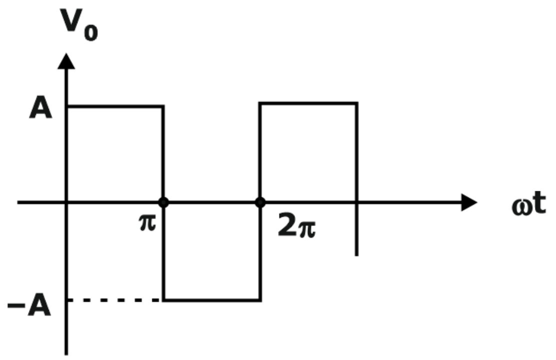
POWER ELECTRONICS (FORMULA NOTES)

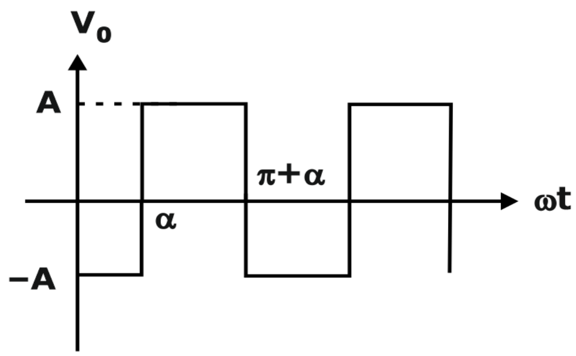
Important Fourier series representations :

1)



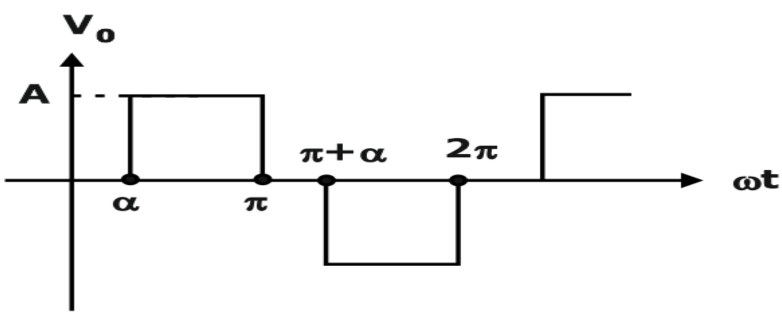
$$V_0 = \sum_{n=1,3,5} \frac{4A}{n\pi} \text{Sin}\, n\omega t$$

2)



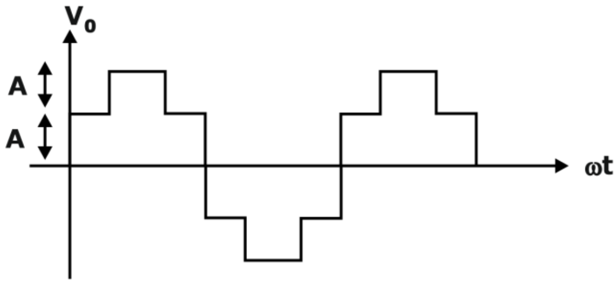
$$V_0 = \sum_{n=1,3,5} \frac{4A}{n\pi} \text{Sin}(n\omega t - n\alpha)$$

3)



$$V_0 = \sum_{n=1,3,5} \frac{4A}{n\pi} \text{Cos}\left(\frac{n\alpha}{2}\right) \text{Sin}(n\omega t - \frac{n\alpha}{2})$$

4)



$$V_0 = \sum_{n=1,5,7} \frac{6A}{n\pi} \text{Sin}\, n\omega t$$

Each pulse width of 60^0 Duration

5) For output of half wave uncontrolled rectifier Fourier series expression is

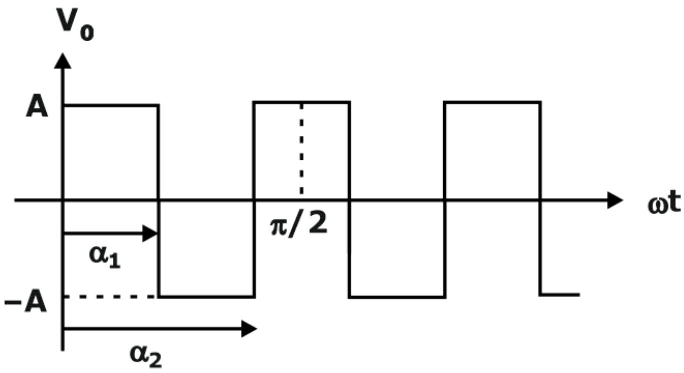
$$V_0 = \frac{A}{\pi} + \frac{A}{2}\text{Sin}\,\omega t + \sum_{n=2,4,6} \frac{2A}{\pi(1-n^2)} \cos n\omega t$$

Where A= Amplitude of signal

6) For output of Full wave uncontrolled rectifier Fourier series expression is

$$V_0 = \frac{2A}{\pi} + (\,fundamental = 0) + 2\sum_{n=2,4,6} \frac{2A}{\pi(1-n^2)} \cos n\omega t$$

7)

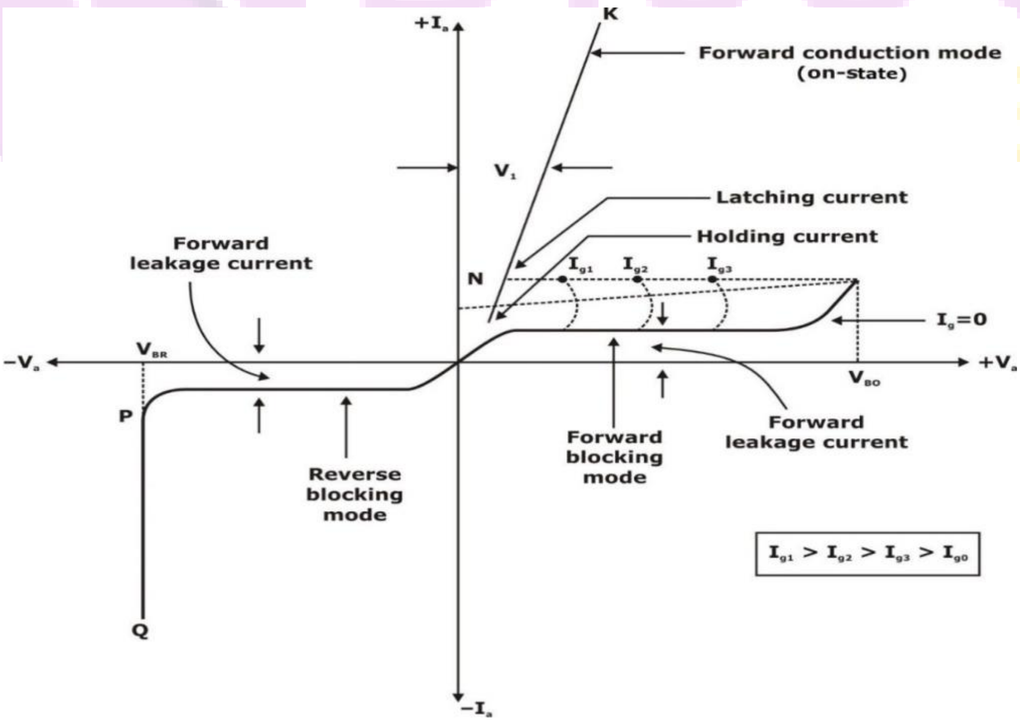


$$V_0 = \frac{4A}{n\pi}\left(1 - 2\cos n\alpha_1 + 2\cos n\alpha_2\right)$$

Diode	BJT/MOSFET/IGBT	SCR and GTO	Triac
• Uncontrolled device • Unipolar and unidirectional	• Fully controlled • Unipolar and unidirectional (Without body diodes)	• SCR-Semi controlled • GTO- Fully controlled • Both are Bipolar and unidirectional	• Semi controlled device • Bipolar and Bidirectional

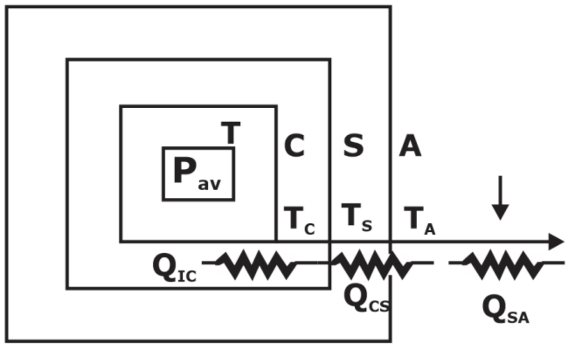
- Conduction losses in BJT is less than MOSFET.
- Switching time in MOSFET is less than BJT.
- Majority carrier devices: MOSFET and Schottky diode are having Positive Temperature Coefficient property.
- Minority carrier devices: SCR, BJT, GTO, IGBT, Power diode and having Negative Temperature Coefficient property.

Static V-I characteristics of SCR



- Latching Current: Minimum current required for conduction even after the gate pulse is removed.
- Holding Current: Minimum Current below which SCR is turned off
- Usually Latching current is 1.5 to 3 times of Holding current

Heat Sink Model:



$$P_{avg} = \frac{T_J - T_C}{Q_{JC}} = \frac{T_C - T_S}{Q_{CS}} = \frac{T_S - T_A}{Q_{SA}}$$

Rating of thyristor $\propto \sqrt{P_{avg}}$

Charge stored in depletion region:

Let Q_R be the charge stored in depletion region of power diode.

$$Q_R = \frac{1}{2}.I_{RM}.t_{rr}$$

$$I_{RM} = \frac{2Q_R}{t_{rr}} = t_a.\frac{di}{dt}$$

$$\text{If } t_a \approx t_{rr},\ t_{rr} = \sqrt{\frac{2Q_R}{di/dt}}$$

$$I_{RM} = t_{rr}.\frac{di}{dt} = \sqrt{2Q_R\left(\frac{di}{dt}\right)}$$

$$t_{rr} \propto \sqrt{Q_R}$$

$$Q_R \propto I_f$$

$$t_{rr} \propto \sqrt{I_f}$$

$$I_{RM} \propto \sqrt{I_f}$$

Relation Between α and β:

$$\boxed{\alpha = \frac{I_C}{I_E}} \quad \boxed{\beta = \frac{I_C}{I_B}}$$

$$\frac{I_E}{I_C} = 1 + \frac{I_B}{I_C}$$

$$\frac{1}{\alpha} = 1 + \frac{1}{\beta}$$

$$\boxed{\alpha = \frac{\beta}{\beta + 1}} \quad \boxed{\beta = \frac{\alpha}{1 + \alpha}}$$

Design of Snubber circuit:

For Inductor (L):

$$\left(\frac{di}{dt}\right)_{max} = \frac{V_s}{L}$$

$$L = \frac{V_s}{(di/dt)_{max}}$$

For resistor (R_s):

$$\left(\frac{dv_a}{dt}\right)_{max} = R_s\left(\frac{di}{dt}\right)_{max}$$

$$\text{or } R_s = \frac{L}{V_s}\left(\frac{dv_a}{dt}\right)_{max}$$

For Capacitor (C_s):

$$C_s = \left(\frac{2\xi}{R_s}\right)^2 L \qquad \text{where } 0.5 < \xi < 1$$

Design of Snubber circuit:

$$\text{String efficiency} = \frac{\text{Actual voltage/current rating of string.}}{n \times \text{individual voltage/current rating of SCR}}$$

Where n is the number of SCR in string.

Derating factor, DRF=1- string efficiency.

Parallel Operation of Thyristors:

When current required by the load is more than the rated current of a single thyristor, SCR's are connected in parallel in a string

Series Operation of Thyristors:

Consider n thyristor connected in series, Let SCR_1 has minimum leakage current I_{bmn}. SCR with lower leakage current blocks more voltage.

Remaining (n-1) SCRs have the same leakage current I_{bmx}

$I_{bmx} > I_{bmn}$

$$\boxed{R_s = \frac{nV_{bm} - V_s}{(n-1)\Delta I_b}}$$

Here V_{bm} is the maximum permissible blocking voltage as SCR1.

R_s is the static equalizing resistance

Similarly Static equalizing capacitance $C = \dfrac{(n-1)\Delta Q_T}{nV_{bm} - V_s}$

ΔQ_T = difference in recovery charge

Ratings of Thyristors:

1)I_{Trms} Rating: The actual Thyristor rms in a converter must always be less than thyristor RMS ratings.

$[(I_T]_{rms}$ value in a converter] < $(I_f)_{rms}$ rating.

2)(I_{Tavg}) Rating: (average on-state current ratings)

$$(I_{Tavg})\text{ rating} = \frac{(I_T)_{rms}\text{ Rating}}{\text{Form Factor of thyristor current waveform}}$$

Average rating of a thyristor depends on:

- Conduction angle of thyristor increases which decrease the form factor and then increase the average thyristor rating.
- Type of load: Smoothness of thyristor current waveform increase the FF decreases and therefore $(I_{Tavg})_{Rating}$ increases.
- Type of converter: because FF of thyristor waveform depends on average value of converter.

11.3. I^2t Rating of thyristor: specified to select a proper fuse for overcurrent protection.

I^2t current Rating of thyristor > I^2t current Rating of Fuse.

11.4. Surge current rating of thyristor:

General values

$(I_T)_{rms}$ = 35A

$(I_S)_{rms}$ = 2000A for one cycle and 3000A for 2MW

N-cycle surge current rating: (I_m): It is the surge current that the SCR can withstand for n-cycles.

$\boxed{(I_{Sn})^2\left(\frac{nT}{2}\right) = I^2t\text{ rating of thy}}$ from the equation, we can find the value of 'I_{sn}'

One-cycle surge current rating (Is): It is the surge current that the SCR can withstand for a cycle. $I_{s1}^2 = (I_{sn})^2$ $\boxed{I_{S_1} = \sqrt{n}\,I_{sn}}$

Sub-cycle surge current rating: It is the surge current that the SCR can withstand for 1/nth period of a cycle. $(I_{s/n})^2 = (I_{sn})^2$ $\boxed{\frac{I_s}{n} = \sqrt{n}\,I_{s1}}$

RECTIFIERS

- For n-pulse converter:

  Source current has nk ± 1 Harmonics k=1,2,3…….

  Output voltage has nk Harmonics.

Single Phase Half Wave controlled rectifier:

R-load:

Average output voltage $V_{o,avg} = \frac{V_m}{2\pi}(1+\cos\alpha)$

RMS output voltage $V_{o,rms} = \sqrt{\frac{V_m^2}{4\pi}\left((\pi-\alpha)+\frac{\sin 2\alpha}{2}\right)}$

RL-Load:

Average output voltage $V_{o,avg} = \frac{V_m}{2\pi}\left(\cos\alpha - \cos\beta\right)$

RMS output voltage $V_{o,avg} = \frac{V_m}{2\sqrt{\pi}}\left(\sqrt{(\beta-\alpha)+\frac{1}{2}(\sin 2\alpha - \sin 2\beta}\right)$

Circuit Turnoff time: $t_c = \frac{2\pi - \beta}{\omega}$

For a constant output current refer below table:

	1-ϕ Full conv.	3ϕ Full conv.	1ϕ Semi conv.	3ϕ semi conv
Output voltage	$\frac{2V_m}{\pi}\cos\alpha$	$\frac{3V_{m,line}}{\pi}\cos\alpha$	$\frac{V_m}{\pi}(1+\cos\alpha)$	$\frac{3V_{m,line}}{2\pi}(1+\cos\alpha)$
Fundamental source current RMS(I_{s1})	$\frac{2\sqrt{2}}{\pi}I_0$	$\frac{\sqrt{6}}{\pi}I_0$	$\frac{2\sqrt{2}}{\pi}I_0\cos\left(\frac{\alpha}{2}\right)$	$\frac{\sqrt{6}}{\pi}I_0\cos\left(\frac{\alpha}{2}\right)$
Source current RMS(I_s)	I_0	$\sqrt{\frac{2}{3}}I_0$	$I_0\sqrt{\frac{\pi-\alpha}{\pi}}$	$\alpha \le 60^0 \rightarrow I_0\sqrt{\frac{2}{3}}$ $\alpha > 60^0 \rightarrow I_0\sqrt{\frac{\pi-\alpha}{\pi}}$
Displacement power factor(DPF)	$\cos\alpha$	$\cos\alpha$	$\cos\left(\frac{\alpha}{2}\right)$	$\cos\left(\frac{\alpha}{2}\right)$

- Distortion factor(DF) = $\frac{I_{s1}}{I_s}$
- Input power Factor= DF*DPF
- Total Harmonic distortion = $\sqrt{\left(\frac{1}{DF}\right)^2 - 1}$

For R-Load refer below table:

	3ϕ Half wave rectifier	3ϕ full wave rectifier
Continuous	$\alpha < 30^0$ $\frac{3V_{m,line}}{2\pi}\text{Cos}\,\alpha$ **Hint:** Integrate from $30+\alpha$ to $150+\alpha$ and Time period $T=120^0$ and function take in phase, You will get above formula Like this $\frac{1}{\left(\frac{2\pi}{3}\right)}\int_{30+\alpha}^{150+\alpha} V_{m,phase}\,\text{Sin}\,\omega t\, dwt$	$\alpha < 60^0$ $\frac{3V_{m,line}}{\pi}\cos\alpha$ **Hint:** Integrate from $60+\alpha$ to $120+\alpha$ and Time period $T=60^0$ and function take in line, You will get above formula
Discontinuous	$\alpha \geq 30^0$ $V_0 = \frac{3V_{m,phase}}{2\pi}\left(1+\cos\left(\alpha+\frac{\pi}{6}\right)\right)$ **Hint:** Integrate from $30+\alpha$ to 180^0 and Time period $T=120^0$ and function take in phase, You will get above formula	$\alpha \geq 60^0$ $V_0 = \frac{3V_{m,line}}{\pi}\left(1+\cos\left(\alpha+\frac{\pi}{3}\right)\right)$ **Hint:** Integrate from $60+\alpha$ to 180^0 and Time period $T=60^0$ and function take in line, You will get above formula

Effect of Source Inductance:

- **1** ϕ Half wave:

$$V_0 = \frac{V_m}{2\pi}(1+\cos\alpha) - fL_sI_0$$

$$I_0 = \frac{V_m}{\omega L_S}(\cos\alpha - \cos(\alpha+\mu))$$

- **1** ϕ Full wave:

$$V_0 = \frac{2V_m}{\pi}\cos\alpha - 4fL_sI_0$$

$$I_0 = \frac{V_m}{2\omega L_S}(\cos\alpha - \cos(\alpha+\mu))$$

Regulation $= \frac{\cos\alpha - \cos(\alpha+\mu)}{2\cos\alpha}$

Displacement power factor: $\cos\left(\alpha+\frac{\mu}{2}\right)$

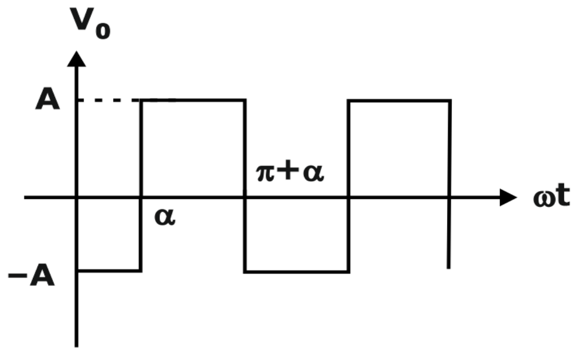
- 3ϕ Full Wave:

$$V_0=\frac{3V_{m,line}}{\pi}\cos\alpha-6fL_sI_0$$

$$I_0=\frac{V_{m,line}}{2\omega L_S}\left(\cos\alpha-\cos(\alpha+\mu)\right)$$

Single Phase Full converter:

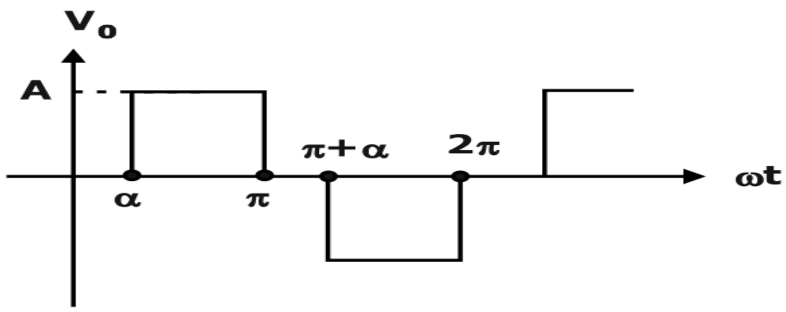
Source current waveform:



Fundamental source current is $i_{s1}=\frac{2\sqrt{2}}{\pi}I_0$

Single phase half controlled or Semi converter:

Source current waveform:



- In this there are two configurations:

Symmetrical configuration: On one leg one thyristor and one diode

Unsymmetrical configuration: on one leg two thyristors or two diodes

$\gamma = represents\ conduction$ in below table

D- Diode, T-Thyristor, α -Firing angle

Symmetrical configuration	Unsymmetrical configuration	Full converter with Freewheeling diode
$\gamma_T = \pi$ $\gamma_D = \pi$	$\gamma_T = \pi - \alpha$ $\gamma_D = \pi + \alpha$	$\gamma_T = \pi - \alpha$ $\gamma_D = 2\alpha$

3-Phase:

Phase Voltage reference:

$$V_{an} = V\angle 0$$
$$V_{bn} = V\angle -120$$
$$V_{cn} = V\angle +120$$

Line Voltage reference

$$V_{AB} = V\angle 0$$
$$V_{BC} = V\angle -120$$
$$V_{CA} = V\angle +120$$

3 Phase half wave controlled Rectifiers:

- Take phase voltage reference in the integration function for the below mentioned limits for calculations
- For R-Load $\alpha < 30^0$

  $\alpha < 30^0$: Continuous conduction: 30+ α to 150+ α

  $\alpha \geq 30^0$: Discontinuous conduction: 30+ α to 180^0
- For current stiff load:

  Without Freewheeling diode: 30+ α to 150+ α

  With Freewheeling diode: $\alpha < 30^0$:30+ α to 150+ α

  $\alpha \geq 30^0$: 30+ α to 180^0

✓ Mentioned limits are useful while calculating output voltage average or RMS values for those particular conditions. Use phase as reference while doing calculations of average and RMS

3 Phase Full wave controlled Rectifiers:

- Take Line voltage reference in the integration function for the below mentioned limits for calculations
- Limits are 60+ α to 120+ α for calculating output voltage average or RMS values
- Circuit Turnoff time:

$$\alpha \leq 60^0, t_c = \frac{240^0 - \alpha}{\omega}$$

$$\alpha > 60^0, t_c = \frac{180^0 - \alpha}{\omega}$$

3 Phase Semi converter:

- $\alpha < 60^0$ it is 6 Pulse converter

  $\alpha \geq 60^0$ it is 3 Pulse converter

- Freewheeling Action Duration:

  $\alpha < 60^0$, Duration = zero (No freewheeling action)

  $\alpha \geq 60^0$, Duration= $3\left(\alpha - \frac{\pi}{3}\right)$

- Limits for calculating output voltage average or RMS values(Line voltages are reference)

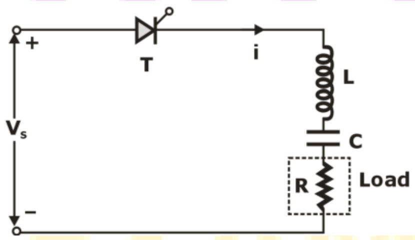
  $\alpha < 60^0: \; 60^0 + \alpha \; to \, 120^0 \rightarrow V_{AB} \; reference$

  $: \; 120^0 \; to \, 180^0 + \alpha \rightarrow V_{AC} \; reference$

  $\alpha \geq 60^0: \; 60^0 + \alpha \; to \; 240^0 \rightarrow V_{AC} \; reference$

Commutation Techniques

1) **Class A Commutation (Load Commutation/self-commutation)**



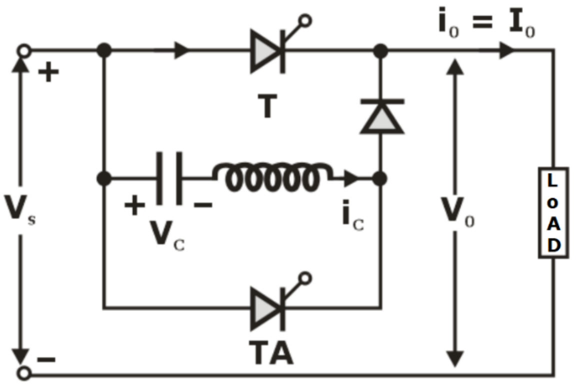
- For successful Load commutation, circuit must be under damped
- For under damped $I = I_p e^{-\alpha t} \, \text{Sin}\, \omega_r t$

$$I_p = \frac{V_s}{\omega_r L}, \alpha = Dampingfactor = \frac{R}{2L}, \; \omega_r^2 = \omega_0^2 - \alpha^2$$

ω_r is ringing frequency, $\omega_0 = Naturalfrequency = \frac{1}{\sqrt{LC}}$

- Conduction time of thyristor, $t_c = \frac{\pi}{\omega_r}$

2) Class B Commutation/Current Commutation/Resonant pulse commutation:



- Voltage across capacitor $V_c = V_s \cos \omega_0 t$
- Circuit turn-off time for the main thyristor (T_1); $tc = C\frac{V_{ab}}{I_0}$

$$V_{ab} = V_s \cos \omega_0 (t_3 - t_2)$$

Where t_3 = time when the main thyristor is turned off

t_2= time when auxiliary thyristor is turned off

$$\omega_0 (t_3 - t_2) = \sin^{-1}\left(\frac{I_0}{I_p}\right)$$

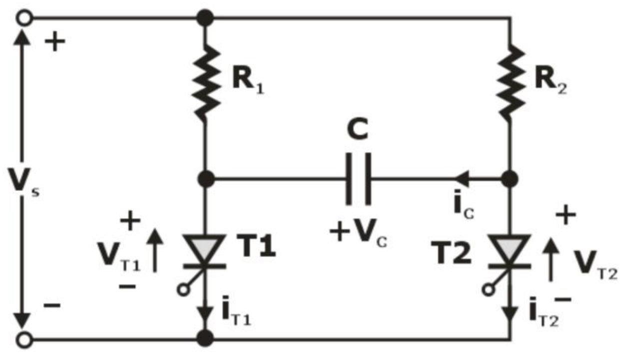
- Main thyristor peak current $=I_0$
- Auxiliary Thyristor peak current= $V_s\sqrt{\frac{C}{L}}$
- Conduction time of auxiliary thyristor= $\pi\sqrt{LC}$
- Conduction time of main thyristor = $\pi\sqrt{LC} + \sqrt{LC}\sin^{-1}\left(\frac{I_0}{I_p}\right)$

Minimum Conduction time of main thyristor= $\pi\sqrt{LC}$

Maximum Conduction time of main thyristor = $\frac{3}{2}\pi\sqrt{LC}$

- Time for which capacitor current exists = $\frac{C}{I_0}\left(V_{ab} + V_s\right)$

Class C Commutation (Impulse/Complementary commutation):



When T_1 is turned on at t=0

- The charging current $I_s = \frac{V_s}{R_z} . e^{-t/R_2 C}$
- Voltage across capacitor

  $V_c(t) = V_s(1 - e^{-t/R_2 C})$

  When T_1 is to be turned-off, T_2 is turned-on at T_1

- The charging current $I_c(t) = -\frac{2V_S}{R_1} . e^{-t/R_1 C}$
- The Voltage across capacitor

  $V_c(t) = V_s[2e^{-t/R_1 C} - 1]$

- Maximum current though thyristor T_1

  $$I_{T_1(max)} = V_s\left[\frac{1}{R_1} + \frac{2}{R_2}\right]$$

- Maximum current though thyristor T_2,

  $$I_{T_2(max)} = V_s\left[\frac{2}{R_1} + \frac{1}{R_2}\right]$$

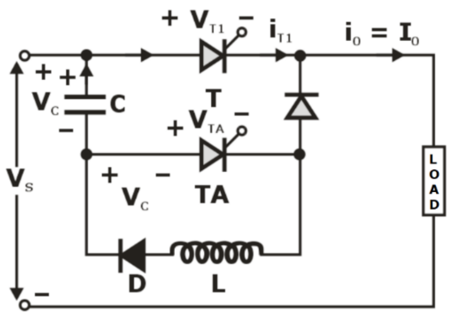
  Circuit turn-off time t_{c_1} for thyristor T_1

  $t_{c_1} = R_1 C \ln(2)$

  Circuit turn-off time t_{c_2} for thyristor T_2

  $t_{c_2} = R_2 C \ln(2)$

Class D Commutation (Voltage commutation):



- Maximum thyristor current Peak= $I_0 + V_s\sqrt{\frac{C}{L}}$
- Auxiliary Thyristor peak current= I_0
- Capacitor peak current = $V_s\sqrt{\frac{C}{L}}$

- Circuit turn-off time for main thyristor T_1 is $t_c = C\frac{V_s}{I_0}$

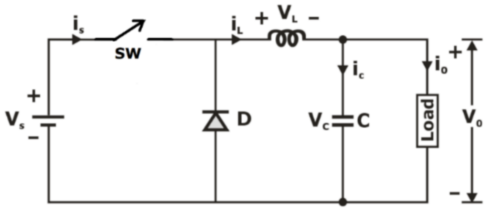
For R-load it t_c= RC ln2

- Circuit turn-off time for main thyristor (TA)

$$t_{c_1} = \frac{\pi}{2\omega_0}$$

DC-DC Converters

Buck Converter:



In Buck regulator, the average output voltage V_0 is less than the input voltage V_S.

$$\Delta I = \frac{(V_S - V_0)T_{ON}}{L}$$

$$\Delta I = \frac{V_0 T_{OFF}}{L}$$

$$V_0 = V_S\frac{T_{ON}}{T} = V_S\alpha$$

Where $\Delta I = I_2 - I_1$ is the peak to peak current ripple of the inductor L.

The peak to peak ripple current is $\Delta I = \frac{V_S\alpha(1-\alpha)}{fL}$

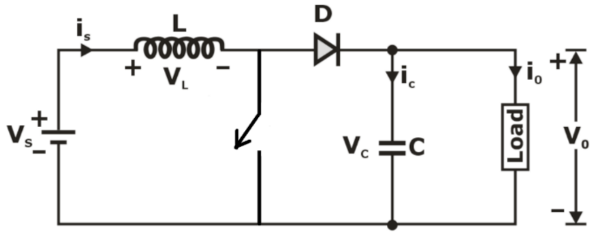
The peak to ripple voltage of the capacitor is $\Delta V_C = \frac{V_S\alpha(1-\alpha)}{8LCf^2}$

Condition for continuous inductor current and capacitor voltage:

If I_L is average inductor current, the inductor ripple current $\Delta I = 2I_L$, which gives the critical value of the inductor L_C as $L_C = L = \frac{(1-\alpha)R}{2f}$

If V_C is the average capacitor voltage, the capacitor ripple voltage $\Delta V_C = 2V_0$, which gives the critical value of capacitor C_C as $C_C = C = \frac{1-\alpha}{16Lf^2}$

Boost Converter:



$$\Delta I = \frac{V_S T_{ON}}{L} = \frac{(V_0 - V_S) T_{OFF}}{L}$$

where $\Delta I = I_2 - I_1$ is peak to peak ripple current of the inductor L.

The average output voltage,

$$V_0 = V_S \frac{T}{T_{OFF}} = \left(\frac{1}{1-\alpha}\right) V_S$$

The peak to peak current ripple is, $\Delta I = \frac{V_S \alpha}{fL}$

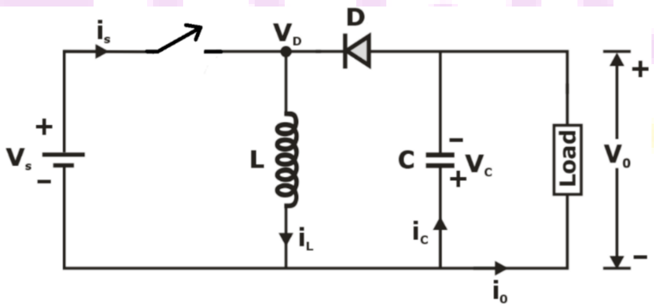
The peak to peak ripple voltage of capacitor, $\Delta V_C = \frac{I_0 \alpha}{fC}$

Condition of continuous inductor current and capacitor voltage:

If V_C is the average capacitor voltage, the capacitor ripple voltage $\Delta V_C = 2V_0$, which gives the critical value of the capacitor C_C as $C_C = \frac{\alpha}{2fR}$

If I_L is average inductor current, the inductor ripple current $\Delta I = 2I_L$, which gives the critical value of the inductor L_C as $L_C = L = \frac{\alpha (1-\alpha)^2 R}{2f}$

Buck Boost Converter:



$$\Delta I = \frac{V_S T_{ON}}{L} = \frac{-V_0 T_{OFF}}{L}$$

where $\Delta I = I_2 - I_1$ is the peak to peak ripple current of inductor L.

The average output voltage is, $V_0 = -\frac{V_S \alpha}{1-\alpha}$

The peak to peak current ripple is, $\Delta I = \frac{V_S \alpha}{fL}$

peak to peak ripple voltage of the capacitor is, $\Delta V_C = \frac{I_0 \alpha}{fC}$

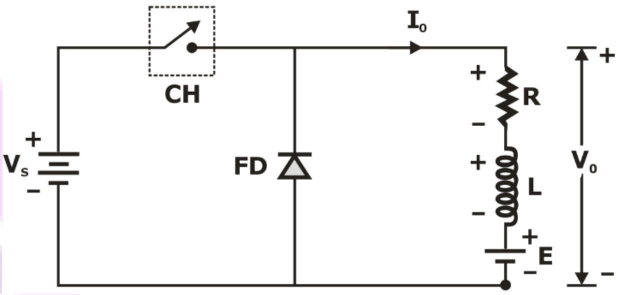
Condition of continuous inductor current and capacitor voltage:

If V_C is the average capacitor voltage, the capacitor ripple voltage, $\Delta V_C = 2V_0$, which gives the critical value of the capacitor C_C as $C_C = \frac{\alpha}{2fR}$.

If I_L is average inductor current, the inductor ripple current $\Delta I = 2I_L$, which gives the critical value of the inductor L_C as $L_C = L = \frac{(1-\alpha)^2 R}{2f}$

Expression for V_0	BUCK	BOOST	BUCK BOOST
In CCM	$V_0 = \alpha V_s$	$V_0 = \frac{V_s}{1-\alpha}$	$V_0 = -\frac{\alpha V_s}{1-\alpha}$
In DCM	$V_0 = \frac{\alpha}{\beta} V_s$	$V_0 = \frac{\beta V_s}{\beta - \alpha}$	$V_0 = -\frac{\alpha V_s}{\beta - \alpha}$

Steady State analysis of Type A Chopper:



Average output voltage

$$V_0 = \alpha V_s$$

$V_{or} = \sqrt{\alpha}\, V_S$ (Rms value of output voltage)

$$I_{max} = \frac{V_S}{R}\left[\frac{1 - e^{-T_{on}/T_a}}{1 - e^{-T/T_a}}\right] - \frac{E}{R}$$

$$I_{min} = \frac{V_S}{R}\left[\frac{e^{T_{on}/T_a} - 1}{e^{T/Ta} - 1}\right] - \frac{E}{R}$$

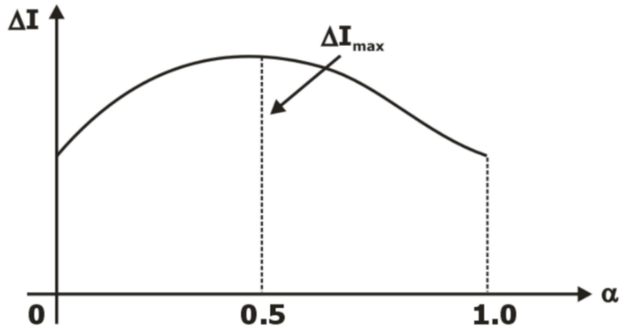
Where, T_a = load time constant

$$T_a = \frac{L}{R}$$

Current ripple,

$$(\Delta I) = I_{max} - I_{min}$$

$$\Delta I = \frac{V_S}{R}\left[\frac{\left(1 - e^{-T_{on}/T_a}\right)\left(1 - e^{-T_{off}/T_a}\right)}{\left(1 - e^{-T/T_a}\right)}\right]$$



$T_{on} = \alpha T$

$T_{off} = (1 -) \alpha T$

Per unit ripple (or) Ripple is a function of duty cycle 'α'. Ripple is minimum at α = 0, increases maximum at α = 0.5 and decrease at α = 1.0. For α = 0.5, ripple would be maximum.

$$(\Delta I)_{max} = \frac{V_S}{R}\left(\frac{\left(1-e^{-0.5x}\right)\left(1-e^{-0.5x}\right)}{1-e^{-x}}\right) \qquad \left(\text{Let, } \frac{T}{T_a} = x\right)$$

$$(\Delta I)_{max} = \frac{V_S}{R}\tanh\left(\frac{R}{4fL}\right)$$

Inverters

Series Inverters: In a series inverter, the commutating elements L and C are connected in series with the load resistance R. The load resistance R can also be in parallel with C. The value of L and C are such that those form an underdamped circuit i.e.

$$R^2 < \frac{4L}{C}$$

$f = \left[\frac{1}{2\left(\frac{T}{2} + T_{off}\right)}\right]$ is the frequency of output voltage.

Where, $\frac{T}{2}$ is the time period of oscillations.

T_{off} is the time gap between turn-off one thyristor and turn-on of the second thyristor.

$$\frac{T}{2} = \frac{\pi}{\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}}$$

The period of oscillation

Bridge Inverter: Bridge circuits are commonly used in DC-AC conversion. Moreover, an output transformer is not essential in a bridge circuit.

1φ Half Bridge Inverter - The output voltage volt $V_0 = \sum_{n-1,3,5...}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t$

1φ Full Bridge Inverter- The output voltage

$$V_0 = \sum_{n-1,3,5,...}^{\infty} \frac{4V_s}{n\pi} \sin n\omega t$$

Where, n = order of harmonic

ω=2πf, is frequency of the output voltage in red/sec

Key points:

- The load impedance (Z_n) is

$$Z_n = \left[R^{2+}\left(n\omega L - \frac{1}{n\omega C}\right)^2\right]^{1/2}$$

- Phase angle, $\phi_n = \tan^{-1}\frac{\left[n\omega L - \frac{1}{n\omega C}\right]}{R}$

3 phase Full Bridge VSI:

	180^0 Conduction	120^0 Conduction
Line Voltage RMS	$V_s\sqrt{\frac{2}{3}}$	$V_s\frac{1}{\sqrt{2}}$
Phase voltage RMS	$V_s\frac{\sqrt{2}}{3}$	$V_s\frac{1}{\sqrt{6}}$
Fundamental line voltage RMS	$V_s\frac{\sqrt{6}}{\pi}$	$V_s\frac{3}{\pi\sqrt{2}}$
Fundamental phase voltage RMS	$V_s\frac{\sqrt{2}}{\pi}$	$V_s\frac{1}{\pi}\sqrt{\frac{3}{2}}$

180^0 Conduction:

1) Pole Voltages = $V_{A0} = \sum_{n-1,3,5...}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t$

2) Line Voltages = $\sum_{n=1,3,5}\left(\frac{4V_s}{n\pi}\cos\left(n\frac{\pi}{6}\right)\right)\sin(n(\omega t + 30^0))$

When n=3, 9, 15 Line voltage= 0, So Line voltages are free from Triplet harmonics

3) Phase Voltage= $\sum_{n=6k\pm1}\frac{2V_{dc}}{n\pi}\sin n\omega t$

$n = 6k \pm 1$ is due to stepped waveform

120^0 Conduction:

1) Pole and Phase Voltage are of same waveform
2) Triplet harmonics are absent in Phase and pole voltages
3) Line voltage contains $n = 6k \pm 1$ Harmonics

Pulse Width Modulation:

- Let N= number of pulses per half cycle

  Each pulse width = $\frac{2d}{N}$

  Then Output voltage Expression is

$$V_0 = \sum_{n=1,3,5}\left(N\frac{4V_s}{n\pi}\sin n\gamma \sin\frac{nd}{N}\right)\sin n\omega t$$

Where $\gamma = \frac{\pi - 2d}{N+1} + \frac{d}{N}$

- Number of pulses per half cycle $N = \frac{f_c}{2f}$

  f = reference input frequency

  f_c = Carrier input frequency

- Modulation Index $m_a = \frac{V_{Ref}}{V_{carrier}}$
- Relation between Pulse width and modulation index

$$\frac{2d}{N} = \frac{\pi}{N}(1-m_a)$$

Amplitude Modulation Depth:

$$m_0 = \frac{\hat{V}_m}{\hat{V}_c}$$

Where V_m, V_c are the modulating and carrier signal voltage, respectively.

For sinusoidal PWM, the amplitude modulation depth must be less than 1.0

Output Voltages by Sinusoidal PWM:

- In single phase half bridge VSI

  Fundamental peak pole voltage $= \hat{V}_{Ao1} = m_a \frac{V_s}{2}$

- In single phase Full bridge VSI

  Fundamental peak pole voltage $= \hat{V}_{Ao1} = m_a V_s$

- In Three phase Full bridge VSI

Peak Fundamental Phase voltage $\hat{V}_{an1} = m_a \frac{V_{DC}}{2}$

The fundamental line-line rms voltage is given by

$$V_{LLO_1} = \frac{\sqrt{3}}{2\sqrt{2}} m_a V_{DC}$$

- If peak value of carrier input and zero crossing of reference sinusoidal coincidence then, Number of Pulses per half cycle will be $N = \frac{f_c}{2f}$
- If Zero Crossing of carrier input and reference sinusoidal coincidence then, Number of Pulses per half cycle will be $N = \frac{f_c}{2f} - 1$
- If N is the number of pulses per half cycle then the predominant harmonics in the output is $2N \pm 1$

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