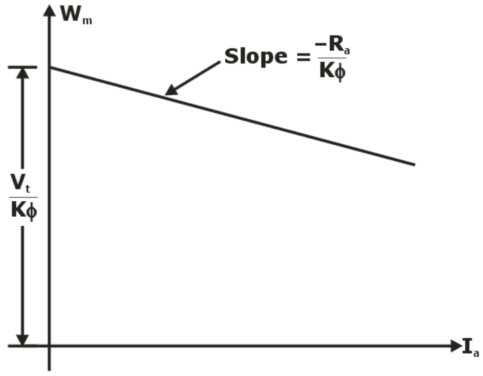
$\boxed{T \propto I_a}$ $\varnothing$ = constant



2. Speed-Armature current characteristics

$E_b = V_t - I_a R_a$

$$\omega_m = \frac{E_b}{K\phi} = \frac{(V_t - I_a R_a)}{k\phi}$$



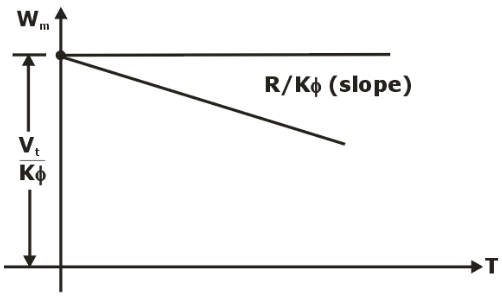
3. Speed-Torque characteristics:

As we know that,

$$\omega_m = \frac{V_t - I_a R_a}{k\phi} \quad \ldots (1)$$

$$T \propto I_a \quad \ldots (2)$$

$$\therefore \ \omega_m \propto \frac{V_t}{k\phi} - T\frac{R_a}{k\phi}$$



Note:

- For shunt excited DC motor all characteristics are same as that of separately excited DC motor.

B. Series excited DC motor:

1. Speed-Armature current characteristics

For series motor, flux is proportional to armature current.

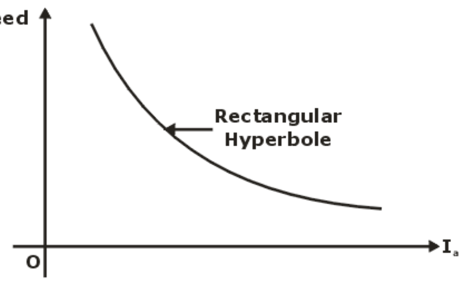
$\phi \propto I_a$

$E_b = V_t - I_a(r_a + r_{se})$

$$\omega_m = \frac{V_t}{k\phi} - \frac{I_a(r_a + r_{se})}{k\phi}$$

$\therefore \quad \boxed{E_b = k\phi\omega_m}$

$$\boxed{\omega_m = \frac{V_t}{kI_a} - \frac{(r_a + r_{se})}{k}}$$



2. Speed-Torque characteristic

$T = k_\varphi I_a$

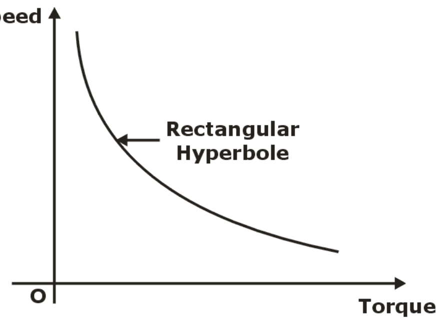
$\boxed{T = kI_a^2}$

$\therefore \ \phi \propto I_a$

$$I_a = \sqrt{\frac{T}{K}} \Rightarrow I_a \propto \sqrt{T}$$

$$\therefore \ \omega_m = \frac{V_t}{kI_a} - \frac{(r_a + r_{se})}{k}$$

$$\boxed{\omega_m = \frac{V_t}{k\sqrt{T}} - \frac{(r_a + r_{se})}{k}}$$



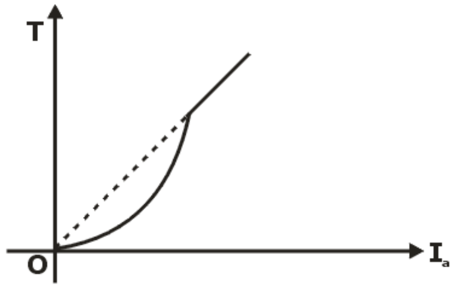
3. Torque-Armature Current characteristics

$\therefore$ flux is proportional to armature current in series excited DC motor.

$\therefore \ T = k\varphi I_a$

$\boxed{T = kI_a^2}$

For large I_a as $\phi \simeq$ constant so $T \propto I_a$



C. DC compound Motor

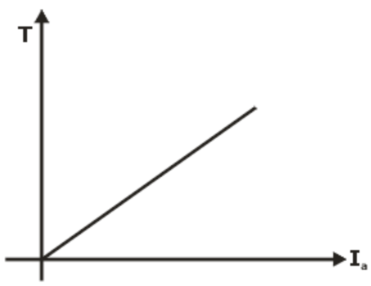
- Cumulatively compound motor

The series field aids the armature field in the motor,

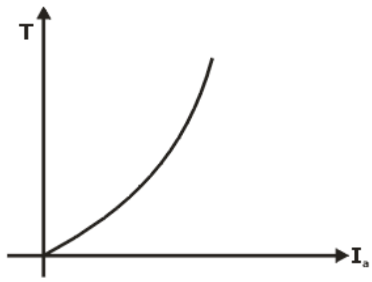
$\emptyset_r = \emptyset_{se} + \emptyset_{sh}$

If series field will dominates then characteristics will look like series motor otherwise shunt motor in case of dominant shunt field.

1. Torque-Armature current characteristics

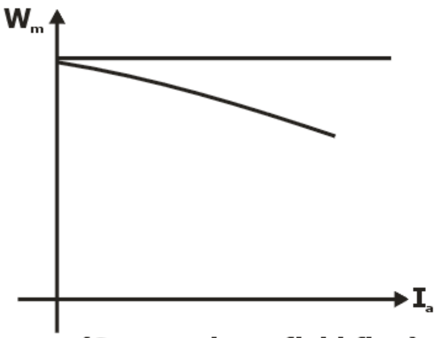


(Strong shunt field flux)

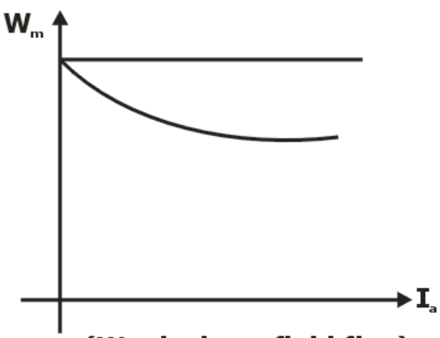


(Weak shunt field flux)

2. Speed-Armature current characteristics

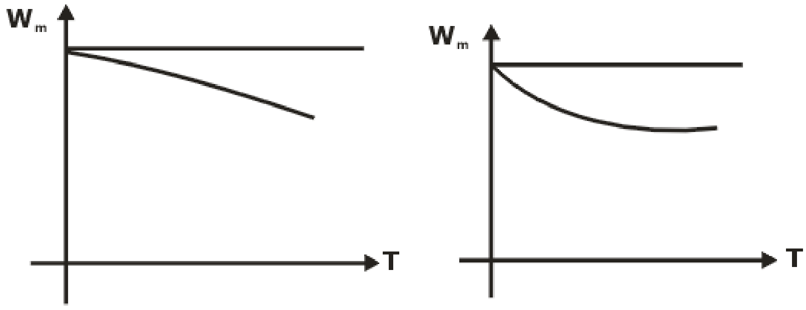


(Strong shunt field flux)



(Weak shunt field flux)

3. Speed-torque characteristics

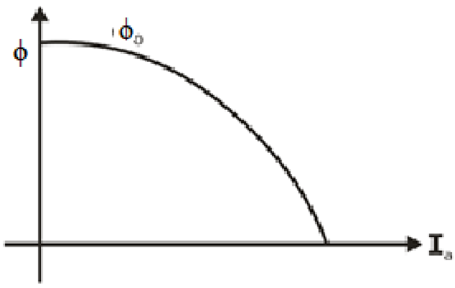


- Differentially compound motor

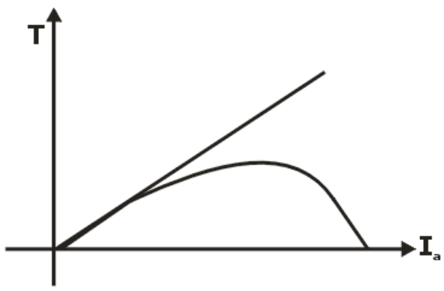
The series flux will oppose the shunt field flux.ø

$$\boxed{\phi_r = \phi_{sh} - \phi_{se}}$$

The flux decreased as the armature current or field current is increased.

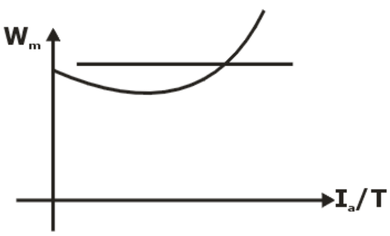


1. Torque-Armature current characteristics



$$\boxed{T \propto \Phi_r}$$

2. Speed-armature current/Speed-Torque characteristics



$$\boxed{\omega \propto \frac{1}{\phi}}$$

$$\boxed{\phi \propto I_a \propto T}$$

The only DC motor which can have zero speed regulation.

SPEED CONTROL OF DC MOTOR:

For variable speed DC drive the speed control of DC motor is necessary. As the speed of DC motor is given by:

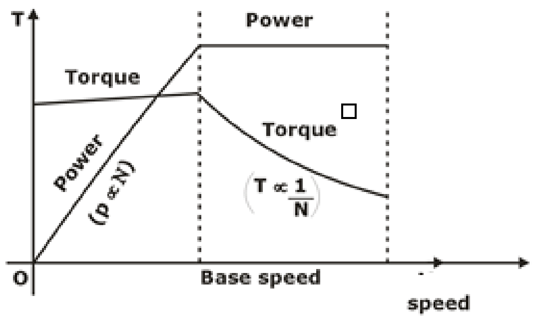
$$\boxed{\omega_m = \frac{V_t - I_a r_a}{k\phi}} \qquad \ldots(1)$$

Based on the above expression, there are 3 methods of speed control.

1. Variation of resistance in armature circuit.
2. Variation of field flux
3. Variation of armature terminal voltage

Ward Leonard system

- With armature voltage controlled method constant torque and variable power drive is obtained from speed below the base speed. (constant torque drive)
- With the field control method, constant power and variable torque is obtained for speed above base speed. (constant power drive)



Constant power and Torque Drive

- Below base speed machine acts as constant torque drive. ($P \propto N$)
- Above base speed machine acts as constant power drive. $\left(T \propto \frac{1}{N}\right)$

S.No.		Lap winding	Wave winding
1.	Coil span:	$Y_{cs} = s/p$	$Y_{cs} = s/p$
2.	Back pitch	$Y_b = UY_{cs}$	$Y_b = UY_{cs}$
3.	Commutator pitch	$Y_c = 1$ for progressive winding $Y_c = -1$ for retrogressive winding	$Y_c = \frac{2(c+1)}{p}$ for progressive winding $Y_c = \frac{2(c+1)}{p}$ for retrogressive winding (Y_c must be integer)
4.	Front pitch	$Y_f = Y_b + 2$ for progressive winding $Y_f = Y_b - 2$	$Y_f = 2Y_b - Y_b$

		for retrogressive winding	
5.	Parallel paths	A = P	A = 2
6.	Conductor current	$I_c = \frac{I_a}{A}$	$I_c = \frac{I_a}{A}$
7.	No. of brushes	No. of brushes = A = P	No. of brushes = 2

Where,

S = No. of commutator segments

P = No. of poles

U = No. of coil sides/No. of poles = $\frac{2c}{p}$

C = No. of coils on the rotor

A = No. of armature parallel paths

I_a = Armature current

Induction Machines

Types of IM:

There are two types of rotor construction in induction machine:

(i) Squirrel Cage rotor IM

(ii) Slip ring rotor IM

IMPORTANT DEFINITIONS

1. **Slip:**

   It is the difference of speed between the synchronous speed and the rotor speed expressed as the fraction of synchronous speed N_s.

   $$s = \left(\frac{N_s - N}{N_s}\right)$$

   Note: A well-designed induction motor runs at very low value of slip at no-load. Its operating range is also a low value of around 1% to 5% from no load to full load.

2. **Slip Speed:**

   It is the difference of speed between the stator (synchronous speed) and the rotor speed.

   Slip speed, $sN_s = N_s - N$

3. **Frequency of Rotor Current or E.M.F:**

   Rotor frequency, $f_r = sf$

   Where, s = slip

   f = supply frequency

4. **Rotor E.M.F:**

   The rotor emf is directly proportional to the frequency. At standstill, this frequency is equal to supply f but at running condition it is equal to (sf).

   Hence, induced emf in the rotor at running condition will be s-times the induced emf at standstill.

   $$E'_2 = \sqrt{2}\,\pi\,(sf)k_w\,f = s\,E_2$$

5. **Rotor inductance (X_2):**

   At starting, $f_r = f$

   $X_2 = 2\pi f L_2$

   At running, $f_r = sf$

   $X'_2 = 2\pi sfL_2 = sX_2$

$$X_2' = sX_2$$

6. **Rotor power factor:**

At starting,

$$\text{Cos}\phi = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

At running,

$$\text{Cos}\phi' = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

pf is better in running condition.

7. **Current in Rotor:**

At starting,

$$I_2 = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$

At running.

$$I_2' = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

COMPARISON BETWEEN INDUCTION MACHINE AND TRANSFORMER

Transformer	**Induction Machine**
1. Electrical to electrical energy conversion through magnetic medium is high permeability core.	1. Electrical to mechanical energy conversion through magnetic medium in the air gap of high reluctance.
2. Low magnetising current required therefore I_0 is equal to 2 – 6% of rated current.	2. High magnetising current required therefore I_0 is equal to around 30% of rated current.
3. Concentrated winding does not produce RMF.	3. Distributed winding produce RMF.
4. Induce emf depends on turn ratio.	4. Induce emf depends on turns ratio only at stand still.
5. Frequency of primary & secondary is same.	5. frequency of stator & rotor same only at stand still.
6. Constant flux device under steady state.	6. Constant flux Machine
7. Mutual induction present	Mutual induction present
8. Secondary is not short circuit but connected to electrical load.	8. Rotor should be essentially short circuit and the load is mechanical across the shaft.
9. No mechanical loss	9. Contains mechanical losses

TORQUE IN INDUCTION MOTOR: (NEGLECTING STATOR IMPEDANCE)

The relative speed between stator field and rotor field is zero. A steady torque is generated by the interaction of both fields.

$$T_e \propto F_1 F_2 \cos \emptyset$$

where, $F_1 \rightarrow$ Stator field produced by induced voltage in stator (E_2)

$F_2 \rightarrow$ rotor field produced by induced current in rotor (I_2')

$\emptyset \rightarrow$ angle between stator and rotor field

Running torque:

Let the motor run at speed N with the slip (s) & delivering full load torque 'T_f'

$T_f \propto E_2 I_2' \cos \emptyset_2$

$$T_f = kE_2 \times \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$T_f = K\frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$$

For given voltage,

$$T_f \propto \frac{sR_2}{R_2^2 + (sX_2)^2}$$

Condition for maximum running torque:

$$\frac{dT_s}{ds} = 0$$

$R_2 = sX_2$

Slip at which maximum torque (T_m) occurs s_m:

$$s_m = \frac{R_2}{X_2}$$

By putting the value of s_m in running torque.

$$T_{max} \propto \frac{1}{2X_2}$$

Note: T_{max} is independent to rotor resistance (R_2), varying R_2 will shift the point of maximum torque.

Starting Torque:

At starting, slip is 1

By putting slip value equal to 1 in running torque

$$T_{st} = K\frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Condition for maximum starting torque:

$$\frac{dT_{st}}{dR_2} = 0$$

$$\Rightarrow R_2 = X_2$$

If the rotor resistance is made equal to its reactance at stand still than the motor starts with maximum starting torque.

Under such conditions the rotor power factor will be 0.707 lag.

Motor Torque in terms of T_{em}:

- The torque expression of an induction motor can also be expressed in terms of maximum torque T_{em} and dimension less ratio $\frac{S}{S_{mT}}$. In order to get a simple and approximate expression, stator resistance r_1 or the stator equivalent resistance R_e is neglected.

$$\therefore \frac{T_e}{T_{em}} = \frac{2X}{\left(\frac{r_2}{s}\right)^2 + X^2} \times \frac{r_2}{s}$$

- The slip at which maximum torque occurs is

$$S_{mT} = \frac{r_2}{X} \qquad \therefore r_2 = S_{mT}X$$

$$\therefore \frac{T_e}{T_{em}} = \frac{2X}{\left[\frac{S_{mT}X}{s}\right]^2 + X^2} \times \frac{S_{mT}X}{s}$$

$$\Rightarrow \frac{T_e}{T_{em}} = \frac{2}{\frac{S_{mT}}{s} + \frac{S}{S_{mT}}}$$

$$T_e = \frac{2T_{em}}{\frac{S_{mT}}{s} + \frac{S}{S_{mT}}}$$

TORQUE SLIP CHARACTERISTIC

Torque equation is given by:

$$T_f = K\frac{sE_2^2R_2}{R_2^2 + (sX_2)^2}$$

Low slip region:

$$T_f \propto \frac{sR_2}{R_2^2 + (sX_2)^2}$$

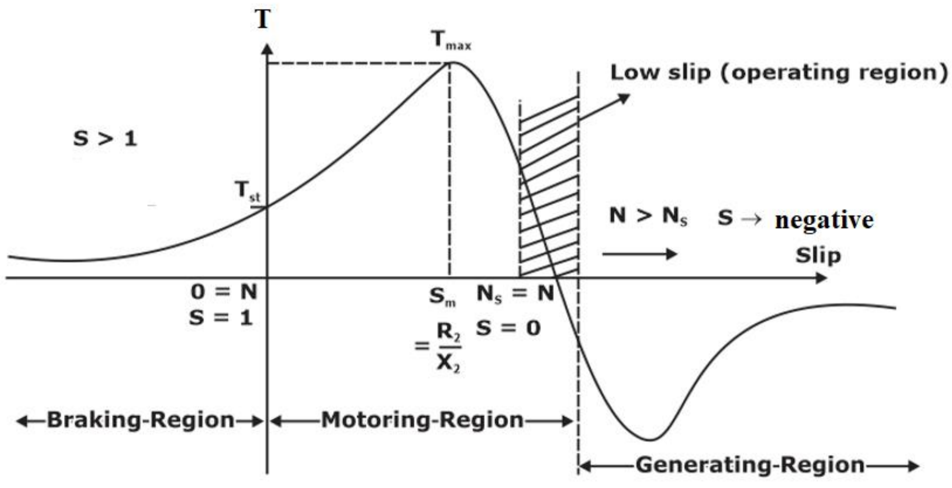
For low slip, $(sX_2)^2 <<< R_2^2$

$$T_f \propto \frac{s}{R_2} \Rightarrow T_f \propto s$$

For high slip region:

$(sX_2)^2 >>> R_2^2$

$$T_f \propto \frac{R^2}{sX_2^2} \Rightarrow T_f \propto \frac{1}{s}$$



Torque-slip characteristic on induction machine

In an operating region, the torque-slip characteristic is essentially a straight line. It has good speed regulation.

Speed Regulation: It is the change in speed when the full load across shaft is disconnected. It is expressed in % of full load speed.

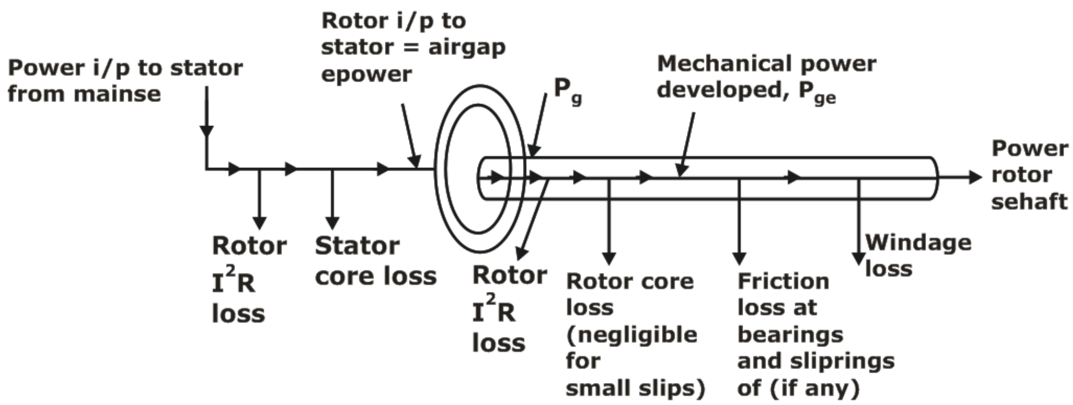
$$S.R\% = \frac{N_o - N}{N} \times 100$$

Note: As the rotor resistance increases under operating region, to maintain constant torque, slip also increases correspondingly.

Constant torque, $T_f \propto \frac{\uparrow s}{\uparrow R_2}$

POWER FLOW IN INDUCTION MOTOR

The Power Flow Diagram of an Induction Motor is shown below.



Power Flow Diagram of Induction Motor

- The power flow diagram of $3-\phi$ induction motor is:

- Per phase power input to rotor is

  $P_g = E_2I_2 \cos\theta_2$

  $$\cos\theta_2 = \frac{\text{Per phase rotor resistance}}{\text{Per phase rotor impedance}} = \frac{r_2/s}{\sqrt{(r_2/s)^2+(x_2)^2}}$$

  $$\therefore P_g = E_2I_2 \times \frac{r_2/s}{\sqrt{(r_2/s)^2+(x_2)^2}} = \frac{E_2}{\sqrt{(r_2/s)^2+(x_2)^2}} \times I_2\frac{r_2}{s} = I_2^2\frac{r_2}{s}$$

- P_g is the power transferred from stator to rotor across the air gap. Therefore, P_g is called air gap power

  $$P_g = I_2^2\frac{r_2}{s} = I_2^2r_2 + I_2^2r_2\left[\frac{1-s}{s}\right]$$

  P_g = (Rotor ohmic loss) + Internal mechanical power developed in rotor (P_m) = $sP_g + (1 - s)P_g$

  $$\therefore P_m = (1-s)P_g = I_2^2r_2\left[\frac{1-s}{s}\right]$$

  Rotor ohmic loss = $\left[\frac{s}{1-s}\right]P_m = sP_g$

- Internal (or gross) torque developed per phase is given by

  $$T_e = \frac{\text{Internal mechanical power developed in rotor}}{\text{Rotor speed in mechanical radian per sec}}$$

  $$T_e = \frac{P_m}{\omega_r} = \frac{(1-S)P_g}{(1-S)\omega_s} = \frac{P_g}{\omega_s}$$

- Electromagnetic torque T_e can also be expressed as

  $$T_e = \frac{P_g}{\omega_s} = \frac{1}{\omega_s} \times \frac{I_2^2r_2}{s} = \frac{\text{Rotor ohmic loss}}{(\omega_s)\,\text{slip}}$$

  $$\therefore T_e = \frac{\text{Rotor ohmic loss}}{(\omega_s)\,\text{slip}}$$

- Power available at the shaft can be obtained from P_g as follows. Output or shaft power,

  $P_{sh} = P_m -$ Mechanical losses

- Mechanical losses imply frication and windage losses.

  $P_{sh} = P_g -$ Rotor ohmic loss – Friction and windage losses = Net mechanical power output or net power output output or shaft power

  $$T_{sh} = \frac{P_{sh}}{\text{Rotor speed}} = \frac{P_{sh}}{(1-s)\omega_s}$$

- If the stator input is known. Then air gap power P_g is given by P_g = stator power input – stator I^2R loss – stator core loss.
- Ratio of Rotor input power, rotor copper losses and gross mechanical output is

$$\frac{I_r^2R_2}{s} : I_r^2R_2 : I_r^2R_2\left[\frac{1}{s}-1\right]$$

$$\Rightarrow 1 : s : (1-s)$$

$\therefore$ Rotor copper losses = s × Rotor input

Gross Mechanical output = (1−s) × Rotor input.

Rotor copper losses = (Gross mechanical output) × $\frac{s}{1-s}$

Efficiency of the rotor is approximately equal to

$$\eta_{rotor} = \frac{\text{Gross mechanical power output}}{\text{Rotor input}}$$

$$\frac{(1-s)\text{Rotor input}}{\text{Rotor input}} = 1-s = 1-\frac{N_S-N}{N_S} = \frac{N}{N_s}$$

$$\eta_{rotor} \simeq \frac{N}{N_s}$$

DETERMINATION OF EQUIVALENT CIRCUIT PARAMETERS

It requires two tests to be conducted on a given induction motor to find out its parameter.

(1) No-load test

(2) Block Rotor test

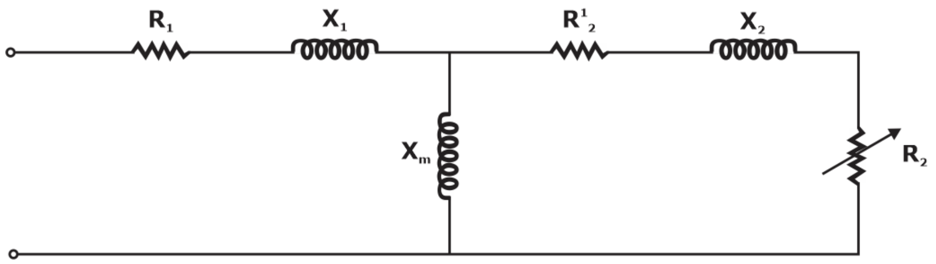
1. **No-Load Test**
   - This test is used to determine the core losses.
   - The motor is made to run at rated voltage and frequency on no load and stator voltage V_{nl}, input current I_{nl} and input power P_{nl} are recorded.
   - Rotor core loss and rotor copper loss are very less and can be neglected because current in rotor at no load is very less.
   - $W_{NL} - 3I_0^2R_1$ = Constant losses
2. **Blocked Rotor Test:**

   This test is done by blocking the rotor and applying small voltage around (5 to 10%) across stator winding while ensuring rated current drawn by the motor.

   Note: As voltage is low, core loss is neglected on stator and rotor.

   W_{BR} = stator copper loss + Rotor copper loss

   $W_{BR} = I_1^2R_{01} = I_{BR}^2R_{BR}$



Equivalent circuit at Blocked rotor test

$$Z_{BR} = \frac{V_{BR} / \sqrt{3}}{I_{BR}}$$

$$X_{BR} = \sqrt{Z_{BR}^2 - R_{BR}^2}$$

On calculating:

$$X_1 = X'_2 = X_{BR} / 2$$

$$R'_2 = \left(R_{BR} - R_1\right)\left(\frac{X_2 + X_m}{X_m}\right)^2$$

Hence, all the parameters of the equivalent circuit can be calculated by conducting the these tests.

STARTING OF THREE PHASE INDUCTION MOTORS

1. Direct-on line (across the line) starting:

- The relation between starting torque and full load torque is:

$$T_e = \frac{1}{\omega_s} \times I_2^2 \frac{r_2}{s}$$

$$\therefore \frac{T_{e,st}}{T_{e,fl}} = \frac{I_{2,st}^2 \dfrac{r_2}{r_1}}{I_{2,fl}^2 \dfrac{r_2}{s_{r_1}}} = \left[\frac{I_{2,st}}{I_{2,fl}}\right]^2 \times s_{fl}$$

The above equation valid for rotor resistance remains constant.

$$\frac{T_{e,st}}{T_{e,fl}} = \left[\frac{I_{st}}{I_{fl}}\right]^2 \times s_{fl}$$

Where,

$$\frac{I_{st}}{I_{fl}} = \frac{(\text{Effective rotor to stator turns ratio})\, I_{2,st}}{(\text{Effective rotor to stator turns ratio})\, I_{2,sl}}$$

- Per phase short – circuit current at stand still (or at starting) is,

$$I_{sc} = \frac{V_1}{Z_{sc}}$$

Where, $Z_{sc} = (r_1 + r_2) + j(X_1 + X_2)$

Here shunt branch parameters of equivalent circuit are neglected.

- Therefore, for direct switching.

$$I_{st} = I_{sc} = \frac{V_1}{Z_{sc}}$$

$$\therefore \frac{T_{est}}{T_{efl}} = \left[\frac{I_{sc}}{I_{fl}}\right]^2 s_{fl}$$

2. Stator resistor (or reactor) starting:

Since per phase voltage is reduced to xV, the per phase starting current I_{st} is given by

$$I_{st} = \frac{xV_1}{Z_{sc}} = xI_{sc}$$

As before $\therefore \frac{T_{est}}{T_{efl}} = \left[\frac{I_{sc}}{I_{fl}}\right]^2 s_{f1} = \left[\frac{xI_{sc}}{I_{fl}}\right]^2 s_{fl}$

- In an induction motor, torque $\propto (\text{Voltage})^2$

$$\therefore \frac{\text{Starting torque with reactor starting}}{\text{Starting torque with direct switching}} = \left[\frac{xV_1}{V_1}\right]^2 = x^2$$

3. Auto transformer starting:

- Per phase starting current from the supply mains is $I_{st} = x^2 I_{sc}$

$$\frac{T_{e,st}}{T_{e,fl}} = \frac{\text{Per phase starting current in motor winding}}{\text{Per phase motor full load current}} \times s_{fl}$$

$$\frac{T_{e,st}}{T_{e,fl}} = \frac{I_{st} I_{sc}}{I_{fl}^2} \times s_{fl}$$

$$\frac{T_{e,st}\text{ with an auto transformer}}{T_{e,st}\text{ with direct switching}} = \left[\frac{xV_1}{V_1}\right]^2 = x^2$$

4. Star – Delta method of Starting:

- $$\frac{\text{Starting torque with star delta starter}}{\text{Starting torque with direct switching in delta}} = \frac{\left[\frac{V_L}{\sqrt{3}}\right]^2}{\left[V_L\right]^2} = \frac{1}{3}$$

  $\therefore$ Star delta starter also reduces the starting torque to one-third of that produce by direct switching in delta.

- With star–delta starter, a motor behaves as if it were started by an auto transformer starter with $x = \frac{1}{\sqrt{3}} = 0.577$ i.e. with 57.7% tapping.

- $$\frac{\text{Starting torque with star delta starter, } T_{e,st}}{\text{Starting torque with direct switching in delta, } T_{efl,d}} = \frac{\frac{1}{\omega_s}\left(I_{st,y}\right)^2 \frac{r_2}{1}}{\frac{1}{\omega_s}\left(I_{st,d}\right)^2 \frac{r_2}{s_{fl}}}$$

$$\frac{\left[\frac{1}{\sqrt{3}} I_{st,d}\right]^2}{\left(I_{fl,d}\right)^2} \times S_{fl} = \frac{1}{3}\left[\frac{I_{sc}d}{I_{fl}d}\right]^2 \times s_{fl}$$

SPEED CONTROL OF 3 – Φ INDUCTION MOTORS

Running speed of an induction motor is given as:

$$N = \frac{120\,f}{P}(1 - s)$$

The method involved in speed control of induction motor is not simple and efficient.

It does not offer wide range of speed control.

1. **Line Voltage Control:**

   As we known,

   $T \propto sV^2$

   - If voltage is reduced by keeping constant torque, the slip had to be increased. Hence, the speed decreases.
   - It draws more current and does not offer wide range of speed control.

2. **V/f control:**

   It is basically a frequency control.

   To keep V/f ratio constant, it requires variable voltage also otherwise stator and rotor core gets saturated.

   $$N_s = \frac{120f}{P} \Rightarrow N_s \propto f$$

   $$s_m = \frac{R_2}{X_2} \Rightarrow s_m \propto \frac{1}{f}$$ (as reactance depends on frequency)

   $$T_{max} = \frac{3 \times 60}{2\pi N_s}\frac{E^2}{2X_2}$$

   $$T_{max} \propto \frac{V^2}{f \times f} \Rightarrow \text{constant}$$

   (as $\frac{V}{f}$ is constant)

3. **Rotor resistance control:**

   - Speed control by this method is used when speed is to be reduced for a short period.
   - Because increase in rotor resistance efficiency becomes less and poor speed regulations.
   - It is possible in slip wound rotor only as rotor resistance cannot be changed in squirrel cage winding.

4. **Speed control by Cascade Arrangement:**

   The two motors may be mechanically coupled together to drive a common load.

   Generally, the rotor output of first machine is connected to the stator of second machine in such a way that the revolving field of both the machines are in the same direction.

   Under this condition, the resulting speed of field will be

   $$N_{s_1} = \left(\frac{120\,f}{P_1 + P_2}\right)$$ (When the machines are cumulatively coupled)

$N_{s_2} = \left(\frac{120\ f}{P_1 - P_2}\right)$ (When the machines are differentially coupled)

Where, f = supply frequency

P_1 and P_2 = number of poles of machines I and II, respectively.

5. Slip power Recovery:

In this method, external voltage is injected into the rotor through the slip ring at slip frequency.

As,

$T_e \propto sV_2^2$

(i) Injecting the voltage to add the existing rotor voltage which will reduce slip to keep torque constant.

(ii) Injecting the voltage to oppose the existing rotor voltage which will increase slip to keep torque constant.

Synchronous Machines

A three-phase synchronous machine is a doubly excited ac machine because its field winding is energized from a DC source and its armature winding is connected to an AC source. It rotates with speed of revolving field i.e. synchronous speed.

Synchronous machines are divided in two parts based on their rotor construction:

1. Cylindrical Rotor Machines
2. Salient pole Rotor Machines

- The synchronous speed N_s and synchronous angular speed of a machine with p-pole pairs running on a supply of frequency f_s are:

  $\omega_s = \frac{2\pi f_s}{p}$

- Slip S = $\frac{N_S - N}{N_S}$

  Where $N_S = \frac{120f}{p}$ = synchronous speed

EMF EQUATION OF SYNCHRONOUS GENERATOR OR ALTERNATOR

The emf equation of Synchronous Generator or Alternator is given as:

Let,

Φ = Flux per pole, in Wb

P = Number of poles

N_{ph} = Turns per phase

f = Frequency of induced emf in Hz

Z = Total number of conductors

Z_{ph} = Conductors per phase connected in series = Z/3 as number of phases = 3

Consider a single conductor placed in a slot.

$$E_{ph} = \sqrt{2}\pi f \phi N_{ph} \text{ volts}$$

This is the general emf equation for an induced emf per phase for full pitch, concentrated type of winding.

1. Pitch Factor or Coil Span Factor (K_c)

It is defined as the ratio of resultant emf when the coil is short pitch to the result emf when the coil is full pitched. It is always less than one.

$$K_c = \frac{E_R \text{ when coil is short pitched}}{E_R \text{ when coil is full pitched}} = \frac{2E\cos\left(\frac{\alpha}{2}\right)}{2E}$$

$$K_c = \cos\left(\frac{\alpha}{2}\right)$$

- Chording angle to eliminate n^{th} harmonics $(\alpha) = \frac{180°}{n}$.

- Coil span to eliminate n^{th} harmonics, $(\beta) = 180\left(\frac{n-1}{n}\right)$.

2. Distribution Factor (K_d)

The distribution factor is defined as the ratio of the resultant emf when coils are distributed to the resultant emf when coils are concentrated. It is always less than one.

$$K_d = \frac{E_R \text{ when coils are distributed}}{E_R \text{ when coils are concentrated}} = \frac{2E\sin\left(\frac{m\beta}{2}\right)}{2mE\sin\left(\frac{\beta}{2}\right)}$$

$$K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)}$$

Where, m = Slots per pole per phase

β = Slot angle = 180°/n

n = Slots per pole

When β is very small and m is large then the total phase spread is (mβ). So, generalised expression for the derivation of emf equation of Synchronous generator or Alternator can be written as

$E_{ph} = \sqrt{2}\pi K_c K_d f \Phi N_{ph}$ Volts

For full pitch coil, $K_c = 1$

For concentrated winding $K_d = 1$

Note:

- **Coil span :** It is the distance between two sides of the coil. It is expressed in terms of degrees, pole pitch, no. of slots/pole etc.
- **Pole pitch:** It is the distance between two identical points on two adjacent poles. Pole pitch is always 180° electrical = slots / pole.
- $\theta_{elec} = \frac{P}{2}\theta_{mech}$
- **Slot pitch or slot angle:** (β) Slot angle is the angle for each slot.
- Speed of space harmonics of order $(6k \pm 1)$ is $\frac{1}{(6k \pm 1)}N_s$

  Where, N_s = Synchronous speed = $\frac{120f}{p}$
- The order of slot harmonics is $\left(\frac{2s}{p} \pm 1\right)$

  where s = No. of slots, P = No. of poles
- Slot harmonics can be eliminated by skewing the armature slots and fractional slot winding. The angle of skew = θ_s = β (slot angle) = 2(harmonic pole pitches) = 1 slot pitch.

- Distribution factor for slot harmonics,

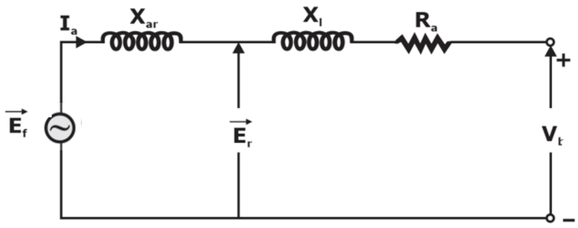
$$k_d\left(\frac{2s}{p}\pm 1\right)$$

Is $k_{d_1} = \dfrac{\sin\frac{my}{2}}{m\sin\frac{y}{2}}$ i.e., same that of fundamental one.

- Pitch factor for slot harmonics, $k_p\left(\frac{2s}{p}\pm 1\right) = k_{p_1} = \cos\frac{\propto}{2}$

Synchronous Generator:

For a synchronous generator with stator induced voltage E_s, stator current I_s and synchronous impedance Z_s, the terminal voltage V is:



Equivalent circuit of synchronous generator

$$\vec{V}_t = \vec{E}_f\angle\delta - \vec{I}_a(R_a + jX_s)$$

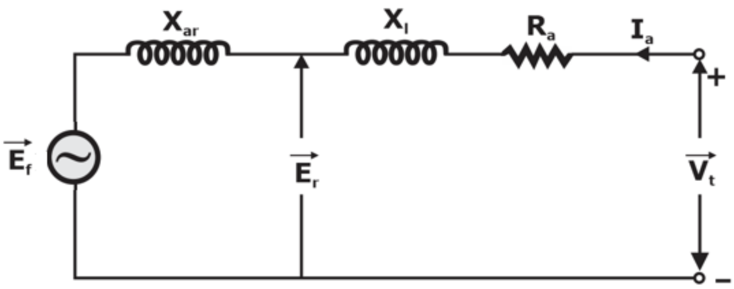
Where, R_s is the stator resistance and X_s is the synchronous reactance.

OR

$$E_f = \sqrt{(V\cos\phi + I_aR_a)^2 + (V\sin\phi \pm I_aX_s)^2}$$
$$+ \Rightarrow \text{lag p.f.}$$
$$- \Rightarrow \text{leading p.f.}$$

Synchronous Motor:

For a synchronous motor with stator induced voltage E_s, stator current I_s and synchronous impedance Z_s, the terminal voltage V is:



Equivalent circuit of synchronous motor

$$\vec{V}_t = \vec{E}_f\angle -\delta + \vec{I}_a(R_a + jx_s)$$

Where, X_s is synchronous reactance.

$$E = \sqrt{(V\cos\phi - I_a R_a)^2 + (V\sin\phi \mp I_a X_s)^2}$$

$- \Rightarrow$ lag p.f.

$+ \Rightarrow$ leading p.f.

where R_s is the stator resistance and X_s is the synchronous reactance

Voltage Regulation:

- % regulation $= \dfrac{|E| - |V|}{|V|} \times 100$

  $E - V = I_a Z_s$

  $\therefore$ % regulation $= \dfrac{E - V}{V} = \dfrac{I_a Z_s}{V} \times 100$

  $\therefore$ regulation $\propto Z_s$

  $\therefore$ As Z_s increases, voltages regulation increases.

- Condition for zero/minimum voltage regulation is, $\cos(\theta + \varphi) = -\dfrac{I_a Z_s}{2V}$
- Condition for max. Voltage regulation is, $\varphi = \theta$

Short circuit ratio (SCR)

It is defined as the ratio of field current required to produce rated voltage on open circuit to the field current required to produce rated armature current.

As SCC is linear,

$$SCR = \frac{I_{SC}}{I_a(\text{rated})} = \frac{\dfrac{\left(V_t(\text{rated})/\sqrt{3}\right)}{X_s(\text{saturated})}}{I_a(\text{rated})} = \frac{Z_{base}}{X_s(\text{saturated})}$$

$$SCR = \frac{1}{X_s(pu)}$$

$$SCR \propto \frac{1}{X_a} \propto \frac{1}{\text{Armature reaction}}$$

Voltage regulation $\propto$ Armature reaction

$$\therefore\ SCR \propto \frac{1}{\text{Voltage regulation}}$$

$\therefore$ Small value of SCR represents poor regulation.

$$\emptyset_a = \frac{\text{Armature mmf}}{\text{Reluctance}}$$

But reluctance $\propto$ Air gap

$$\therefore \emptyset_a = \frac{\text{Armature mmf}}{\text{Airgap}}$$

$$\phi_a \propto \frac{1}{\text{Airgap length}}$$

$$\text{Armature reaction} \propto \phi_a \propto \frac{1}{\text{Airgap length}}$$

$$\therefore \text{SCR} \propto \frac{1}{\text{Armature reaction}} \propto \text{Airgap length}$$

Air gap length $\propto$ SCR

$\therefore$ Machine size $\propto$ SCR , Cost $\propto$ SCR

$$\text{Power} = \frac{EV}{X_a}\sin\delta \Rightarrow P \propto \frac{1}{X_s} \propto \text{SCR}$$

Power $\propto$ SCR

$\therefore$ Large value of SCR represents more power output.

POWER FLOW IN CYLINDRICAL ROTOR SYNCHRONOUS MACHINE

Power flow in generator:

$$S_{out} = \vec{V}_t \vec{I}_a *$$

$$S_{out} = P_{out} + jQ_{out}$$

$$P_{out} = \frac{E_f V_t}{Z_s}\cos(\theta - \delta) - \frac{V_t^2}{Z_s}\cos\theta$$

$$Q_{out} = \frac{E_f V_t}{Z_s}\sin(\theta - \delta) - \frac{V_t^2}{Z_s}\sin\theta$$

P_{out} will be maximum when $\theta = \delta$

$$\boxed{P_{out(max)} = \frac{E_f V_t}{Z_s} - \frac{V_t^2}{Z_s}\cos\theta}$$

Neglecting armature resistance, R_a

Then, $Z_s = X_s$ and $\theta = 90°$

By putting the value in power equation,

$$\boxed{P = \frac{V_t E_f}{X_s}\sin\delta}$$

Reactive power:

$$\boxed{Q_{out} = \frac{V_t}{X_s}(E_f\cos\delta - V_t)}$$

Power Flow in motor:

$$S_{input} = P_{input} + jQ_{input} = V_t I_a^*$$

$$\therefore P_{in} = \frac{V_t^2}{Z_s}\cos\theta - \frac{V_t E_f}{Z_s}\cos(\theta+\delta)$$

$$\therefore Q_{in} = \frac{V_t^2}{Z_s}\sin\theta - \frac{V_t E_f}{Z_s}\sin(\theta+\delta)$$

Neglecting armature resistance, R_a

Then, $Z_s = X_s$ and $\theta = 90°$

By putting the value in power equation,

$$\boxed{P = \frac{V_t E_f}{X_s}\sin\delta}$$

Reactive power:

$$\boxed{Q_{out} = \frac{V_t}{X_s}(V_t - E_f\cos\delta)}$$

Conclusion:

For generator:

Case 1: When $E_f\cos\delta = V_t$ i.e. normally excited, then $Q_{out} = 0$ and therefore operating at unity factor.

Case 2: When $E_f\cos\delta > V_t$ i.e. overexcited, then Q_{out} = +ve, that is supplying lagging VARS and therefore operating at lagging p.f.

Case 3: When $E_f\cos\delta < V_t$ i.e. under excited, then Q_{out} = –ve, that is supplying leading VARs and therefore operating at leading p.f.

For motor:

Case 1: When $E_f\cos\delta = V_t$ i.e. normally excited motor than $Q_{in} = 0$ and therefore operating at unity power factor.

Case 2: When $E_f\cos\delta > V$ i.e. overexcited motor than Q_{in} = –ve i.e. taking leading VARs and therefore operating at leading p.f.

Case 3: When $E_f\cos\delta < V$ i.e. under excited motor than Q_{in} = +ve i.e. taking lagging VARs and therefore operating at lagging p.f.

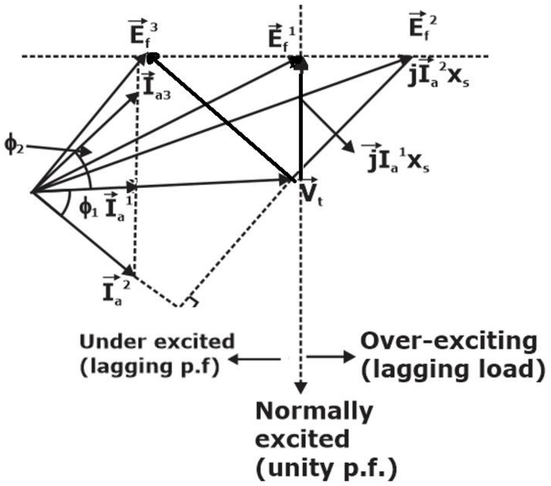
EFFECT OF CHANGE IN EXCITATION AT CONSTANT (KW) OUTPUT

Power can be expressed as:

$$P = \frac{VE_f}{X_s}\sin\delta \qquad\qquad P = VI_a\cos\phi$$

$$P \propto E_f\sin\delta \qquad\qquad P \propto I_a\cos\phi$$

A) Generator:

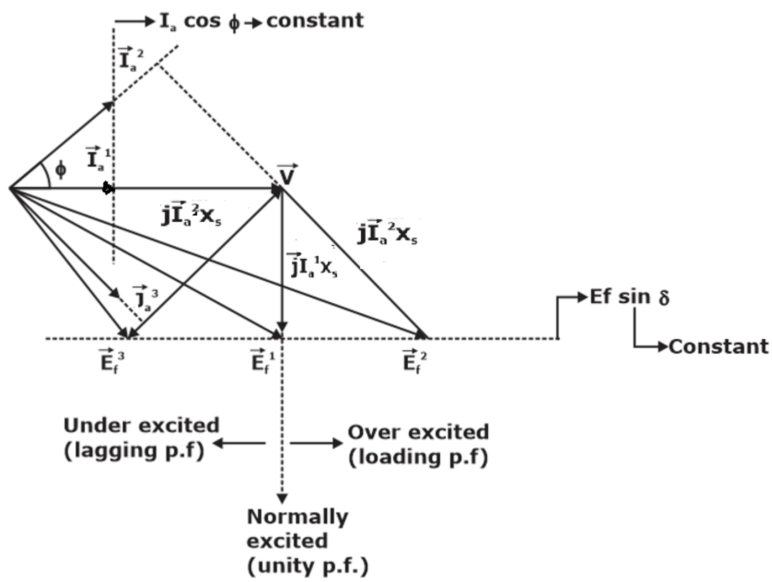


Effect of change in excitation on alternator

Conclusion:

1. For unity power factor load (I_a^1), generator is normally excited (E_f^1)
2. For lagging power factor load (I_a^2), generator is over-excited (E_f^2)
3. For leading power factor load (I_a^3), generator is under-excited (E_f^3)

B) Motor:



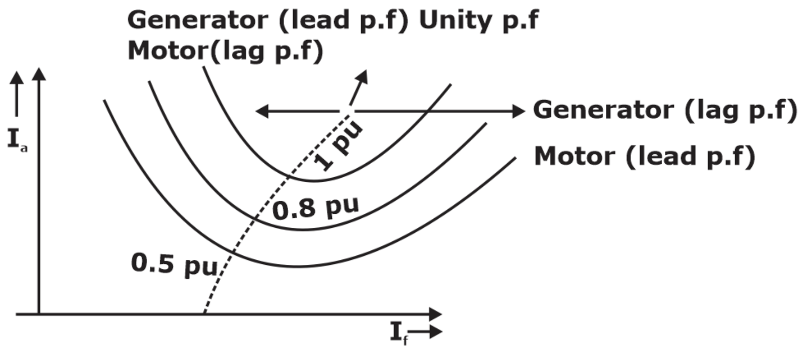
Effect of change in excitation on synchronous motor

Conclusion:

1. For unity power factor load (I_a^1), motor is normally excited (E_f^1)
2. For leading power factor load (I_a^2), motor is over-excited (E_f^2)
3. For lagging power factor load (I_a^3), motor is under-excited (E_f^3)

V curve:

It is the plot of armature current (I_a) versus field current (I_f) at constant load.

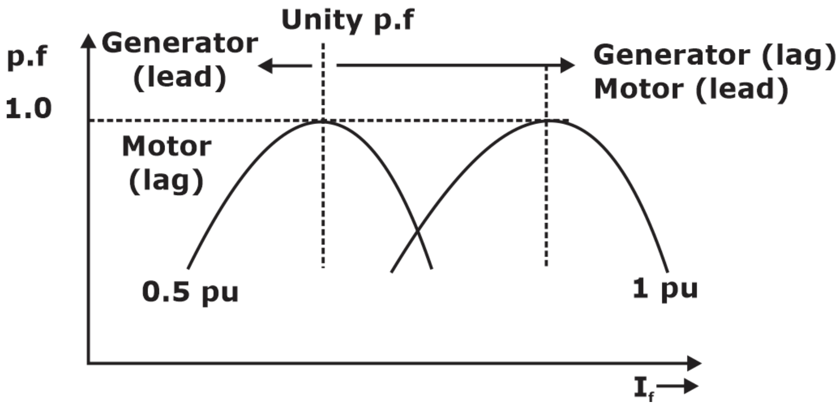


V-curves

Note: Armature current is minimum at normally excited machine which is at unity p.f load.

Inverted 'V' curve:

These are the curves plotted between power factor (p.f) and field current (I_f).



Inverted V-curves

Note: p.f is unity at normally excited machine at unity p.f. load.

Synchronizing power coefficient or stability factor

Synchronizing power coefficient or stability factor P_{sy} is given as:

$$P_{sy} = \frac{dp}{d\delta} = \frac{d}{d\delta}\left(\frac{EV}{X_s}\sin\delta\right) = \frac{EV}{X_s}\cos\delta$$

P_{sy} is a measure of stability.

$\therefore$ Stability $\propto P_{sy}$

But, $P_{sy} \propto \frac{1}{X_s} \propto \text{SCR}$

$\therefore$ Stability $\propto$ SCR

Stability $\propto$ SCR $\propto$ air gap length

$\therefore$ Stability $\propto$ air gap length

$\therefore$ Synchronizing Power coefficient, S_P:

$$\boxed{S_P = \frac{\Delta P}{\Delta \delta} = \frac{VE_f}{X_s}\cos\delta \text{ Watts / Electric radian}}$$

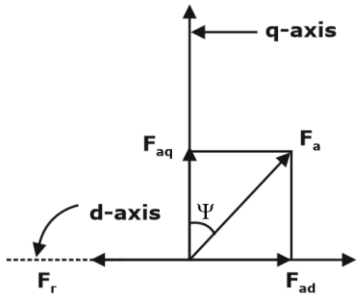
$S_p = P_{max} \cos\delta$

Synchronizing power increases with decrease in load angle. This is the reason to synchronize generator at no-load.

SALIENT POLE SYNCHRONOUS MACHINE

Two reaction theory

According to two reaction theory, the sinusoidal armature mmf, F_a is resolved into two sinusoidal components:



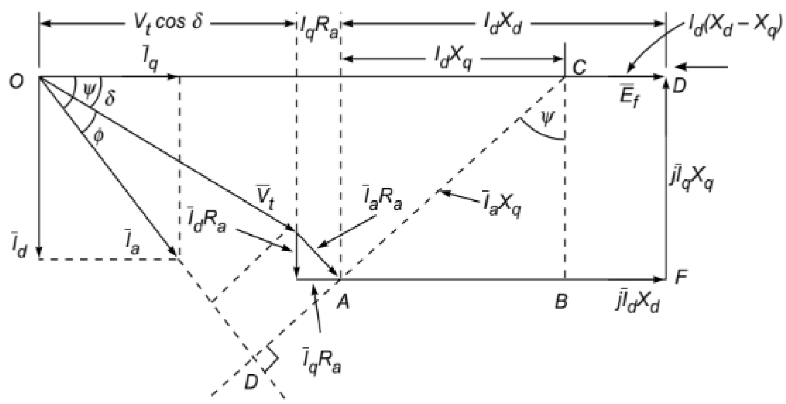
Phasor diagram explaining two-reaction theory

- F_{ad} along d-axis
- F_{aq}, along q-axis

Where, $F_d = F_a \sin\varphi$

$F_q = F_a \cos\varphi$

The d-axis F_{ad} component is magnetizing or demagnetizing depending upon, Ψ (angle of lead or lag). The q-axis components F_{aq} produces only distortion of the field flux wave. F_{ad} and F_{aq} are in time quadrature with E_f, (produced by I_d and I_a).



Phasor diagram of salient pole synchronous generator at lagging load

where,

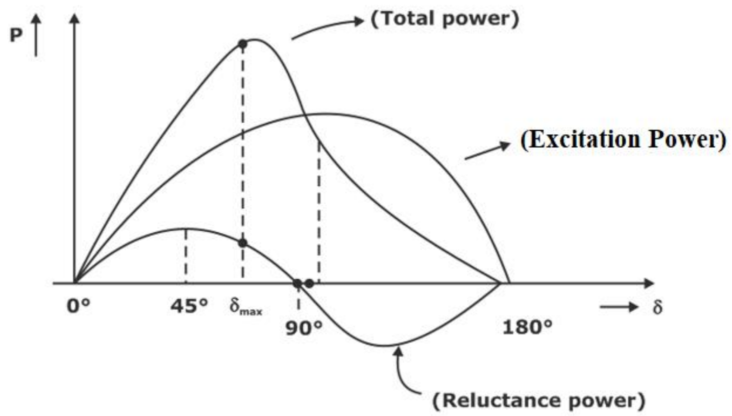
$$\vec{E_f} = \vec{V_t} + \vec{I_a}R_a + j\vec{I_d}X_q + j\vec{I_q}X_q$$

d-axis current, $I_d = I_a \sin \phi$

q-axis current, $I_q = I_a \cos \phi$

Power flow in salient pole synchronous generator

$$P = \underbrace{\frac{V E_f \sin\delta}{X_d}}_{\text{Excitation power}} + \underbrace{\frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)}_{\substack{\text{power due to saliency} \\ \text{or} \\ \text{reluctance power.}}} \sin 2\delta$$



Power charactertics of salient pole synchronous generator

Note:

- Cylindrical rotor Synchronous machine,

  The per phase power delivered to the infinite bus is given by $P = \frac{E_f V_t}{X_s} \sin \delta$

- Salient pole synchronous machine,

  The per phase power delivered to the infinite bus is given by

  $$P = \frac{E_f V_t}{X_d} \sin \delta + \frac{V_t^2}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right] \sin 2\delta$$

Condition for max. power:

- For cylindrical rotor machine:

  At constant V_t and E_f, the condition for max. power is obtained by putting $\frac{dp}{d\delta} = 0$

  $$\therefore \frac{dp}{d\delta} = \frac{E_f V_t}{X_s} \operatorname{cod}\delta = 0$$

  $\cos\delta = 0$

  $\delta = 90°$

  Hence maximum power occurs at $\delta = 90°$

- For salient – pole synchronous machine:

$$\frac{dp}{d\delta} = 0$$

$$\Rightarrow \frac{V_t E_f}{X_d}\cos\delta + V_t^2\left(\frac{1}{x_q} - \frac{1}{x_d}\right)\cos 2\delta = 0$$

Øthan 90°.

$\therefore$ Maximum power occurs at δ < 90°

- Synchronizing power = $P_{sy}.\Delta\delta$. = $\frac{EV}{X_s}\cos\delta\Delta\delta$
- Synchronizing torque = $\frac{\text{Synchronizing power}}{\omega}$

SLIP TEST

- When the stator mmf is aligned with the d – axis of field poles then flux $Ø_d$ per pole is set up and effective reactance offered by the alternator is X_d.

  $X_d = \frac{\text{Maximum voltage}}{\text{Minimum current}} = \frac{(V_t)\text{line}(\text{at min}, I_a)}{\sqrt{3}I_a(\text{min})}$ = Direct axis reactance

- When the stator mmf is aligned with the q – axis of field poles then flux $Ø_q$ per pole is set up and the effective reactance offered by the alternator is X_q.

  $X_q = \frac{\text{Minimum voltage}}{\text{Maximum current}} = \frac{(V_t)\text{line}(\text{at max}, I_a)}{\sqrt{3}I_a(\text{max})}$ = Quadrature axis reactance

Hunting

- A synchronous machine operates satisfactorily if the mechanical speed of the rotor is equal to the stator field speed i.e., if relative speed between rotor and stator fields is equal to zero. any departure from these conditions gives rise to synchronizing forces, which tend to maintain this equality.
- The phenomena of oscillation of rotor about its final equilibrium position is called hunting. The orientation of phasor E_f changes relative to V_t and because of this reason, hunting is called phase swinging.

METHODS OF STARTING OF SYNCHRONOUS MOTOR

- Motor starting with an external prime Mover
- Damper winding

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