BYJU'S EXAM PREP

The Most Comprehensive Preparation App For All Exams

IMPORTANT FORMULAS TO REMEMBER

CHAPTER-1 FORCE ANALYSIS AND EQUILIBRIUM

INTRODUCTION

Engineering mechanics is the branch of science that considers the motion of bodies and the effects of forces on that motion. It may be divided into two parts

Statics:

Statics deals with the special case of a body at rest or a body that moves with a constant velocity.

Dynamics:

Dynamics is further divided into Kinematics and Kinetics. Kinematics is concerned with rates of change of geometrical quantities in a moving system; it does not involve the concept of force. Kinetics treats the causes and the nature of motion that results from specified forces.

Note:

- A particle has a mass, but a size that can be neglected. For example, the size of the earth is insignificant compared to the size of its orbit and thereafter the earth can be modeled as a particle when studying its orbital motion.
- A rigid body can be considered as a combination of a large number of particles in which particles remain at a fixed distance from one another both before and after applying a load.

FORCE

An agent which produces or tends to produce, destroy or tends to destroy motion.

SI Unit: Newton (N)

System of forces

When two or more than two forces of different magnitude and direction act upon a body, they constitute a system of forces.

Concurrent forces

Two or more forces that act at the same point are called concurrent forces. Concurrent forces need not have the same direction. They simply act at the same point.

Collinear forces

If concurrent forces do have the same direction, they are collinear forces.

Coplanar forces

Two or more forces whose directed arrows lie in the same plane are called coplanar forces.

- Since two concurrent forces always lie in a common plane, they are always coplanar.
- Three or more concurrent forces are not necessarily coplanar.

PRINCIPLE OF TRANSMISSIBILITY

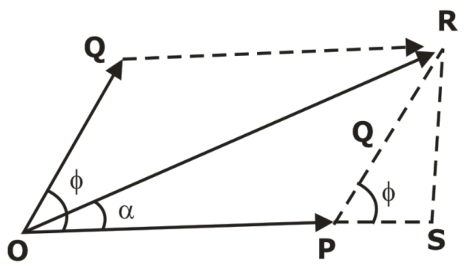
The state of rest or of motion of a rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the applied forces.

RESULTANT OF FORCE

It is possible to find a single force which will have the same effect as that of a number of forces acting on a body. The single force is called resultant force and the process of finding out the resultant force is called composition of forces. The reverse of the composition of forces is called resolution of force.

PARALLELOGRAM LAW OF FORCES

If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram, which passes through their point of intersection, their resultant force is represented, both in magnitude and in direction, by the diagonal of the parallelogram drawn through their point of intersection. Let two forces P and Q are represented by OP and OQ respectively, two sides of the parallelogram. Now the parallelogram OPRQ is completed and the resultant is represented by diagonal OR.



$$R = \sqrt{\left(P^2 + Q^2 + 2PQ\cos\phi\right)}$$

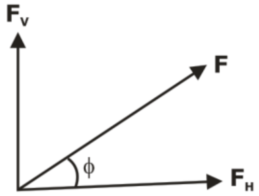
$$\tan\alpha = \frac{Q\sin\phi}{P + Q\cos\phi}$$

ANALYTICAL METHOD

Resolve all forces horizontally and vertically, then determine

ΣH = algebraic sum of all horizontal component.

ΣV = algebraic sum of all vertical component.



$$R^2 = \sqrt{\sum H^2 + \sum V^2}$$

$$\tan\phi = \frac{\sum V}{\sum H}$$

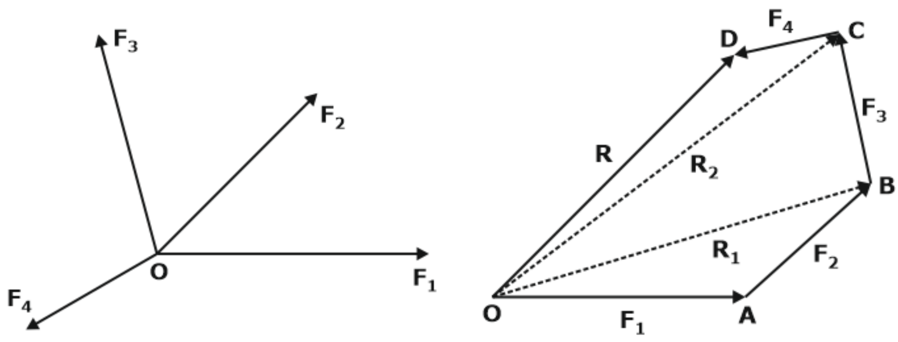
TRIANGLE LAW OF FORCES

If two forces acting simultaneously on a body are represented in magnitude and direction by two sides of a triangle in order, then the third side will represent the resultant of the two forces in the direction and magnitude taken in opposite order.

POLYGON LAW OF FORCES

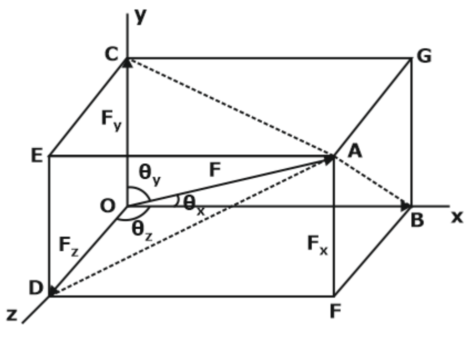
When the forces acting on a body are more than two, the triangle law can be extended to polygon law. Polygon Law states that if a number of coplanar concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in order, then their resultant can be represented by closing side of the polygon in magnitude and direction in the opposite order.

Let forces F_1, F_2, F_3 and F_4 are acting at a point O as shown in figure and can be represent by sides of polygon OA, AB, BC, CD respectively. Vector OD represents the resultant force $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4$ in magnitude and direction.



RESOLUTION AND RESULTANT OF FORCE IN SPACE

Consider a force **F** acting at the origin O of the system of rectangular coordinates x, y, z as shown in Fig. The direction of force **F** is defined by three angles θ_x, θ_y, and θ_2. Let F_x, F_y, and F_z be the components of force F in x, y, and z directions, respectively.



From the triangle OAB $\cos\theta_x = \dfrac{OB}{OA} = \dfrac{F_x}{F}$, component of **F** along x-direction $F_4 = F\cos\theta_x$

From the triangle OAC $\cos\theta_y = \dfrac{OC}{OA} = \dfrac{F_y}{F}$, component of **F** along y-direction $F_y = F\cos\theta_x$

From the triangle OAD $\cos\theta_x = \dfrac{OC}{OA} = \dfrac{F_z}{F}$, component of **F** along z-direction $F_z = F\cos\theta_z$

The cosines of θ_x, θ_y and θ_z are known as the direction cosine of the force **F**.

The angles θ_x, θ_y and θ_z are not independent, they are related as $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

.

We may express **F** in the vector form $\mathbf{F} = F_x i + F_y j + F_x k$

When the components F_x, F_y and F_z given, the magnitude of force **F** is $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

and direction of force $\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right), \theta_y = \cos^{-1}\left(\frac{F_y}{F}\right)$, and $\theta_z = \cos^{-1}\left(\frac{F_z}{F}\right)$

MOMENTS

The turning effect produced by a force, on the body, on which it acts.

SI Unit: Nm

Moment M = F × L, Where F = Force acting on the body, and L = Perpendicular distance of the point, about which the moment is determined and the line of action of the force.

Force F and position vector r in rectangular components may be written as

$$\mathbf{F} = F_x i + F_y j + F_z k$$

$$\mathbf{r} = xi + yj + zk$$

Thus,

$$\mathbf{M}_o = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} i & j & k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$M_o = i(yF_z - zF_y) - (xF_z - zF_x)j + (xF_y - yF_x)k$$

In case of problems involving only two dimensions, the force F may be assumed to lie in the xy-plane. Carrying $z = 0$ and $F_z = 0$, we obtain

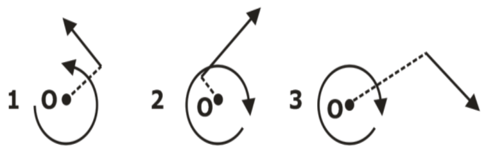
$$\mathbf{M}_o = (xF_y - yF_x)k$$

Note.

Just as force has a tendency to translate the body, moment has a tendency to rotate the body about the point.

DETERMINATION OF DIRECTIONS

Assume perpendicular distance as hand of clock. Keep the point (one end of perpendicular distance), about which the moment is to be determined, hinged like hinged end of hands of a clock. Move the other end of perpendicular distance in the direction of action of force.



1. Anticlockwise
2. Clockwise
3. Clockwise

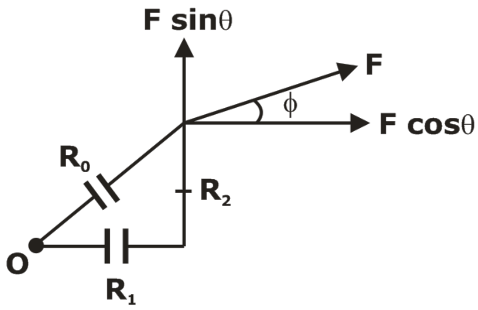
Note.

Clockwise whose effect is to turn, in the same direction in which the hands of clock move.

Anticlockwise whose effect is to turn, in the opposite direction in which the hands of clock move.

VARIGNON'S THEOREM

Moment of a force about any point is equal to the algebraic sum of the moments of the components of that force about the same point.



$$M_o = F\cos\theta \times R_2 - F\sin\theta \times R_1$$

Couples

- The moment produced by two equal, opposite and non-collinear forces is called couple.
- It does not produce any translation but produces only rotation.
- The resultant force of a couple is zero.
- The moment of a couple is the product of the magnitude of one of the forces and the perpendicular distance between their lines of action.

CONDITIONS OF EQUILIBRIUM

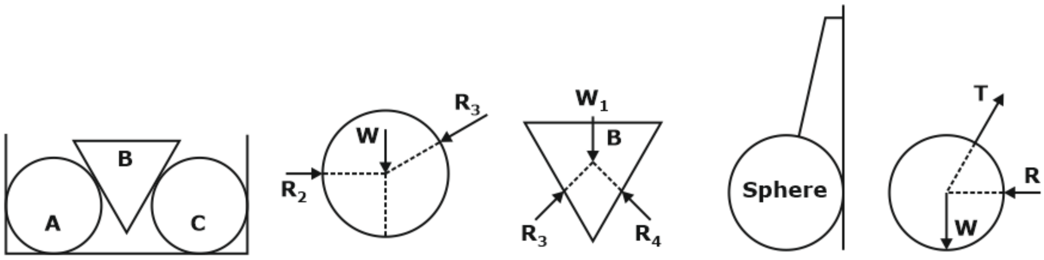
- The term equilibrium implies that either the body is at rest or it moves with a constant velocity.
- A body is said to be in static equilibrium when the resultant force in it must be zero and the body must have no tendency to rotate.

i.e. $\Sigma F = 0$ means that $\Sigma H = 0$ And $\Sigma V = 0$

and $\Sigma M = 0$ about any point.

FREE BODY DIAGRAM

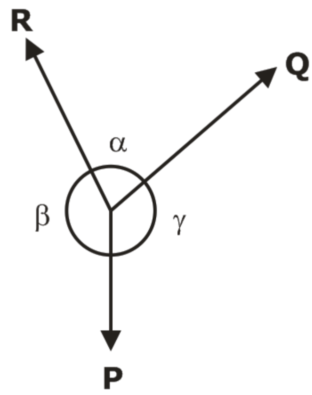
A free body diagram is a sketch of the body that shows the body (by itself, free of the other part of the system) and all the forces applied to it, that is, all forces acting on the body. Fig shows free body diagrams.



FBD of A FBD of B FBD of sphere

LAMI'S THEOREM

- If three coplanar forces acting at a point be in equilibrium, then each force is proportional to sine of the smaller angle Between other two forces.
- Here P, Q & R are the three coplanar forces and α, β, γ are three angles. According to Lami's theorem $\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$



CHAPTER-2 ANALYSIS OF TRUSSES

INTRODUCTION

A structure refers to a system of connected parts used to support a load. Important examples include buildings, towers, ship, aircraft frames, tanks, pressure vessels, mechanical systems and electrical supporting structures.

TRUSSES

- A framework composed of straight members joined at their ends to form a structure is called a truss. Truss is used for supporting moving or stationary load. Bridges, roof supports, and other such structures are common example of trusses.
- When the members of the truss lie essentially in a single plane, the truss is called a plane truss.
- When the members of the truss lie in three-dimension, the truss is called a space truss.
- The simple plane truss is built up from an elementary triangle by adding two new members for each new pin.

CLASSIFICATION OF TRUSS

1) **EFFICIENT OR PERFECT TRUSS**
   - A truss is said to be perfect when the number of members in the truss is just sufficient to prevent distortion of its shape when loaded with an external load.
   - A perfect truss has to satisfy the following equation $m = 2j-3$, where m is the number of members and j is the number of joints.
2) **DEFICIENT OR COLLASIBLE TRUSS**
   - A deficient truss is an imperfect truss, in which the number of members is NOT sufficient to prevent distortion of its shape when loaded with an external load.
   - It is also known as collapsible truss as under the action of collapsible truss the truss tends to collapse.
   - It satisfies the following equation $m < 2j-3$
3) **REDUNDANT TRUSS**
   - A redundant truss is an imperfect truss in which the number of members is more than $2j-3$ i.e. $m > 2j-3$

ASSUMPTIONS OF IDEALIZE TRUSS ANALYSIS

- Members are connected at their ends by smooth frictionless pins.
- The weight of members is negligible.
- Loads and reactions are applied to truss at joints only.
- The centroidal axis of each member is straight and coincides with the line connecting the joint centres at the end of the member.
- All members function as two-force members i.e. they are subjected to either tension or compression.

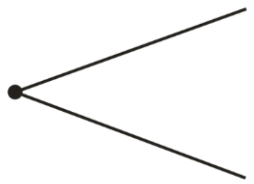
ANALYTICAL METHOD OF TRUSS ANALYSIS

1) METHOD OF JOINTS

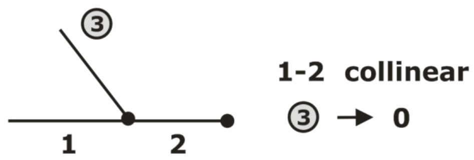
- In order to analyze or design a truss, we must obtain the
- Force in each of its members can be obtained by making free body diagram of respective joint and applying equation of equilibrium analysis.
- At each joint the forces in the members meeting at the joint and the loads at the joint, if any, constitute a system of concurrent forces.
- For starting analysis, a joint is selected where there are only two unknown forces.

Note.

- If two non collinear members are in static equilibrium, then these two members must have zero force.



- When three members are meeting at an unloaded joint and out of them two are collinear, then the force in third member will be zero.



2) METHOD OF SECTIONS

- When we need to find the force in only a few members of a truss, we can analyze the truss using method of sections.
- It is based on the principle that if a body is in equilibrium then any part of the body is also in equilibrium.
- In the method of sections, a section is of the truss is cut, such that not more than three unknown forces are required to be computed. Free body diagram of either of its two parts is drawn and unknown obtain by applying equation of equilibrium analysis.

Note.

- Firstly, the reactions are need to be find.

CHAPTER-3 FRICTION

INTRODUCTION

Friction is the force distribution at the surface of contact between two bodies that prevents or impedes sliding motion of one body relative to the other. It is the frictional force, which causes body to stop.

DRY FRICTION

Dry friction is said to occur when there is relative motion between two completely unlubricated surfaces. It is further divided into two types:

- **Sliding Friction**: When the two surfaces have a sliding motion relative to each other.
- **Rolling Friction**: Friction due to rolling of one body over another.

FILM FRICTION

When two surfaces in contact are completely separated by lubrication, friction will occur due to the shearing of different layers of the lubricant. This is known as film friction or viscous friction.

BOUNDARY FRICTION

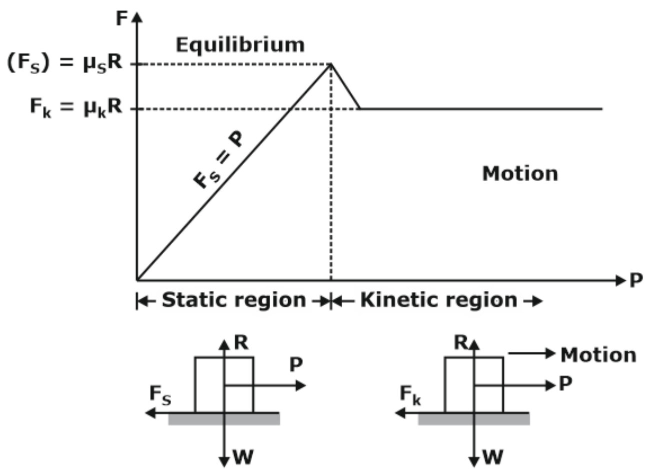
When two surfaces in contact have a minute thin layer of lubricant between them, it is known as boundary or skin friction.

Note.

- Friction force is self-adjusting in nature. The friction force cannot increase beyond a limit. Thus, there is a limiting value of friction, known as **limiting friction**.
- When the applied force is less than limiting friction, the body remains at rest and such frictional force is called **static friction**, which may have any value between zero to limiting friction.
- If the value of the applied force exceeds the limiting friction, the body starts moving over the other body and the frictional resistance experienced by the body while moving is known as **Dynamic friction** or **Kinetic friction.**
- It is experimentally found that the magnitude of limiting friction bears a Constant ratio to the normal reaction between the two surfaces and this ratio is called coefficient of Friction

$$\text{Coefficient of friction} = F_s/R$$

where F_s is limiting friction and R, the normal reaction between the contact surfaces.



- The graph between the applied load P and the friction force F is linear up to the point when limiting friction is reached.

- The force of static friction between any two surfaces in contact is opposite the applied force and can have values given by $F_s \le \mu_s R$, where μ_s is called coefficient of static friction.
- When the block is on the verge of slipping, force of static friction is given by $F_s = (F_s)_{max} = \mu_s R$
- The force of kinetic friction acting on an object is opposite to the direction of motion of the object and is given by $F_k = \mu_k R$, where μ_k is a constant called coefficient of kinetic friction.
- The values of coefficients of friction μ_s and μ_k do not depend upon the area of the surfaces in contact. Both the coefficients, however, depend strongly on the nature of the surface in contact i.e. roughness/smoothness.
- μ_k is generally less than μ_s

SIGNIFICANCE OF FRICTION

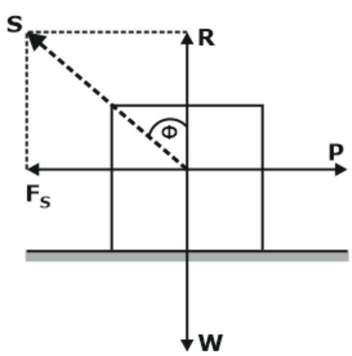
Friction is generally undesirable force because additional power is required to overcome the friction. However, it is a desirable feature in certain situation, For example

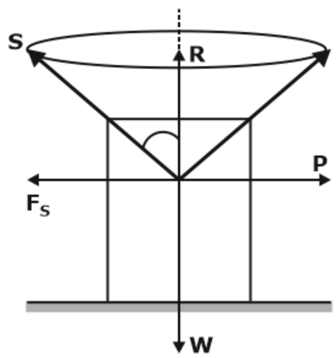
- friction helps in walking and moving on road
- friction forces are used to stop moving vehicle by applying brakes
- to transmit power by belt drives
- facilitates use of ladder when friction force prevents slipping of the ladder

ANGLE OF FRICTION

- Angle θ which resultant S subtends with the normal to the plane, when the body just starts sliding over the horizontal plane. This is also known as limiting angle of reaction or friction.
- It is sometimes found convenient to replace the normal force R and the friction force F_s by their resultant S. From the geometry of Fig, we note that

$$\tan\phi = \frac{F_s}{R} = \frac{\mu R}{R} = \mu$$



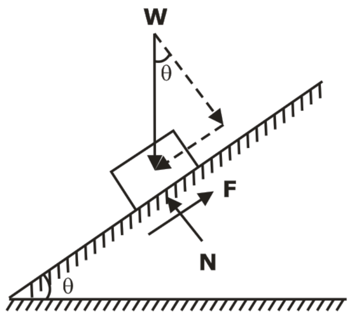


- limiting angle of friction $\phi = \tan^{-1}\mu$
- This indicates that the friction coefficient is tangent of angle of friction.

- We know F_s vary from 0 to limiting value, So if F_s form a circle, then resultant S will describe a right circular cone of apex angle 2ϕ about the line of action R as its axis, then this cone is called a **Cone of friction**.
- The body will be stationary, if resultant is within cone of friction.

ANGLE OF REPOSE

- The maximum inclination of the plane on which a body, free from external forces, experiences repose (sleep) is called Angle of Repose.
- Applying Equilibrium equation to the block shown above, we get $\mu = \tan\theta$
- In terms of angle of friction φ, $\theta = \phi$



Direction of friction on DRIVING AND DRIVEN WHEELS OF AN AUTOMOBILE

- On a smooth surface driving wheels will rotate about axis of axle but vehicle will not move.
- Frictional torque on driving wheels is overcome by engine torque i.e. frictional torque direction opposite to engine torque. It provides driving force or tractive force to the vehicle.

$$0 \le (F = T / r) \le F_s$$

  where F_s is limiting friction, T is engine torque and r is radius of wheel.
- Friction on driven wheels provides motion to driven wheels i.e. friction direction is along the motion of wheel.

CHAPTER-4 DYNAMIC

INTRODUCTION

Dynamics is part of mechanics that deals with the analysis of bodies in motion. Its subparts are

- **Kinematics** is the study of the geometry of motion. It is used to related to displacement, velocity, acceleration and time, without reference to the cause of the motion.
- **Kinetics** is the study of the relation existing between the forces acting on a body, the mass of the body and the motion of the body.

REST

When a body occupies a fixed position for any length of time, it is said to be at rest.

MOTION

A body is said to be in motion when it changes its position with respect to some other fixed object.

- **RECTILINEAR OR TRANSLATORY MOTION**
  Motion in which a particle or point mass body is moving along a straight line.
- **CIRCULAR OR ROTARY MOTION**
  A circular motion is that motion in a particle or a point mass body is moving on a circle.
- **OSCILLATORY OR VIBRATORY MOTION**
  Oscillatory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point (called mean position) in a definite interval of time.

FRAME OF REFERANCE

Frame of reference is a system of coordinate axes attached to an observer having a clock with him, with respect to which, the observer can describe position, displacement, acceleration etc. of a moving object.

- **Inertial frame** of reference is one in which newton's first law of motion holds good. For example, a frame of reference attached to a person in a bus at rest or moving with a uniform velocity along a straight line.
- **Non-Inertial frame** of reference is one in which newton's first law of motion does not hold good. For example, a frame of reference attached to a person in a bus moving with uniform velocity along circular curve or moving with acceleration along a straight line.

DISPLACEMENT

- The change in position of a body in a definite direction is known as displacement and it is measured by straight distance between the initial and the final positions of the moving body.
- The displacement represents both magnitude and direction followed by the body.
- It is a vector quantity.

SPEED

The speed of a moving body is the rate of change of its position.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time Taken}}$$

UNIFROM SPEED

An object is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time.

VARIABLE SPEED

An object is said to be moving with a variable speed if it covers equal distances in unequal intervals of time or unequal distances in equal intervals of time.

AVERAGE SPEED

Average speed of the object is that constant speed with which the object covers the same distance in a given time as it does while moving with variable speed during the given time.

$$\text{Average Speed} = \frac{\text{Total Distance travelled}}{\text{Total Time Taken}}$$

INSTANTANEOUS SPEED

The speeds of an object at a given instant of time is called its instantaneous speed.

VELOCITY

The velocity of a moving body is the time rate of change of its displacement.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time Taken}}$$

UNIFORM VELOCITY

Velocity of object with which, it undergoes equal displacements in equal intervals of time. An object is said to be in uniform motion if its velocity is uniform.

VARIABLE VELOCITY

An object is said to moving with a variable velocity, if either its speed (magnitude) or its direction of motion or both change with time.

AVERAGE VELOCITY

Average velocity is that uniform velocity with which the object will cover the same displacement in a given interval of time as it does with its actual velocity during the given interval of time.

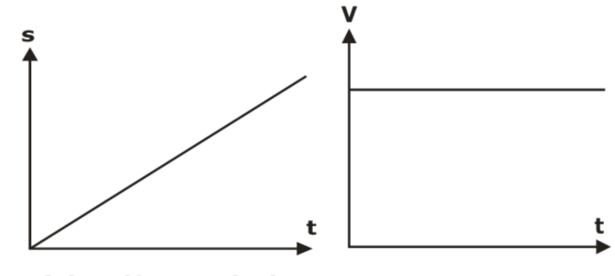
INSTANTANEOUS VELOCITY

The velocity of an object at a given instant of time is called its instantaneous velocity.

GRAPHICAL REPRESENTATION OF VELOCITY, TIME AND DISTANCE TRAVELLED

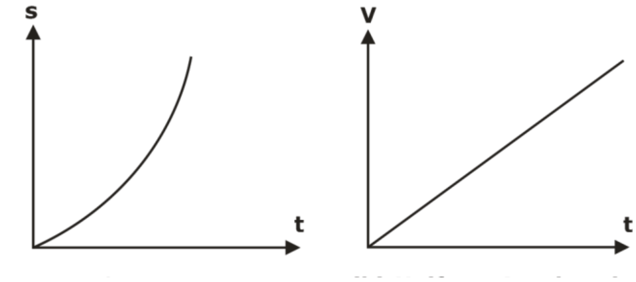
- The slope of the position-time graph gives velocity of the object. Straight line parallel to position axis represent infinite velocity, which is not possible.
- The slope of velocity-time graph gives acceleration of object.
- In velocity-time graph, for the given interval of time, the positive area enclosed with the time axis gives positive displacement and the negative area enclosed with the time axis gives negative displacement.

- Distance travelled by a body in a given interval of time is equal to total area of velocity-time graph, without considering sign.

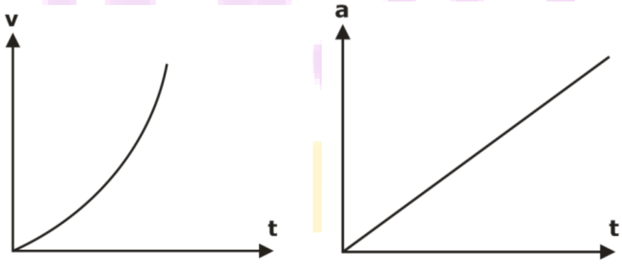


Uniform Motion (Uniform Velocity)

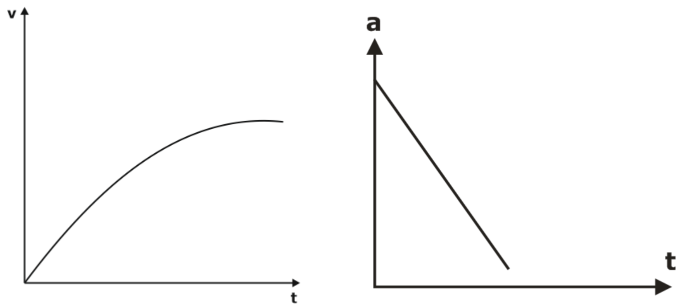
.



Uniform Acceleration



Uniformly Increasing Acceleration



Uniformly Decreasing acceleration

MOTION UNDER VARIABLE ACCELERATION

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{ds}{dt} \times \frac{dv}{ds} = v\frac{dv}{ds}$$

$$ads = vdv$$

SCALAR & VECTORS

- Scalars are those physical quantities which have magnitude only but no direction i.e. mass, length.
- Vectors are those physical quantities which have both magnitude and direction i.e. velocity, acceleration etc.
- A vector can be represented by a straight line with arrow head on it. The length of line drawn on suitable scale represents the magnitude of vector and arrow head tells the direction of the vector.
- The starting point of arrowed line is called tail or origin of vector and the end of the arrowed line is called tip or head of the vector.

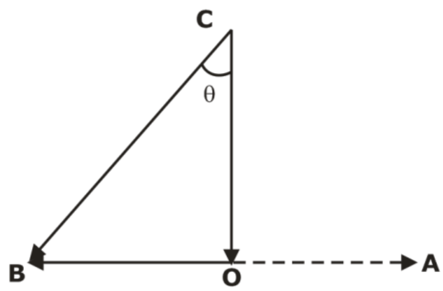
VECTOR ADDITION

The general rule of vector addition is used which states that the vectors to be added are arranged in such a way so that the head of first vector coincides with the tail of second vector, whose head coincides with the tail of third vector and so on, then the single vector drawn from the tail of the first vector to the head of the last vector represents their resultant vector.

RELATIVE VELOCITY OF RAIN WRT THE MOVING MAN

The relative velocity of rain with respect to moving man is the velocity of rain which the man will feel.

- STEP 1. Draw Lines showing actual velocity of Rain and Man... (OC and OA in the Fig.)
- STEP 2. Now draw a Line in opposite direction of actual velocity of Man... (OB in Fig.)
- STEP 3. The Resultant velocity CB in triangle COB is relative velocity of the rain and man. It will give the magnitude and direction of rain which a man will feel.



Equation of PLANE MOTION under uniform acceleration

$v = u + at$

$s = ut + \frac{1}{2} at^2$

$s_{nth} = u + \frac{1}{2} a(2n - 1)$

$v^2 - u^2 = 2as$

where u = Initial velocity

v = Final velocity

t = Time taken by the particle to change its velocity from u to v

a = Uniform +ve acceleration

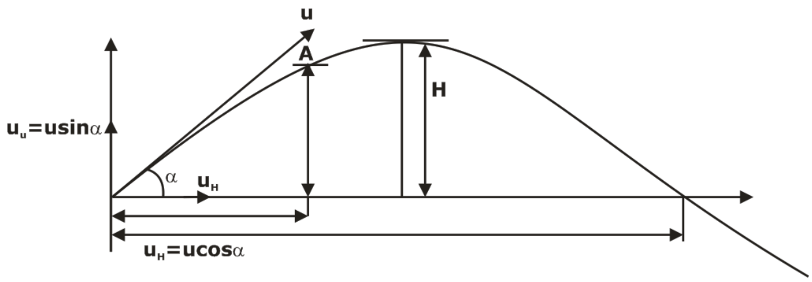
s = distance travelled in t seconds

n = n is integer number for which second we study

PROJECTILE

A Particle, moving under combined effect of horizontal and vertical forces is called a projectile.

- Horizontal component: remains constant if air resistance is ignored.
- Vertical component: always subjected to gravitational acceleration. A rocket is not a projectile.



Here u = Velocity of projection

α = Angle of projection with horizontal

Initial Horizontal component of velocity = u_H = u cosα

Initial Vertical Component of velocity = u_v = u sinα

Applying Equation of plane motion along horizontal and vertical direction we get

- Trajectory of projectile $y = x\tan\alpha - \dfrac{gx^2}{2u^2\cos^2\alpha}$
- Time for which the projectile remained in air $T = \dfrac{2u\sin\alpha}{g}$
- Horizontal Range of a Projectile $R = \dfrac{u^2\sin2\alpha}{g}$, $R_{max} = \dfrac{u^2}{g}$ at $\alpha = 45°$
- Maximum height of a projectile $H = \dfrac{u^2\sin^2\alpha}{2g}$

MOTION OF ROTATION

Motion of rotation which takes place about the geometric axis of the body.

- **ANGULAR VELOCITY**
  - Rate of change of angular displacement of a body.
  - Denoted by ω.
  - Expressed in rad/s.
  - For N rpm, ω = 2π radian/ minute.
  - For N rpm, ω = 2π N/60 r.p.m = 2π N/60 rad/sec
- **ANGULAR ACCELERATION**
  - Rate of change of angular velocity.
  - Denoted by α.
  - Expressed in rad/sec^2.
- **ANGULAR DISPLACEMENT**

- Total angle, through which a body has rotated.
- Denoted by θ.
- Expressed in radian.
- $\theta = \omega \times t$ for uniform velocity

- **MOTION UNDER UNIFORM ACCELERATION**

$\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \alpha t^2/2$

$$\theta_{nth} = \omega_1 + (2n-1)\frac{\alpha}{2}$$

$$\omega_0^2 = \omega^2 + 2\alpha\theta$$

Where ω_0 =initial velocity

ω= Final velocity

t =Time taken by the particle to change its velocity from ω_0 to ω

α =Uniform angular acceleration

θ= Total angular displacement in t seconds

- **LINEAR (TANGENTIAL) VELOCITY OF A ROTATING BODY**

ω =Angular velocity of the body in rad/sec

r =Radius of circular path

v= Linear velocity of particle on the periphery in m/sec.

θ =Total angular displacement in t seconds

l =Total length of arc covered in t seconds = $r\theta$

Linear Velocity =Length of arc/ time taken

$$v = \frac{d}{dt}(r\theta) \rightarrow v = \omega r$$

- **LINEAR (TANGENTIAL) ACCELERATION OF A ROTATING BODY**

α =Uniform angular acceleration

a =Linear acceleration

Linear acceleration (a) = dv/dt and v = r*ω

$$\frac{dv}{dt} = r\frac{d\omega}{dt} \rightarrow a = r\alpha$$

- **RADIAL AND NORMAL VELOCITIES IN CYLINDRICAL COORDINATES**

Resultant velocity of particle at $P(r,\theta)$ $\quad v = \sqrt{v_r^2 + v_\theta^2}$

$$v = \sqrt{\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\theta}{dt}\right)^2}$$

- **RADIAL AND NORMAL ACCELERATIONS IN CYLINDRICAL COORDINATES**

Resultant acceleration, $a = \sqrt{a_r^2 + a_\theta^2}$

Where $a_\theta = r\frac{d^2\theta}{dt^2} + 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right) = \frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)$

And $a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$

Special case: When the path of particle is circular, radius of circle, r, is constant.

Radial acceleration, $a_r = 0 - r\left(\frac{d\theta}{dt}\right)^2 = -r\omega^2$, where $\omega = \frac{d\theta}{dt}$

Transverse acceleration, $a_\theta = r\left(\frac{d^2\theta}{dt^2}\right) = r\alpha$ where $\alpha = \frac{d^2\theta}{dt^2}$

$$a_r = -\omega^2 r \text{ and } a_\theta = r\alpha$$

MOTION ALONG A CIRCULAR PATH

A body, moving in a circle or along a circular path, with a constant velocity

- Suffers a continuous change in its direction at every point of motion
- But, magnitude of its speed remains the same.
- Since velocity involves both magnitude and direction, hence velocity is continuously changing.
- Some force must act at right angle to the direction of motion at every point, which should change the direction of the motion of the body.
- The force, which acts along the radius of the circle at every point, and is always directed towards the centre of the circle along which the body moves, is known as Centripetal Force.
- A pseudo force centrifugal force act opposite to centripetal force.

r = radius of the circular path in m, and

v = Linear velocity of the particle in m/s

ω = Angular velocity in rad/s

a = centripetal or centrifugal acceleration.

$a = v^2/r = \omega^2 r$

LAWS OF MOTION

- **NEWTON'S FIRST LAW OF MOTION**

  "Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force."

- **NEWTON'S SECOND LAW OF MOTION**

  "The rate of change of momentum is directly proportional to the impressed force and takes place, in the same direction in which the force acts."

m = mass of the body

u = Initial velocity of the body

v = Final velocity of the body

a = Constant acceleration

t = time required to change velocity from u to v

Initial momentum = mu

Final momentum = mv

Rate of change of momentum = (mv – mu)/t = m (v – u)/t = ma

Acc. To law, Impulse force $P \alpha ma \rightarrow P = ma$

- **NEWTON'S THIRD LAW OF MOTION**

  The mutual forces of action and reaction between two particles are equal, opposite and collinear or "To every action, there is always an equal and opposite reaction."

D'ALEMBERT'S PRINCIPLE

D'Alembert proposed a method of analysis that would transform a dynamic problem into equivalent problem in statics. The essence of his idea, which is known as D'Alembert's principle is to consider ma as the force acting on particle called inertial force. Newton's second law F-ma=0 can then be treated as an equilibrium condition that involves the applied forces together with inertial force.

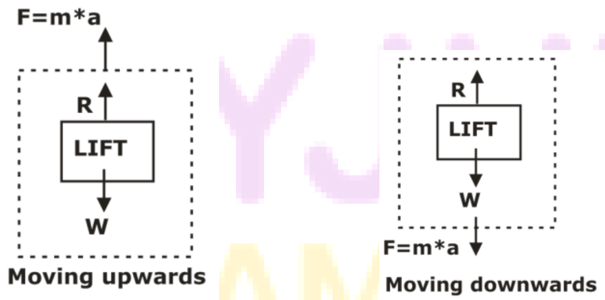
MOTION OF A LIFT

Consider a lift carrying some mass m and moving with a uniform acceleration a.

W = Weight carried by Lift

R = Reaction of the lift or tension in the cable, supporting the lift.

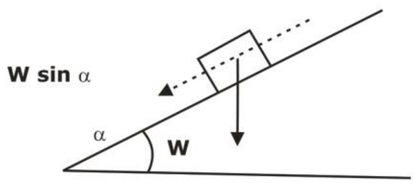
- **WHEN THE LIFT IS MOVING UPWARDS** Accelerating force $F = ma = R - mg$
- **WHEN THE LIFT IS MOVING DOWNWARDS** Accelerating force $F = ma = mg - R$



MOTION ON AN INCLINED SMOOTH SURFACE

Consider a body having weight W = mg sliding downwards with acceleration a.

Accelerating Force $= ma = mg \sin \alpha$ i.e. $a = g \sin \alpha$



WORK

- Whenever a force acts on a body, and the body undergoes a displacement.

  Work done = Component of the force in the direction of motion x Distance

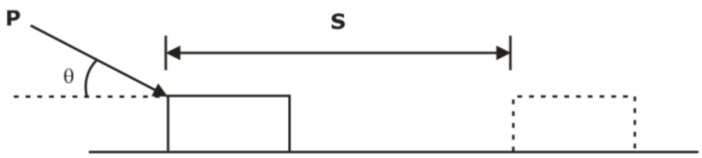
  $W = P \cos \theta S$

  Where P = Applied force

  S = Displacement of the body

  θ = Angle between applied force and direction of motion

- Unit of work: 1 J or Nm
- Work done is equal to Area under curve on Force displacement diagram.



POWER

- Rate of doing work
- Indicated Power: The actual power generated in engine cylinder.
- Brake Power: Power available for useful work.
- Unit: watt (W) on Nm/s.

ENERGY

- Capacity to do work.
- **Potential Energy**. Energy possessed by a body by virtue of its position. P.E. = mgh

  Where m =mass of a body

  h = position of body above the datum level

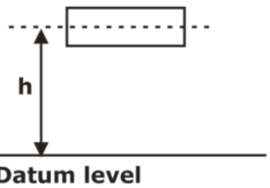


Fig:40

- **Kinetic Energy**: Energy possessed by a body by virtue of its velocity. K.E. = ½ (mv^2)

  Where m = mass of body

  v = velocity of body
- **Law of Conservation of Energy:** "The energy can neither be created nor destroyed, though it can be transformed from one form into any of its forms, in which energy can exist."

Corresponding term used in Linear motion and Rotation motion

Linear Motion	**Motion of Rotation**
1. Mass(m)	Moment of Inertia (I)
2. Distance travelled (s)	Angular displacement (θ)
3.Linear motion (v)	Angular motion (ω)
4. Linear acceleration (a)	Angular acceleration (α)
5.Linea Momentum = mv	Angular Momentum = Iω
6.Force (F)	Torque (T)
7. Force F = ma	Torque T = Iα
8.Linear K.E. = $mv^2/2$	Angular K.E. = $I\omega^2/2$
9. Work done = Fs	Work done = Tθ
10. Conservation of momentum	Conservation of angular momentum

CHAPTER-6-CONSERVATION OF MOMENTUM PRINCIPLE

COLLISION

- The bodies after collision come momentarily to rest.
- The two bodies tend to compress each other, so long as they are compressed to the maximum value. The time taken is called time of compression.
- The two bodies attempt to regain its original shape due to their elasticity. This process is known as restitution and time taken during restitution is called the time of restitution.
- The line of impact is the common normal to both the bodies. The plane of impact is the common tangent plane of bodies.
- ***Time of collision = time of compression + time of restitution***
- **Direct collision** is the collision in which the two bodies, before impact are moving along the line of impact.
- **Indirect collision** if the two bodies, before impact are not moving along the line of impact.

LAW OF CONSERVATION OF MOMENTUM

"The total momentum of two bodies remains constant after their collision."

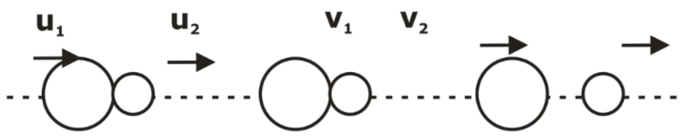
m_1 = mass of first body

u_1 = Initial velocity of first body

v_1 = Final velocity of first body

m_2, u_2, v_2 = corresponding values for the second body.

$$m_1u_1+m_2 u_2 = m_1v_1+ m_2 v_2$$



NEWTON'S LAW OF COLLISION

"When two bodies collide with each other, their velocity of separation bears a constant ratio to their velocity of approach."

m_1 = mass of first body

u_1 = velocity of first body before impact

v_1 =velocity of first body after impact

m_2, u_2, v_2 = Corresponding values for the second body.

Let us assume that $u_1 > u_2$ and $v_2 > v_1$

$$v_2 - v_1 = e(u_1 - u_2)$$, Where e = coefficient of restitution

- e varies between 0 and 1.
- Perfectly elastic collision: When e = 1, velocity of approach = velocity of separation
- Inelastic collision: when e = 0, $v_2 = v_1$

LOSS OF KINETIC ENERGY

- Only in Inelastic collision.
- Difference between kinetic Energies of system before and after collision.

$$E_L = E_f - E_i = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 - \frac{1}{2}m_1{u_1}^2 - \frac{1}{2}m_2{u_2}^2$$

CHAPTER-7 CENTROID, CENTRE OF GRAVITY, MOMENT OF INERTIA

INTRODUCTION

- In engineering structures, the members of various cross-sections are used to withstand loads.
- The load carrying capacity depends on type of material used as also cross-section of the members.

CENTROID

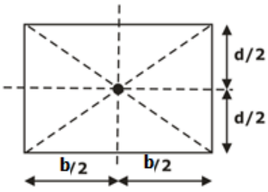
- Centroid is the point where the whole area is assumed to be concentrated.
- It is related to shape or the geometry of object.
- Centroid of line $\bar{x} = \frac{\int xdL}{L}$, $\bar{y} = \frac{\int ydL}{L}$ and $\bar{z} = \frac{\int zdL}{L}$. where $\int dL = L$
- Centroid of Area $\bar{x} = \frac{\int xdA}{\int dA}$, $\bar{y} = \frac{\int ydA}{\int dA}$, $\bar{z} = \frac{\int zdA}{\int dA}$, where $\int dA = A$
- Centroid of Volume $\bar{x} = \frac{\int xdV}{V}$, $\bar{y} = \frac{\int ydV}{y}$ and $\bar{z} = \frac{\int zdV}{V}$ where $\int dV = V$

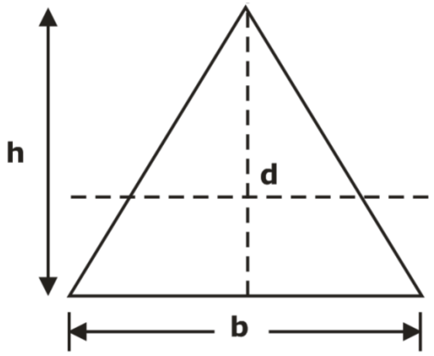
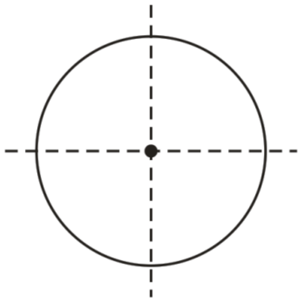
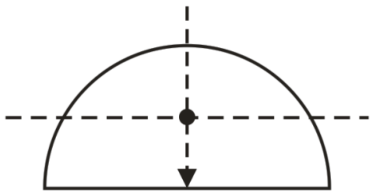
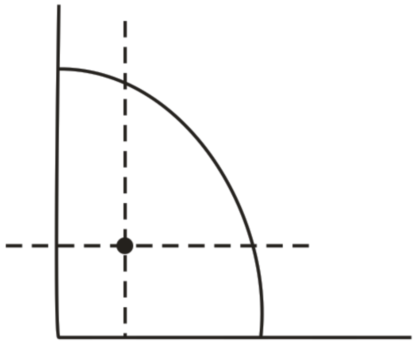
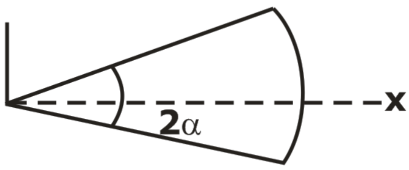
CENTRE OF GRAVITY

- Centre of gravity of the body or the system of particles rigidly connected together is that point where the weight of body is concentrated and gravitational force acts through it.
- $\bar{x} = \frac{\int xdm}{\int dm}$, $\bar{y} = \frac{\int ydm}{\int dm}$, $\bar{z} = \frac{\int zdm}{\int dm}$, where $\int dm = M$

Note.

When density is uniform i.e. the body material is homogenous, centroid and C.G are same but when density is not same, two point will not be some. C.G lies where density is more, i.e. weight more.

SHAPE	FIGURE	$\bar{x}$	$\bar{y}$
Rectangle		$\frac{b}{2}$	$\frac{d}{2}$

Triangle		$\frac{b}{2}$	$\frac{h}{3}$
Circle		$\frac{r}{2}$	$\frac{r}{2}$
Semi-Circle			$\frac{4R}{3\pi}$
Quarter of Circle		$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$
Sector of a Circle		$\frac{2R}{3\alpha}\sin\alpha$	0

AREA MOMENT OF INERTIA

Moment of Inertia for an area: - This is also known as second moment of area.

$I_{xx} = \int dA\, y.y$ = second moment of area about x axis

$I_{yy} = \int dA\, x.x$ = second moment of area about y axis.\

THEOREM OF PARALLEL AXIS

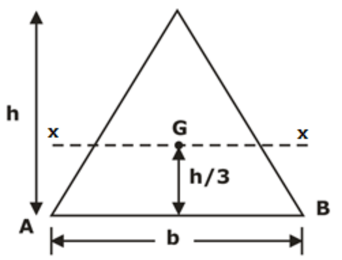
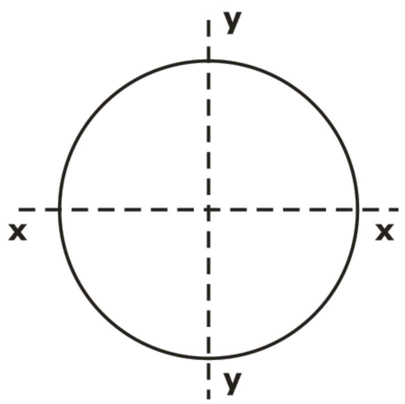
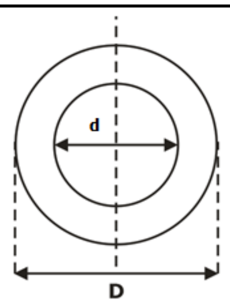
The moment of Inertia about any axis parallel to centroidal axis at a distance (h) is equal to sum of moment of Inertia about centroidal axis and product of area and square of distance B/w two axis.

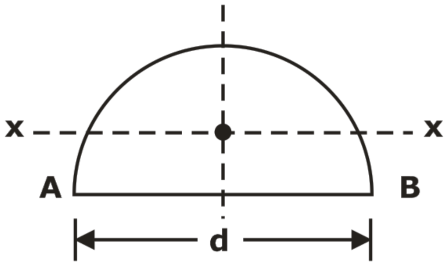
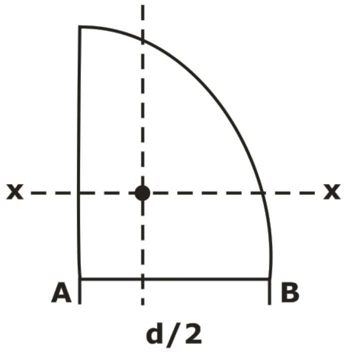
$$I_{parallelAxis} = I_{Centroid} + Ah^2$$

THEOREM OF PERPENDICULAR AXIS

If moment of Inertia about two perpendicular centroidal axis in a plane are given (I_1, I_2) then moment of Inertial about third axis which is perpendicular to both mutually perpendicular axis passing through the point of intersection is equal to sum of moment of Inertia about two centroidal axis.

$$I_{zz} = I_{xx} + I_{yy}$$

SHAPE	AXIS	Moment of Inertia
	x-x AB	$\frac{bh^3}{36}$ $\frac{bh^3}{12}$
	x-x y-y	$\frac{\pi}{64}D^4$ $\frac{\pi}{64}D^4$
	x-x y-y	$\frac{\pi}{64}\left(D^4 - d^4\right)$ $\frac{\pi}{64}\left(D^4 - d^4\right)$

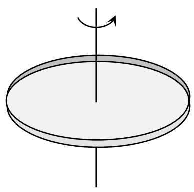
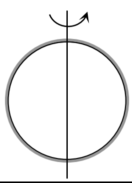
	x-x AB	$0.0068598\ d^4$ $\frac{\pi d^4}{128}$
	x-x AB	$0.00343\ d^4$ $\frac{\pi d^4}{256}$

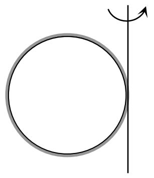
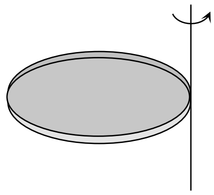
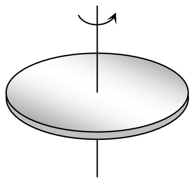
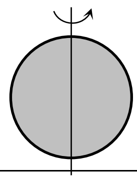
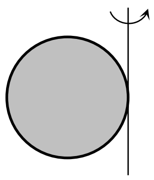
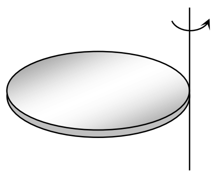
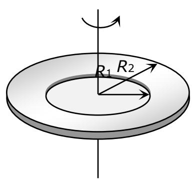
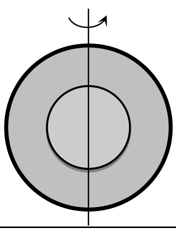
MASS MOMENT OF INERTIA

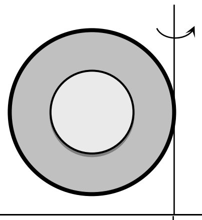
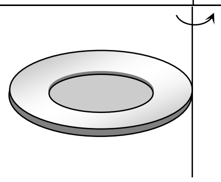
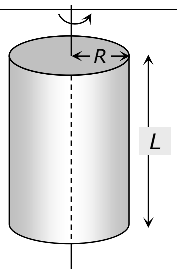
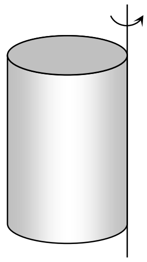
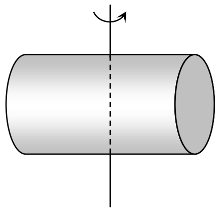
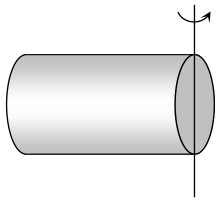
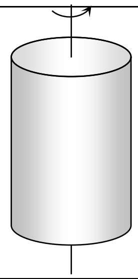
Moment of moment of mass. Also called second moment of mass. Theorem of parallel axis and perpendicular axis also applies in it.

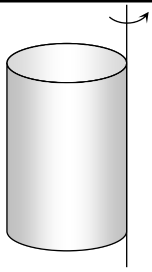
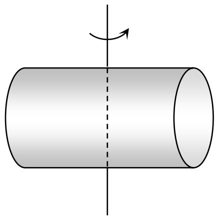
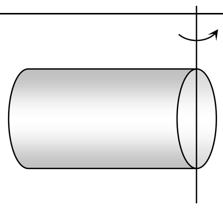
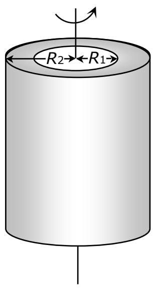
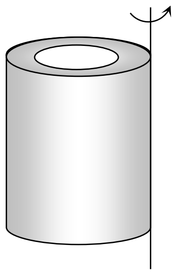
RADIUS OF GYRATION

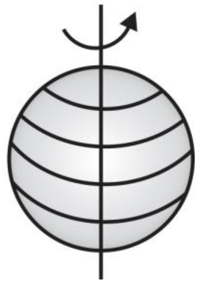
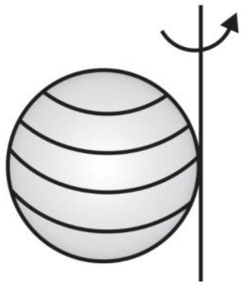
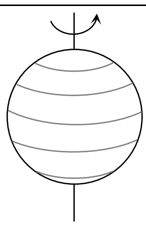
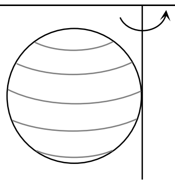
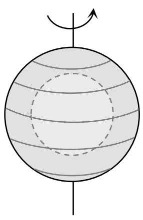
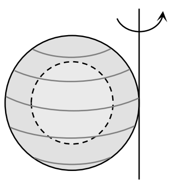
- The radius of gyration k describes the way in which the area of a cross-section is distributed around its centroidal axis.
- If the area is concentrated far from the centroidal axis, it will have a greater value of k and a greater resistance to buckling (or bending).
- The radius of gyration is defined as: $k = \sqrt{\frac{I}{A}}$, where k is the radius of gyration, I is the moment of inertia, and A is the area of cross-section.

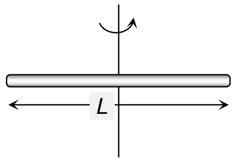
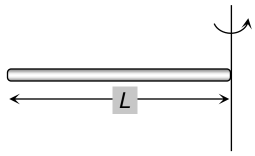
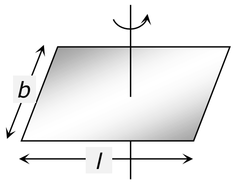
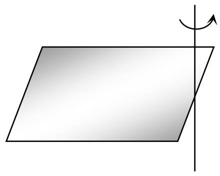
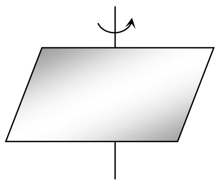
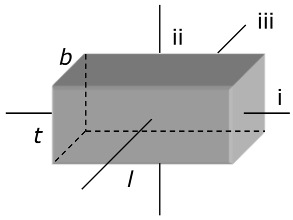
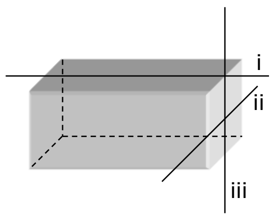
Body	Axis of Rotation	Figure	Moment of inertia	*k*	*k^2/R^2*
Ring	About an axis passing through C.G. and perpendicular to its plane		MR^2	*R*	1
Ring	About its diameter		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$

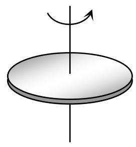
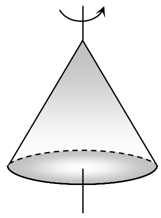
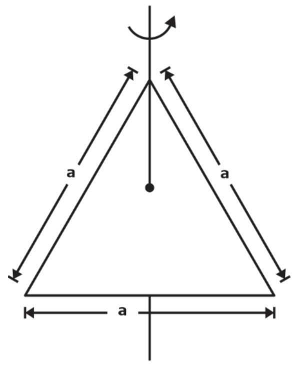
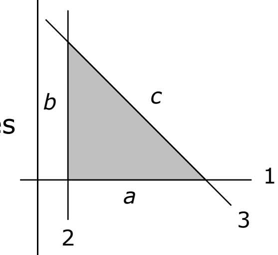
Body	**Axis of Rotation**	**Figure**	**Moment of inertia**	***k***	***k²/R²***
Ring	About a tangential axis in its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Ring	About a tangential axis perpendicular to its own plane		$2MR^2$	$\sqrt{2}R$	2
Disc	About an axis passing through C.G. and perpendicular to its plane		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Disc	About its Diameter		$\frac{1}{4}MR^2$	$\frac{R}{2}$	$\frac{1}{4}$
Disc	About a tangential axis in its own plane		$\frac{5}{4}MR^2$	$\frac{\sqrt{5}}{2}R$	$\frac{5}{4}$
Disc	About a tangential axis perpendicular to its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Annular disc inner radius = R_1 and outer radius = R_2	Passing through the centre and perpendicular to the plane		$\frac{M}{2}[R_1^2 + R_2^2]$	–	–
Annular disc	Diameter		$\frac{M}{4}[R_1^2 + R_2^2]$	–	–

Body	**Axis of Rotation**	**Figure**	**Moment of inertia**	***k***	***k²/R²***
Annular disc	Tangential and Parallel to the diameter		$\frac{M}{4}[5R_1^2 + R_2^2]$	–	–
Annular disc	Tangential and perpendicular to the plane		$\frac{M}{2}[3R_1^2 + R_2^2]$	–	–
Solid cylinder	About its own axis		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Solid cylinder	Tangential (Generator)		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Solid cylinder	About an axis passing through its C.G. and perpendicular to its own axis		$M\left[\frac{L^2}{12} + \frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{12} + \frac{R^2}{4}}$	
Solid cylinder	About the diameter of one of faces of the cylinder		$M\left[\frac{L^2}{3} + \frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{3} + \frac{R^2}{4}}$	
Cylindrical shell	About its own axis		MR^2	R	1

Body	**Axis of Rotation**	**Figure**	**Moment of inertia**	***k***	**k^2/R^2**
Cylindrical shell	Tangential (Generator)		$2MR^2$	$\sqrt{2}R$	2
Cylindrical shell	About an axis passing through its C.G. and perpendicular to its own axis		$M\left[\frac{L^2}{12}+\frac{R^2}{2}\right]$	$\sqrt{\frac{L^2}{12}+\frac{R^2}{2}}$	
Cylindrical shell	About the diameter of one of faces of the cylinder		$M\left[\frac{L^2}{3}+\frac{R^2}{2}\right]$	$\sqrt{\frac{L^2}{3}+\frac{R^2}{2}}$	
Hollow cylinder with inner radius = R_1 and outer radius = R_2	Axis of cylinder		$\frac{M}{2}(R_1^2+R_2^2)$		
Hollow cylinder with inner radius = R_1 and outer radius = R_2	Tangential		$\frac{M}{2}(R_1^2+3R_2^2)$		

Body	Axis of Rotation	Figure	Moment of inertia	*k*	k^2/R^2
Solid Sphere	About its diametric axis		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$	$\frac{2}{5}$
Solid sphere	About a tangential axis		$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$	$\frac{7}{5}$
Spherical shell	About its diametric axis		$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$	$\frac{2}{3}$
Spherical shell	About a tangential axis		$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$	$\frac{5}{3}$
Hollow sphere of inner radius R_1 and outer radius R_2	About its diametric axis		$\frac{2}{5}M\left[\frac{R_2{}^5 - R_1{}^5}{R_2{}^3 - R_1{}^3}\right]$		
Hollow sphere	Tangential		$\frac{2M[R_2^5 - R_1^5]}{5(R_2^3 - R_1^3)} + MR_2^2$		

Body	Axis of Rotation	Figure	Moment of inertia	*k*	k^2/R^2
Long thin rod	About on axis passing through its centre of mass and perpendicular to the rod.		$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$	
Long thin rod	About an axis passing through its edge and perpendicular to the rod		$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$	
Rectangular lamina of length *l* and breadth *b*	Passing through the centre of mass and perpendicular to the plane		$\frac{M}{12}[l^2 + b^2]$		
Rectangular lamina	Tangential perpendicular to the plane and at the mid-point of breadth		$\frac{M}{12}[4l^2 + b^2]$		
Rectangular lamina	Tangential perpendicular to the plane and at the mid-point of length		$\frac{M}{12}[l^2 + 4b^2]$		
Rectangular parallelopiped length *l*, breadth *b*, thickness *t*	Passing through centre of mass and parallel to (i) Length (x) (ii) breadth (z) (iii) thickness (y)		(i) $\frac{M[b^2 + t^2]}{12}$ (ii) $\frac{M[l^2 + t^2]}{12}$ (iii) $\frac{M[b^2 + l^2]}{12}$		
Rectangular parallelepiped length *l*, breath *b*, thickness *t*	Tangential and parallel to (i) length (x) (ii) breadth (y) (iii) thickness(z)		(i) $\frac{M}{12}[3l^2 + b^2 + t^2]$ (ii) $\frac{M}{12}[l^2 + 3b^2 + t^2]$ (iii) $\frac{M}{12}[l^2 + b^2 + 3t^2]$		

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Elliptical disc of semimajor axis = *a* and semi minor axis = *b*	Passing through CM and perpendicular to the plane		$\frac{M}{4}[a^2+b^2]$		
Solid cone of radius R and height h	Axis joining the vertex and centre of the base		$\frac{3}{10}MR^2$		
Equilateral triangular lamina with side a	Passing through CM and perpendicular to the plane		$\frac{Ma^2}{6}$		
Right angled triangular lamina of sides *a*, *b*, *c*	Along the edges		(1) $\frac{Mb^2}{6}$ (2) $\frac{Ma^2}{6}$ (3) $\frac{M}{6}\left[\frac{a^2b^2}{a^2+b^2}\right]$		

****