CHAPTER-1: FUNDAMENTALS OF CONDUCTION

1. Modes of Heat Transfer:

- Conduction
- Convection
- Radiation

2. Conduction:

- Conduction is a mode of heat transfer which generally occurs in solids and static fluids by ***molecular lattice vibration energy transfer (30%) & also by free electron transfer (70%).***
- The highest conductivity (2100 W/m-K) of diamond is due to its perfect crystalline molecular lattice arrangement.
- The thermal conductivity of gases increases with increase of their temperature.
- ***Thermal conductivity of liquids decreases with increasing temperature*** as the liquid expands and the molecules move apart.
- In the case of solids, because of lattice distortions, higher temperatures make it difficult for electrons to flow, hence the **thermal conductivity of metals decreases with increase in temperature.**
  Exception – Aluminium & Uranium

3. Convection:

- Convection is a mode of heat transfer which generally occurs between a solid & a fluid due to temperature difference associated ***with macroscopic bulk displacement of the fluid*** transporting thermal energy (heat energy).
- In case of forced convection heat transfer this macroscopic bulk motion of the fluid is provided by an external agency like far or a blower or a pump.
- In case of free convection heat transfer, this motion is provided by buoyancy forces arising out of density changes of fluid due to its temperature change.

4. Radiation:

- Radiation is a mode of heat transfer which do not require any material medium for its propagation & hence occurs by electromagnetic waves.
- All bodies emit radiation above the 0K.

5. Governing Laws of Heat Transfer:

5.1. Fourier's Law of Conduction:

The law states that "the rate of heat transfer by conduction in a given direction is directly proportional to the temperature gradient along that direction & the area of heat transfer lying perpendicular to the direction of heat transfer".

It is given as

$$Q_x = -kA\left(\frac{dT}{dx}\right)\text{Watts}$$

'k' is thermal conductivity (thermo physical property) of material of slab.

A is the area perpendicular to direction of the heat flow.

Thermal conductivity - It is a thermophysical property of material which tells about its ability to allow the heat energy to get conducted through the material more rapidly.

Here are the list of some material and its thermal conductivity

Materials	Thermal Conductivity
Silver	405 W/mK
Copper	385 W/mK
Gold	319 W/mK
Aluminium	200 W/mK
Steels	17 to 45 W/mK
Asbestos	0.2 W/mK
Refractory bricks	0.9 W/mK
Glass wool	0.075 W/mK
Polyurethane Foam	0.02W/mK
Air	0.026W/mK
Water	0.63 W/mK
Mercury	8.34 W/mK

NOTE- Conductivity of Pure metal is higher than the conductivity of Alloy.

Conductivity in Gases - Conductivity in Gases is due to molecular momentum transfer.

NOTE- Conductivity of gases increases with the increase in temperature.

5.2 Newton's Law of Cooling of Convection:

$$\boxed{Q = hA\left(T_w - T_\infty\right)}$$

Where:

T_∞: Ambient temperature

h is convection heat transfer coefficient.

A is the area of contact.

It is not the property of the material but depends on some of the thermo physical properties of fluid.

5.3. Stefan – Boltzman law of radiation:

The law states that "Radiation Energy emitted from the surface of a black body per unit time & per unit area is directly proportional to the fourth power of the absolute temperature of the body".

$$\boxed{Q_b = \sigma \varepsilon A T^4}$$

Where σ is Stefan-Boltzman constant.

A = Area of Contact

ϵ = emissivity of the material

$\sigma = 5.67 \times 10^{-8}\ W/m^2k^4$

For Black Body: $\epsilon = 1$

$$\boxed{Q_b = \sigma A T^4}$$

Where σ is Stefan-Boltzman constant

$\sigma = 5.67 \times 10^{-8}\ W/m^2k^4$

6. CONDUCTION EQUATION

6.1. Conduction Heat Transfer Through a Slab:

Generalised conduction equation for 3-Dimensional body in cartesian coordinate:

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \left(\frac{\rho C_p}{K}\right)\frac{\partial T}{\partial \tau}}$$

Thermal diffusivity:

- It is given by: $\alpha = \dfrac{K}{\rho C_p}$.
- Its unit is m^2/sec.
- Thermal diffusivity (ɑ) of a medium signifies the rate at which heat energy can diffuse through the medium.

Case-1: If conditions are steady: $\dfrac{\partial T}{\partial \tau} = 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = 0$$

$$\boxed{\nabla^2 T + \frac{q}{K} = 0}$$ Poisson equation

Case-2: If there is no heat generation (q = 0):

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\left(\frac{\rho C_P}{K}\right)\frac{\partial T}{\partial \tau}$$

$$\boxed{\nabla^2 T=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}}$$ Diffusion equation

Case-3: When there is no heat generation and conditions are steady:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0$$

$$\boxed{\nabla^2 T=0}$$ Laplace equation

6.2. Radial conduction heat transfer through a cylinder:

Generalised conduction equation:

$$\boxed{\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{K}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}}$$

6.3. Radial conduction heat transfer through a sphere:

Generalised conduction equation:

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\psi}\frac{\partial}{\partial \psi}\left(\sin\psi\frac{\partial T}{\partial \psi}\right)+\frac{1}{r^2\sin^2\phi}\left(\frac{\partial^2 T}{\partial \phi^2}\right)+\frac{\dot{q}}{K}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}}$$

General Equation,

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)=\left(\frac{\rho C_P}{K}\right)\frac{\partial T}{\partial \tau}$$

For plane slab, n = 0

For cylinder, n = 1

For sphere, n = 2

Conduction with Heat Generation,1D

Slab

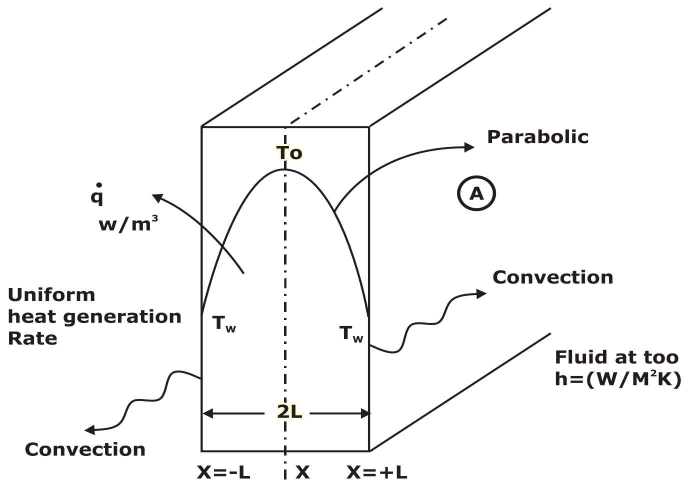
$$\frac{\partial^2 T}{\partial x^2}+\frac{\dot{q}}{K}=0$$

$$T(\text{at } x)=T_O-\frac{\dot{q}x^2}{2k}$$ (Parabolic Profile)

Temperature Distribution, $\frac{T_O-T}{T_O-T_W}=\frac{x^2}{L^2}$

Wall temperature: $T_w=\frac{\dot{q}L}{h}+T_\infty$

Maximum temperature: $T_0=T_w+\frac{\dot{q}}{k}\frac{L^2}{2}=\frac{\dot{q}L^2}{2k}+\frac{\dot{q}L}{h}+T_\infty$



In case, if one side of sab is completely insulated:

Boundary condition: $\left(\frac{dT}{dx}\right) = 0$ at the insulated surface.

Cylinder (cable wire)

Generalised conduction equation for radial conduction:

$$\boxed{\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{\dot{q}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}}$$

Wall temperature: $T_w = \frac{\dot{q}R}{2h} + T_\infty$

Maximum temperature: $T_{max} = T_w + \frac{\dot{q}R^2}{4K} = \frac{\dot{q}R^2}{4K} + \frac{\dot{q}R}{2h} + T_\infty$

Temperature Distribution, $\frac{T_O - T}{T_O - T_W} = \frac{r^2}{R^2}$

NOTE – Suppose current 'I' is flowing through a wire having length, 'L' radius 'R" Electrical resistance 'R_{elec}' for a time period of 't'

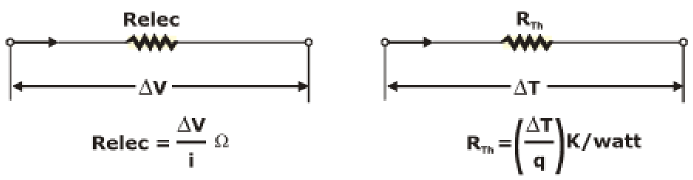
Then, heat generation inside the cylinder per unit Volume is given by $\dot{q} = \frac{I^2 R_{elec}}{\pi R^2 L}$

Sphere

Wall temperature: $T_w = \frac{\dot{q}R}{3h} + T_\infty$

Maximum temperature: $T_{max} = T_w + \frac{\dot{q}R^2}{6K} = \frac{\dot{q}R^2}{6K} + \frac{\dot{q}R}{3h} + T_\infty$

7. Electrical Analogy of Heat Transfer:



7.1. Plane slab with steady state, no heat generation:

Heat flow rate: $\boxed{Q = \dfrac{KA(T_1 - T_2)}{b}}$

$$\boxed{R_{Th} = \frac{T_1 - T_2}{Q} = \frac{b}{KA}} \text{ K / watt}$$

Temperature at a distance of 'x' is given as $T = T_1 - \dfrac{(T_1 - T_2)x}{L}$ $\quad T_1 > T_2$

Linear Temperature Profile

7.2. Cylinder with steady state, no heat generation:

Heat flow is given by: $q = \dfrac{2\pi kL(T_1 - T_2)}{\ell n \dfrac{r_2}{r_1}}$

Thermal resistance: $(R_{Th})_{cylinder} = \dfrac{(T_1 - T_2)}{Q} = \dfrac{\ell n\left(\dfrac{r_2}{r_1}\right)}{2\pi KL}$ K / watts

Logarithmic Temperature profile

7.3. Sphere with steady state, no heat generation:

Heat flow: $Q = \dfrac{4\pi k[T_1 - T_2]r_1 r_2}{(r_2 - r_1)}$ watt

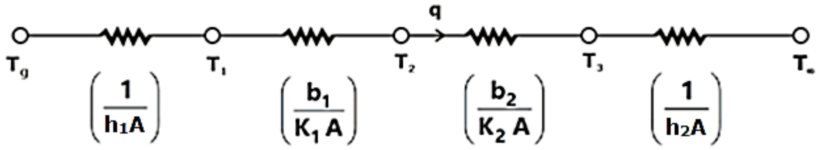
Thermal resistance: $(R_{Th})_{spherical} = \dfrac{(r_2 - r_1)}{4\pi k r_1 r_2}$ K / watt

Hyperbolic temperature Profile

7.4. Convection thermal resistance:

- Heat flow: $Q = hA\Delta T$
- ☐ $\boxed{(R_{Th})_{convection} = \left(\dfrac{1}{hA}\right) \text{ K / watt}}$
- ☐ Higher the convection heat transfer coefficient h or more the convection heat transfer area, smaller the convection thermal resistance.

7.5. Conduction – Convection Heat Transfer Through A Composite Slab:



7.6. Overall Heat Transfer Coefficient (U):

Overall heat transfer coefficient is a parameter which consider all the types of heat transfer into single entity.

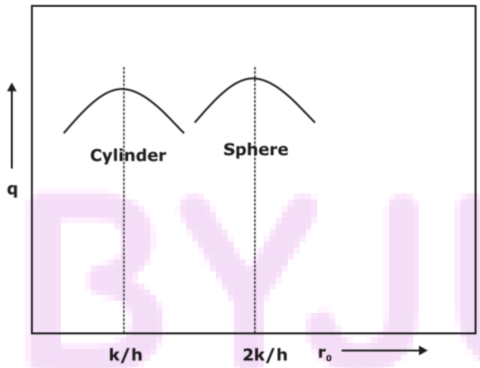
Unit – W/m^2K

- ☐ $q = UA.\Delta T$ watts
- ☐ $\boxed{(R_{Th}) = \left(\frac{1}{UA}\right)}$ K / watt

8. Critical Radius of Insulation:

It is the radius at which heat transfer rate from the surface is maximum.

For sufficiently thin wires, whose radius is lesser than critical radius of insulation, any insulation wrapped around the wire may result in increase of heat transfer rate (left hand side domain).



For cylinder: $\boxed{r_o = \frac{K_{ins}}{h_{surrouding}}}$

For sphere: $\boxed{(R_{critical})_{sphere} = \frac{2K_{ins}}{h_{surrouding}}}$

9. FINS (EXTENDED SURFACES):

Fins are projection protruding from the hot surface into ambient fluid to increase the heat transfer rate from the surface.

Fin efficiency: it is defined as the ratio of actual heat transfer from the fin to maximum possible heat transfer from the fin.

$$\text{Efficiency of fin}(\eta) = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,max}}$$

Effectiveness of fin: it is defined as the ratio of heat transfer rate when the fin is attached to the surface to the heat transfer rate hen there is no fin.

$$\text{Effectiveness}(\varepsilon_{fin}) = \frac{\dot{Q}_{fin}}{\dot{Q}_{withoutf\ fin}}$$

Case 1: Fin is infinitely long (or) very long fin:

Boundary condition, At x= 0, $T = T_O$, & At x = ∞, $T = T_\infty$

Temperature distribution: $\frac{\theta}{\theta_o} = \frac{T - T_\infty}{T_0 - T_\infty} = e^{-mx}$

m, $m = \sqrt{\frac{hp}{kA_c}}$

where, A_c is the cross-sectional Area

P is the perimeter

Heat transfer through Fin: $Q_{fin} = \sqrt{hPKA}\,\theta_0$ watts

Fin efficiency: $\eta_{fin} = \frac{1}{mL}$

Fin effectiveness: $\epsilon_{fin} = \frac{1}{\sqrt{\frac{hA}{kP}}}$

Case 2: Fins is finite in length & its tip is insulated:

Boundary condition, At x= 0, $T = T_O$, & At x = L, $\frac{dT}{dx} = 0$

Temperature distribution: $\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL}$

Heat transfer through Fin: $q_{thro\ fin} = \sqrt{hPkA}\,\theta_0 \tanh(mL)$ watt

Fin efficiency: $\eta_{fin} = \frac{\tanh(mL)}{mL}$

10. Relation between efficiency and effectiveness of the fin:

$$\text{Effectiveness}(\varepsilon) = \frac{\text{Surface area } (A_s)}{\text{Cross} - \text{section area } (A_c)} \times \text{Efficiency}(\eta)$$

For Fin being effective:

For fin being effective: $\epsilon > 1$

$$\varepsilon = \frac{1}{\sqrt{B_i}}$$

Thus, fin will be more effective if Biot No. is less than 1.

Addition of extended surfaces will not always increase in the heat transfer rate. For effective heat transfer rate $h < mk$.

CHAPTER-2: TRANSIENT HEAT CONDUCTION

1. Transient Heat Conduction:

T = f (time) or T = f (τ) not the function of space when a body is kept in environment whose temperature is either less or more than body. This type of conduction is called transient heat conduction.

2. Lumped heat capacity analysis:

If in any material, the internal temperature gradient within the body are neglected i.e. the temperature of the body is assumed to be uniform throughout it mass at any instant of time. Such analysis is called **lumped Heat capacity analysis**.

T = f (time) but T ≠ f (space)

$$\boxed{Bi = \frac{\text{Internal conductive resistance offered by body}}{\text{External convective resistance}} = \frac{ICR}{ECR}}$$

- Biot Number: Bi < 0.1
- Biot Number: $\boxed{Bi = \frac{hL_c}{K_{solid}}}$

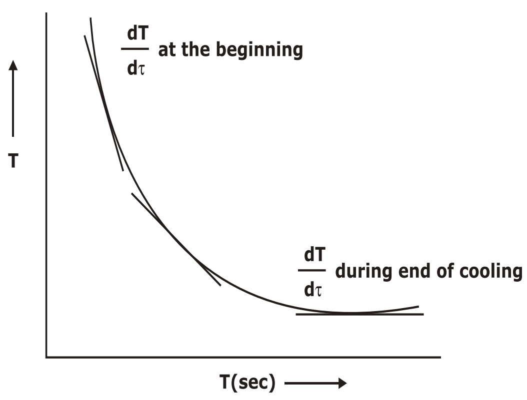
Where L_c = characteristic dimension of body

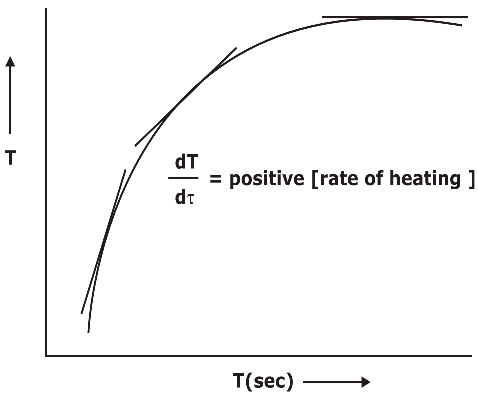
$$\boxed{L_c = \left(\frac{V}{A}\right) = \frac{\text{volume}}{\text{Surface Area}}}$$

For spherical Body: $\boxed{L_c = \frac{R}{3}}$

When a body having an initial temperature T is immersed in a fluid having temperature T_∞ then, temperature of body after time "τ" will be T and can be calculated by given relation,

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\left(\frac{hA}{\rho V C_P}\right)\tau}$$





Fourier Number = It is the ratio of amount of heat conduction to the capacity of heat storage in given volume of element.

It signifies the degree of penetration of heating & cooling effect through a solid.

Fourier Number: $F_o = \frac{\alpha\tau}{L_c^2} = \frac{k}{\rho C}\frac{\tau}{L_c^2}$

Product of Biot number and Fourier number = $Bi \times Fo = \frac{hL_c}{k} \times \frac{k}{\rho C}\frac{\tau}{L_c^2} = \frac{h\tau}{\rho C L_c}$

Thus: $\frac{T - T_\infty}{T_i - T_\infty} = e^{-\left(\frac{hA}{\rho v C_p}\right)\tau} = \frac{T - T_\infty}{T_i - T_\infty} = e^{-Bi \times Fo}$ $\boxed{\left[\frac{(T - T_\infty)}{(T_i - T_\infty)}\right] = e^{-Bi \times Fo}}$

Time constant (τ) is given by:

$$\boxed{\tau = \frac{\rho V C_p}{hA}}$$

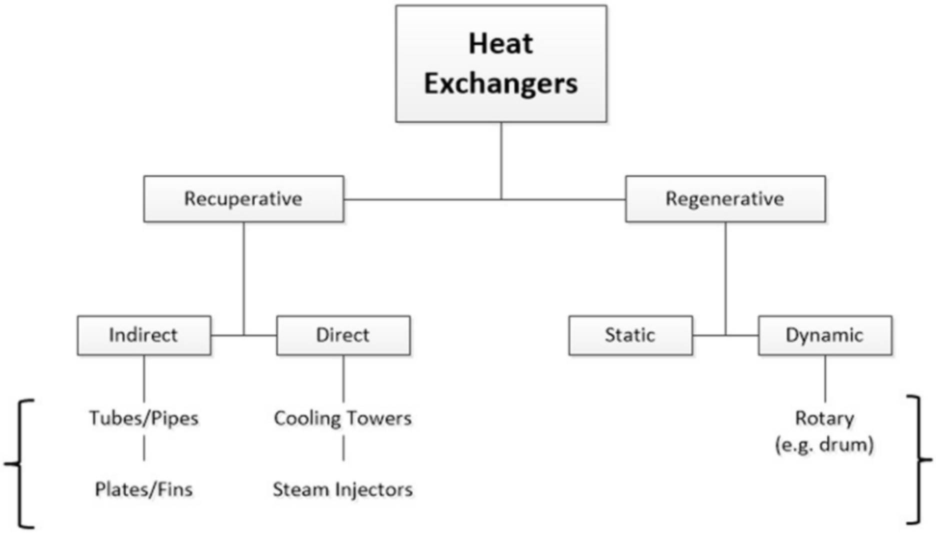
It signifies that larger masses ρV and larger heat capacities c_p lead to slower changes in temperature, while larger surface areas As and better heat transfer h lead to faster temperature changes.

CHAPTER-3: HEAT EXCHANGER

1. Introduction

Heat exchanger is a steady flow adiabatic device (open system) in which two flowing fluids exchange or transfer heat between them without using or gaining any heat from the ambient.

Heat exchanger classification by construction:

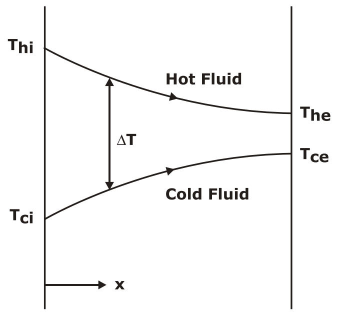


Base of Heat Exchanger

Rate of decrease of Enthalpy of hot fluid = Rate of increase of Enthalpy of cold fluid

$$m_h C_{ph}\left(T_{hi} - T_{he}\right) = m_c C_{pc}\left(T_{ce} - T_{ci}\right)$$

2. **Parallel flow heat exchanger:** Flow of hot and cold fluid are in same direction parallel to each other.



Logarithmic mean temperature difference is given by

$$\text{LMTD} = \left(\Delta T_m\right)_{\text{parallel}} = \frac{\Delta T_i - \Delta T_e}{\ln \dfrac{\Delta T_i}{\Delta T_e}}$$

Where, $\left(\Delta T_i\right) = T_{hi} - T_{ci}$

$(\Delta T_e) = T_{he} - T_{ce}$

Heat transfer rate

$$Q = m_h C_{ph}(T_{hi} - T_{he}) = m_c C_{pc}(T_{ce} - T_{ci}) = UA(\Delta T_m)_{parallel}$$

Effectiveness – Effectiveness of a heat exchanger is given as the ratio actual rate of heat transfer to the maximum possible rate of heat transfer.

$$\text{Effectiveness, } \in = \frac{Q_{actual}}{Q_{max.possible}} = \frac{m_h c_{ph}(T_{hi} - T_{he})}{(mc_p)_{small}(T_{hi} - T_{ci})} = \frac{m_c c_{pc}(T_{ce} - T_{ci})}{(mc_p)_{small}(T_{hi} - T_{ci})}$$

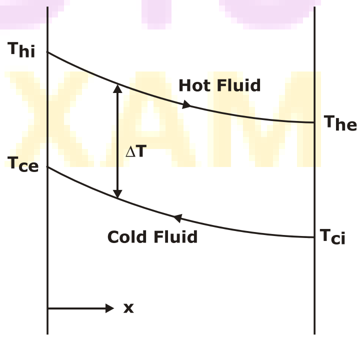
Effectiveness in terms of NTU and heat capacity ratio,

$$\text{Effectiveness: } \in_{\text{parallel flow}} = \left[\frac{1 - e^{-(1+c)NTU}}{1 + C}\right]$$

Where, $C\,(\text{Heat Capacity}) = \dfrac{(mC_p)_{small}}{(m\,C_p)_{large}}$ $\quad 0 \le C \le 1$

$$\text{NTU (Number of Transfer unit)} = \frac{UA}{(mC_p)_{small}}$$

3. **Counter flow heat exchanger:** Hot & Cold Fluid travel in the opposite direction parallel to each other.



$$(\text{LMTD})_{counter} = \frac{\Delta T_i - \Delta T_e}{\ln \dfrac{\Delta T_i}{\Delta T_e}}$$

Where: $(\Delta T_i) = T_{hi} - T_{ce}$

$(\Delta T_e) = T_{he} - T_{ci}$

Heat transfer rate

$$Q = m_h C_{ph}(T_{hi} - T_{he}) = m_c C_{pc}(T_{ce} - T_{ci}) = UA(\Delta T_m)_{counter}$$

Effectiveness in terms of NTU and heat capacity ratio,

$$\text{Effectiveness: } \boxed{\in_{\text{counterl flow}} = \left[\frac{1 - e^{-(1-c)NTU}}{1 - Ce^{-(1-c)NTU}}\right]}$$

LMTD FOR CROSS FLOW HEAT EXCHANGER:

LMTD for cross-flow heat exchanger can be obtained as

$$\left(\Delta T_m\right)_{\text{cross flow}} = \left(\Delta T_m\right)_{\text{counter flow}} \times F$$

Where, F = correction factor (To be obtained from databook)

NOTE - For the same inlet & exit temperature of hot & cold fluids:

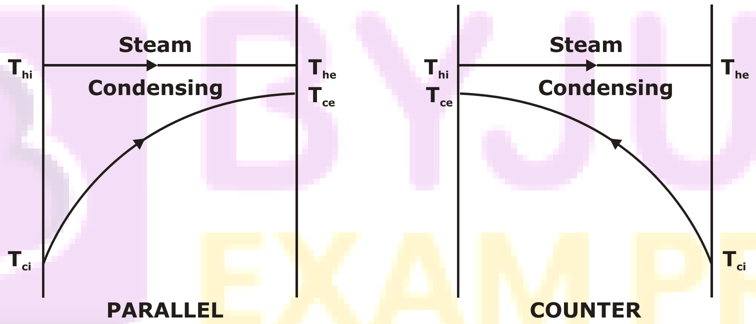
(ΔT_m) counter flow > (ΔT_m) shell & tube with one fluid condensing (surface condenser) > (ΔT_m) cross flow > (ΔT_m) parallel flow

F= < 1

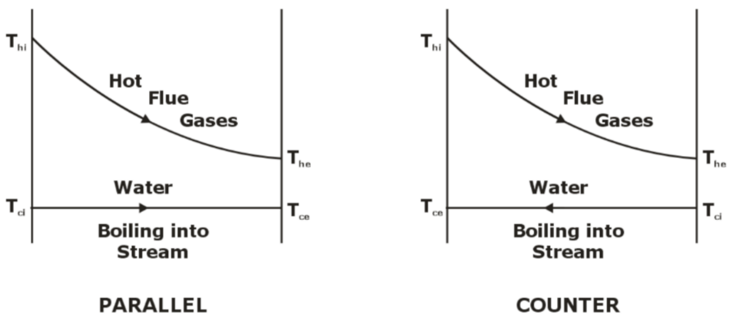
4. Special cases:

Case-1: when the one of the fluids is changes its phase i.e one of the fluid id having infinite heat capacity.

$$C\,(\text{Heat Capacity}) = \frac{\left(mC_p\right)_{\text{small}}}{\left(m\,C_p\right)_{\text{large}}} = \frac{\left(mC_p\right)_{\text{small}}}{\infty} = 0$$



Condenser



Evaporator

Effectiveness: $\in_{\text{parallel flow}} = \left[\dfrac{1 - e^{-(1+c)NTU}}{1 + C}\right] = 1 - e^{-NTU}$

Effectiveness: $\in_{\text{counterl flow}} = \left[\dfrac{1 - e^{-(1-c)NTU}}{1 - Ce^{-(1-c)NTU}}\right] = 1 - e^{NTU}$

Thus,

$$(\Delta T_m)_{\text{parallel flow}} = (\Delta T_m)_{\text{counter flow}}$$

Case-2: when both hot & cold fluids have equal capacity rates ($m_h Cp_h = m_c C_{pc}$):

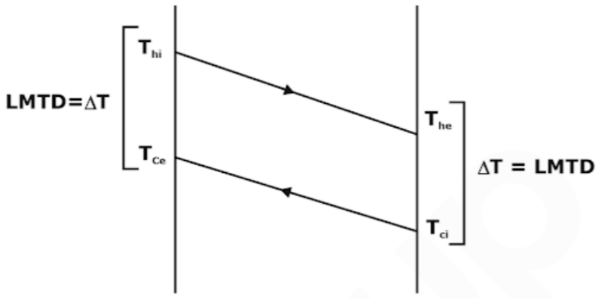
i.e. C = 1

Parallel Flow heat exchanger:

$$\epsilon_{\text{parallel flow}} = \frac{1 - e^{-2NTU}}{2}$$

Counter flow Heat exchanger:

$$\epsilon_{\text{counterl flow}} = \frac{NTU}{1 + NTU}$$



Then the temperature profiles in such a case will be two parallel lines.

Effectiveness in Terms of Temperatures Only –

CASE -1 When $m_h C_{ph} > m_c C_{pc}$

Thus, $(mC_p)_{small} = m_c C_{pc}$

$$\text{Effectiveness}, \in = \frac{Q_{actual}}{Q_{max.possible}} = \frac{m_c c_{pc}(T_{ce} - T_{ci})}{(mc_p)_{small}(T_{hi} - T_{ci})} = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}$$

CASE -2 When $m_c C_{pc} > m_h C_{ph}$

Thus, $(mC_p)_{small} = m_h C_{ph}$

$$\text{Effectiveness}, \in = \frac{Q_{actual}}{Q_{max.possible}} = \frac{m_h c_{ph}(T_{hi} - T_{he})}{(mc_p)_{small}(T_{hi} - T_{ci})} = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}$$

NOTE- Only one of the above two formulae given for effectiveness hold good at a time.

NOTE - In case if both the capacity rates are equal, any one of the above two formulae can be used.

5. Fouling Factor:

Fouling factor 'F' is the factor which takes into account the thermal resistance offered by any scaling or chemical deposit that is formed on the heat transfer surfaces over a period of time. Unit of Fouling Factor is (m^2 K/watt).

$$\frac{1}{U_{\text{with fouling}}} = \frac{1}{h_1} + \underset{\text{Hot side}}{F_1} + \frac{1}{h_2} + \underset{\text{Cold side}}{F_2}$$

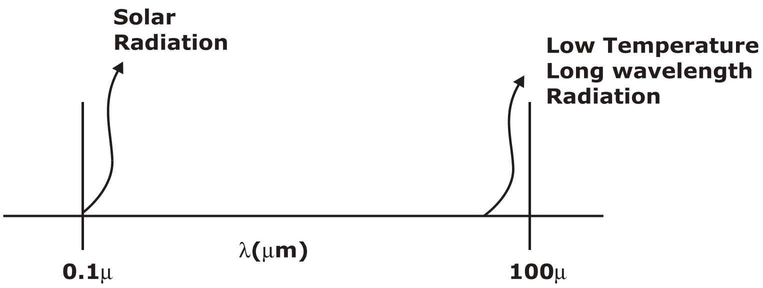
6. Industries and Applications of Heat Exchangers by Type:

Type of heat exchanger	Applications
Shell and Tube	Oil refining, Preheating, Steam generation, Boiler blowdown heat recovery, Vapour recovery systems, Industrial paint systems
Double pipe	Industrial cooling processes, Small heat transfer area requirements
Plate type	Cryogenic, food processing, Chemical processing, Furnaces, Closed loop to open loop water cooling
Condensers	Power plants, Refrigeration, HVAC, Chemical processing, Distillation and refinement processes
Evaporators and boilers	distillation and refinement processes, Steam trains, Refrigeration, HVAC
Compact	Limited space requirements (e.g., aircrafts and automobiles), cryogenics, Electronics cooling

CHAPTER-4: RADIATION HEAT TRANSFER

1. Introduction:

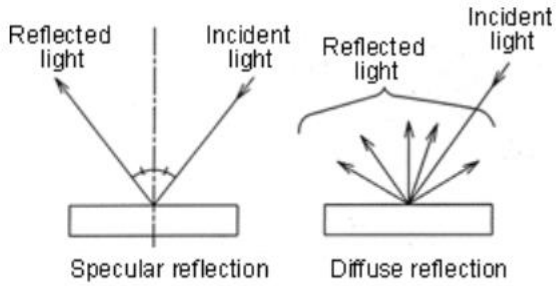
- ☐ Radiation is the heat transfer by electromagnetic waves.
- ☐ Fastest method of heat transfer.
- ☐ No medium required.
- ☐ All bodies above temperatures absolute zero emits thermal radiations.



Electromagnetic Spectrum

Diffuse and Specular radiation:

Incident light reflected symmetrically with respect to the normal line is called "specular reflection," while incident light scattered in different directions is called "diffuse reflection."



2. Basic definitions:

- Emissive Power is defined as the radiation energy emitted from the surface of a body per unit time & per unit area in all possible hemispherical directions integrated over all the wavelengths (in J/s m^2 or W/m^2).
- ☐ Black body is the body which absorbs all the thermal radiation incident or falling upon it. It is also the perfect emitter.
- ☐ **Emissivity (ϵ):**

  The total hemispherical emissivity of a surface is simply the average emissivity over all directions and wavelengths:

  $$\epsilon = \frac{E(T)}{E_b(T)}$$

- ☐ Monochromatic Emissive power is emissive power per unit wavelength. Spectral emissivity or monochromatic emissivity is defined in a similar manner:

  $$\epsilon_\lambda(T) = \frac{E_\lambda(T)}{E_{b\lambda}(T)}$$

where E_λ (T) is the spectral or monochromatic emissive power of the real surface.

☐ A diffuse surface shall have the same intensity of radiation along al the directions.

- When the emissivity of the material does not change with temperature, it is called a gray body. A Gray body has a constant monochromatic emissivity, i.e. with respect to wavelength.

3. Absorptivity (α), Reflectivity (ρ) & Transmissivity (τ):

Absorptivity (α): It is the fraction of radiation energy incident upon a surface which is absorbed by it.

Reflectivity (ρ) is the fraction of radiation energy incident upon a surface which is reflected by it.

Transmissivity (τ) is the fraction of radiation energy incident upon a surface which is transmitted by it.

$$\boxed{\alpha + \rho + \tau = 1}$$

For opaque surface (τ =0): $\boxed{\alpha + \rho = 1}$

Note.

Air at atmospheric pressure and temperature is transparent to thermal radiation for which τ = 1 and α = ρ = 0.

For black body: $\alpha = \epsilon = 1$

There are some surfaces which absorb nearly all incident radiation, yet do not appear black. ***Ice, snow, white-washed walls*** have absorptivities greater than 0.95.

For white body: ρ = 1

3. Laws of thermal radiation:

3.1. Kirchhoff's law of radiation

The law states that "Whenever a body is in thermal equilibrium with its surrounding's its emissivity is equal to its absorptivity".

For a body in thermal equilibrium: $\alpha = \epsilon$

Kirchhoff's law also holds for monochromatic radiation: i.e. $\alpha_\lambda = \epsilon_\lambda$

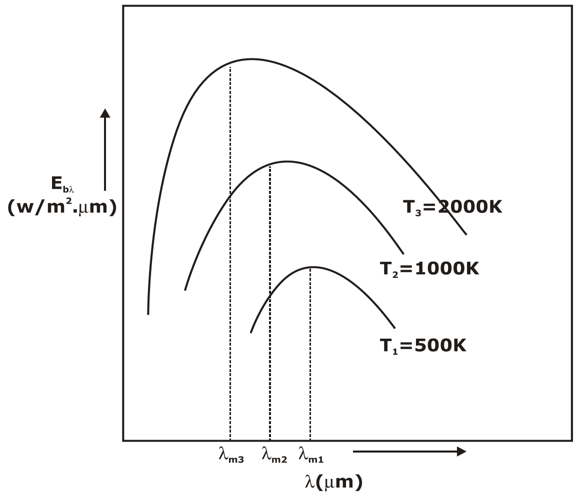
3.2. Planck's law of thermal radiation

The law states that the monochromatic emissive power of black body is dependent on both absolute temperature of black body & also on wavelength of emission.

Planck deduced by his quantum theory the expression for $E_{b\lambda}$ as a function of temperature and Wavelength:

$$\boxed{E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1}}$$

$E_{b\lambda} = f(\lambda, T)$, T is in kelvin.



Planck's law is the basic law of thermal radiation.

3.3. Rayleigh Jeans's Law ($C_2/\lambda T \lll 1$):

$$E_{b\lambda} = \frac{C_1 T}{C_2 \lambda^4}$$

This equation is known as Rayleigh–Jeans law which is found to be valid for large wavelengths.

3.4. Wein's Law ($C_2/\lambda T \ggg 1$):

$$E_{b\lambda} = \frac{C_1}{\lambda^5} e^{-C_2/\lambda T}$$

This equation is known as Wien's law, which is found to be valid for short wavelengths.

3.5. Wein's displacement law:

This law tells about the wavelength at which $E_{b\lambda}$ will be maximum at a specified temperature

$$\frac{C_2}{\lambda_{max} T} = 4.965$$

$$\lambda_m T = 2898 \ \mu\text{m-K}$$

The value of $(E_{b\lambda})_{max}$ shifts towards the shorter wave lengths with increasing temperature.

$$(E_{b\lambda})_{max} = C_3 T^5$$

The magnitude of $(E_b)_{max}$ is proportional to the fifth power of the absolute temperature of the body.

3.6. Stefan Boltzmann's law:

The law states that the total hemispherical emissive power of a black body is directly proportional to fourth power of the absolute temperature of black body.

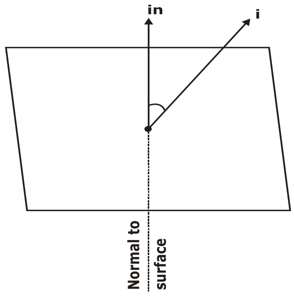
$$E = \sigma \varepsilon T^4 \ W/m^2$$

σ = Stefan Boltzmann Constant= 5.67×10^{-8} W/m² K⁴

For black body's: $\epsilon = 1$

$$\boxed{E_b = \sigma T^4} \ W/m^2$$

3.7. Lambert's cosine law:



As per this law, Intensity of radiation in a direction θ from the normal to a diffuse emitter is proportional to cosine of the angle θ and given as,

$$i = i_n \cos\theta$$

Where, i_n = normal intensity of radiation

i = Intensity of Radiation along a direction making an angle θ w.r.t normal

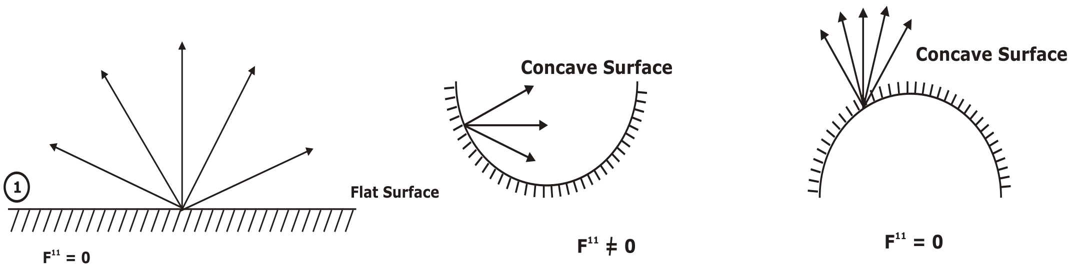
3.8. Intensity of radiation:

The intensity of radiation, I is defined as the rate of heat radiation in a given direction from a surface per unit solid angle per unit area of the projection of the surface on a plane normal to the direction of radiation.

$$I = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi}$$

4. Shape factor or view factor or configuration factor:

The radiation heat transfer between bodies depends on their relative orientation. F_{12} (Shape factor of 1 relative to 2) is Fraction of radiation energy leaving surface 1 that reaches surface 2.



SOLID ANGLE

The relation of an angle and arc length to three dimensions is defined by solid angle.

4.1. SHAPE FACTOR OR VIEW FACTOR OR CONFIGURATION FACTOR

Radiation heat transfer between surfaces depends upon the orientation of the surfaces relative to each other. To account for the effects of orientation on radiation heat transfer between the surfaces, a new parameter VIEW FACTOR is defined which is purely geometric quantity and is independent of radiation properties and temperatures.

NOTE – Shape factor for concave surface is always zero.

Shape factor for convex surface is always less than one.

Shape factor for plane or Flat surface is always zero.

4.1.1. Reciprocity relation:

Reciprocity Relation is valid between any two surfaces even when there are more than two number of surfaces involved in Radiation Heat Exchange & it is given as,

$$A_1F_{12} = A_2F_{21}$$

4.1.2. Summation rule:

If there are n number of surfaces involved in any radiation heat exchange then.

$$F_{11} + F_{12} + F_{13} + \ldots\ldots\ldots\ldots\ldots + F_{1n} = 1$$

$$F_{21} + F_{22} + F_{23} + \ldots\ldots\ldots\ldots\ldots + F_{2n} = 1$$

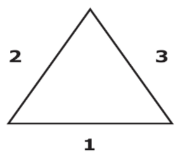
$$F_{n1} + F_{n2} + F_{n3} + \ldots\ldots\ldots\ldots\ldots + F_{nn} = 1$$

NOTE - If there are Total n surfaces of an enclosure, then total numbers of view factors that need to be evaluated directly $= {}^nC_2 = \frac{n(n-1)}{2}$

4.1.3. Symmetry rule:

Two or more surfaces that possess symmetry about a third surface, will have identical view factors from both the surface.

For equilateral triangle: $F_{12} = F_{13}$



4.1.4. Enclosure:

Total number of shape factors in "n" surface enclosure is $\mathbf{n^2}$.

5. Radiation Networks:

Irradiation (G) is defined as the total thermal radiation incident upon a surface per unit time per unit area (in W/m^2).

The total thermal radiation leaving a surface per unit time & per unit area is called **Radiosity**(J) of the surface.

Suppose a surface having emissivity ϵ & Irradiation is G, then Radiosity will be expresses as

$$J = \varepsilon E_b + (1-\varepsilon)\, G$$

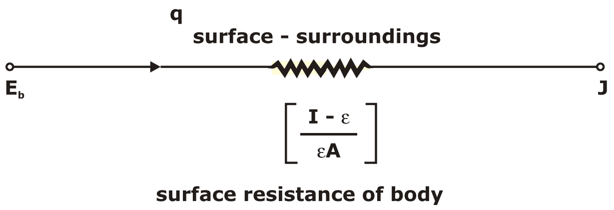
- The net radiation heat liberated from the surface:

$$q_{net} = (J - G)\,A \quad \text{Watts}$$

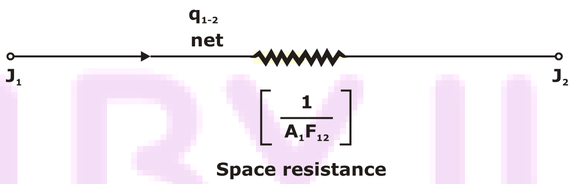
$$q_{net} = \frac{E_b - J}{\left(\frac{1-\varepsilon}{\varepsilon A}\right)} \text{ watts}$$

Comparing it with Current voltage analogy,

$\text{Resistance} = \frac{1-\varepsilon}{\varepsilon A}$, It is known as surface resistance.



☐ Net radiation heat exchange between two black surface 1 and 2 having radiosity J_1 & J_2



$$q_{1-2} = J_1A_1F_{12} - J_2A_1F_{21}$$

Since $A_1F_{12} = A_2F_{21}$

$$q_{1-2} = \frac{J_1 - J_2}{\frac{1}{A_1F_{12}}} = \frac{J_1 - J_2}{\frac{1}{A_2F_{21}}}$$

$\frac{1}{A_1F_{12}} = \frac{1}{A_2F_{21}}$ is called the space resistance.

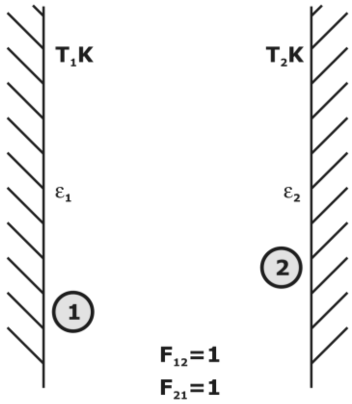
- Net radiation heat exchange between two gray surface 1 and 2 having radiosity E_{b1} & E_{b2}

$$\left(q_{(1-2)}\right)_{net} = \frac{E_{b1} - E_{b2}}{\sum R_{Th}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}} \text{ Watts}$$

Thus, two surface exchanging heat will consist two surface resistance (one on each) and one space resistance because of the gap prevails between them.

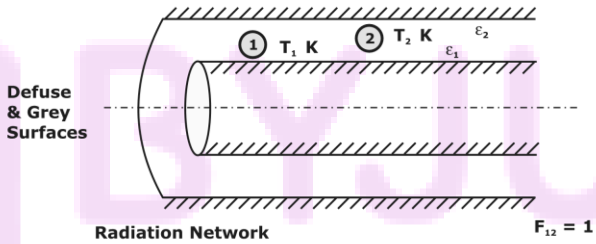
6. APPLICATIONS OF RADIATIONS NETWORKS

(a). Heat transfer rate between two infinitely large plane surfaces having emissivity ϵ_1 & ϵ_2



$$\text{Radiation Heat Flux} = \frac{E_{b1} - E_{b2}}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)} \text{ watt / m}^2$$

(b). Two infinitely long cylinders having emissivity ϵ_1 & ϵ_2



$$(q_{1-2})_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right)A_1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left[\frac{1}{\varepsilon_2} - 1\right]} \text{ watts}$$

(c). A very small body is Kept in Very large room, i.e. $A_2 \gg A_1$

$$A_1 <<< A_2 \Rightarrow \frac{A_1}{A_2} \approx 0$$

$$(q_{1-2})_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right)A_1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left[\frac{1}{\varepsilon_2} - 1\right]} \Rightarrow (q_{1-2})_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right)A_1}{\frac{1}{\varepsilon_1} + 0 \times \left[\frac{1}{\varepsilon_2} - 1\right]}$$

$$(q_{1-2})_{net} = \sigma\left(T_1^4 - T_2^4\right)A_1\varepsilon_1$$

7. Radiation shields: It is simply a plane surface, which is place in between two surface exchanging heat such that the heat transfer rate can be reduced.

CASE – 1 if two infinitely long surfaces having emissivity ϵ_1 & ϵ_2 exchanging heat and a third surface having emissivity ϵ_3 is introduced between them then net radiation between them will be given as

$$\left((q/A)_{(1-2)}\right)_{\text{net with screen}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right) + \left(\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1\right)} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \left(\frac{2}{\varepsilon_3} - 2\right)}$$

NOTE - In case if all the surfaces including n screens have equal emissivity ϵ then

$$\left(q_{1-2}\right)_{\text{Net with 'n' screen}} = \left(\frac{1}{n+1}\right) q_{\text{without any screen}}$$

NOTE - If there are n number of screens being used between the planes, there will be total (2n + 2) surface resistances in the entire radiation network drawn with the screens thus each surface brings two space resistance and 1 space resistance.

CHAPTER-5: CONVECTION

1. Convection heat transfer: Convection is a mode of heat transfer which generally occurs between a solid & a fluid due to temperature difference associated **with macroscopic bulk displacement of the fluid** transporting thermal energy (heat energy).

The rate of heat transfer is governed by Newton's Law of cooling:

$$q = hA(\Delta T)$$

Where A = heat transfer surface area

h = heat transfer coefficient

CONVECTION	
Forced	**Free or Natural**
velocity is evident i.e. heat transfer due to bulk motion of the fluid.	No velocity is evident, but flow occurs due to buoyant forces arising out of density changes of fluid).

Various dimensionless number used in Convection

(a). Reynold's number (Re) – it is defined as the ratio of inertia force to viscous force.

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho VD}{\mu} = \frac{VD}{\nu}$$

Where, D = characteristics dimensions, $D = \frac{4 \times \text{cross sectional Area}}{\text{Perimeter}} = \frac{4A}{P}$

For circular pipe, D = d = diameter of pipe

For rectangular duct having dimension a×b, $D = \frac{4 \times \text{cross sectional Area}}{\text{Perimeter}} = \frac{2ab}{a+b}$

It tells about the behaviour of the fluid whether it is laminar or turbulent.

For Laminar Flow

Through Pipe, Re < 2000

Through plate, $Re < 5\times10^5$

For channel, Re < 1000

For turbulent Flow

Through Pipe, Re < 4000

Through plate, $Re < 7\times10^5$

(b) Prandtl Number (Pr) = Prandtl Number (Pr) is the only dimensionless number which is a properly of the fluid defined as the ratio between kinematic viscosity of fluid & its thermal diffusivity.

$$Pr = \frac{\text{Kinematic viscosity}}{\text{Thermal Diffusivity}} = \frac{\upsilon}{\alpha} = \frac{(\mu/\rho)}{(k/\rho C_p)} = \frac{\mu C_p}{K}$$

Prandtl number signifies the relative magnitude of momentum diffusion rate & thermal diffusion rate.

NOTE - (i). Pr of air = 0.65 to 0.71

(ii). Pr or water = 2 to 6

(iii). Pr of Mercury [Liquid Metal] is very Low, since k is very high.

(iv). Pr of Lubricating oils is high [may be up to 100].

(c). Nusselt number - It is the ratio of convective heat transfer to conductive heat transfer.

$$Nu = \frac{\text{conductive resistance of fluid}}{\text{Surface convective resistance}} = \frac{\frac{D}{Ak_f}}{\frac{1}{hA}} = \frac{hD}{k_f}$$

$$D = \frac{4 \times \text{cross sectional Area}}{\text{Perimeter}} = \frac{4A}{P}$$

(d). Peclet Number – It is the product of Reynold number and Prandtl number.

$$Pe = Re \times Pr$$

(e). Stanton Number – It is the ratio of heat transfer coefficient to the flow of heat per unit temperature rise due to the velocity of the fluid.

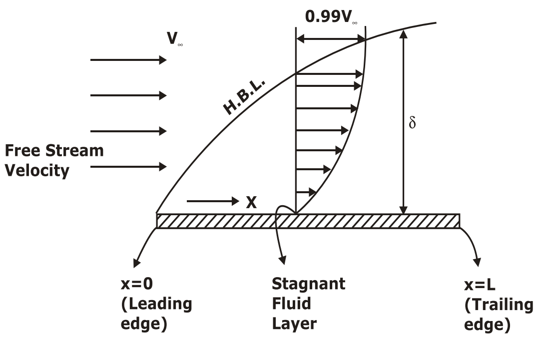
It is also the ratio of Nusselt number and Peclet number.

$$St = \frac{Nu}{Pe} = \frac{Nu}{Re \times Pr} = \frac{h}{\rho V C_P}$$

2. Boundary layer:

2.1. Hydrodynamic boundary layer:

Hydrodynamic Boundary Layer is defined as a thin region formed on the plate inside which velocity gradients are seen in a normal direction to the plate.



2.2. Thermal boundary layer: Thermal boundary Layer is a region inside which temperature gradients are seen in the normal direction to the plate.

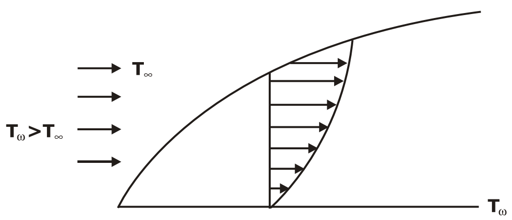
T = f (x,y)

δ_t = Thickness of thermal Boundary Layer

$\delta_t = f(x)$

The Thermal boundary layer thickness is defined as the distance measured form the solid boundary in y direction at which $\frac{T_s - T}{T_s - T_\infty} = 0.99$

$T_\infty > T_w$



Relation between the Prandtl Number (Pr), thermal boundary layer thickness (δ_t) and velocity boundary layer thickness (δ):

$$\frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-1/3}$$

Case-1: If Pr > 1 then $\frac{\delta_t}{\delta} < 1$

Case-2: If Pr < 1 then $\frac{\delta_t}{\delta} > 1$

Case-3: If Pr ≈ 1 then $\delta \approx \delta_t$

ENERGY BALANCE FOR THERMAL BOUNDARY LAYER-

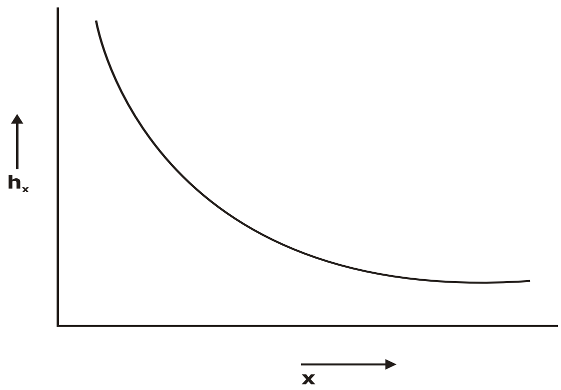
In a thermal boundary layer, first heat energy gets conducted through plate and then convected through fluid.

$$dq = -K_f \, dA \left(\frac{\partial T}{\partial y}\right)_{at\, y=0} = h_x \, dA \left(T_w - T_\infty\right)$$

FOURIER's Law NEWTON's LAW

$$h_x = \frac{-K_f \left(\frac{\partial T}{\partial y}\right)_{at\, y=0}}{\left(T_w - T_\infty\right)} \, W/m^2K$$

Where, h_x = Local convective heat transfer coefficient at that x



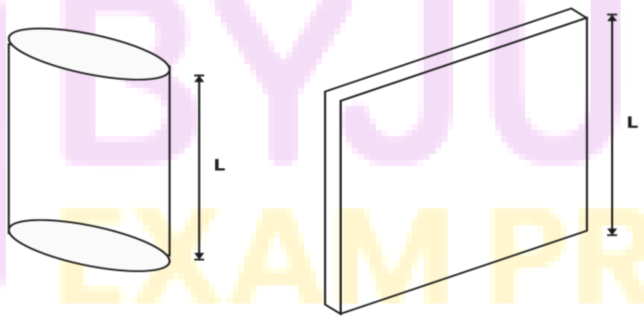
variation of h_x with x

3. Free convection or natural convection:

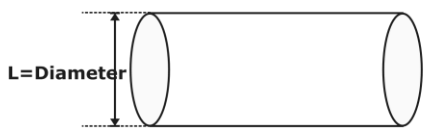
Heat transfer takes place due to temperature difference present in the fluid in motion & the motion is fluid occurs due to buoyancy forces arising out of density changes.

Characteristic dimension for various geometry in Free convection:

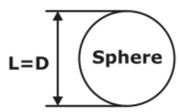
For vertical plates & for vertical cylinder



For horizontal cylinder & sphere -

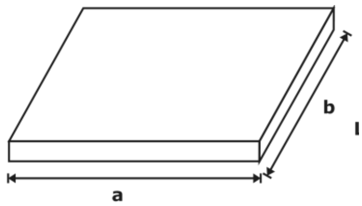


For Sphere



For horizontal plate

$$L = \frac{\text{Area}}{\text{perimeter}} = \frac{ab}{2(a+b)}$$



Grashoff number (Gr):

$$Gr = \frac{\text{Inertia Force} \times \text{Buoyancy Force}}{(\text{viscous force})^2}$$

$$Gr = \frac{g\beta\Delta TL^3}{\left[\frac{\mu}{\rho}\right]^2} = \frac{g\beta\Delta TL^3}{\upsilon^2}$$

Where, β = Isobaric volume expansion coefficient of fluid $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{P=C}$ [per kelvin]

In any free convection heat transfer,

Nu = f(Gr Pr)

Rayleigh Number – It is the product of Grashoff Number and Prandtl Number.

Rayleigh number tells weather the flow is laminar or turbulent.

If Ra < 10^9 then the flow is laminar

If Ra > 10^9 then the flow is turbulent

Relation between Nusselt Number and Grashoff Number-

Nu = C $(Gr\ Pr)^m$

C & m are constants which vary from case to case.

m = 1/4 for Laminar Flow

m = 1/3 for Turbulent Flow

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