BYJU'S EXAM PREP

AE/JE Foundation

Electrical Engineering

EMT and Signals & System

Top 100 Most Important Questions

1. If the Nyquist rate for the signal x(t) is $2\omega_0$. Then the Nyquist rate for signal x(t) $\cos\omega_0 t$ is-

A. ω_0

B. $2\omega_0$

C. $3\omega_0$

D. $4\omega_0$

Ans. D

Sol.: $x(t) \xrightarrow{NR} 2\omega_0$

Maximum frequency present in the signal $x(t) = \frac{2\omega_0}{2} = \omega_0$

(Because NR is twice the maximum frequency)

Maximum frequency present in the signal $[x(t)\cos\omega_0 t]$ is = $(\omega_0 + \omega_0) = 2\omega_0$

So, NR (Nyquist rate) for the signal $x(t)\cos\omega_0 t = 4\omega_0$

2. What will be the value of x(2), if X(z) is the z-transform of x[n] and ROC is |z| > 1/5.

$$X(z) = \frac{1+2z^{-1}}{1+\frac{1}{5}z^{-1}}$$

A. $-\frac{9}{25}$

B. $\frac{20}{11}$

C. 1

D. $\frac{9}{5}$

Ans. A

Sol.: Arrange numerator and denominator in descending order because ROC: $|z| > \frac{1}{5}$

Therefore, x[n] will be right sided.

$$X(z) = \frac{1+2z^{-1}}{1+\frac{1}{5}z^{-1}}$$

$$1+\frac{1}{5}z^{-1}\Big)\,1+2z^{-1}\,\Big(1+\frac{9}{5}z^{-1}-\frac{9}{25}z^{-2}$$

$$\begin{array}{r}
1+\frac{1}{5}z^{-1} \\
\hline
\frac{9}{5}z^{-1} \\
\frac{9}{5}z^{-1}+\frac{9}{25}z^{-2} \\
\hline
-\frac{9}{25}z^{-2} \\
-\frac{9}{25}z^{-2}-\frac{9}{125}z^{-3} \\
\hline
\frac{9}{125}z^{-3}
\end{array}$$

$$X(z) = 1 + \frac{9}{5}z^{-1} - \frac{9}{25}z^{-2} + ...$$

x(0) = 1

x(1) = 9/5

x(2) = −9/25

3. Which one most appropriate dynamic system?

A. $y(n) + y(n-1) + y(n+1)$

B. $y(n) + y(n-1)$

C. $y(n) + y(n-1) + y(n+3) = 0$

D. all of the above

Ans. D

Sol.: Because present output of y(n) depend upon past y(n - K) and future y(n + K).

4. A signal $x_1(t) = 20\cos(200\pi t)$ is sampled at 150 Hz and $x_2(t) = 20\cos(100\pi t)$ is sampled at f_s. What is the value of f_s so that both the sequences of samples will be identical.

A. 50 Hz

B. 200 Hz

C. 150 Hz

D. 100 Hz

Ans. C

Sol.: For sequences to be similar, the samples must have same values.

General form of cosine: $\cos(2m\pi \pm \alpha)$

$$2m\pi \pm \frac{200n\pi}{150} = \frac{100\pi}{f_s}$$

At m = 1 and n = 1

$$2 \pm \frac{4}{3} = \frac{100}{f_s}$$

f_s = 150 Hz and 300 Hz

5. What is the even part of the signal x(n), whose Fourier transform is $X(e^{j\omega}) = 1 + \cos\omega$?

A. $\left\{\frac{1}{2}, 1, \frac{1}{2}\right\}$

B. $\{1, \underset{\uparrow}{2}, 1\}$

C. $\left\{\underset{\uparrow}{\frac{1}{2}}, 1, \frac{1}{2}\right\}$

D. $\left\{-\frac{1}{2}, \underset{\uparrow}{1}, \frac{1}{2}\right\}$

Ans. A

Sol.: $x\left(e^{j\omega}\right) = 1 + \frac{1}{2}e^{j\omega} + \frac{1}{2}e^{-j\omega}$

$$x(n) = \left\{\frac{1}{2}, \underset{\uparrow}{1}, \frac{1}{2}\right\}$$

$$x(-n) = \left\{\frac{1}{2}, \underset{\uparrow}{1}, \frac{1}{2}\right\}$$

$$\text{Even part} = \frac{x(n) + x(-n)}{2} = \frac{\{1, 2, 1\}}{2} = \left\{\frac{1}{2}, 1, \frac{1}{2}\right\}$$

$$\Rightarrow \text{Even part of } x(n) = \left\{\frac{1}{2}, \underset{\uparrow}{1}, \frac{1}{2}\right\}$$

6. One complex multiplication takes 2μs and assume that the amount of time it takes to perform all of the multiplications. Then the time taken to compute a 512 points DFT is?

A. 0.52 s

B. 0.26 s

C. 0.78 s

D. 1.04 s

Ans. A

Sol.: N-point DFT requires N^2 complex multiplications.

Time taken = $(512)^2 \times 2 \times 10^{-6}$ s = 0.52 s

7. The expression of trigonometrical Fourier series coefficient b_n in terms of exponential Fourier series coefficient C_n is

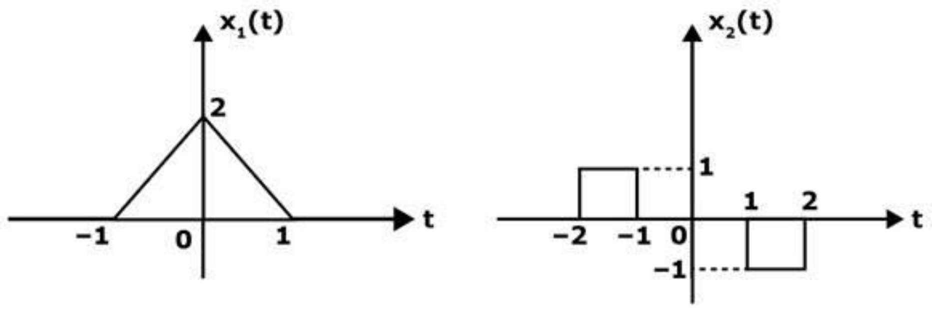
A. $j(C_n + C_{-n})$ B. $j\left(\frac{C_n + C_{-n}}{2}\right)$

C. $j(C_n - C_{-n})$ D. $j\left(\frac{C_n - C_{-n}}{2}\right)$

Ans. C

Sol.: $b_n = \frac{2}{T}\int_T x(t)(\sin\omega_0 nkt)\,dt = j \times \frac{1}{T}\left[\left[\int_T x(t)\left(-e^{j\omega_- nk}\right)dt\right] + \left[\int_T x(t)\left(+e^{j\omega -nkt}\right)dt\right]\right] = j(C_n - C_{-n})$

8. Consider the signals $x_1(t)$ & $x_2(t)$ as shown below,



Which of the following statements are true.

S_1 : Spectrum of $x_1(t)$ is real & odd.

S_2 : Spectrum of $x_1(t)$ is real & even.

S_3: spectrum of $x_2(t)$ is imaginary & odd.

S_4: Spectrum of $x_2(t)$ is real & odd.

A. Only S_1 & S_4 B. Only S_1 & S_3

C. Only S_2 & S_3 D. Only S_2 & S_4

Ans. C

Sol.: $x_1(t)$ is real and even signal. Its spectrum $X_1(\omega)$ will be also real & even.

$x_2(t)$ is real & odd signal. Its spectrum $X_2(\omega)$ will be imaginary & odd.

∴ Statements S_2 & S_3 are true.

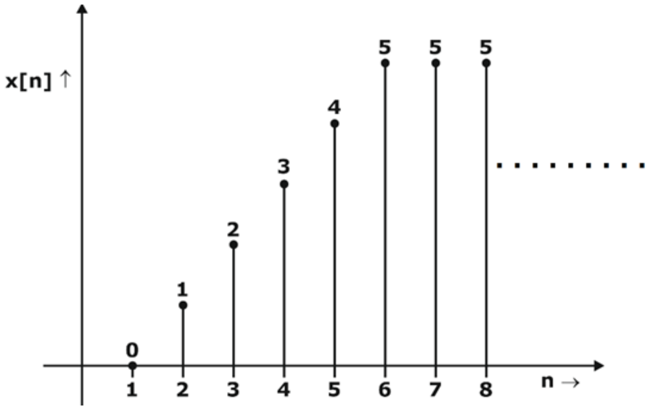
9. What is the power of the given signal, $x[n] = r[n - 1] - r[n - 6]$

A. 25 W B. 12.5 W

C. 55 W D. 0 W

Ans. B

Sol.: Signal is broken into two parts, (i). $1 \le n \le 6$ & (ii). n > 6



Power of non-periodic signal is given as

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} |x[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \left[\sum_{1}^{6} |x[n]|^2 + \sum_{7}^{N} (5)^2 \right]$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[0+1+4+9+16+25+5^2[N-7] \right]$$

$$= \lim_{N\to\infty} \left[\frac{55}{2N+1} + \frac{25[N-7]}{2N+1} \right] = 0 + \frac{25}{2} = 12.5W$$

$$= \lim_{N\to\infty} \left[\frac{55}{2N+1} + \frac{25[N-7]}{2N+1} \right] = 0 + \frac{25}{2} = 12.5W$$

10. The signal $x(t) = 3\cos\left(5t + \frac{\pi}{6}\right)$ is a periodic signal with a period of:

A. $\frac{2\pi}{3}$

B. $\frac{\pi}{5}$

C. $\frac{2\pi}{5}$

D. $\frac{2\pi}{7}$

Ans. C

Sol.: The time period of continuous time signals is given as:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5} \text{sec}$$

11. The odd part of the signal x(n) = α^n u[n] is-

A. $\frac{1}{2}\alpha^{|n|} + \frac{1}{2}\delta(n)$

B. $\frac{1}{2}\alpha^{|n|}\text{sign}(n)$

C. $\frac{1}{2}\alpha^{|n|} - \frac{1}{2}\delta(n)$

D. $\frac{1}{2}\alpha^{|n|}\text{sign}(-n)$

Ans. B

Sol.: Odd part of the signal

$x(n) = \alpha^n u(n)$

$$x_0(n) = \frac{x(n) - x(-n)}{2}$$

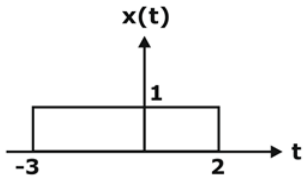
$$x_o(n) = \frac{\alpha^n u(n) - \alpha^{-n} u(-n)}{2}$$

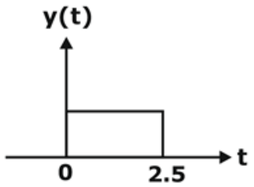
$$\boxed{x_0(n) = \frac{1}{2}\alpha^{|n|}\,\text{sgn}(n)}$$

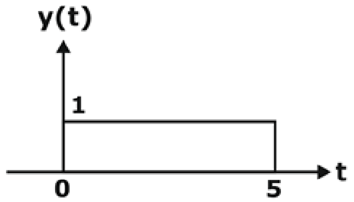
where,

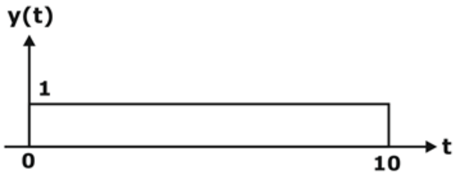
sgn(n) = u(n) – u(–n)

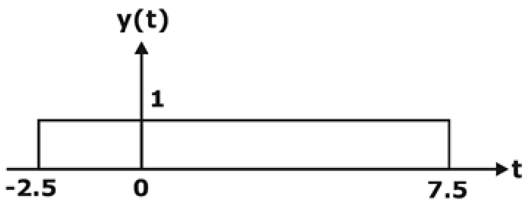
12. If x(t) is as shown below, then $y(t) = x\left(\frac{t}{2} - 3\right)$ will be



A. 

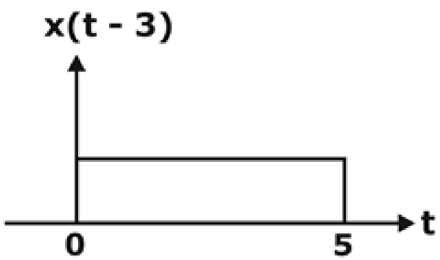
B. 

C. 

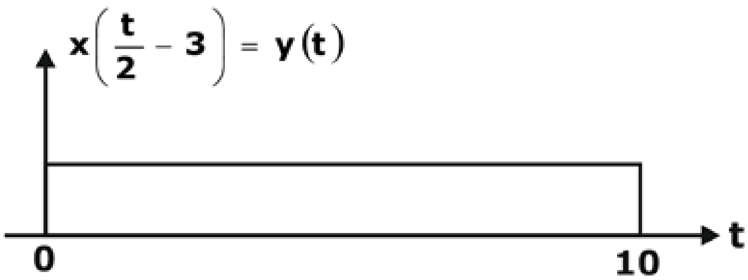
D. 

Ans. C

Sol.: Shifting → t → (t – 3)



Expanding → t → $\frac{t}{2}$



13. Find the fourier transform of

$$x(t) = \begin{cases} \pi, & |t| \le \pi \\ 0, & |t| > \pi \end{cases}$$

where, Sa(ωπ) represents the sampling function.

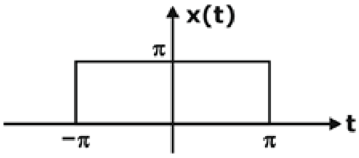
A. $2\pi^2 Sa(\omega\pi)$

B. $\pi^2 Sa(\omega\pi)$

C. $2\pi Sa(\omega\pi)$

D. $\pi Sa(\omega\pi)$

Ans. A

Sol.: x(t) = 

$$A\,\text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{\text{F.T.}} AT.Sa\left(\frac{\omega T}{2}\right)$$

$$\pi\text{rect}\left(\frac{t}{2\pi}\right) \xleftrightarrow{\text{F.T.}} 2\pi^2 Sa(\omega\pi)$$

14. The inverse Fourier transform of u(ω). (where, u(ω) is unit step function)

A. $\frac{j}{t} + \pi\delta(-t)$

B. $\frac{j}{2\pi t} + \frac{\delta(-t)}{2}$

C. $\frac{-1}{2\pi t} + \frac{\delta(-t)}{2}$

D. $\frac{1}{jt} + \pi\delta(t)$

Ans. B

Sol.: We know that,

$$u(t) \rightleftharpoons \frac{1}{j\omega} + \pi\delta(\omega)$$

Using time reversal property,

$$u(-t) = \frac{-1}{j\omega} + \pi\delta(-\omega)$$

Using property of duality,

$$\frac{-1}{jt} + \pi\delta(-t) \rightleftharpoons 2\pi u(\omega)$$

$$\frac{-1}{2\pi jt} + \frac{\delta(-t)}{2} \rightleftharpoons u(\omega)$$

$$u(\omega) = \frac{j}{2\pi t} + \frac{\delta(-t)}{2}$$

15. Consider the following properties of Fourier transform. If Fourier transform of x(t) is X(ω).

1) $x^*(t) \rightleftharpoons X^*(-\omega)$

2) $x(at) \rightleftharpoons \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$

3) $\int_{-\infty}^{t} x(\tau)\,d\tau \rightleftharpoons \frac{X(\omega)}{j\omega}$

4) $t^n x(t) \rightleftharpoons (-j)^n \frac{d^n X(\omega)}{d\omega^n}$

Which of the stated properties is/are correct.

A. 1, 2 and 4

B. 1 and 2

C. 3 and 4

D. 1, 2, 3 and 4

Ans. B

Sol.: 3 and 4 properties are wrong.

$$\int_{-\infty}^{t} x(\tau)d\tau \rightleftharpoons \frac{X(\omega)}{j\omega} + \pi \cdot X(0) \cdot \delta(\omega)$$

$$t^n x(t) \rightleftharpoons (j)^n \frac{d^n X(\omega)}{d\omega^n}$$

16. The relationship between Fourier series coefficient X_k and Y_k of signals x(n) and y(n) is $Y_k = (1 - (-1)^k)\, X_k$. Then which one of the following difference equations represents the relationship between x[n] and y[n]?

A. y(n) = x(n - N) – x(n)

B. y(n) = x(n) – x(n – N/2)

C. y(n) = x(n) – x(n - N)

D. y(n) = x(n – N/2) – x(n)

Ans. B

Sol.:

$$X_k = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega k}$$

$$x(n) \leftrightarrow X(k)$$

$$x\left(n - \frac{N}{2}\right) \leftrightarrow X(k) e^{-j\frac{N}{2}\cdot\frac{2\pi}{N}k}$$

$$x\left(n - \frac{N}{2}\right) \leftrightarrow x(k)(-1)^k$$

$$y(n) \leftrightarrow Y(k)$$

$$Y(k) = x(k)\left(1-(-1)^{k}\right)$$

$$\Rightarrow y(n) = x(n) - x\left(n - \frac{N}{2}\right)$$

17. Identify the linear system(s) from the following list:

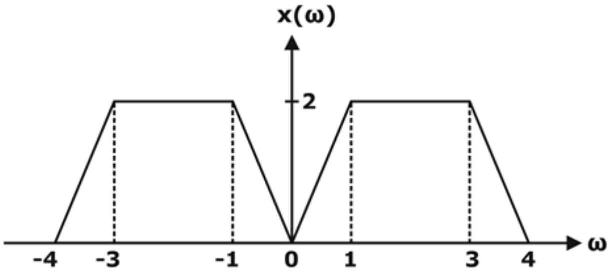
1) y(t) = x(t2)
2) y(t) = x2(t)
3) y(t) = Ax(t) + B
4) y(t) = ex(t)

A. 2 and 3
B. 3 and 4
C. 1 only
D. 1 and 3

Ans. C

Sol.: Only y(t) = x(t^2) is linear system.

18. The spectrum X(ω) of a continuous time signal x(t) is shown as



The value of $\int_{-\infty}^{\infty} x(t)e^{-j2t} + \int_{-\infty}^{\infty} x(t) + \int_{-\infty}^{\infty} x(t)e^{j3t}$ is

A. 2
B. 6
C. 8
D. 4

Ans. D

Sol.: $\int_{-\infty}^{\infty} x(t)\, e^{-j2t} = X(2) = 2$

$$\int_{-\infty}^{\infty} x(t) = X(0) = 0$$

$$\int_{-\infty}^{\infty} x(t)\, e^{j3t} = X(-3) = 2$$

So, $\int_{-\infty}^{\infty} x(t)e^{-j2t} + \int_{-\infty}^{\infty} x(t) + \int_{-\infty}^{\infty} x(t)e^{j3t} = 2 + 0 + 2 = 4$

19. Which of the following is a non-causal system?

A. y(t) = 2x(t) + dx(t)/dt
B. y(t) = x(t) – x(t –1)
C. y(t) = t.x(t)
D. y(t) = x(t) + 2x(3 – t)

Ans. D

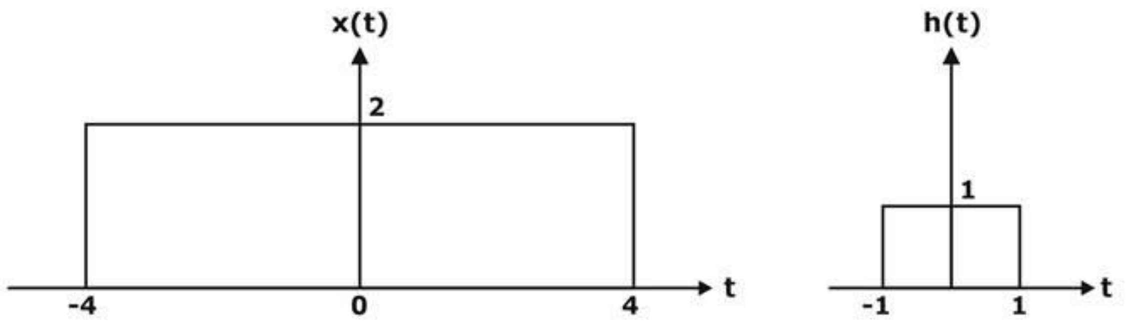
Sol.: **Non-causal system:** A system that has some dependence on input values from the future (in addition to possible dependence on past or current input values) is termed as non-causal system or acausal system.

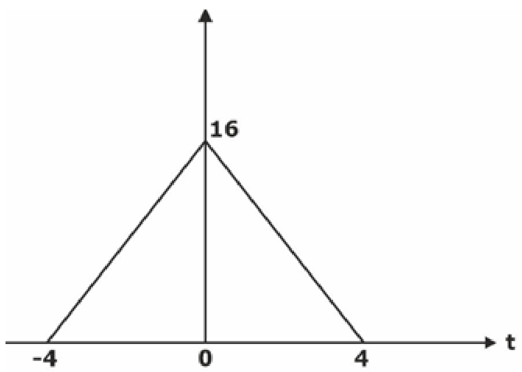
$y(t) = x(t) + 2x(3 - t)$

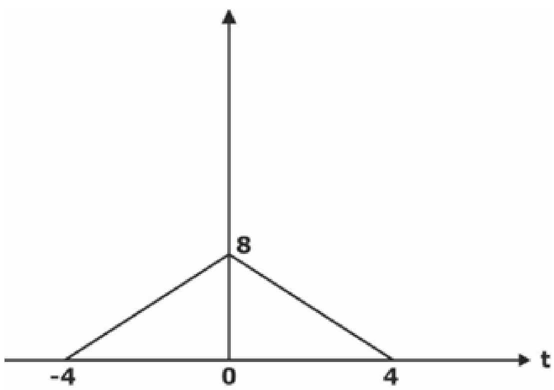
$y(1) = x(1) + 2x(3 - 1) = x(1) + 2x(2)$

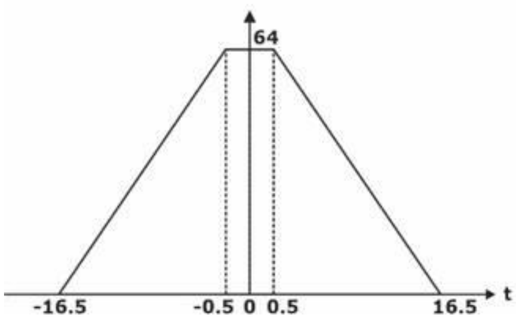
The system depends on future values, given system is non-causal.

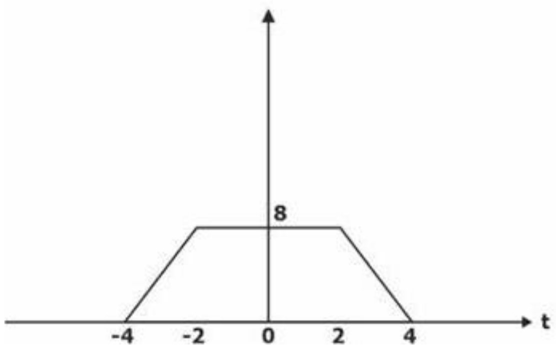
20. The two signals are shown below. If x(2t) is convolved with $2h\left(\frac{t}{2}\right)$, then the result is



A.


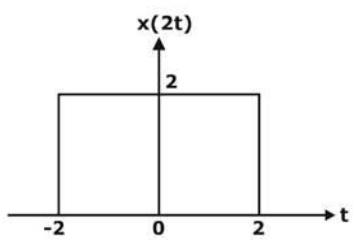
B.


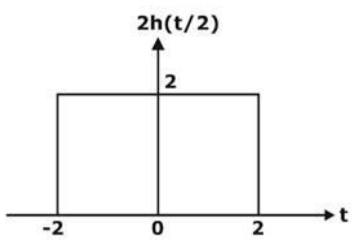
C.


D.


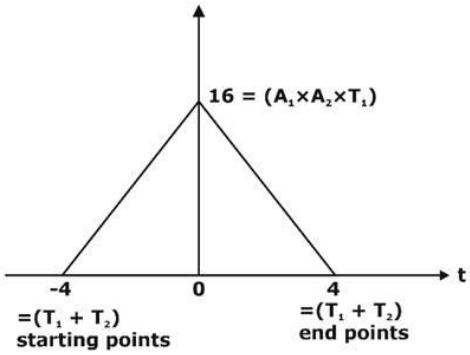
Ans. A

Sol.:





When two rectangular pulses of same length are convolved then a triangular pulse is obtained.



21. A system is said to be BIBO stable if Bounded Inputs always yield Bounded Outputs. By this criterion,

As u(t) is bounded but its derivative impulse function is unbounded and its integration is also unbounded at t equal to infinity.

A. an integrator is stable but a differentiator is not

B. a differentiator is stable but an integrator is not

C. neither the differentiator nor the integrator is stable

D. both the differentiator and integrator are stable

Ans. C

Sol.: Both integrator and differentiator yields infinite gain at low and high frequencies respectively....regardless of the fact that input is bounded so both are unstable with BIBO criteria

22. The input x(t) and output y(t) of a system are related by

$$y(t) = \begin{cases} -10, & t < -1 \\ 10x(t), & |t| \le 1 \\ 10, & t > 1 \end{cases}$$

The system is

A. Linear, time invariant and stable

B. Linear, time varying and stale

C. Non-linear, time invariant and stable

D. Non-linear, time varying and unstable

Ans. D

Sol.: $y(t) = -10 \; t < -1 = 10.x(t) \; |t| \le 1 = 10 \; t > 1$

$y(t) = -10u(-t-1) + 10x(t)\,[u(t+1) - u(t-1)] + 10u(t-1).$

$y(t) = 10x(t)\,[u(t+1) - u(t-1)] + 10\,[u(t-1) - u(-t-1)]$

$y_1(t) = 10x_1(t)\,[u(t+1) - u(t-1)] + 10\,[u(t-1) - u(-t-1)]$

$y_2(t) = 10x_2(t)\,[u(t+1) - u(t-1)] + 10\,[u(t-1) - u(-t-1)]$

$y(t) = 10[a_1x_1(t) + a_2x_2(t)]\,[u(t+1) - u(t-1)] + 10[u(t-1) - u(-t-1)]$

$\le \ne a_1y_1(t) + a_2y_2(t)$ – Non-linear

$y_1(t) = 10x(t - \tau)\,[u(t + 1) - u(t - 1)] + 10\,[u(t - 1) - u(-1 - t)]$

$y(t-\tau) = 10x(t-\tau)[u(t-\tau+1) - u(t-1-\tau)] + 10[u(t-1-\tau) - u(-1-t+\tau)] \neq y_1(t)$,

So, Time Variant.

Input is bounded and output is unbounded, so unstable.

23. If R_1 is the region of convergence of x(n) and R_2 is the region of convergence of y(n), then the region of convergence of x(n) convoluted y(n) is:

A. $R_1 + R_2$
B. $R_1 - R_2$
C. $R_1 \cap R_2$
D. $R_1 \cup R_2$

Ans. C

Sol.: $x(n) \to R_1$

$y(n) \to R_2$

$x(n) \otimes y(n) \to R_1 \cap R_2$

24. If the Nyquist rate of sampling of x(t) is 8 kHz and y(t) is 13 kHz. What would be the Nyquist rate of sampling of $[x^5(t)*y^2(t)]$ is

A. 40 kHz
B. 26 kHz
C. 66 kHz
D. 52 kHz

Ans. B

Sol.: Maximum frequency component of x(t) = 4 kHz

Maximum frequency component of y(t) = 13/2 kHz

Maximum frequency of $x^5(t) = 4 \times 5 = 20$ kHz

Maximum frequency of $y^2(t) = 13/2 \times 2 = 13$ kHz

Nyquist rate of $[x^5(t)*y^2(t)] = 2 \times [\text{min. of } x^5(t) \text{ and } y^2(t)] = 2 \times 13 = 26$ kHz

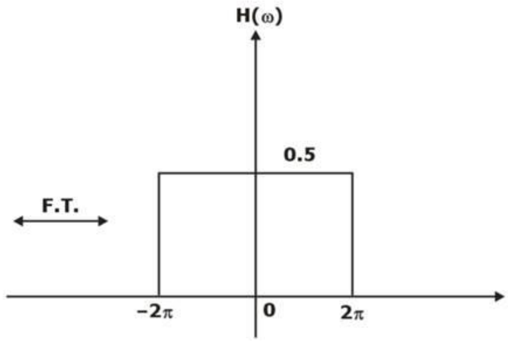
25. A signal $x(t) = 5\sin(4\pi t) - 6\cos(8\pi t)$ is applied as the input to the filter with impulse response $h(t) = \text{Sinc}(2t)$. Then the output is.

A. $0.8 + 0.2\cos(4\pi t)$
B. $0.8 + 0.8\cos(4\pi t)$
C. $4t\cos(4\pi t)$
D. 0

Ans. D

Sol.:

$$h(t) = \text{sinc}(2t) = \frac{\sin 2\pi t}{2\pi t}$$



H(w) is an ideal low pass filter which blocks the frequencies above 2π

The input signal frequencies are 4п, 8п

So output is y(t) = 0

26. Consider the following statements about Fourier series:

1) Fourier series expansion exists only for periodic signals.

2) For Fourier series expansion, signal should be absolutely integrable over its time period.

3) Fourier series expansion is done using orthogonal functions.

Which of the above statement is/are correct?

A. 1 and 2 only

B. 1 and 3 only

C. only 2

D. 1, 2 and 3

Ans. D

Sol.: 1) Fourier series expansion is done for periodic signals only.

2) One of the Drichlet's condition is signal should be absolutely integrable over its time period.

3) Sine and cosine are orthogonal to each other.

Hence, all statements are correct.

27. For the system represented by the given differential equation, if the system is neither stable nor causal, the value of h(t) is

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 3x(t)$$

A. $3[e^{-3t} - e^{-2t}]u(t)$

B. $3[e^{-2t} - e^{-3t}]u(t)$

C. $3[e^{-3t} - e^{-2t}]u(-t)$

D. $3[e^{-2t} - e^{-3t}]u(-t)$

Ans. C

Sol.: Taking Laplace transform both sides, $s^2Y(a) + 5sY(s) + 6Y(s) = 3X(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{3}{s^2 + 5s + 6} = \frac{3}{(s+2)(s+3)}$$

$$H(s) = \left[\frac{3}{s+2} - \frac{3}{s+2}\right]$$

Now, poles are s = −2, −3

If system is neither causal nor stable,

h(t) will be left sided and ROC: σ < −3

$$h(t) = -3e^{-2t}u(-t) + 3e^{-3t}u(-t)$$

$$h(t) = 3e^{-3t} - e^{-2t}u(-t)$$

28. Z-transform of x(n) = 2^n u(−n − 2) is-

A. $\frac{0.25z}{2z^{-1}-1}, |z| < 2$ (ROC)

B. $\frac{0.25z}{2z^{-1}-1}, |z| > 2$ (ROC)

C. $\frac{0.5z}{2z^{-1}-1}, |z| < 2$ (ROC)

D. $\frac{0.5z}{2z^{-1}-1}, |z| > 2$ (ROC)

Ans. C

Sol.: $a^n u[n] \xrightarrow{Z.T.} \frac{1}{1-az^{-1}}, |z| > a$

$$\left(\frac{1}{2}\right)^n u[n] \xrightarrow{Z.T.} \frac{1}{1-\frac{1}{2}z^{-1}}, |z| > \frac{1}{2}$$

$$\left(\frac{1}{2}\right)^{-n} u[-n] \xrightarrow{Z.T.} \frac{1}{1-\frac{1}{2}z}, |z| < 2$$

$$\left(\frac{1}{2}\right)^{-n} u[-n] \xrightarrow{Z.T.} \frac{1}{2-z}, |z| < 2$$

$$2^n u[-(n+2)] \xrightarrow{Z.T.} \frac{2z^2}{2-2}, |z| < 2$$

$$2^{n+2} u[-(n+2)] \xrightarrow{Z.T.} \frac{2z^2}{2-2}, |z| > 2$$

$$4 \cdot 2^n u[-(n-2)] \xrightarrow{Z.T.} \frac{2z^2}{2-z}, |z| < 2$$

$$2^n u(-n-2) \xrightarrow{Z.T.} \frac{1}{2}\left(\frac{z}{2z^{-1}-1}\right), |z| < 2$$

29. If the Laplace transform of x(t) is X(s). Then the Laplace transform of $x\left(\frac{t+2.5}{8}\right)$ is

A. $\frac{1}{8}e^{5s/2}X(8s)$

B. $16e^{-5s/2}X(8s)$

C. $8e^{-5s/2}X(8s)$

D. $8e^{5s/2}X(8s)$

Ans. D

Sol.: $x\left(\frac{t+2.5}{8}\right) = x\left(\frac{t}{8}+\frac{2.5}{8}\right) = x\left(\frac{t}{8}+\frac{5}{16}\right)$

$$x(t) \longleftrightarrow X(s)$$

$$x\left(t+\frac{5}{16}\right) \longleftrightarrow e^{\frac{5}{16}s}X(s)$$

$$x\left(\frac{t}{8}+\frac{5}{16}\right)\longleftrightarrow\frac{1}{|1/8|}e^{\frac{5}{16}\frac{s}{1/8}}X\left(\frac{s}{1/8}\right)$$

$$x\left(\frac{t+2.5}{8}\right)\longleftrightarrow 8e^{\frac{5s}{2}}X(8s)$$

30. There are two sequences x[n] and y[n]. Given $x[n] = (0.4)^n u(n)$ and $y[n] = x[n] * x[n]$.

Then the value of $\sum_{-\infty}^{\infty} y[n]3^{-n}$ is

A. $\frac{50}{18}$

B. $\frac{225}{169}$

C. $\frac{169}{225}$

D. None of the above

Ans. B

Sol.: $X(z)=\frac{1}{1-0.4z^{-1}}$

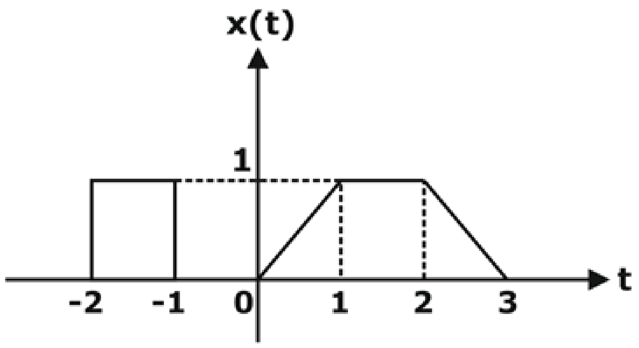
$$Y(z)=X(z)\cdot X(z)=\frac{1}{\left(1-0.4z^{-1}\right)^2}$$

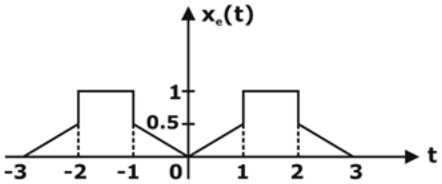
$$Y(z)=\sum y[n]\,z^{-n}$$

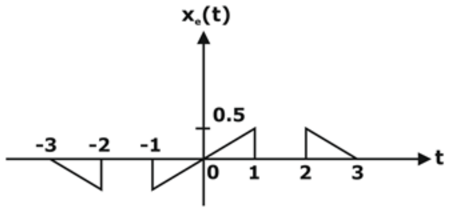
$$Y(3)=\sum_{-\infty}^{\infty}y[n]\,3^{-n}=Y(z)\Big|_{z=3}=\frac{1}{\left(1-\frac{0.4}{3}\right)^2}$$

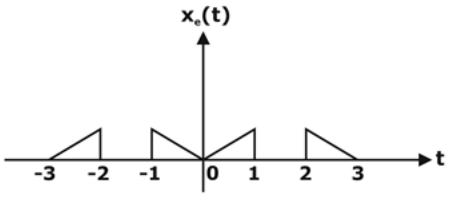
$$Y(3)=\frac{1}{\left(\frac{26}{30}\right)^2}=\left(\frac{30}{26}\right)^2=\frac{225}{169}$$

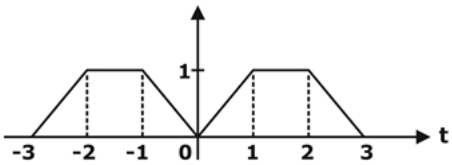
31. The even part of signal shown in the figure is



A. 

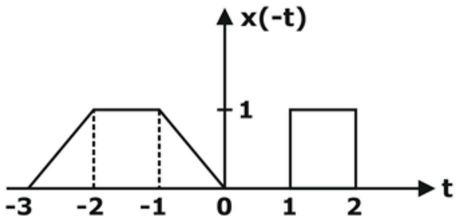
B. 

C. 

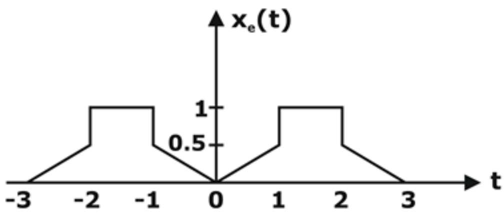
D. 

Ans. A

Sol.: $x_e(t) = \dfrac{x(t) + x(-t)}{2}$



Hence, $x_e(t)$ →



32. A signal is defined as $x(t) = \begin{cases} 5e^{j2\omega_0 t} & ; -3 \le t \le 4 \\ 0 & ; \text{otherwise} \end{cases}$. Then the energy of the signal, x(t) is

A. 35 joules

B. 50 joules

C. 100 joules

D. 175 joules

Ans. D

Sol.: Energy, $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$

$$|x(t)| = \begin{cases} 5; & -3 \le t \le 4 \\ 0; & \text{otherwise} \end{cases}$$

$$E = \int_{-\infty}^{\infty} (5^2) dt = \int_{-3}^{4} 25 \, dt = 25[t]_{-3}^{4}$$

= 175 joules

33. Which of the following signal is/are non-periodic?

1) $\cos\left(2\pi t+\frac{\pi}{4}\right)+\sin\left(2t-\frac{\pi}{3}\right)$

2) $e^{j18\pi t}-e^{j9\pi t}$

3) $\sin\left(2\sqrt{2}t\right)-\sin\left(3\sqrt{2}t\right)$

4) $\cos(2t)+e^{4t}$

A. 1 and 3

B. 2, 3 and 4

C. 1 and 4

D. 3 and 4

Ans. C

Sol.: For sum of two periodic signals to be periodic ratio of their time periods should be a rational number.

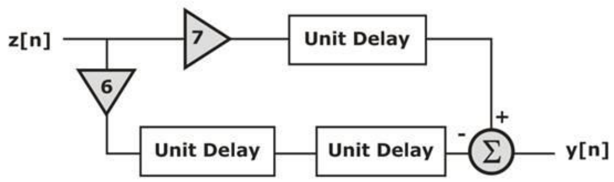
(1) $\frac{T_1}{T_2}=\frac{2\pi}{2}=\pi\rightarrow$ irrational

(2) $\frac{T_1}{T_2}=\frac{18\pi}{9\pi}=2\rightarrow$ Rational

(3) $\frac{T_1}{T_2}=\frac{2\sqrt{2}}{3\sqrt{2}}=\frac{2}{3}\rightarrow$ Rational

(4) e^{4t} is non-periodic, as it is an increasing function hence (4) is non-periodic.

34. The direct form structure of a FIR filter is shown below. The filter approximately represents a



A. Low pass filter

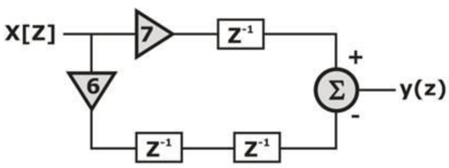
B. High pass filter

C. Band pass filter

D. Band stop filter

Ans. B

Sol.:



$Y(z) = [7z^{-1} - 6z^{-2}]\, X(z)$

$$\frac{Y(z)}{X(z)}=7z^{-1}-6z^{-2}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = 7e^{-j\omega} - 6e^{-2j\omega}$$

$$\left|H(e^{j\omega})\right|_{\omega=0} = 1$$

$$\left|H(e^{j\omega})\right|_{\omega=\pi} = 13$$

Hence, it is approximated as high pass filter.

35. If $y(t) + \int_{-\infty}^{\infty} y(\tau)\, x(t-\tau)\, d\tau = \delta(t) + x(t)$, then y(t) is

A. δ(t)
B. u(t)
C. r(t)
D. can't be determined

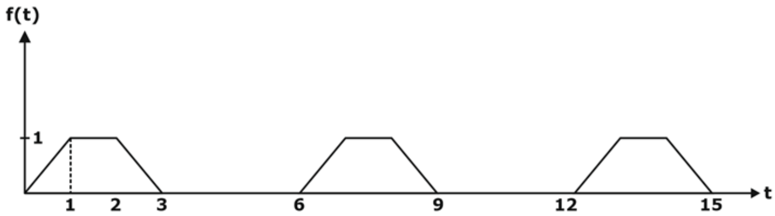
Ans. A

Sol.: Let y(t) = δ(t)

$$\int_{-\infty}^{\infty} y(\tau)\, x(t-\tau)\, d\tau = \int_{-\infty}^{\infty} \delta(\tau)\, x(t-\tau)\, d\tau = x(t)$$

Hence, y(t) = δ(t)

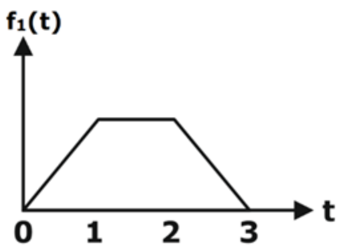
36. What is the Laplace transform of the periodic waveform shown for t > 0:



A. $\frac{1}{s^2(1-e^{-3s})}\left[1-e^{-s}-e^{-2s}+e^{-3s}\right]$
B. $\frac{1}{s^2(1-e^{-3s})}\left[1-e^{-s}-e^{-2s}\right]$
C. $\frac{1}{s^2(1-e^{-6s})}\left[1-e^{-s}-e^{-2s}+e^{-3s}\right]$
D. $\frac{1}{s^2(1-e^{-6s})}\left[1-e^{-s}-e^{-2s}\right]$

Ans. C

Sol.:



$$f_1(t) \longleftrightarrow F_1(s)$$

$$f(t) \longleftrightarrow F(s)$$

Then,

$$F(s)=\frac{1}{1-e^{-Ts}}F_1(s),\quad T=6$$

$$f_1(t)=r(t)-r(t-1)-r(t-2)+r(t-3)$$

$$F_1(s)=\frac{1}{s^2}\left[1-e^{-s}-e^{-2s}+e^{-3s}\right]$$

$$F(s)=\frac{1}{s^2\left(1-e^{-6s}\right)}\left[1-e^{-s}-e^{-2s}+e^{-3s}\right]$$

37. A signal f(t) is represented as $f(t)=\sum_{n=-\infty}^{\infty}\frac{(2n)^2}{30+\left(\frac{4n}{\pi}\right)^2}e^{jn\pi t}$. The value of a_{10}?

A. $\frac{800}{3+160/\pi^2}$

B. 0

C. $\frac{40}{3+160/\pi^2}$

D. $\frac{80}{3+160/\pi^2}$

Ans. D

Sol.:

$$C_n=\frac{(2n)^2}{30+\left(\frac{4n}{\pi}\right)^2}=C_{-n}$$

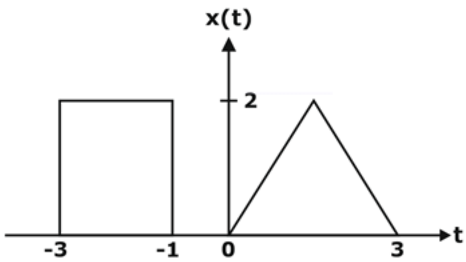
It is real and even.

Hence, f(t) is also real and even.

For real signals, $a_n = 2\,\mathrm{Re}[C_n] = 2\,C_n$

$$a_{10} = 2C_{10} = 2\times\frac{(2\times10)^2}{3+\left(\frac{4\times10}{\pi}\right)^2}=\frac{800}{30+\frac{1600}{\pi^2}}=\frac{80}{30+\frac{160}{\pi^2}}$$

38. A signal x(t) is shown in the figure below. It's Fourier transform is X(jω). The value of 3 X(jω) at ω = 0 is



A. 7

B. 0

C. 14

D. 21

Ans. D

Sol.: $x(j\omega)\big|_{\omega=0} = \int_{-\infty}^{\infty} x(t)\,dt$ = Area under the curve $= 2\times 2 + \frac{1}{2}\times 3\times 2 = 7$

$3X(j\omega)\big|_{\omega=0} = 3\times 7 = 21$

39. A discrete time signal is given as $x[n] = \left[\cos^2\left(\frac{n\pi}{4}\right) + \sec^2\left(\frac{n\pi}{4}\right)\right]\left(u[n] - u[n-4]\right)$. The energy of x[n] is

A. 13

B. 4

C. 9

D. x[n] is not an energy signal

Ans. D

Sol.: Energy of $x[n] = \sum_{-\infty}^{\infty} |x(n)|^2 = \sum_{0}^{3}\left[\cos^2\left(\frac{n\pi}{4}\right) + \sec^2\left(\frac{n\pi}{4}\right)\right]^2$

$$= \left[\cos(0) + \sec(0)\right]^2 + \left[\cos\left(\frac{\pi}{4}\right) + \sec\left(\frac{\pi}{4}\right)\right]^2 + \left[\cos\left(\frac{\pi}{2}\right) + \sec\left(\frac{\pi}{2}\right)\right]^2 + \left[\cos\left(\frac{3\pi}{4}\right) + \sec\left(\frac{3\pi}{4}\right)\right]^2$$

$$= 4 + \frac{9}{2} + (0+\infty)^2 + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$= \infty$$

Hence, it is not an energy signal.

40. What is the fundamental period of the signal 'x(t)',

$$x(t) = 3 + e^{j3\pi n/18} - e^{jn\pi/5} + \sin\left(2\pi n + \frac{\pi}{6}\right)$$

A. 30

B. 60

C. 90

D. signal is not periodic

Ans. B

Sol.: 3 → constant

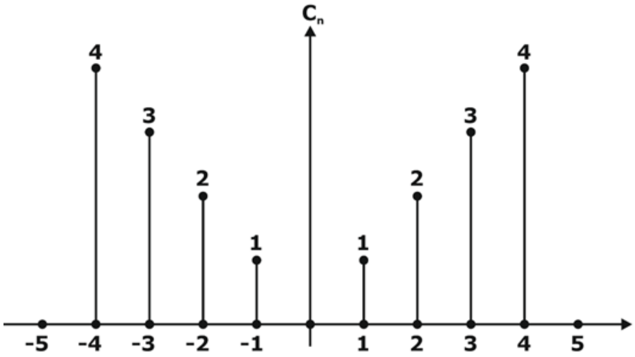
$$e^{j3\pi n/18} \rightarrow \omega_1 = \frac{3\pi}{18} = \frac{\pi}{6}, N_1 = \frac{2\pi}{\pi/6} = 12$$

$$e^{jn\pi/5} \rightarrow \omega_2 = \frac{\pi}{5}, N_2 = \frac{2\pi}{\pi/5} = 10$$

$$\sin\left(2\pi n + \frac{\pi}{6}\right) \rightarrow \omega_3 = 2\pi, N_3 = \frac{2\pi}{2\pi} = 1$$

Fundamental time period = LCM (1, 10, 12) = 60

41. The power of signal x(t) is whose Fourier series coefficients C_n is represented below is



A. 20
B. 15
C. 30
D. 60

Ans. D

Sol.: Power of signal x(t) by Parseval's theorem,

$$P = \sum_{n=-\infty}^{\infty} |C_n|^2 = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt$$

$P = 4^2 + 3^2 + 2^2 + 1^2 + 1^2 + 2^2 + 3^2 + 4^2 = 19 + 9 + 4 + 1 + 1 + 4 + 9 + 16 = 60$

42. Which of the following statement is/are correct?

1) The convolution is used to find the impulse response of a system.

2) The convolution operator is associative, commutative and distributive.

3) The step response of a continuous LTI system with impulse response h(t) is $\int_{-\infty}^{t} h(\tau)d\tau$.

A. 1 and 2
B. 2 and 3
C. 1 and 3
D. 1, 2 and 3

Ans. B

Sol.: Convolution is used to find the output of a system using impulse response.

Convolution properties:

(1) Commutative: $x_1(t) * x_2(t) = x_2(t) * x_1(t)$

(2) Associative: $x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$

(3) Distributive: $x_1(t) * [x_2(t) + x_3(t)] = [x_1(t) * x_2(t)] + [x_1(t) * x_3(t)]$

Step response of system,

$$y(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau = \int_{-\infty}^{t} h(\tau)d\tau$$

43. If a signal x(n)=x(−n), then it is called ____ signal:

A. Odd
B. Energy
C. Power
D. Even

Ans. D

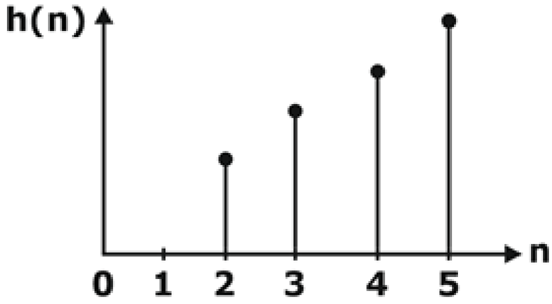
Sol.: Even signals are generally consistent around vertical axis, and a particular signal can be considered as an even signal if the time reversed version of it is similar to its original version. So, answer is option (d)

44. A system having impulse response $h(n) = e^{2n}u(n-2)$, the system is

A. Causal and stable
B. Non-causal and stable
C. Causal and unstable
D. Non-causal and unstable

Ans. C

Sol.: $h(n) = e^{2n}u(n-2)$



$h(n) = 0$ for $n < 0$

∴ System is causal.

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} e^{2n}u(n-2) = \sum_{n=2}^{\infty} e^{2n} = \infty \quad \text{(Infinite)}$$

∴ System is unstable.

45. The exponential Fourier series coefficient C_0 of $x(t) = \sin^2 t$ is:

A. 0
B. $\frac{1}{2}$
C. –1
D. 1

Ans. B

Sol.: The value of C_0 is given as:

$$C_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

$$x(t) = \sin^2 t$$

$$x(t) = \frac{1}{2}\left(2\sin^2 t\right) = \frac{1}{2}(1-\cos 2t) = \frac{1}{2} - \frac{1}{2}\cos 2t$$

$$C_0 = \frac{1}{2}$$

46. If Laplace transform of x(t) is X(s) and $X(s) = \int_{-\infty}^{s} \frac{2}{s^2+1}\,ds$, then x(0) is

A. -1
B. 0
C. 1
D. 2

Ans. A

Sol.: $X(s) = \int_{-\infty}^{s} \frac{2}{s^2+1}\,ds$

$$\frac{dX(s)}{ds} = \frac{2}{s^2+1}$$

$x(t) \Leftrightarrow X(s)$

$$tx(t) \longleftrightarrow \frac{dX(s)}{ds}$$

$$L^{-1}\left(\frac{2}{s^2+1}\right) = -t\,x(t)$$

2sin(t) u(t) = –tx(t)

$$\Rightarrow x(t) = \frac{-2\sin(t)u(t)}{t}$$

$$\Rightarrow x(t)u(t) = -tx(t)$$

At t = 0,

$$x(0) = \lim_{t\to 0}\frac{-2\sin(t)}{t}u(t) \quad \left[\because \lim_{t\to 0}\frac{\sin(t)}{t}\right]$$

$$= 2\lim_{t\to 0}u(t) = -2\times\frac{1}{2} = -1$$

47. Let $x(n) = 16^n u(n) - b^{2n}u(-n-1)$. If the z-transform of x(n) exists. Then condition on 'b' is

A. b > 4

B. – 4 < b < 4

C. b < 4

D. No values of 'b' is possible

Ans. A

Sol.: $x(n) = 16^n u(n) - b^{2n} u(-n-1)$

$x_1(n) = 16^n u(n) - b^{2n} u(-n-1)$

$$X_1(z) = \frac{1}{1-16z^{-1}};\ |z| > 16 \quad \text{(causal signal)}$$

$x_2(n) = -b^{2n} u(-n-1)$

$$X_2(z) = \frac{1}{1-b^2z^{-1}};\ |z| < b^2 \quad \text{(non-causal signal)}$$

For z-transform to exist, ROC should be in ring form i.e. non-causal signal.

$16 < |z| < b^2$

Hence, $b^2 > 16$

$\Rightarrow b > 4$

48. The nature of the spectrum of continuous time Fourier series is

A. Continuous and periodic

B. Discrete and continuous

C. Discrete and periodic

D. Discrete and Aperiodic

Ans. D

Sol.: Continuous time Fourier series is applied if the signal is continuous and periodic.

∴ The spectrum if Aperiodic and discrete.

49. The impulse response of a system is $h(n) = a^n u(n)$. The condition for the system to be BIBO stable is:

A. a is real and positive

B. a is real and negative

C. [a] > 1

D. [a] < 1

Ans. D

Sol.: $h(n) = a^n u(n)$

$$H(\tau) = \frac{Z}{Z-a}$$

When $|a| < 1$, then pole of the system lies in the unity circle which makes system stable.

50. Find i(t) if $I(s) = \frac{s+1}{(s+4)(s+3)}$:

A. $3e^{-4t} - e^{-5t}$

B. $3e^{-4t} - 2e^{-3t}$

C. $3e^{-4t} - e^{-2t}$

D. None of these

Ans. B

Sol.: Given,

$$I(s) = \frac{s+1}{(s+4)(s+3)}$$

Using partial fractions, we can write,

$$I(s) = \frac{3}{s+4} - \frac{2}{s+3}$$

As we know,

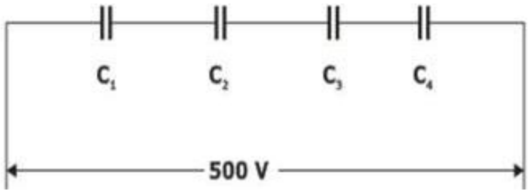
$$L[e^{-at}] = \frac{1}{s+a}$$

Hence, taking inverse Laplace transform, we get,

$f(t) = 3e^{-4t} - 2e^{-3t}$

Electromagnetic Fields

51. Four capacitor C_1, C_2, C_3 And C_4 connected in series as shown. Total charge on these capacitors is 20MC and connected to 500V supply. The ratio of capacitances is 1 : 2 : 3 : 4 respectively. The voltage across C_4 is



A. 60 V B. 200 V

C. 50 V D. 150 V

Ans. A

Sol. In series charge on each capacitor is same

$Q_1 = Q_2 = Q_3 = Q_4$

$C_1V_1 = C_2V_2 = C_3V_3 = C_4V_4$

$C_1 : C_2 : C_3 : C_4 = 1 : 2 : 3 : 4$

$\Rightarrow V_1 = 2V_2 = 3V_3 = 4V_4$

Also, $V_1 + V_2 + V_3 + V_4 = 500$ V

$4V_4 + 2V_4 + \frac{4}{3}V_4 + V_4 = 500$

$\frac{25}{3}V_4 = 500 \Rightarrow V_4 = 60$ V

52. A potential distribution is given as $V = 9x^2 + 8y + 5z^3$. If density in c/m^3 at (1, 1, 1) is $-96\ \varepsilon_0$ then the permittivity of medium is

A. 1 ε_0 B. 5 ε_0

C. 3 ε_0 D. 2 ε_0

Ans. D

Sol. By Laplace equation, $\nabla^2 V = -\frac{\rho_V}{\varepsilon}$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{-\rho_V}{\varepsilon}$$

$$18 + 0 + 30z = \frac{-\rho_V}{\varepsilon}$$

At (1, 1, 1)

$$48 = \frac{-\delta v}{\varepsilon}$$

$48\varepsilon = +96\varepsilon_0$

$\varepsilon = +2\varepsilon_0$

53. Consider the following statements:

1. Gradient of a scalar is a vector.
2. Divergence of a vector is a vector
3. Laplacian of a scalar is a vector
4. Curl of a vector is a vector

Which of the above statements are correct?

A. 1, 2, and 3 B. 1,3 and 4

C. 2 and 4 only D. 1, 2, 3and 4

Ans. B

Sol. 1. $\nabla A = \frac{\partial A}{\partial x}\hat{a}_z + \frac{1}{\partial y}\hat{a}_2 + \frac{\partial A}{\partial z}\hat{a}_2$

2. $\nabla \vec{A} = \frac{\partial A_X}{\partial x} + \frac{\partial A_S}{\partial y} + \frac{\partial A_z}{\partial z}$

3. $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

4. $\nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

54. Consider z = 0 is boundary between two media with $\mu_1 = 2\mu_o$ and $\mu_2 = 4\mu_o$. If magnetic field intensity $\vec{H}_1 = 2\hat{a}_x + 4\hat{a}_y + 2\hat{a}_z$ and the current at boundary is zero. The value of $\vec{H}_2$ is

A. $2\hat{a}_x + 4\hat{a}_y + 2\hat{a}_z$ B. $2\hat{a}_x + 4\hat{a}_y + 4\hat{a}_z$

C. $4\hat{a}_x + 8\hat{a}_y + 2\hat{a}_z$ D. $\hat{a}_x + 2\hat{a}_y + 2\hat{a}_z$

Ans. A

Sol. $\vec{H}_{t_1} = 2\hat{a}_x + 4\hat{a}_y \quad \vec{H}_{N_1} = 2\hat{a}_z$

We know that, $\vec{H}_{t_1} - \vec{H}_{t_2} = \vec{a}_{N_{12}} \times \vec{K}$

But K = 0

So, $\vec{H}_{t_1} = \vec{H}_{t_2}$

Also $\mu_1 \vec{H}_{N_1} = \mu_2 \vec{H}_{N_2}$

$2\mu_0 \times 2\hat{a}_z = 4\mu_0 \times \vec{H}_{N_2}$

$\Rightarrow \vec{H}_{N_2} = \vec{a}_z$

$= 2\hat{a}_z + 4\hat{a}_z + \hat{a}_z$

55. Which of the following statements doesn't represent any Maxwell equation?

A. Total magnetic flux crossing the closed surface is zero.

B. MMF around a closed path is equal to the sum of flux conduction current density and rate of displacement flux density over the open surface.

C. Charges can neither be created nor be destroyed, although equal amount of positive and negative charges can be simultaneously created.

D. Emf around a closed path is equal to the negative of time rate of magnetic flux density over the open surface.

Ans. C

Sol. A. Represents $\nabla^2\vec{B} = 0$

B. Represents $\nabla^2\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$

C. Represents $\nabla \cdot \vec{J} = -\frac{\partial PV}{\partial t}$ [equation of continuity]

D. Represents $\nabla \times \vec{E} = \frac{-\partial\vec{B}}{dt}$

56. A conducting coil is moving with uniform velocity along x-axis in a time varying magnetic field $\vec{B} = B_m \sin\omega_0 t\hat{a}_z Wb / m^2$, the emf induced in the coil is

A. Zero

B. Motional emf

C. Transformer emf

D. Combination of motional and transformer emf.

Ans. D

Sol. Transformer emf: stationary loop/bar in time varying magnetic field.

Motional emf: By having line varying loop area or non-stationary bar in a static magnetic field.

57. A time varying magnetic flux linked to a 100-turn coil is given by $\Phi = 5t^3 - 2t$. The emf induced at t = 5 second is

A. −373 V

B. 37.3 KV

C. -37.3 KV

D. None of the above

Ans. C

Sol. $e = -\frac{d\phi}{dt} = -\left[15t^2 - 2\right]$

At t = 5 sec, e = −373 V

emf induced in 100-turns = 373 × 100

= −37.3 kV

58. The two perfect conductors spaced apart 0.5m with 10 V maintained between them, such that $\vec{E}$ is in $\hat{a}_z$ direction. If conductivity of material between them is

σ = 25 s/m, the value of J is

A. $500\hat{a}_s A / m^2$

B. $250\hat{a}_s A / m^2$

C. $250\hat{a}_s A / m^2$

D. $500\hat{a}_s A / m^2$

Ans. D

Sol. Electric field intensity, $\vec{E} = \frac{\vec{V}}{d}\hat{a}_z = \frac{10}{0.5} = 20V / m$

$\vec{J} = \sigma\vec{E} = 25 \times 20\hat{a}_z A / m^2$

$= 500\hat{a}_z A / m^2$

59. A test charge q = 5 nC has a velocity of 3 × 10^4 m/s in $\hat{a}_x - \hat{a}_y + \hat{a}_z$ direction. The force exerted on the charge by an electric field, E = $4\hat{a}_x - 2\hat{a}_x + 9\hat{a}_z$ is

A. 0 N

B. $\left(20\hat{a}_x - 10\hat{a}_y + 9\hat{a}_z\right)$

C. $45\hat{a}_z$ nN

D. $\left(20\hat{a}_x - 10\hat{a}_y + 45\hat{a}_z\right)$nN

Ans. D

Sol. Force exerted by electric field is independent of velocity

$\vec{F} = Q\vec{E}$

$= 5 \times 10^{-9} \times \left(4\hat{a}_x - 2\hat{a}_x + 9\hat{a}_z\right)$

$= \left(20\hat{a}_x - 10\hat{a}_y + 45\hat{a}_z\right)$nN

60. Consider the following statements for a dielectric statement for a dielectric-dielectric boundary surface with different permittivity:

1. The tangential compound of electric field is always a continuous across the boundary.

2. Normal component of electric field density vector ($\bar{D}$) is continuous across charge free interface only.

Which of the following statements is/are not correct?

A. Only 1

B. Only 2

C. Both 1 and 2

D. Neither 1 nor 2

Ans. D

Sol. Both statements are correct.

At dielectric - dielectric boundary

$E_{t1} = E_{t2}$

and, D_{n1} - D_{n2} = δ_s, [δ_s = surface charge density]

If δ_s =0 then,

$D_{n1} = D_{n2}$

61. The total charge on a cylinder whose volume charge density is $\rho_v = \frac{50}{\sqrt{x^2 + y^2}} \times 10^{-6} C / m^3$

Whose radius is 5 m and height is 25 m $\hat{a}_z$ is

A. 62.5 C

B. 12.5π × 10^{-3} C

C. 12.5 × 10^{-3} C

D. 62.5π × 10^{-3} C

Ans. B

Sol. For a cylinder

$dV = r dr d\varphi dz =$

$r^2 = x^2 + y^2$

$$\rho_v = \frac{50}{\sqrt{x^2+y^2}} \times 10^{-6}\ C/m^3$$

$dq = r_v dV$

$$Q = \int_z \int_\phi \int_\rho \rho_v\ \rho d\rho\ d\phi\ dz$$

$$= \int_0^{25} \int_0^{2\pi} \int_0^5 \frac{50}{\rho} \times \rho\ d\rho\ d\phi\ dz \times 10^{-6}$$

$$Q = \int_0^{25} \int_0^{2\pi} \int_0^5 50 \times d\rho\ d\phi\ dz \times 10^{-6}\ C$$

$= 50 \times 5 X 2\pi \times 25 \times 10^{-6}$ C

$= 12.5\pi \times 10^{-3}$ C

62. A parallel plate capacitor is charged to 100 μC at 200 V. If it is connected to another capacitor resulting in net charge of first capacitor is 25×10^{-6}C then the capacitor of the new capacitor is

A. 0.167 μF

B. 3 μF

C. 1.5 μF

D. 0.333 μF

Ans. C

Sol. $Q = CV$

$$C_1 = Q/V = \frac{100 \times 10^{-6}}{200} = 0.5\mu F$$

After redistribution

$Q_1 + Q_2 = Q$

$Q_1 = 25 \times 10^{-6}$ C.

$Q_2 = 75 \times 10^{-6}$ C

Since voltage across capacitors is same

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$C_2 = \frac{Q_2 C_1}{Q_1} = \frac{75 \times 10^{-6}}{25 \times 10^{-6}} \times 0.5 \times 10^{-6}$$

$C_2 = 1.5$ μF

63. A point charge of 5μC is located at origin and another point charge of −5μC is a located at (1, 1, 1). The potential at (0, 1, 0) is

A. 76.185×10^3 V

B. 13.18×10^3 V

C. 13.18 V

D. 76.185 V

Ans. B

Sol. Potential at a point $= \dfrac{1}{4\pi\varepsilon_0} \dfrac{q}{r}$

Distance between (0, 0, 0) and (0, 1, 0) = 1

Distance between (1, 1, 1) and (0, 1, 0) = $\sqrt{2}$

Potential at (0, 1, 0) = $V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$

$= 9\times10^9\left[\frac{5\times10^{-6}}{1} - \frac{5\times10^{-6}}{\sqrt{2}}\right]$

$= 45\times10^3\left[1-\frac{1}{\sqrt{2}}\right]$

$= 45 \times 10^3\,[0.293] = 13.18 \times 10^3$ V

64. Image theory is applicable to

A. Electrostatic field only

B. Magneto static field only

C. Both electrostatic and magnetic static fields

D. None of the above

Ans. A

Sol. The electrostatic field has a tangential and normal component. The tangential component displaces electrons on the conductor surface hence charges are periodically accumulated on the entire conductor surface. This can be calculated by image charge.

65. Find the divergence of $\vec{P} = \frac{\sin\phi}{\rho}\hat{a}_p + \rho z^2\hat{a}_\phi + \frac{z^2}{\rho}\hat{a}_z$ at point

$(10, \frac{\pi}{3}, 4)$

A. $\frac{-\sqrt{3}}{200} + \frac{4}{5}$

B. 0

C. $\frac{-\sqrt{3}}{200}$

D. $\frac{4}{5}$

Ans. D

Sol. $\nabla\cdot\vec{p} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho P_p\right) + \frac{1}{\rho}\frac{\partial P_\phi}{\partial\phi} + \frac{\partial P_z}{\partial Z}$

$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\frac{(\rho\cdot\sin\phi)}{\rho} + \frac{1}{\rho}\frac{\partial}{\partial\phi}\left(\rho Z^2\right) + \frac{\partial}{\partial Z}\left(\frac{z^2}{\rho}\right)$

$= 0 + 0 + \frac{2z}{\rho} = \frac{8}{10} = \frac{4}{5}$

66. A vector $\vec{A} = y\hat{a}_x + (x+z)\hat{a}_y$, the value of A_ρ at point P (−2, 6, 3) is

A. $\frac{-38}{\sqrt{40}}$

B. $\frac{-18}{\sqrt{40}}$

C. $\frac{1}{6}$

D. $\frac{-6}{\sqrt{40}}$

Ans. D

Sol. At P,

$$\rho = \sqrt{x^2 + y^2} = \sqrt{4+36} = \sqrt{40}$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{6}{-2} = \tan^{-1}(-3)$$

Z = 3

$$\begin{vmatrix} A_\zeta \\ A_\phi \\ A_z \end{vmatrix} = \begin{vmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix}\begin{vmatrix} y \\ x+z \\ 0 \end{vmatrix}$$

As = y cos φ + (x + z) sin φ

= 6 cos φ + sin φ

$$= 6 \times \frac{-2}{\sqrt{40}} + \frac{6}{\sqrt{40}} = \frac{-6}{\sqrt{40}}$$

67. Loss-tangent in plane waves in lossy dielectrics will be

A. proportional to the Y component of the magnetic field intensity(H_Y)

B. inversely proportional to the Y component of the magnetic field intensity (H_y)

C. inversely proportional to the X component of the magnetic field intensity(H_x)

D. proportional to X component of the magnetic field intensity (H_X)

Ans. B

Sol. inversely proportional to the Y component of the magnetic field intensity H.

68. A transmission line of characteristic impedance 50Ω is terminated at one end by j50 Ω . The VSWR produced by the line is

A. +1

B. 0

C. ∞

D. +j

Ans. C

Sol. Z_c = 50

Z_L = j50

$$|\rho| = \left|\frac{50 - j50}{50 + j50}\right| = 1$$

$$\text{VSWR} = \frac{1+|\rho|}{1-|\rho|} = \infty$$

69. If the current density inside a straight conductor is uniform over its cross-section, the flux density variation inside the conductor at different distances from its centre is

A. linear

B. square of the distance

C. inverse of the distance

D. exponential

Ans. A

Sol. For 0 < r < R:

$\oint H \cdot dl = \int J \cdot ds$

$H \cdot 2\pi r = J \cdot \pi r^2$

$H = \frac{J \cdot r}{2}$

$B = \frac{\mu J \cdot r}{2}$

70. The electric field intensity $\vec{E}$ at a point p is given by $10\hat{i}9 + 10\hat{j} + 10\hat{k}$ where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vector in x, y and z directions respectively if α, β, γ are respectively the angle the $\vec{E}$ vector makes with x, y, and z axis respectively. They are given by which of the following?

A. $\alpha = \beta = \gamma = 30^\circ$

B. $\alpha = \beta = \gamma = 60^\circ$

C. $\alpha = \beta = \gamma = \cos^{-1}\frac{1}{\sqrt{3}}$

D. $\alpha = \beta = \gamma = \cos^{-1}\frac{1}{3}$

Ans. C

Sol. $\alpha = \beta = \gamma$

$= \cos^{-1}\frac{10}{\sqrt{10^2 + 10^2 + 10^2}}$

$= \cos^{-1}\frac{10}{10\sqrt{3}} = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

71. If the potential function V_1 and V_2 satisfy Laplace's equation within a closed region and assume the same values on its surface, then which of the following is correct?

A. V_1 and V_2 are identical

B. V_1 is inversely proportional toV_2

C. V_1 has the same direction as V_2

D. V_1 has the same magnitude as V_2 but has different direction

Ans. A

Sol. Laplace equation gives a unique solution, Therefore if potential function V_1 and V_2 assume same values on its surface satisfying Laplace's equation, then V_1 and V_2 are identical.

72. An electric charge of q coulombs is located at the origin. Consider electric potential V and electric field intensity E at any point (x, y, z). Then

A. E and V are both scalar

B. E and V are both vectors

C. E is a scalar V is vector

D. E is a vector and is scalar

Ans. D

Sol. Charge comes in multiples of an indivisible unit of charge, represented by the letter e. In other words, charge comes in multiples of the charge on the electron or the proton. These things have the same size charge, but the sign is different. A proton has a charge of +e, while an electron has a charge of -e. Electrons and protons are not the only things that carry charge. Other particles (positrons, for example) also carry charge in multiples of the electronic charge. Those are not going to be discussed, for the most part, in this course, however.

Putting "charge is quantized" in terms of an equation, we say:

q = n e

q is the symbol used to represent charge, while n is a positive or negative integer, and e is the electronic charge, 1.60 x 10-19 Coulombs.

The flux is the sum or total over that whole area. The flux density is the magnitude (and direction) of the flux at a particular point in the flow. For some fluxes there is a different flow in different directions. For others such as a hosepipe, there is only one direction. If we imagine holding up a hoop into the flow the flux is the total passing through the hoop. If we surround our source of flow with a sphere the total amount passing through the sphere must be the same as the total amount passing through any other surrounding sphere, even if the density is distributed differently over the sphere or decreases simply because the second sphere has a larger radius. This important fact is the basis of the inverse square law, Gauss' theorem and other theorems.

73. A varying magnetic flux linking a coil is given by $\phi = \frac{1}{3}\lambda t^3$. If at time t=3 s, the emf induced is 9 V then the value of λ is

A. Zero

B. 1 Wb/s^2

C. -1 Wb/s^2

D. 9Wb/s^2

Ans. C

Sol. Given $\phi = \frac{1}{3}\lambda t^3$

Emf at t = 3s s 9V

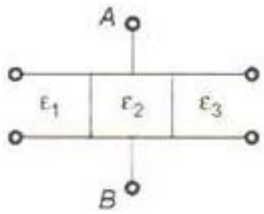
Induced emf = $-\frac{d\phi}{dt} = -\frac{d}{dt}\left[\frac{1}{3}\lambda t^3\right]$

$\Rightarrow \text{emf} = -\lambda t^2$

$9 = -\lambda(3)^2$

$\Rightarrow \lambda = -1\,Wb/s^2$

74. The space between the plates of a parallel plate capacitor of capacitance C is filled with three dielectric slabs of identiacal size as shown in the below figure. If dielectric constants are ε_1, $\mathcal{E}_2$ and $\mathcal{E}_3$ the new capacitance is



A. $\frac{C}{3}$

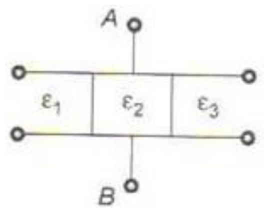
B. $\frac{(\varepsilon_1+\varepsilon_2+\varepsilon_3)C}{3}$

C. $(\varepsilon_1+\varepsilon_2+\varepsilon_3)C$

D. $\frac{9(\varepsilon_1+\varepsilon_2+\varepsilon_3)}{\varepsilon_1\mathcal{E}_2\mathcal{E}_3}$

Ans. B

Sol.



The circuit shoen is connected in parallel with different permittivities of dielectric slabs.

$C_1 = \frac{(A/3)\varepsilon_1\varepsilon_0}{d}$ Also, $C = \frac{A\varepsilon_o}{d}$

$C_2 = \frac{(A/3)\varepsilon_2\varepsilon_0}{d}$

$C_3 = \frac{(A/3)\varepsilon_3\varepsilon_0}{d}$

So, Since all the three slabs are connected in parallel so net capacitance is

$= C_1 + C_2 + C_3$

$= \left(\frac{A\varepsilon_1}{3d} + \frac{A\varepsilon_2}{3d}\frac{A\varepsilon_3}{3d}\right)\varepsilon_0$

$= \frac{A\varepsilon_1}{3d}(\varepsilon_1+\varepsilon_2+\varepsilon_3)$

$= \frac{C}{3}(\varepsilon_1+\varepsilon_2+\varepsilon_3)$

75. Electric displacement current density D at any point on a spherical surface of radius r centred at the isolated charge q is

A. $\frac{q^2}{r^2}$

B. $\frac{q}{r^2}$

C. $\frac{q}{4\pi r^2}$

D. $\frac{q}{4\pi^2 r^2}$

Ans. C

Sol. Electric field intensity

$E = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}$

$\therefore D = \varepsilon E = \frac{q}{4\pi r^2}$

76. What is the force developed pet meter length between two current-carrying conductor 10cm apart and carrying 1000A and 1500 A currents, respectively?

A. 3 N

B. $\frac{N}{3}$

C. 2 N

D. $\frac{N}{2}$

Ans. A

Sol. I_1 = 1000A

I_2 = 1500A

R = 10 cm

$F = \frac{\mu_0 I_1 I_2 l}{2\pi R}$

$\Rightarrow \frac{F}{l} = \frac{4\pi \times 10^{-7} \times 15 \times 10^{5}}{2\pi \times 0.1}$

F = 3N

77. The potential (scalar) distribution is given as $V=10y^4 + 20x^3$. If ϵ_0 is the permittivity of free space what is the charge density ρ at the point (2, 0) ?

A. – 200 ϵ_0

B. – 200/ ϵ_0

C. 200 ϵ_0

D. – 240 ϵ_0

Ans. D

Sol. Poisson's equation is $\nabla^2 V = \frac{-\rho}{\epsilon_0}$

$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial z^2}\right)\left(10y^4 + 20x^3\right) = \frac{\rho}{\epsilon_0}$

$\Rightarrow 20 \times 3 \times 2x + 10 \times 4 \times 3y^2 = \frac{-\rho}{\epsilon_0}$

At point (2,0)

$20 \times 3 \times 2 \times 2 = \frac{-\rho}{\epsilon_0}$

$\Rightarrow \rho = -240 \ \epsilon_o$

78. Equipotential surface about a point charge are in which one of the following forms?

A. Spheres

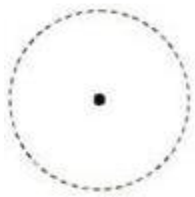
B. Planes

C. Cylinders

D. Cubes

Ans. A

Sol. Equipotential surface about a potential charge is sphere



79. Which one of the following statements is correct?

A. Both Laplce's and poisson's equations are non-linear equation differential equation

B. Laplace's equation is non-linear but Poisson's equation is linear differential equation

C. Laplace's equation is linear but Poisson's equation is non- linear differential equation

D. Both Laplace's and Poisson's equation are linear differential equation

Ans. D

Sol. $\nabla^2 V = -\frac{\rho}{\varepsilon}$

The above equation is known as Poisson's Equation which states that the Potential distribution in a region depend upon the local charge distribution.

In some cases the charge distribution is involved on the surface of the conductor for which the free charge volume density is zero, i.e; **ρ = 0**., in that case, the Poisson equation reduces to **$\nabla^2 V = 0$** ⇒ This equation is known as Laplacian equation.

& the above equation Poisson & Laplacian Equation both are Linear Differential Equation.

80. Two charges are placed at a small distance apart. If glass slab is placed between them. The force between the charges will

A. not change

B. increase

C. decrease

D. reduce to zero

Ans. C

Sol. Force between the two charges is given by Coulomb's law

$F = \frac{1}{4\pi\varepsilon_r\varepsilon_o}\frac{q_1q_2}{r^2}$

For air $\varepsilon_r = 1$

For glass slab $\varepsilon_r > 1$, hence force will decrease between the two charges.

81. Which of the following represents Maxwell's divergence equation for static electric field?

A. $\nabla \cdot B = 0$

B. $\nabla \cdot H = 0$

C. $\nabla \cdot B = \mu$

D. $\nabla \cdot H = \mu$

Ans. A

Sol. All four of the Maxwell equations, including the divergence ones,

$\nabla \cdot E = \rho/\epsilon 0$ and $\nabla \cdot B = 0$, $\nabla \cdot E = \rho/\epsilon 0$ and $\nabla \cdot B = 0$,

are kinematic and dynamical conditions that must be imposed on electric and magnetic fields for them to be physical. That is, Maxwell's equations *are* the "properties EE and BB must have" to represent the fields we observe in nature.

You might then ask, "well, how do we find fields that satisfy them?" This is indeed a problem and it is the central problem of classical electromagnetism. It boils down to solving a set of (partial differential, linear, coupled) equations, and there are various techniques to do that. Maybe a look in Griffiths will help with that. And, as Lubos says, you do *not* need the magnetic field to be time-independent for Maxwell's equations to hold.

82. A current of 5 passes along the axis of a cylinder of a 5cm radius. The flux density at the surface of the cylinder is

A. 2μT
B. 20μT
C. 200μT
D. 2000μT

Ans. B

Sol. According to Ampere's law

$$H = \frac{I}{2\pi r}$$

Here I = 5A

r = 5 cm

$$\therefore H = \frac{5}{2\pi \times 5} = \frac{1}{2\pi} A/m$$

$$B = \mu H = \frac{4\pi \times 10^{-7}}{2\pi} = 20\mu T$$

83. Consider a uniform sphere of charge density ρ_0 and radius b centered at the origin. The electric field at a radial distance r(r<b). according to gauss's law. Is

A. $\frac{r\rho_0}{3 \in}$
B. $\frac{\rho_0}{4\pi \in r^2}$
C. $\frac{b^2\rho_0}{3 \in r^2}$
D. $\frac{\rho_0}{r}$

Ans. A

Sol. $\int_s \vec{D} \cdot \vec{ds} = \rho_o \times \frac{4}{3}\pi r^3$

$$\Rightarrow D \times 4\pi r^2 = \rho_o \times \frac{4\pi}{3} r^3$$

$$\Rightarrow D = \frac{\rho_o r}{3}$$

$$\therefore \quad E = \frac{\rho_o r}{3 \in}$$

84. Which of the following is not electromagnetic in nature?

A. UV rays
B. X-rays
C. Gamma rays
D. Cathode rays

Ans. D

Sol. Alpha rays and Cathode rays are not electromagnetic in nature.

85. Which one of the following concepts is used to find the expression of radiated $\vec{E}$ and $\vec{H}$ field due to a magnetic urrent element?

A. Concept of vector magnetic potential
B. Concept of scalar electric potential
C. Concept of scalar magnetic potential
D. Concept of vector electric potential

Ans. A

Sol. The magnetic field strength $\vec{H}$ is obtained from $\vec{H}$ through relation

$\vec{E} = -\nabla V - \vec{A}$

86. Which one of the following vector functions represents a magnetic field $\vec{B}$? ($\hat{X}$, $\hat{y}$ and $\hat{z}$ are unit vectors along x-axis, y-axis, and z-axis, respectively).

A. $10x\hat{x} + 20y\hat{y} - 30z\hat{z}$

B. $10x\hat{x} - 30z\hat{y} + 20y\hat{z}$

C. $10y\hat{x} - 20x\hat{y} - 10z\hat{z}$

D. $10z\hat{x} + 20y\hat{y} - 30x\hat{z}$

Ans. A

Sol. $\nabla \cdot \vec{B} = 0$ [Gauss law for magnetic fields]

Applying on options,

10 + 20 – 30 = 0

∴ Option 'A' is the answer.

87. The current to flow in a loop radius of 0.5 m to produce magnetic field of 1.5 mA/m at centre of loop. Calculate current in (mA):

A. 1 mA

B. 1.5 mA

C. 2 mA

D. 1.2 mA

Ans. B

Sol. |H| be at centre due to current at loop is given by = $\frac{I}{2a}$ = 1.5×10^{-3}

$$\frac{I}{2 \times 0.5} = 1.5 \times 10^{-3} = 1.5\text{mA}$$

88. Given that $\vec{D} = \frac{\theta}{\pi r^2}(1 - \cos 3r)\vec{a}_r$, in spherical coordinates the charge density is

A. Zero

B. $\frac{3\theta}{\pi r^2}$

C. $\frac{3\theta}{\pi r^2}\sin 3r$

D. $\frac{2\theta}{\pi r^2}\sin 3r$

Ans. C

Sol. $\nabla \cdot \vec{D} = \rho$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{r^2\theta}{\pi r^2}(1 - \cos 3r)\right] = \rho$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{\theta}{\pi}(1 - \cos 3r)\right] = \rho$$

$$\frac{\theta}{r^2\pi}\left[\frac{\partial}{\partial r}(1 - \cos 3r)\right] = \rho$$

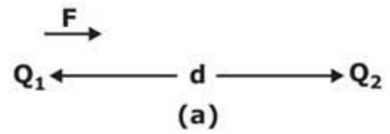
$$\rho = \frac{3\theta}{r^2\pi}\sin 3r$$

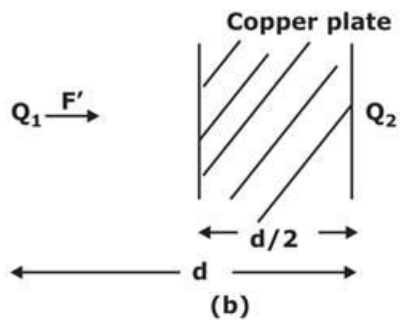
89. The force between two charges placed at distance 'd' apart is 'F'. If a copper plate of thickness $\frac{d}{2}$ is placed between them, then the effective force will be KF. The value of K will be ________.

A. 1
B. 2
C. 0
D. 4

Ans. C

Sol.





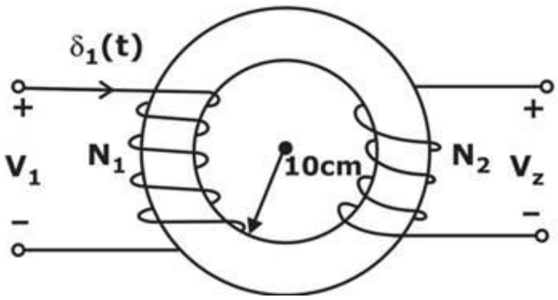
The force between the charges becomes zero.

Because copper (good conductor) plate will not transfer the field lines of one charge to other.

$\Rightarrow$ F' = 0

$\Rightarrow$ K = 0

90. The magnetic circuit of figure has a uniform cross-section of 10^{-3} m^2. If the circuit is energized by a current $i_1(t) = 3 \cdot \sin(100\pi t)$A in the coil of N_1 = 200 turns, then the emf induced in the coil of N_2 = 100 turns will be (Assume that $\mu = 500\ \mu_0$)



A. $-4\pi.\cos(\pi t)$V
B. $6\pi.\cos(\pi t)$V
C. $-6\pi.\cos(100\pi t)$V
D. $4\pi.\cos(100\pi t)$V

Ans. C

Sol. The flux in the circuit is

$$\varphi = \frac{\text{Magnetomotive force (MMF)}}{\text{Reductance (s)}}$$

$$\varphi = \frac{N_1 I_1}{I / \mu A}$$

Here, $l = 2\pi r = 2\pi\left(\frac{10}{100}\right) = 0.2\pi m$

According to faraday's law, the emf induced in the second coil is

$$V_2 = -N_2 \cdot \frac{d\phi}{dt}$$

$$V_2 = -N_2 \cdot \frac{d}{dt}\left(\frac{N_1 I_1 \cdot \mu A}{I}\right)$$

$$V_2 = \frac{-N_1 N_2 \cdot \mu A}{l} \cdot \frac{dI_1}{dt}$$

$$V_2 = \frac{-(200)(100)\left(500 \times 4\pi \times 10^{-7}\right) \times 10^{-3} \times 300\pi \cdot \cos(100\pi t)}{(0.2\pi)}$$

$$V_2 = -6\pi \cdot \cos(100\pi t) \text{ volts}$$

91. Which of the following equation represents the modified Kirchhoff's current law.

A. $\nabla \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t}$

B. $\nabla \cdot \vec{B} = 0$

C. $\nabla \cdot \vec{D} = \rho_V$

D. $\nabla \times \vec{H} = J_C + \frac{\partial \vec{D}}{\partial t}$

Ans. A

Sol. $I = \frac{-dQ}{dt}$ (Q decreasing for definite volume)

$$Q = \int \rho_V \cdot dv$$

$$\Rightarrow I = \frac{-d}{dt}\int \rho_v \cdot dv = -\int \frac{\partial \rho_v}{\partial t} \cdot dv$$

$$I = \oint \cdot \vec{J} \cdot \vec{ds}$$

$$\Rightarrow \oint \vec{J} \cdot \vec{ds} = -\int \frac{\partial \rho_v}{dt} \cdot dv$$

By gauss-diversion theorem

$$\oint \vec{J} \cdot ds = -\int (\Delta . J).dv$$

$$\Rightarrow \int (\nabla \cdot J) \cdot dv = -\int \frac{\partial \rho_v}{\partial t} \cdot dv$$

$$\Rightarrow \nabla \cdot \vec{J} = -\frac{-\partial \rho_V}{\partial t}$$

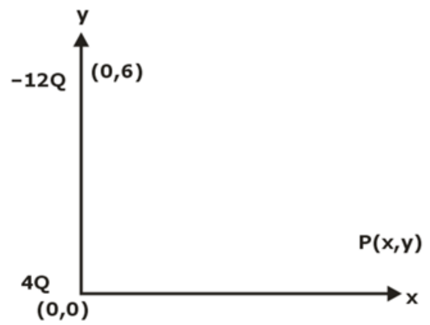
Tips : for steady state, ρ_v is constant then, $\nabla \cdot \vec{J} = 0$ (or) $\int \vec{J} \cdot \vec{ds} = 0$ which is nothing but Kirchhoff's current law.

92. Two electric charges 4Q and -12Q are placed at (0,0) and (0,6) in the x-y plane. The radius of the zero equipotential curve would be

A. 2.25 m
B. 5.2 m
C. 1.25 m
D. 2.98 m

Ans. A

Sol. Let at any point in x-y plane P(x, y) the potential due to 4Q and −12Q charges are V_1 and V_2 respectively then,



$$V_1 = \frac{4Q}{4\times\varepsilon_0\sqrt{x^2+y^2}}$$

and

$$V_2 = \frac{-12Q}{4\times\varepsilon_0\sqrt{x^2+(y-6)^2}}$$

For zero equipartition curve,

$V_1 + V_2 = 0$

$$\frac{4Q}{4\times\varepsilon 0\sqrt{x^2+y^2}} - \frac{12Q}{4\times\varepsilon 0\sqrt{x^2+(y-6)^2}} = 0$$

$$\frac{1}{\sqrt{x^2+y^2}} = \frac{3}{\sqrt{2^2+(y-6)^2}}$$

8x2 + 8y2 + 12y − 36 = 0

$$x^2 + y^2 + \frac{3}{2}y\frac{-9}{2} = 0$$

$$x^2 + \left(y+\frac{3}{4}\right)^2 - \frac{9}{16} - \frac{9}{2} = 0$$

$$x^2 + \left(y+\frac{3}{4}\right)2 = (2.25)^2$$

The above equation is equation of circle on comparing this equation with standard circle equation (x − a)2 + (y − b)2 = r2

∴ r = 2.25m

93. The capacitance of an isolated sphere whose radius (inner) is given by 'a'.

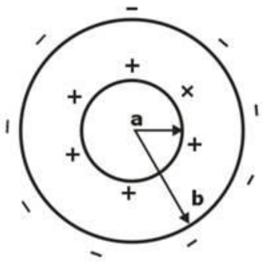
A. $4\pi\varepsilon a$
B. 0
C. $\frac{4\pi\varepsilon}{a}$
D. $2\pi\varepsilon a$

Ans. A

Sol. For sphere

a = inner radius

b = outer radius



$$C_1 = \frac{4\pi\varepsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

It $b \to \infty \Rightarrow$ It is known as isolated sphere.

$$\therefore C_2 = \frac{4\pi\varepsilon}{\left(\frac{1}{a} - \frac{1}{\infty}\right)} = 4\pi\varepsilon a$$

94. Electro statically a typical thunder cloud may be represented by a capacitor model with horizontal plates 10 km^2 in Area separated by a vertical distance of 1 km. the upper plate has a positive charge of 200 coulomb. The electrostatic energy stored in the cloud is:

A. 1.13×10^9 joules

B. 2.26×10^{11} joules

C. 1.13×10^{11} joules

D. 2.26×10^9 joules

Ans. B

Sol. Given

A = 10 km2 = 10 × 106 m2

= 107 m2

d = 1 km = 103 m

Q = 200 C

$$C = \frac{EA}{d}$$

$$C = \frac{8.85 \times 10^{-12} \times 10^7 \times 1}{10^3}$$

= 8.854 × 10–8 F

Energy stored, $E = \frac{1}{2}CV2$

$$\Rightarrow \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} \times \frac{(200)^2}{8.854 \times 10^{-8}}$$

E = 2.26 × 1011 Joules

95. In a certain conducting region. $\bar{H} = -4y^3 \hat{a}_x\hat{a}_x$ A/m. The current through a square of side 2m, with the corner at the origin and the sides coinciding with the positive x and y axis is ______ in (Amps).

A. 24 Amps
B. 32 Amps
C. 64 Amps
D. 72 Amps

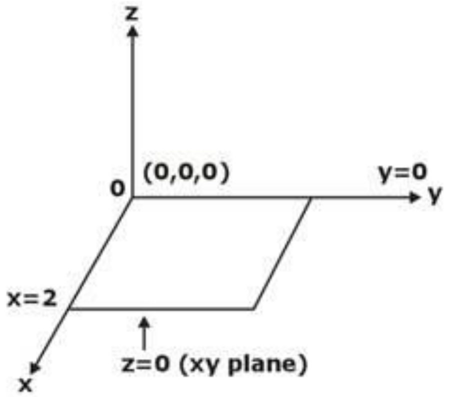
Ans. C

Sol. Given data,

Magnetic field intensity,

$\bar{H} = -4y^3 \hat{a}_x$ A/m

As per the given data closed path (open surface) is defined in Z = 0 plane as shown in fig.



Current, I passing through z = 0 (xy) plane is given by $I = \int_S \bar{I} \cdot \bar{d}_s$

$\bar{d}_S = dxdy\ \hat{a}_z$ ($\because$ z = 0 plane)

$\bar{I} = \nabla \times \bar{H}$

$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -4y^3 & 0 & 0 \end{vmatrix} = 12y2\ \hat{a}_z \text{ A/m2}$$

$\therefore$ Current through the plane is given by

$$I = \int_{x=0}^{2} \int_{y=0}^{2} 12y^2\ dx\ dy$$

$$= 12 \times \frac{y^3}{3}\Big|_0^2 \times \Big|_0^2$$

$= 4 \times 8 \times 2 = 64$ Amps

$\therefore$ I = 64 Amps

96. A potential function $V = x^2yz + Ay^3z$ volts, satisfies the Laplace's equation then what is the value of A.

A. 0.19
B. 0.33
C. −0.33
D. 0.25

Ans. C

Sol. $V = x^2yz + Ay^3z$

$\nabla^2 V = 0$ is Laplace equation

$\nabla^2 V = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}$ .... (1)

$\frac{\partial V}{\partial X} = 2xyz \Rightarrow \frac{\partial^2 v}{\partial x^2} = 2yz$

$\frac{\partial v}{\partial y} = x^2z + 3Ay^2z$

$\Rightarrow \frac{\partial^2 v}{\partial y^2} = 6Ayz$

$\frac{\partial V}{\partial z} = x^2z + Ay^3$

$\Rightarrow \frac{\partial^2 v}{\partial z^2} = 0$

$\therefore$ From (1)

$\nabla^2 V = 2yz + 6Ayz = 0$

$2 + 6A = 0$

$A = \frac{-2}{6} = -0.33$

$\therefore A = -0.33$

97. If a scalar field V defined as $V = xz - x^2y + y^2z^2$, then the value of the div(grad V) is

A. $2\left(y^2 - z^2 + y\right)$

B. $2\left(y^2 - z^2 - y\right)$

C. $2\left(y^2 + 2z - y\right)$

D. $2\left(y^2 + z^2 - y\right)$

Ans. D

Sol. grad V = $\nabla V = \frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}$

grad V = $(z - 2xy)\hat{i} + \left(-x^2 + 2yz^2\right)\hat{j} + \left(x + 2y^2z\right)\hat{k}$

$\therefore$ div (grad V) = $= \nabla \cdot (\nabla V)$

$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left[(z - 2xy)\hat{i} + \left(-x^2 + 2yz^2\right)\hat{j} + \left(x + 2y^2z\right)\hat{k}\right]$

$= \frac{\partial}{\partial x}(z - 2xy) + \frac{\partial}{\partial y}\left(-x^2 + 2yz^2\right) + \frac{\partial}{\partial z}\left(x + 2y^2z\right)$

$= -2y + 2z^2 + 2y^2$

$= 2\left(y^2 + z^2 - y\right)$

98. Consider a wattmeter that is rated at 20 A, 50 V is having the following characteristics:

Parameter	Value
Current coil resistance	(1/10) Ω
Potential coil resistance	(625) Ω

If the potential coil is connected across the load and is purely resistive. Then at rated condition, error in reading is

A. 0.4 %

B. −0.4 %

C. 0.2 %

D. −0.2 %

Ans. A

Sol. Actual load = 20 × 50 = 1000 W

Power consumed by pressure coil is

$$= \frac{(50)^2}{625} = \frac{2500}{625} = 4 \text{ Watts}$$

Total measured power = 1000 + 4 = 1004 Watts

So, error is given as

$$\% \text{ error} = \frac{1004 - 1000}{1000} \times 100 = +0.4\%$$

99. When measuring power in a circuit with low current, the wattmeter current coil should be connected

A. to the load side

B. to the source side

C. anywhere, either load side or source side, does not matter

D. in series with the load along with CT for current amplification

Ans. A

Sol. When measuring power in a circuit with low current, the wattmeter current coil should be connected to the load side.

100. In 2-wattmeter method for measurement of power in a star-connected 3 phase load, magnitude of the two wattmeter readings will be equal

A. at zero power factor

B. at unity power factor

C. 0.5 power factor

D. readings of the two wattmeters will never be equal

Ans. B

Sol. In 2-wattmeter method for measurement of power in a star-connected 3 phase load, magnitude of the two wattmeter readings will be equal at unity power factor.

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