

AE/JE Foundation

Electrical Engineering

Circuit theory

▶ Formula Notes



FORMULA NOTES (CIRCUIT THEORY)

Basic Concepts

Current: Electric current is the time rate of change of charge flow.

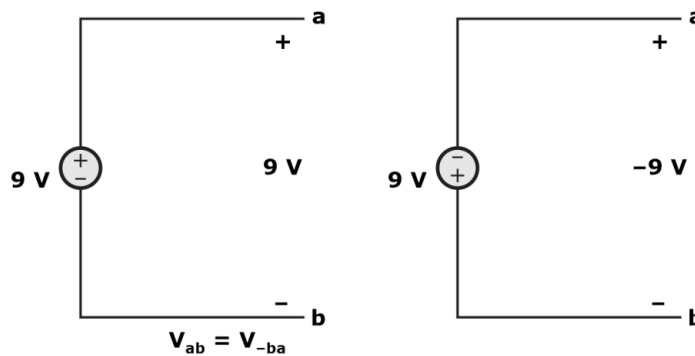
$$i = \frac{dq}{dt} \quad (\text{Ampere})$$

- Charge transferred between time t_0 and t

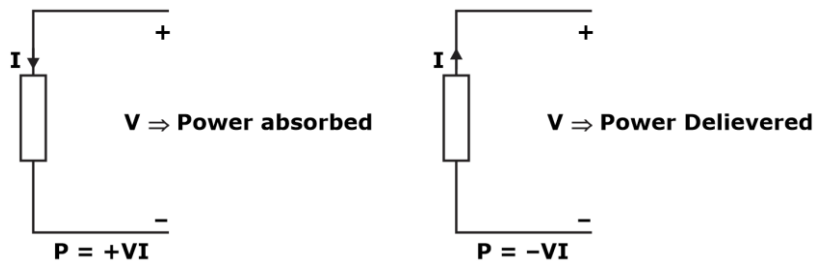
$$q = \int_{t_0}^t i dt$$

Sign Convention: A negative current of -5A flowing in one direction is same as the current of +5A in opposite direction.

Voltage: Voltage or potential difference is the energy required to move a unit charge through an element, measured in volts.



Power: It is time rate of expending or absorbing energy.

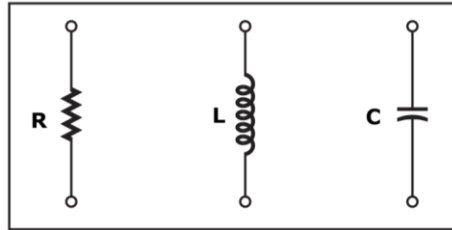


- Law of conservation of energy must be obeyed in any electric circuit.
- Algebraic sum of power in a circuit, at any instant of time, must be zero.

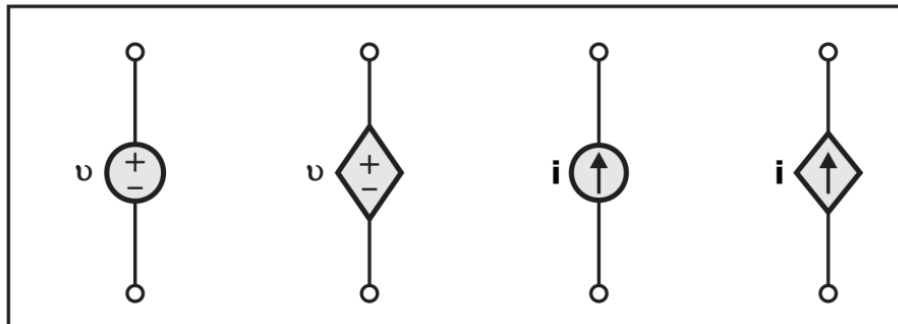
i.e. $\sum P = 0$

Circuit Elements:

Passive element: If it is not capable of delivering energy, then it is passive element. Example: Resistor, Inductor, and capacitor.



Active element: If an element is capable of delivering energy independently, then it is called active element. Example: Voltage source, and current source.



Linear and Non-linear elements: If voltage and current across an element are related to each other through a constant coefficient then the element is called as linear element otherwise it is called as non-linear.

Unidirectional and Bidirectional: When elements characteristics are independent of direction of current then element is called bi-directional element otherwise it is called as unidirectional.

- R, L & C are bidirectional
- Diode is a unidirectional element.
- Voltage and current sources are also unidirectional elements.
- Every linear element should obey the bi-directional property but vice versa as is not necessary.

Resistor: Linear and bilateral (conduct from both direction)

- In time domain $V(t) = I(t)R$
- In s domain: $V(s) = RI(s)$

$$R = \frac{\rho l}{A} \text{ ohm}$$

- l = length of conductor, ρ = resistivity, A = area of cross section
- Extension of wire to 'n' times results in increase in resistance:

$$R' = n^2R$$

- Compression of wire results in decrease in resistance:

$$R' = \frac{R}{n^2}$$

Capacitor: All capacitors are linear and bilateral, except electrolytic capacitor which is unilateral.

- Time Domain:

$$i(t) = \frac{Cdv(t)}{dt} \quad v(t) = \frac{1}{C} \int_{-\infty}^t i(t)dt$$

- In s-domain:

$$I(s) = s CV(s), V(s) = \frac{1}{sC} I(s)$$

- Capacitor doesn't allow sudden change of voltage, until impulse of current is applied.
- It stores energy in the form of electric field and power dissipation in ideal capacitor is zero.
- Impedance:

$$Z_c = -jX_c \Omega \text{ \& } X_c = \frac{1}{\omega C}; X_c \rightarrow \text{Capacitive reactance; } \omega = 2\pi f$$

Inductor:

Linear and bilinear element

- Time Domain:

$$V(t) = L \frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L} \int_{-\infty}^t v(t) dt$$

- Impedance:

$$Z_L = j X_L \Omega \text{ \& } X_L = \omega L$$

- In s-domain:

$$V(s) = sL I(s)$$

$$I(s) = \frac{1}{sL} V(s)$$

- Inductor doesn't allow sudden change of current, until impulse of voltage is applied. It stores energy in the form of magnetic field.
- Power dissipation in ideal inductor is zero.

FORMULAS FOR THE BASIC CIRCUIT COMPONENTS					
CIRCUIT ELEMENT	IMPEDANCE		VOLT- AMP EQUATIONS		ENERGY (dissipated on R or stored in L, C)
	Absolute Value	Complex form	instantaneous values	RMS values for sinusoidal signals	
RESISTANCE	R	R	v=iR	V _{rms} =I _{rms} R	E=I _{rms} ² Rt
INDUCTANCE	2 π fL	jωL	V = L di/dt	V _{rms} =I _{rms} ×2 π fL	E = 1/2 Li ²
CAPACITANCE	1/2πfC	1/jωC	i = C dV/dt	V _{rms} = I _{rms} /2πfC	E = 1/2 CV ²

Notes:
 R- resistance in ohms, L - inductance in Henry, C- capacitance in farad, f – frequency in Hertz, t- time in seconds, π ≈ 3.14159,
 ω = 2 π f – angular frequency
 j – imaginary unit (j²=-1)
 Euler's formula: e^{jx} =cosx + jsinx

EQUATIONS FOR SERIES AND PARALLEL CONNECTIONS				
CIRCUIT ELEMENT	SERIES CONNECTION		PARALLEL CONNECTION	
RESISTORS		$R_{series} = R_1 + R_2 + \dots$		$R_{parallel} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \dots\right)}$
INDUCTORS		$L_{series} = L_1 + L_2 + \dots$		$L_{parallel} = \frac{1}{\left(\frac{1}{L_1} + \frac{1}{L_2} + \dots\right)}$
CAPACITORS		$C_{series} = \frac{1}{\left(\frac{1}{C_1} + \frac{1}{C_2} + \dots\right)}$		$C_{parallel} = C_1 + C_2 + C_3 \dots$

Rules of series	Rules of parallel
$V_{eq} = V_1 + V_2 + V_3$	$i_{eq} = i_1 + i_2 + i_3$
$R_{eq} = R_1 + R_2 + R_3$	$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$
$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$	$C_{eq} = C_1 + C_2 + C_3$

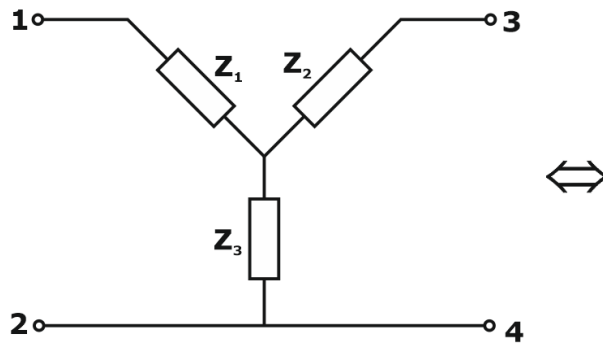
CALCULATIONS OF EQUIVALENT RLC IMPEDANCES		
CIRCUIT CONNECTION	COMPLEX FORM	ABSOLUTE VALUE
 Series	$Z = R + j\omega L + \frac{1}{j\omega C}$	$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$
 Parallel	$Z = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C}$	$Z = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}}$

Faraday Law of electromagnetic induction:

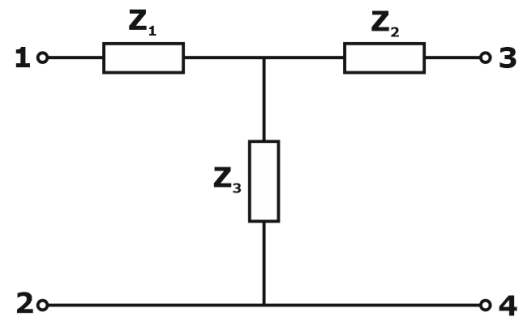
- If a flux linking a coil varies as a function of time then an emf is induced between its terminals (statically induced emf).

- $V = N \frac{d\phi}{dt}$ Also, $V = L \frac{di}{dt}$
- So,
 $N\phi = Li$
 $L = \frac{N\phi}{i}$

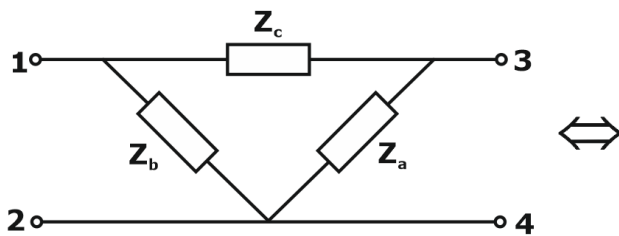
(Δ - γ) And (γ - Δ) Transformations:



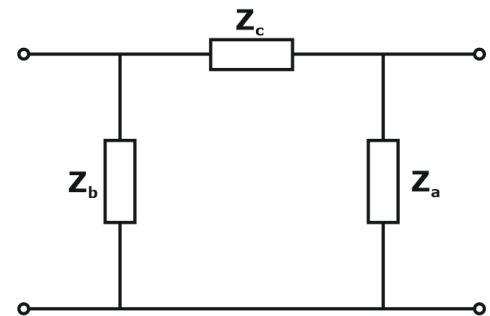
Y network



T network



Δ network



π network

Δ - γ Transformation:

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

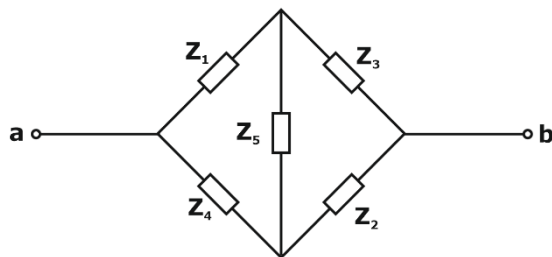
γ → Δ Transformation:

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_1}$$

$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 + Z_3}{Z_2}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_3}$$

Bridge:



$$Z_1 = |Z_1| \angle \theta_1$$

$$Z_2 = |Z_2| \angle \theta_2$$

$$Z_3 = |Z_3| \angle \theta_3$$

$$Z_4 = |Z_4| \angle \theta_4$$

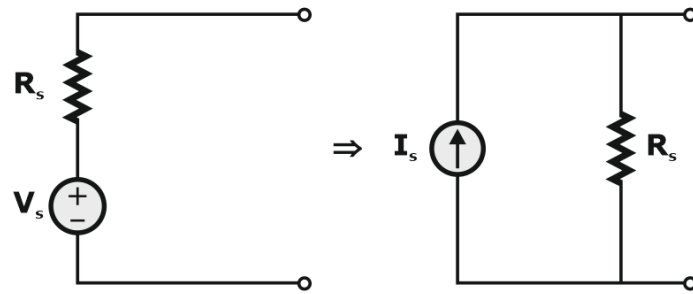
for balance,

$$Z_1 Z_2 = Z_3 Z_4$$

$$\Rightarrow \begin{cases} |Z_1 \parallel Z_2| = |Z_3 \parallel Z_4| \\ \text{and} \\ \angle \theta_1 + \angle \theta_2 = \angle \theta_3 + \angle \theta_4 \end{cases}$$

Source Transformation:

It is a process in which a voltage source connected in series with resistance can be converted into a current source connected in parallel with same resistor and vice-versa.



Where,

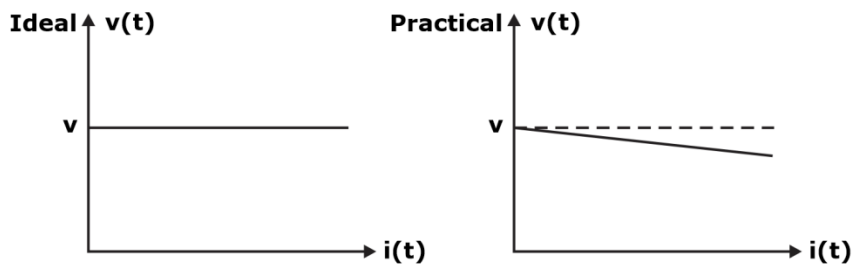
$$V_s = I_s R_s$$

then

$$I_s = \frac{V_s}{R_s}$$

Voltage Source:

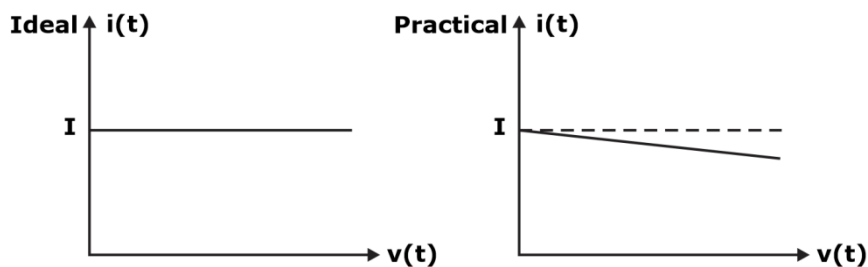
- In practical voltage source, there is small internal resistance, so voltage across the element varies with respect to current.



- Ideal voltmeter, R_v is infinite (Internal resistance)

Current Source:

- In practical current source, there is small internal resistance, so current varies with respect to the voltage across element.



- Ideal Ammeter, R_a is 0 (Internal resistance)
- Internal resistance of voltage source is in series with the source.
- Internal resistance of ideal voltage source is zero.
- Internal resistance of current source is in parallel with the source.
- Internal resistance of ideal current source is infinite.

Independent source: Voltage or current source whose values doesn't depend on any other parameters.

- **Example:** Generator

Dependent Source: Voltage or current source whose values upon other parameters like current, voltage.

Dependent Source: Voltage or current source whose values upon other parameters like current, voltage.

Lumped Network: Network in which all network elements are physically separable is known as lumped network.

Distributed Network: A network in which the circuit elements like resistance, inductance etc, are not physically separate for analysis purpose, is called distributed network. Example: Transmission line.

Mesh and Nodal Analysis

Kirchhoff's current law (KCL):

KCL states that the algebraic sum of the currents entering a node or meeting at a point is zero.

→ It is based on law of conservation of charge.

Kirchhoff's voltage law:

It states that algebraic sum of all the voltages in a closed loop is zero.

It is based on the law of conservation of energy.

Mesh Analysis:

- Path – A set of elements that may be traversed in order, without passing thru the same node twice
- Loop – a closed path
- Mesh – A loop that does not contain any other loop within it.
- Planar Circuit – A circuit that may be drawn on a plane surface in such a way that there are no branch crossovers
- Non-Planar Circuit – A circuit that is not planar, i.e. some branches pass over some other branches (cannot use Mesh Analysis)
- Number of equations required to solve the circuit with the help of mesh analysis are:

$$e = b - (N - 1)$$

Where,

b = number of branches

n = number of nodes

Nodal Analysis:

- identify total number of nodes in the circuit.
- Assign the voltage at each node and one of the nodes is taken as the reference node and the potential of reference node is equal to the ground potential.
- Develop KCL equation for each non – reference node.
- By solving KCL equation, find the node voltages.

The total number of equations required to solve the network by using nodal analysis are:

$$e = N - 1$$

Circuit Theorem

Thevenin’s Theorem:

Any linear network can be replaced by an independent voltage source in series with an impedance such that the current voltage at the terminals is unchanged.

Norton’s Theorem:

Identical to Thevenin’s statement except that the equivalent circuit is an independent current source in parallel with $Z_s = R_{Th}$.

Superposition Theorem:

If a number of voltage or current sources are acting simultaneously in a linear bidirectional network, the resultant response in any branch is the algebraic sum of the responses that would be produced in it, when each source acts alone replacing all other independent sources by their internal resistances.

Max. power Transfer Theorem:

For DC Circuits:

$$R_{Th} = R_L$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

For AC Circuits:

1. When complete Z_L Load is varying i.e. R_L & X_L is varying, then the condition for maximum power to be transferred to the load is $Z_L = Z_S^*$
2. When only R_L is varying & X_L is fixed, then the condition for maximum power to be transferred to the load is $R_L = \sqrt{R_S^2 + (X_S + X_L)^2}$

3. When only R_L is varying and $X_L = 0$, then the condition for maximum power to be transferred to the load is $R_L = |Z_S|$
4. When only X_L is varying and R_L is fixed, then the condition for maximum power to be transferred to the load is $X_L = -X_S$.

$$P_{\max} = \frac{V_{Th}^2}{4R_{Th}}$$

Millman’s Theorem:

As per Millman’s Theorem,

$$V = \frac{\pm \frac{V_1}{R_1} \pm \frac{V_2}{R_2} \pm \frac{V_3}{R_3} + \dots \pm \frac{V_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}}$$

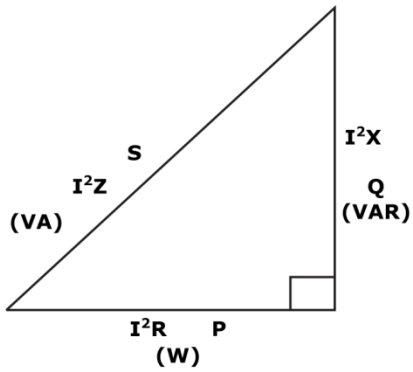
$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}}$$

Mathematically, it is written as

$$V = \frac{\sum_{K=1}^n \frac{V_K}{R_K}}{\sum_{K=1}^n \frac{1}{R_K}}$$

$$R = \frac{1}{\sum_{K=1}^n \frac{1}{R_K}}$$

<p>Average Power</p> $P_{\text{Avg}} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos(\theta)$ $= V_{\text{rms}} I_{\text{rms}} \cos(\theta)$	<p>Power Factor</p> $\text{PF} = \cos(\theta)$ $\theta = \theta_v - \theta_i$
--	--

<p>Reactive Power</p> $Q = V_{rms}I_{rms} \sin(\theta)$ <p>Measured VARs Volt Amperes Reactive</p> $P^2 + Q^2 = (V_{rms}I_{rms})^2$	<p>True Power</p> $P = I^2 R$ 
<p>Apparent Power (s)</p> $S = V_{rms}I_{rms} \text{ VA}$ $S = P + j Q$ $S = I^2 Z$	

TRANSIENT ANALYSIS

DC Transients:

For Inductor,

$$i_L(0^-) = i_L(0) = i_L(0^+)$$

Where,

$i_L(0^-)$ = current at instant just before switch operation

$i_L(0)$ = current at exact instant of switch operation

$i_L(0^+)$ = current at instant just after the switch operation.

For capacitor,

$$V_c(0^-) = V_c(0) = V_c(0^+)$$

Source free circuits:

Source free RL circuit:

$$i(t) = I_0 e^{-t/\tau}$$

Where, τ is time constant

$$\tau = \frac{L}{R_{eq}}$$

$$V_L(t) = -I_0 R e^{-Rt/L}$$

$$V_R(t) = I_0 R e^{-Rt/L}$$

Energy stored in inductor:

$$W_L(t) = \frac{1}{2} Li^2$$

$$\Rightarrow W_L(t) = \frac{1}{2} L [I_0 e^{-Rt/L}]^2$$

$$\Rightarrow W_L(t) = \frac{1}{2} L I_0^2 e^{-2t/\tau}$$

$$W_R(t) = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau})$$

Source Free RC circuit:

$$V_c(t) = V_0 e^{-t/\tau}$$

Where, τ = time constant

$$\tau = RC$$

$$i_c(t) = -\frac{V_0}{R} e^{-t/RC}$$

$$i_R(t) = \frac{V_0}{R} e^{-t/RC}$$

Energy stored in Capacitor:

$$W_c(t) = \frac{1}{2} C V^2$$

$$= \frac{1}{2} C (V_0 e^{-t/\tau})^2$$

$$W_c(t) = \frac{1}{2} C V_0^2 e^{-2t/\tau}$$

$$W_R(t) = \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau})$$

Circuits with Source:

RL circuit with source:

$$i(t) = \underbrace{i_{CF}}_{\substack{\text{due to} \\ \text{circuit element} \\ \text{only} \\ \text{(source} = 0)}} + \underbrace{i_{PI}}_{\substack{\text{due to source} \\ \text{only}}}$$

General equation $\Rightarrow i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$ (where, $\tau = \frac{L}{R}$)

RC circuit with source:

$$V_c(t) = V_{CF}(t) + V_{PI}(t)$$

$$V_c(t) = V_c(\infty) + [V_c(0) - V_c(\infty)] e^{-t/\tau} \quad \text{where, } \tau = RC$$

Second Order System:

Series RLC system:

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

For Underdamped case:

$$i(t) = (C_1 \cos \beta t + C_2 \sin \beta t)e^{-\alpha t}$$

Time constant, $\tau = \frac{1}{\alpha} = \frac{2L}{R}$

$$\omega_d \text{ (damping frequency)} = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\omega_d = \frac{1}{\sqrt{LC}} \sqrt{1 - \left(\frac{R}{2} \sqrt{\frac{C}{L}}\right)^2}$$

$\rightarrow \omega_n = \frac{1}{\sqrt{LC}} \rightarrow$ natural frequency; $\xi = \frac{R}{2} \sqrt{\frac{C}{L}}$

Parallel RLC circuit:

$$s_{1,2} = \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

For Underdamped case:

$$V(t) = (C_1 \cos \beta t + C_2 \sin \beta t)e^{-\alpha t}$$

where, Time constant, $\tau = \frac{1}{\alpha} = 2RC$

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$$

$$\omega_d = \frac{1}{\sqrt{LC}} \sqrt{1 - \left(\frac{1}{2R} \sqrt{\frac{L}{C}}\right)^2}; \quad \omega_d = \omega_n \sqrt{1 - \xi^2}$$

Where, $\omega_n = \frac{1}{\sqrt{LC}}; \xi = \frac{1}{2R} \sqrt{\frac{L}{C}}$

AC Transients:

$$i(t) = -\frac{V_m}{|Z|} \sin(\omega t_0 + \theta - \alpha) e^{-\frac{R(t-t_0)}{L}} + \frac{V_m}{|Z|} \sin(\omega t + \theta - \alpha)$$

For transient free response:

$$\omega t_0 + \theta - \alpha = 0$$

$$\omega t_0 = \alpha - \theta$$

$$\omega t_0 = \tan^{-1} \frac{\omega L}{R} - \theta$$

Two-port Networks

The six parameters thus derived from Two Port Network are as follows:

- Z Parameters
- Y or Z⁻¹ Parameters
- h Parameters
- g or h⁻¹ Parameters
- ABCD or T Parameters
- (ABCD)⁻¹ or T⁻¹ Parameters

Impedance parameter or Z – parameter or open-circuit parameter:

- Z parameter is also referred as open circuit parameters.

$$V_1 = f(I_1, I_2)$$

$$V_2 = f(I_1, I_2)$$

Independent Variables are I₁ & I₂

Dependent variables are V₁ & V₂

- V₁ = Z₁₁I₁ + Z₁₂I₂
- V₂ = Z₂₁I₁ + Z₂₂I₂
- Z Parameters for the symmetrical lattice network are

$$Z = \begin{bmatrix} \frac{Z_a + Z_b}{2} & \frac{Z_b - Z_a}{2} \\ \frac{Z_b - Z_a}{2} & \frac{Z_a + Z_b}{2} \end{bmatrix}$$

Admittance parameter or Y-parameter or short-circuit parameter:

- Y parameter is also referred as open circuit parameters.

$$I_1 = f(V_1, V_2)$$

$$I_2 = f(V_1, V_2)$$

Independent Variables are V₁ & V₂

Dependent variables are I₁ & I₂

- I₁ = y₁₁V₁ + y₁₂V₂

$$I_2 = y_{21}V_1 + y_{22}V_2$$

Condition of Reciprocity and Symmetricity:

Parameter	Reciprocity	Symmetricity
Z parameter	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y parameter	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$

All resistive circuits are reciprocal network and dependent source networks are non-reciprocal. In case of dependent source network, if $Z_{12} = Z_{21}$ then the network will be reciprocal.

Hybrid parameter or h-parameter:

- $V_1 = f(I_1, V_2)$
 $I_2 = f(I_1, V_2)$
 Independent Variables are I_1 & V_2
 Dependent variables are V_1 & I_2
- $V_1 = h_{11}I_1 + h_{12}V_2$
 $I_2 = h_{21}I_1 + h_{22}V_2$
- Relation between h Parameters and Z or Y Parameters:

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = \frac{1}{Y_{11}} \Rightarrow \boxed{h_{11} = \frac{1}{Y_{11}}}$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = \frac{Z_{12}}{Z_{22}} \Rightarrow \boxed{h_{12} = \frac{Z_{12}}{Z_{22}}}$$

$$h_{21} = \left. \frac{I_2}{V_2} \right|_{V_1=0} = \frac{Y_{21}}{Y_{11}} \Rightarrow \boxed{h_{21} = \frac{Y_{21}}{Y_{11}}}$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = \frac{1}{Z_{22}} \Rightarrow \boxed{h_{22} = \frac{1}{Z_{22}}}$$

- Condition for Reciprocity:** $h_{12} = -h_{21}$
- Condition for Symmetry:** Δh or $|h| = 1$ or $h_{11}h_{22} - h_{12}h_{21} = 1$

Inverse Hybrid parameter or g- parameter:

- $I_1 = f(V_1, I_2)$
 $V_2 = f(V_1, I_2)$
 Independent Variables are V_1 & I_2
 Dependent variables are I_1 & V_2
- $I_1 = g_{11}V_1 + g_{12}I_2$
 $V_2 = g_{21}V_1 + g_{22}I_2$

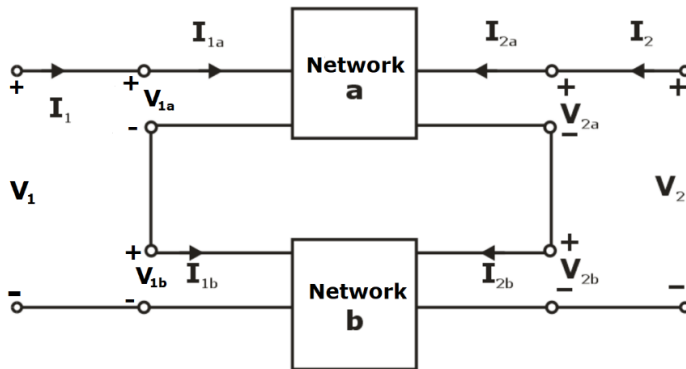
- **Condition for Reciprocity:** $g_{12} = g_{21}$
- **Condition for Symmetry:** $\Delta g = |g| = 1$ or $g_{11}g_{12} - g_{12}g_{21} = 1$

Transmission parameter or ABCD parameter or T parameter:

- $V_1 = f(I_1, I_2)$
 $V_2 = f(I_1, I_2)$
 Independent Variables are I_1 & I_2
 Dependent variables are V_1 & V_2
- $V_1 = AV_2 - BI_2$
 $I_1 = CV_2 - DI_2$
- **Condition for Reciprocity:** $\Delta T = 1$ or $AD - BC = 1$
- **Condition for Symmetry:** $A = D$

INTERCONNECTION OF TWO PORT NETWORKS

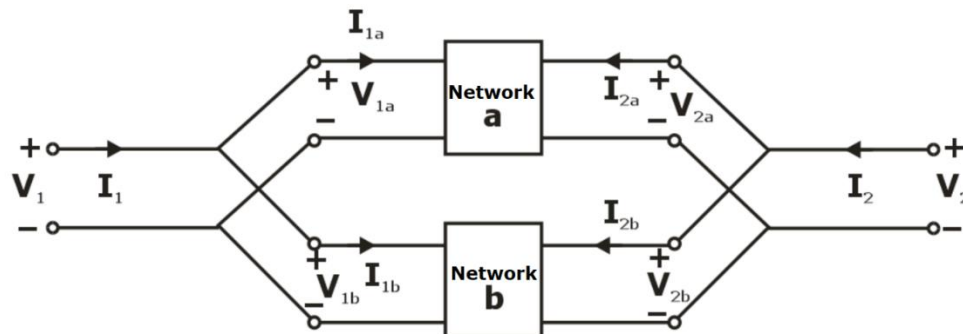
Series Connection:



$$[Z] = [Z]_a + [Z]_b$$

$$[Z] = \begin{bmatrix} Z_{11a} + Z_{11b} & Z_{12a} + Z_{12b} \\ Z_{21a} + Z_{22b} & Z_{22a} + Z_{22b} \end{bmatrix}$$

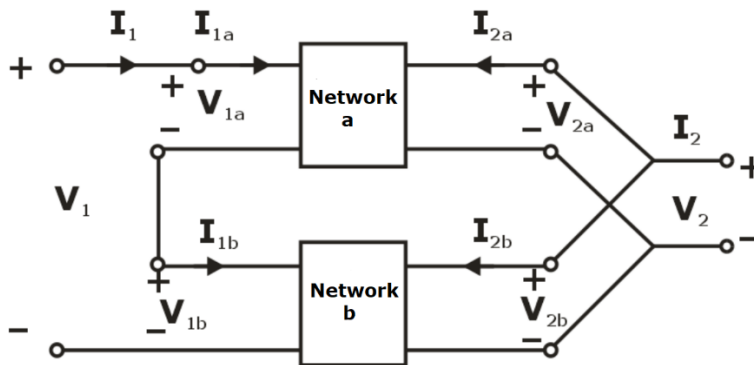
Parallel Connection:



$$[Y] = [Y]_a + [Y]_b$$

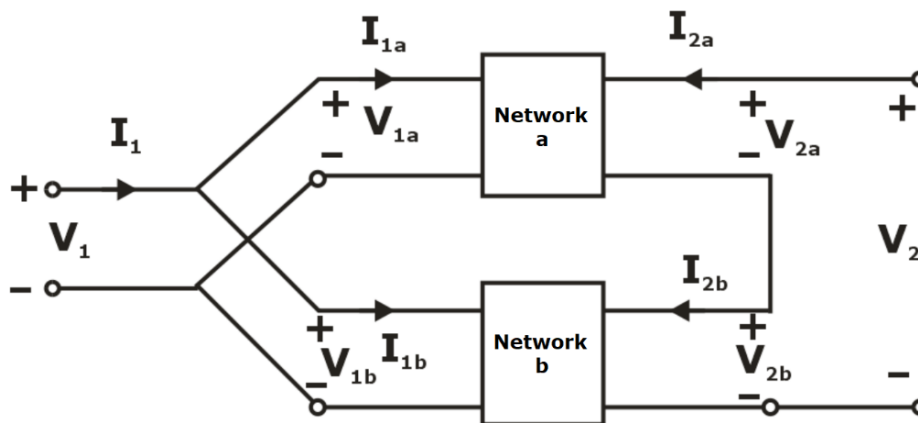
$$[Y] = \begin{bmatrix} Y_{11a} + Y_{11b} & Y_{12a} + Y_{12b} \\ Y_{21a} + Y_{21b} & Y_{22a} + Y_{22b} \end{bmatrix}$$

Series parallel Connection:



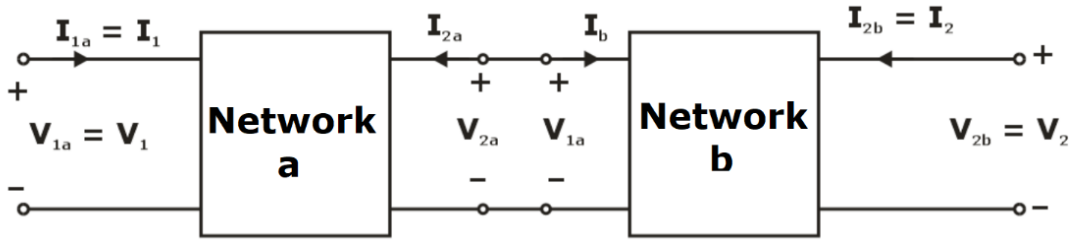
$$[h] = [h]_a + [h]_b$$

Parallel Series Connection:



$$[g] = [g]_a + [g]_b$$

Cascade Connection:

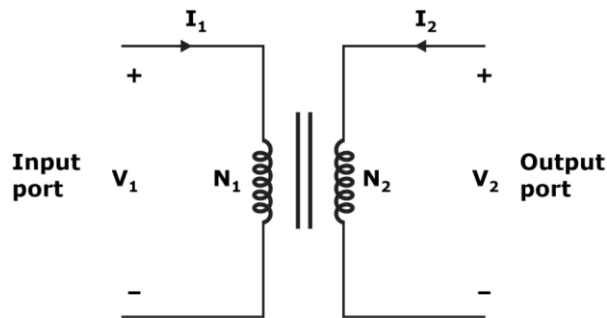


$$[ABCD]_{\text{Total}} = [ABCD]_a \times [ABCD]_b$$

Network Components:

Transformer:

4 terminal or 2-port devices.



- $N_1 > N_2$: Step down transformer

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

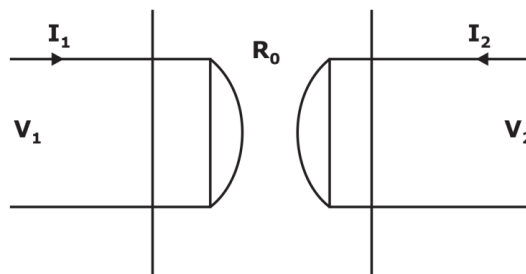
- $N_2 > N_1$: Step up transformer

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}, \quad \text{Where } \frac{N_1}{N_2} = K \rightarrow \text{Turns ratio.}$$

- $[T] = \begin{pmatrix} n & 0 \\ 0 & 1/n \end{pmatrix} \quad [h] = \begin{pmatrix} 0 & n \\ -n & 0 \end{pmatrix}$

- Transformer does not work as amplifier because current decreases in same amount power remain constant.

Gyrator:

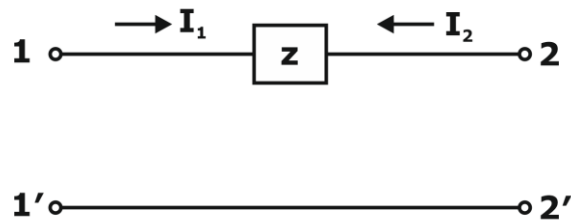


- R_0 = Coefficient of Gyrator
- $V_1 = R_0 I_2$
- $V_2 = R_0 I_1$
- If load is capacitive then input impedance will be inductive and vice versa.

- If load is inductive then input impedance will be capacitive.
- It is used for simulation of equivalent value of inductance.

For Series and Shunt elements:

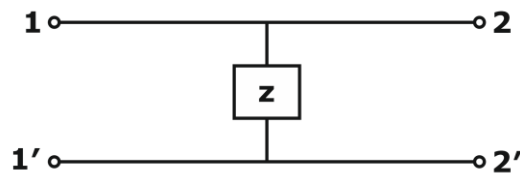
For Series element ($I_1 = -I_2$)



ABCD parameters can be defined by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

For Shunt element ($V_1 = V_2$)



ABCD parameters can be defined by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}$$

AC Circuits

Circuits driven by sinusoidal voltage and current sources are called ac circuits. Both sine and cosine come under sinusoidal sources.

Root Mean Square:

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T V^2 dt}$$

where, T is time period.

Average value:

$$V_{avg} = \frac{1}{T} \int_0^T V dt$$

where, T is time period of equal.

Form factor:

It is the ratio of RMS value of waveform to the average value of waveform.

$$FF = \frac{\text{RMS Value}}{\text{Avg Value}}$$

Peak Factor:

It is the ratio of the max value of waveform to the RMS value of waveform.

$$PF = \frac{\text{Max Value}}{\text{RMS Value}}$$

Sinusoidal steady state analysis:

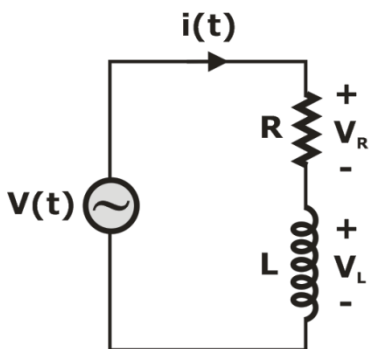
Conditions to compare the phases of two sinusoidal:

- (i) Both functions should be expressed either in sine or cosine.
- (ii) Both functions should have to written with positive amplitude.
- (iii) Both functions should have same frequency.

Representation of Circuit elements:

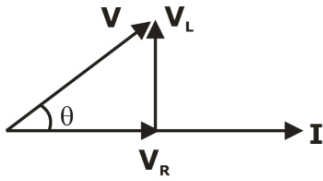
Time Domain		Frequency domain	
	$v = Ri$	$V = RI$	
	$V = L \frac{di}{dt}$	$V = j\omega LI$	
	$v = \frac{1}{C} \int idt$	$v = \frac{1}{j\omega C} I$	

Series RL Circuit:



$$Z = R + jX_L$$

Phasor:



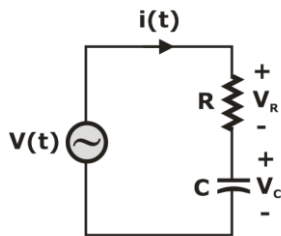
$$V = \sqrt{V_R^2 + V_L^2}$$

$$\theta = \tan^{-1} \frac{V_L}{V_R} \Rightarrow \theta = \tan^{-1} \frac{X_L}{R}$$

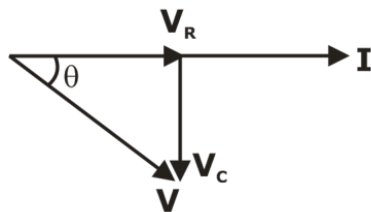
$$\cos \theta = \frac{V_R}{V} \text{ (lagging)}$$

$$P_{\text{avg}} = V_{\text{RMS}} I_{\text{RMS}} \cos \theta$$

Series RC Circuit:



Phasors:



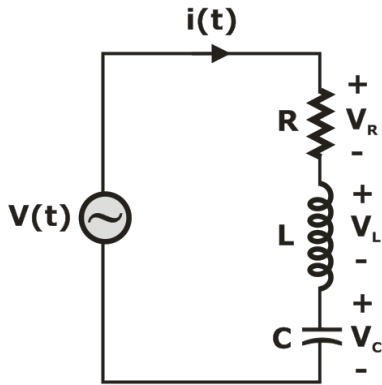
$$V = \sqrt{V_R^2 + V_C^2}$$

$$\theta = -\tan^{-1} \frac{V_C}{V_R} \Rightarrow \theta = -\tan^{-1} \frac{X_C}{R}$$

$$\cos \theta = \frac{V_R}{V} \rightarrow \text{leading}$$

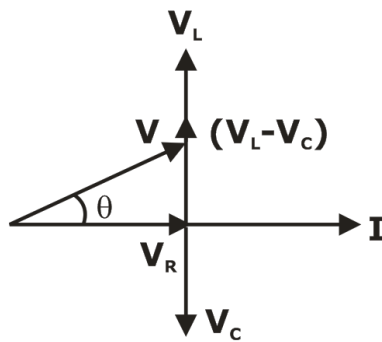
$$P_{\text{avg}} = V_{\text{RMS}} I_{\text{RMS}} \cos \theta$$

Series RLC Circuit:



$$Z = R + j(X_L - X_C)$$

Phasor:

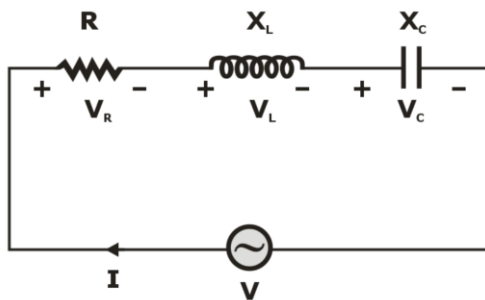


$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

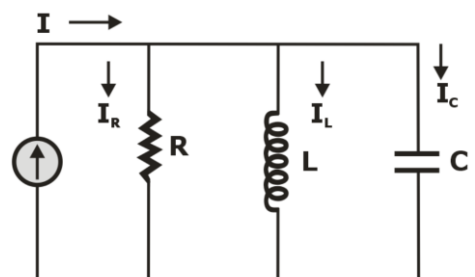
$$\theta = \tan^{-1} \frac{V_L - V_C}{V_R} \Rightarrow \theta = \tan^{-1} \frac{X_L - X_C}{R}$$

$$\cos \theta = \frac{V_R}{V} \rightarrow \text{(lagging)}$$

Series and Parallel Resonance:

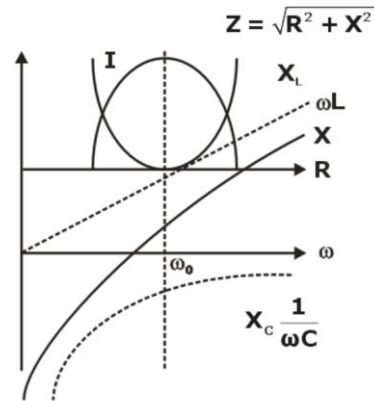


$$Z = R + jX_L - jX_C$$



$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

Variation of circuit parameters for series RLC with frequency:



- a) Reactance of the circuit is zero at resonant frequency.
- b) Impedance of the circuit reaches its minimum at resonant frequency.
- c) Current in a series resonant circuit is maximum at resonant frequency.

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec or } f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

Characteristic	Series Circuit	Parallel Circuit
Resonant Frequency (ω_0)	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
Quality Factor	$\frac{\omega_0 L}{R}$ or $\frac{1}{\omega_0 RC}$	$\frac{R}{\omega_0 L}$ or $\omega_0 RC$
Bandwidth	$\frac{\omega_0}{Q}$	$\frac{\omega_0}{Q}$

Magnetic Coupling Circuits

Self-Inductance:

Self-Inductance is the effect of change in the magnetic flux linking in the circuit due to change in current of the same circuit. It is denoted by 'L'. and the unit is Henry.

$$V_{\text{induced}} = L_1 \frac{di_1}{dt} \text{ (According to Faraday's Law)}$$

L_1 is the self-inductance corresponding to change in current i_1 ,

$$\text{Also, } L_1 = N_1 \frac{d\phi_{11}}{di_1}$$

Mutual-Inductance:

Mutual inductance is the effect of change in magnetic flux linking in a circuit due to change in current of another circuit.

The voltage induced in the first coil due to current in it.

$$V_1 = N_1 \frac{d\phi_{11}}{dt} = L_1 \frac{di_1}{dt}$$

The voltage induced in the second coil due to current in the first coil.

$$V_2 = N_2 \frac{d\phi_{12}}{dt} = M_{21} \frac{di_1}{dt}$$

Coefficient of Coupling:

The coefficient of coupling is defined as fraction of the total flux that links the magnetic coil.

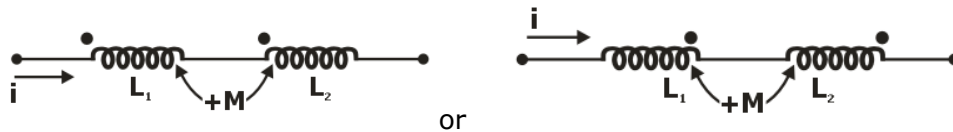
$$K = \frac{\phi_{12}}{\phi_1} \text{ or } K = \frac{\phi_{21}}{\phi_2}$$

$$\therefore L_1 = \frac{N_1 \phi_1}{i_1}; L_2 = N_2 \frac{\phi_2}{i_2}$$

$$M_{12} = N_1 \frac{\phi_{21}}{i_2}; M_{21} = N_2 \frac{\phi_{12}}{i_1}$$

$$M = K \sqrt{L_1 L_2}$$

Magnetically coupled coils in series connection:



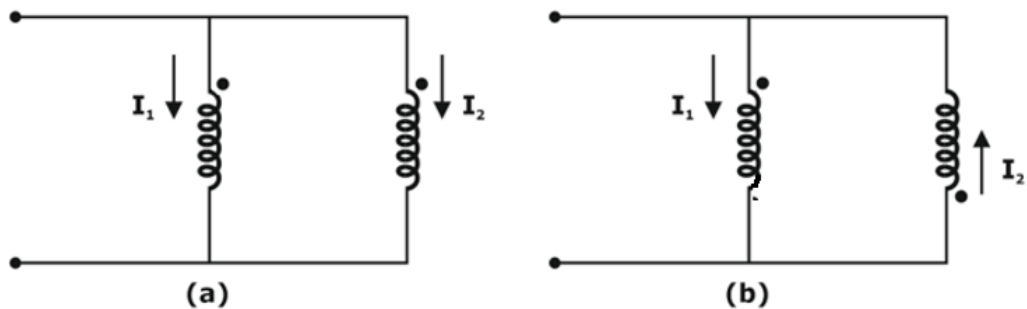
For positively mutually connected series coil: $L_{eq} = L_1 + L_2 + 2M$



For positively mutually connected series coil: $L_{eq} = L_1 + L_2 - 2M$

Magnetically coupled coils in parallel connection:

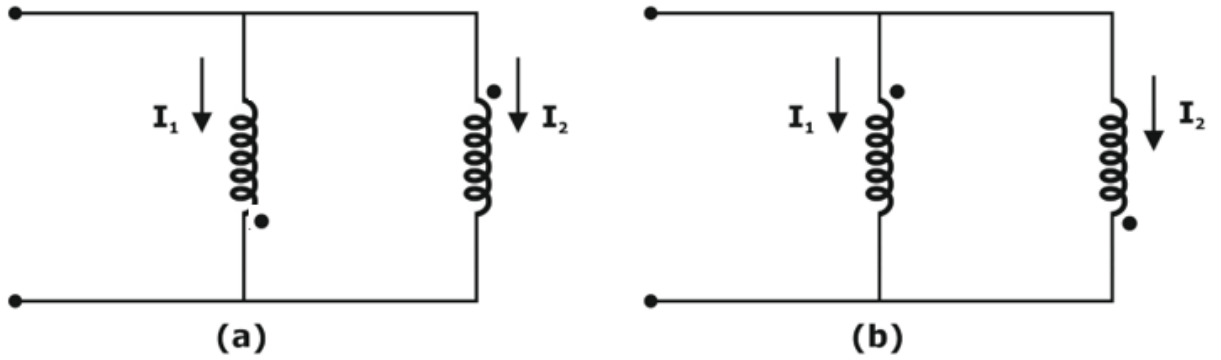
Case-I: Positive naturally connected parallel coil



As current enters through dots in both the coils, they are positive mutually connected coils.

$$L_{eq} = \frac{L_1 L_2 - N^2}{L_1 + L_2 - 2M}$$

Case-II: Negative mutually connected parallel coil



As current enters one coil and leaves through dot in another coil, they are negative mutually connected coils.

$$L_{eq} = \frac{L_1 L_2 - N^2}{L_1 + L_2 + 2M}$$
