BYJU'S
EXAM PREP

AE/JE Foundation

Electrical Engineering

Signals & Systems

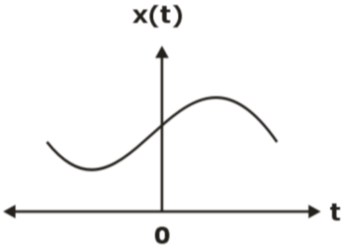
Formula Notes

IMPORTANT FORMULAS TO REMEMBER

CHAPTER 1: BASICS

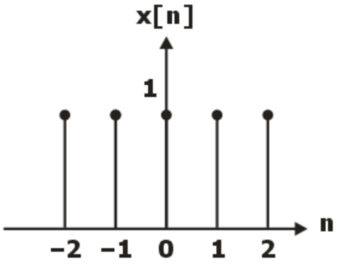
1. Continuous-time signal

A signal x(t) is continuous-time (CT) signal, if t is a continuous variable. A continuous time signal is defined continuously with respect to time.



2. Discrete-time signal

If t is a discrete variable, then it is a discrete-time (DT) signal. A discrete time signal is often identified as a sequence of numbers, denoted by x[n], where n is an integer.

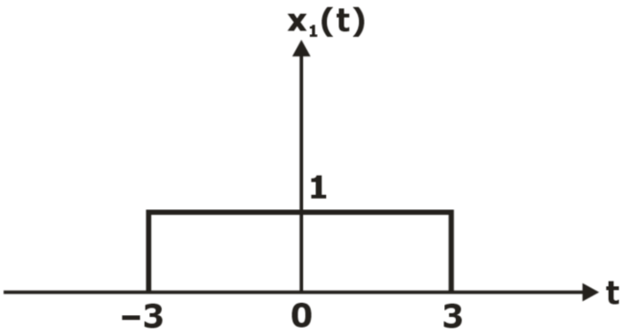


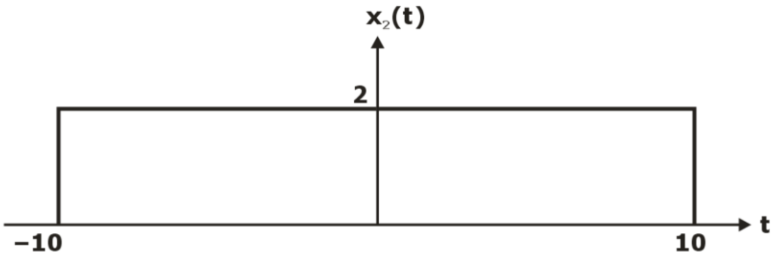
3. BASIC OPERATIONS ON CONTINUOUS TIME SIGNAL

3.1. Addition/Subtraction of signals

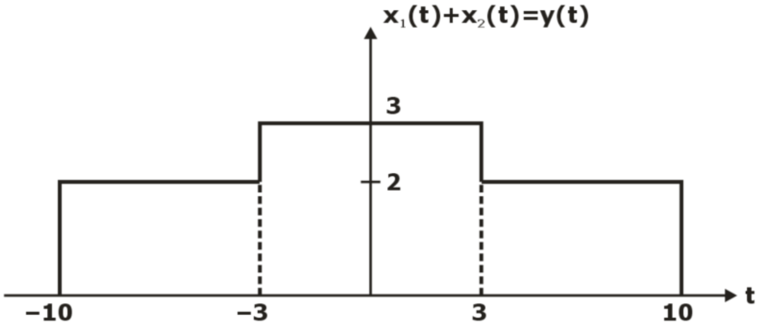
The sum of two continuous-time signals can be obtained by adding their values at every instant of time. Similarly, the subtraction of two continuous-time signals can be obtained by subtracting their values at every instant of time.

Example:





(i) Addition

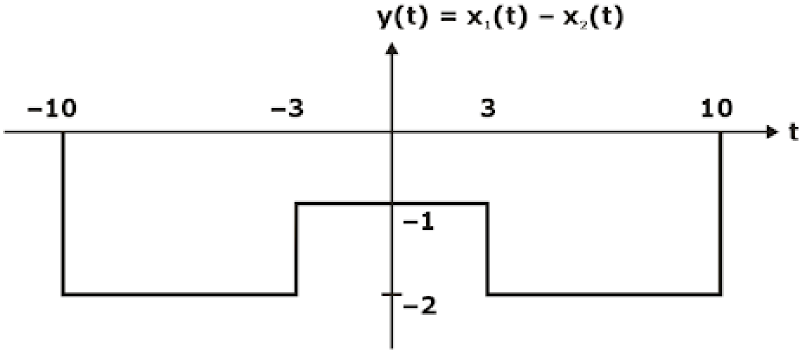


From −10 < t < −3, amplitude of y(t) = $x_1(t) + x_2(t)$ = 0 + 2 = 2

From −3 < t < −3, amplitude of y(t) = $x_1(t) + x_2(t)$ = 1 + 2 = 3

From 3 < t < 10 amplitude of y(t) = $x_1(t) + x_2(t)$ = 0 + 2 = 2

(ii) Subtraction



From −10 < t < −3, amplitude of y(t) = $x_1(t) - x_2(t)$ = 0 − 2 = − 2

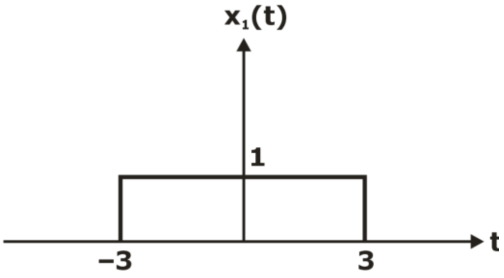
From −3 < t < −3, amplitude of y(t) = $x_1(t) - x_2(t)$ = 1 − 2 = − 1

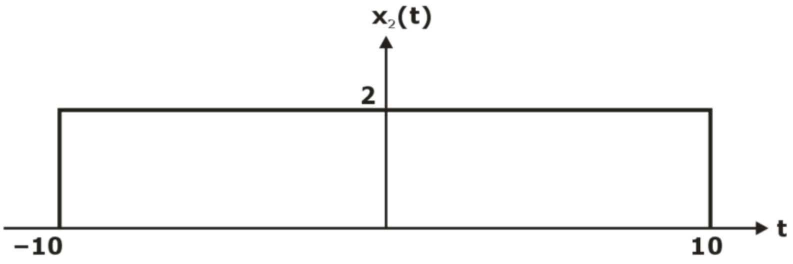
From 3 < t < 10, amplitude of y(t) = $x_1(t) - x_2(t)$ = 0 − 2 = − 2

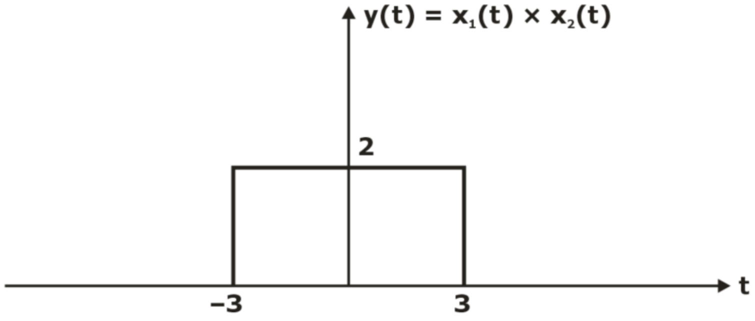
3.2. Multiplication of signals

The multiplication of two continuous signals can be obtained by multiplying their values at every instant.

Example:







From -10 < t < -3, amplitude of y(t) = $x_1(t) \times x_2(t) = 0 \times 2 = 0$

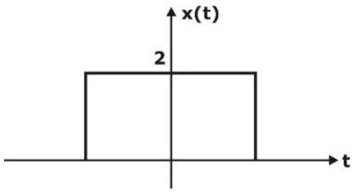
From -3 < t < 3, amplitude of y(t) = $x_1(t) \times x_2(t) = 1 \times 2 = 2$

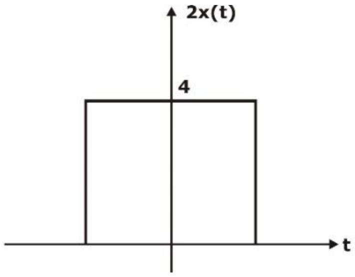
From 3 < t < 10, amplitude of y(t) = $x_1(t) \times x_2(t) = 0 \times 2 = 0$

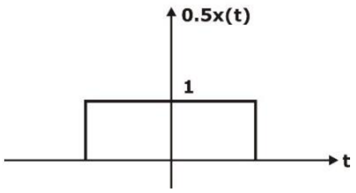
3.3. Amplitude scaling of signals

The amplitude of a signal can be changed by amplitude scaling. If a signal x(t) is multiplied by a factor A, it is expressed as A x(t) which means that, at every instant t, the amplitude of x(t) is multiplied by A.

Example:







Note:

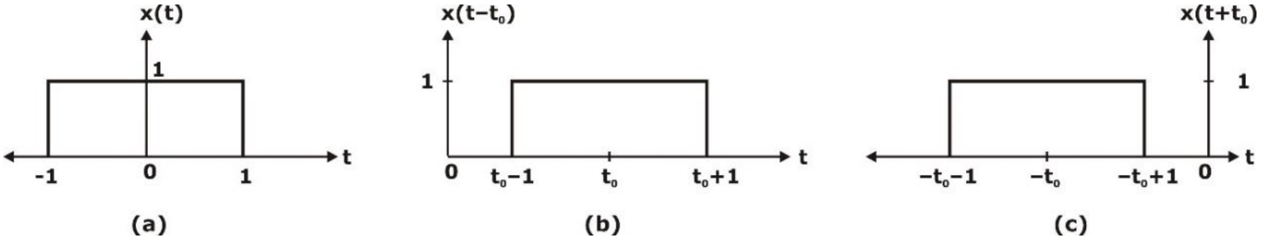
Amplitude scaling signal A x(t) is identical in shape to the original signal x(t) but its amplitude is multiplied by A everywhere.

3.4. Transformation of signal

i. Time-Shifting

Signal $x(t - t_0)$ represents a time shifted version of $x(t)$ by t_0 seconds. If $t_0 > 0$, then the signal is delayed by t_0 seconds. If $t_0 < 0$, then $x(t + t_0)$ represents an advanced version of $x(t)$. The time shifting operation is shown in figure.

The waveform of $x(t - t_0)$ is identical to that of $x(t)$, except for a shift of t_0 time units towards the right-hand side.

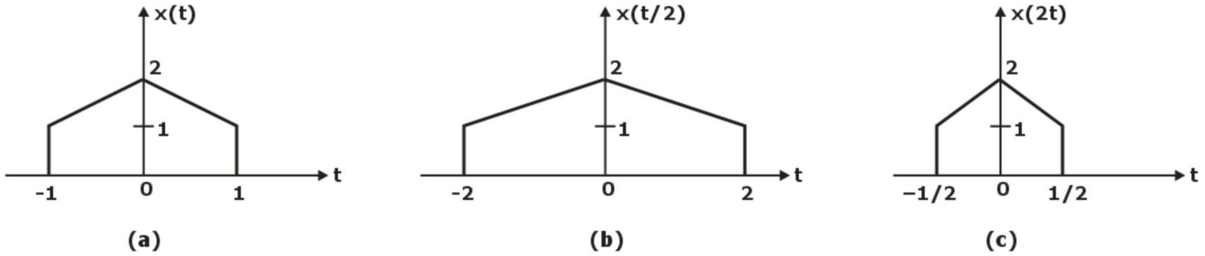


(a) (b) (c)

Figure: Time shifting operation (a) Original signal x(t) (b) Time delayed version of x(t) (c) Time advanced version of x(t)

ii. Time scaling

If the independent variable t is scaled by a parameter a, then $x(at)$ is time scaled version of $x(t)$. It is important to note that time scaling is performed on t-axis such that the values $x(t)$ and $x(at)$ at $t = 0$ are the name for both waveforms.

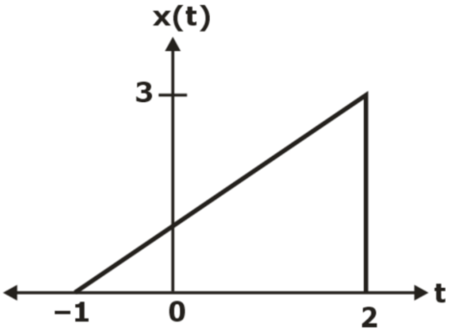


(a) (b) (c)

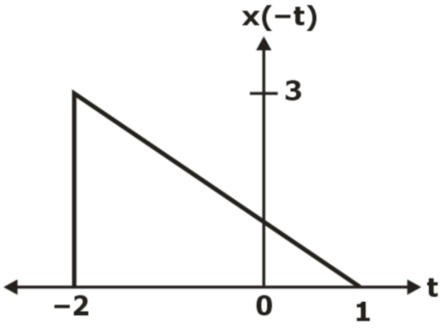
Figure: Time scaling operations, (a) Original signal x(t), (b) Time expanded version, (c) Time compressed version of x(t)

iii. Time-Reversal/Folding

The signal $x(-t)$ is called folded version of signal $x(t)$ and is obtained by taking reflection of $x(t)$ about vertical axis $t = 0$ as shown in figure.



(a) Signal x(t)



(b) Reflection of x(t)

4. MULTIPLE OPERATIONS ON CONTINEOUS-TIME SIGNALS

Consider a signal x(t) with multiple transformation given as

x(t) → A x(bt ± t_0)

where, a and b are assumed to be real numbers. The operations should be performed in the following order

METHODOLOGY 1

Step 1: First multiply signal by a constant A to obtained amplitude scaled version of x(t) that is A x(t).

Step 2: Shift the signal A x(t) to the left or to right by t_0 time units. This will produced shifted signal A x(t ± t_0).

Step 3: Scale the signal A x(t ± t_0) by b, the resulting signal represents Ax(bt ± t_0).

Step 4: If b is negative, reflect the scaled signal Ax(bt ± t_0) about the vertical axis.

The correct sequence for the above transformation is

$$x(t)\xrightarrow{A_{scaling}^{amplitude}} Ax(t)\xrightarrow{t\to t-t_0{}_{shifting}^{time}} x(t-t_0)\xrightarrow{t\to bt_{scaling}^{time}} Ax(bt-t_0)$$

If time scaling is done before time shifting it will produce incorrect results.

$$x(t)\xrightarrow{A_{scaling}^{amplitude}} Ax(t)\xrightarrow{t\to bt_{scaling}^{time}} Ax(bt)\xrightarrow{t\to t-t_0{}_{shifting}^{time}} Ax[b(t-t_0)]\neq Ax(bt-t_0)$$

For a different multiple transformation, different order of sequence is performed. Consider a signal x(t) with multiple transformation given as

$x(t)\to Ax\left(\frac{t-t_0}{a}\right)$

For this sequence the simplest sequence of operation is given as follows.

METHODOLOGY 2

Step 1: First multiply signal by a constant A to obtained amplitude scaled version of x(t) that is Ax(t).

Step 2: Scale the signal Ax(t) by 1/a, the resulting signal represents x(t/a).

Step 3: If a is negative, reflect the scaled signal x(t/a) about the vertical axis.

Step 4: Shift the scaled signal Ax(t/a) by t_0 units to the left or to right by t_0 time units. This will produced signal Ax[(t – t_0) /a].

The correct sequence for the above transformation is

$$x(t) \xrightarrow{A_{scaling}^{amplitude}} Ax(t) \xrightarrow{t\to t/a_{scaling}^{time}} Ax\left(\frac{t}{a}\right) \xrightarrow{t\to t-t_{0}{}_{shifting}^{time}} Ax\left(\frac{t-t_0}{a}\right)$$

If we change the order of sequence (time scaling is done after time shifting), then we would not get correct results.

$$x(t) \xrightarrow{A_{scaling}^{amplitude}} Ax(t) \xrightarrow{t\to t-t_{0}{}_{shifting}^{time}} Ax(t-t_0) \xrightarrow{t\to t/a_{scaling}^{time}} Ax\left(\frac{t}{a}-t_0\right) \neq Ax\left(\frac{t-t_0}{a}\right)$$

It must be noted that the operation of reflecting and time scaling is commutative, whereas the operation of shifting and reflecting or shifting and time scaling is not.

5. Some Important Signals

Name	Continuous	Discrete
Unit Step function	$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$	$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & t < 0 \end{cases}$
Ramp signal	$r[t] = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$	$r[n] = nu(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$
Impulse function	$\delta(t) = 0,\ t \neq 0$	$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & otherwise \end{cases}$
Rectangular pulse function	$rect\left(\frac{t}{\tau}\right) = \begin{cases} 1, & \lvert t\rvert \leq \tau/2 \\ 0, & \lvert t\rvert > \tau/2 \end{cases}$	$rect\left[\frac{n}{2N}\right] = \begin{cases} 1, & \lvert n\rvert \leq N \\ 0, & \lvert n\rvert > N \end{cases}$
Triangular pulse	$tri\left(\frac{t}{\tau}\right) = \begin{cases} 1-\left\lvert\frac{t}{\tau}\right\rvert, & t \leq \lvert\tau\rvert \\ 0, & t > \lvert\tau\rvert \end{cases}$	$tri\left[\frac{n}{N}\right] = \begin{cases} 1-\frac{\lvert n\rvert}{N}, & \lvert n\rvert \leq N \\ 0, & elsewhere \end{cases}$
Signum signal	$sgn(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$	$sgn[n] = \begin{cases} 1, & n > 0 \\ -1, & n < 0 \end{cases}$
Sinusoidal signal	$x(t) = \sin(2\pi f_0 t + \theta)$	$X[n] = \sin(2\pi f_0 n + \theta)$
Sinc function	$sin(\omega_0 t) = \frac{sin(\pi\omega_0 t)}{\pi\omega_0 t}$	$sin(\omega_0 n) = \frac{sin(\pi\omega_0 n)}{\pi\omega_0 n}$

6. Important Properties of Signals

Signals in term of unit step and vice versa	r(t) = tu(t) $u(t) = \frac{d}{dt} r(t)$ $\delta(t) = \frac{d}{dt} u(t)$ $u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$ sgn = u(t) − u(−t) sgn = 2u(t) − 1 $\Pi\left(\frac{t}{\tau}\right) = u\left(t + \frac{t}{\tau}\right) - u\left(t - \frac{t}{\tau}\right)$	Impulse properties	$\int_{-\infty}^{\infty} \delta(t) dt = 1$ $\delta(\alpha t) = \frac{1}{\lvert\alpha\rvert} \delta(t)$ $\delta(\alpha t + b) = \frac{1}{\lvert\alpha\rvert} \delta\left(t + \frac{b}{\alpha}\right)$ $\int_{-\infty}^{\infty} f(t) \delta(t - \lambda) dt = f(\lambda)$ $f(t)\delta(t - \lambda) = f(\lambda)\delta(t - \lambda)$
Time period of linear combination of two signals	Sum of signals is periodic if $\frac{T_1}{T_2} = \frac{m}{n} =$ rational number The fundamental period of g(t) is given by $nT_1 = mT_2$ provided that the values of m and n are chosen such that the greastest common divisor (gcd) between m and n is 1	odd and even & symmetry	$x_e(t) = x_e(-t)$ $x_0(t) = -x_0(-t)$ $x(t) = x_e(t) + x_0(t)$ $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ $x_0(t) = \frac{1}{2}[x(t) - x(-t)]$
Combined operation	x(t) ⇒ Kx(t) + C Scale by K then shift by C …. x(t) ⇒ x(αt − β) Shift by β : [x(t − β)] Then compress by a:[x (t − β) ⇒ x(αt − β)] Or, compress by α: [x(t)⇒x(at)] then shift by $\frac{\beta}{\alpha}$:$x\left\{\alpha\left(t - \frac{\beta}{\alpha}\right)\right\}$= x (αt − β)}]	Derivative of impulse (doublet)	$\frac{d}{dt}\delta(t) = \delta'(t) = \begin{cases} undefined, & t = 0 \\ 0, & otherwise \end{cases}$ $\delta'(\alpha t) = \frac{1}{\alpha\lvert\alpha\rvert} \delta'(t)$ $\int_{-\infty}^{\infty} x(t)\delta'(t - \lambda) dt = -x'(\lambda)$ x(t)δ′(t) = x(0)δ′(t) − x′(0)δ(t)
Energy and power	Periodic signals have infinite energy hence power type signals.		

Properties

1. $x(t) \times \delta(t) = x(t)$
2. $x(t-\alpha) \times \delta(t-\beta) = x(t-\alpha-\beta)$
3. $\delta(t) \times \delta(t) = \delta(t)$
4. $[\delta(t) \times \delta(t) \times \delta(t) \times \text{-------}] = \delta(t)$
5. $\delta(t-\alpha) \times \delta(t-\beta) = \delta(t-\alpha-\beta)$

A. $u(t) \times u(t) = r(t)$

B. $u(t-\alpha) \times u(t-\beta) = r(t-\alpha-\beta)$

C. $u(t) \times u(t) = \rho(t) = \frac{t^2}{2}\, u(t)$

D. $r(t-\alpha) \times u(t-\beta) = \rho(t-\alpha-\beta) = \frac{(t-\alpha-\beta)^2}{2}\, u(t-\alpha-\beta)$

7. Gaussian function

The Gaussian function is defined by the expression

$$g_a(t) = e^{-at^2} \quad -\infty < t < \infty$$

The function is extremely useful in probability theory.

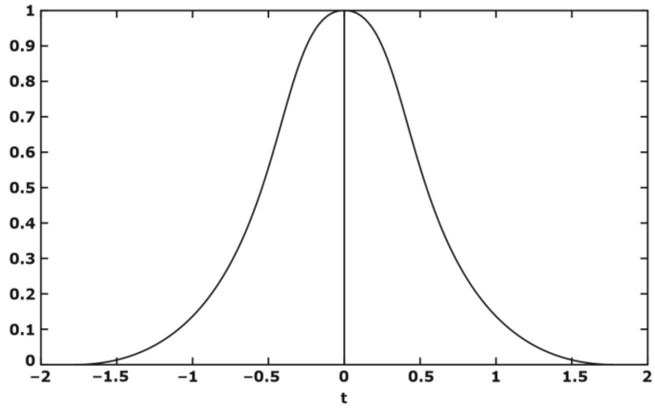


Figure: Gaussian function

8. Sinusoidal signal

A continuous-time sinusoidal signal is given by

$$x(t) = A \sin(\Omega t + \varphi)$$

where A is the amplitude, Ω is the frequency in radians per second and φ is the phase angle in radians.

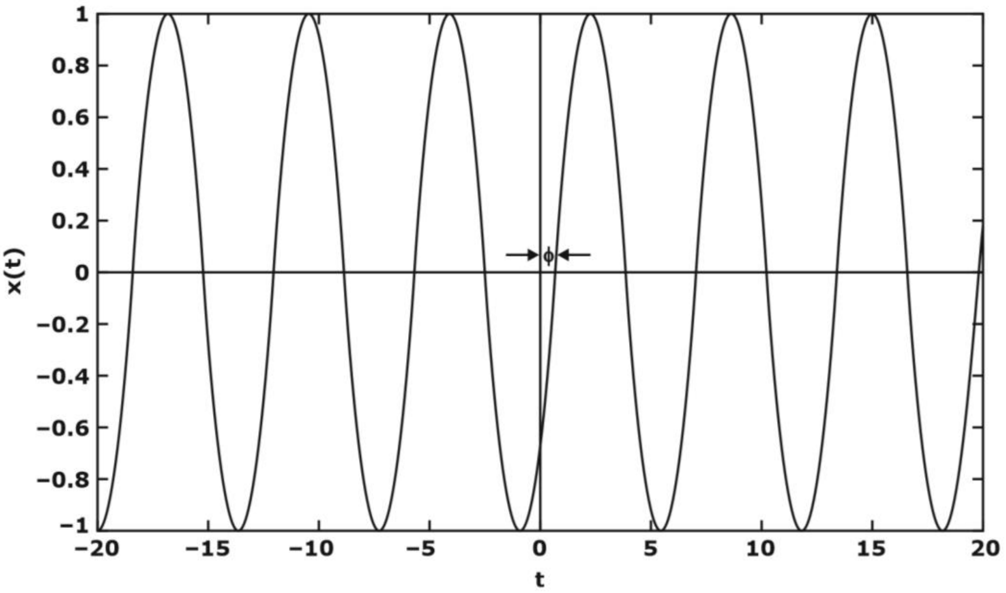


Figure: Sinusoidal signal

9. Real exponential signal

A real exponential signal is defined as

$$x(t) = Ae^{at}$$

where both A and a are real. Depending on the value of ‘a’ we get different signals.

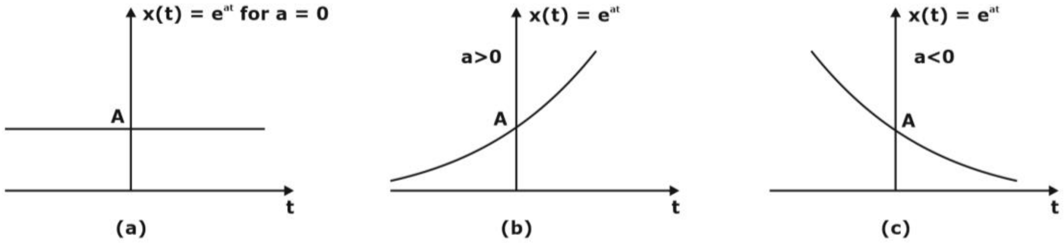


Figure: (a) A dc signal (b) exponentially growing signal (c) exponentially decaying signal.

10. Complex exponential signal

The most general form of complex exponential is given by

$x(t) = e^{st}$

Where, s is a complex variable defined as

$s = \sigma + j\Omega$

Depending on the values of σ and Ω, we get different signals.

A. If $\sigma = 0$ and $\Omega = 0$ then $x(t) = 1$; that is the signal x(t) is a pure DC signal.

B. If $\Omega = 0$, then $s = \sigma$ and $x(t) = e^{\sigma t}$, which decays exponentially for $\sigma < 0$ and grows exponentially for $\sigma > 0$.

C. If $\sigma = 0$ then $s = \pm j\Omega$ gives $x(t) = e^{j\Omega t}$ a sinusoidal signal with $\varphi = 0$.

D. If σ < 0 with finite Ω, then x(t) is a exponentially decaying sinusoidal signal.

E. If σ > 0 with finite Ω, then x(t) is a exponentially growing sinusoidal signal.

11. BASIC OPERATION ON DISCRETE TIME SIGNAL:

11.1. Addition of discrete-time signals

Addition of discrete time sequence is done by adding the signals at every instant of time [n].

11.2. Multiplication of discrete time signal

The multiplication of two discrete time signals $x_1[n]$ and $x_2[n]$ is obtained by multiplying the signal values at each instant of time n.

11.3. Amplitude scaling of discrete time Signals

Amplitude scaling is obtained by multiplying the signal x[n] with a constant A at each instant of time n. The amplitude-scaled is represented as A x[n].

11.4. Time-Scaling of discrete time Signals

Consider a discrete-time signal x[n], if the independent variable n is scaled by a factor of n then x[an] is the time-scaled version of x[n]. There are two types of time scaling

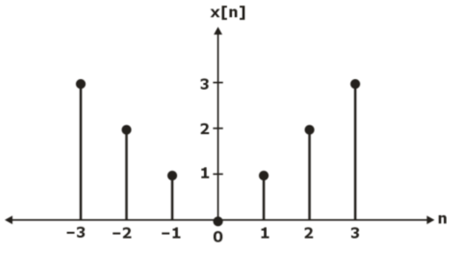
Note: Time-scaling of discrete time signals is different from continuous time signals, since discrete time signals are defined only for integer values of time variable n.

Time Compression: Decimation or Down-sampling

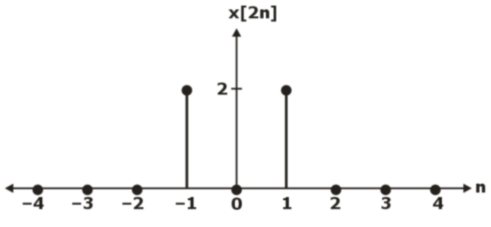
Compression of discrete time signals is also referred to as decimation. If a sequence x[n] is compressed by a factor a, some data samples of x[n] are lost. For example, if we compress x[n] by a factor of 2, the compressed signal y[n] = x[2n] contains only the alternate samples x[0], x[2], x[4] and so on. This operation losses data, and that is why time compression is called decimation or down-sampling.

Time Expansion: Interpolation or Up-sampling

In the discrete time domain, expansion is also referred to as interpolation. Let x[n] is expanded by a factor of 2 and the expanded signal is given as y[n] = x[n/2]. It is known that x[n] is defined only for integer value of n and zero for all non-integer values of n. Therefore, y[n] contains samples y[0] = x[0], y[2] = x[1], y[4] = x[2] and so on. The odd numbered samples y[1], y[3], y[5] all are zero.



(a)



(b)

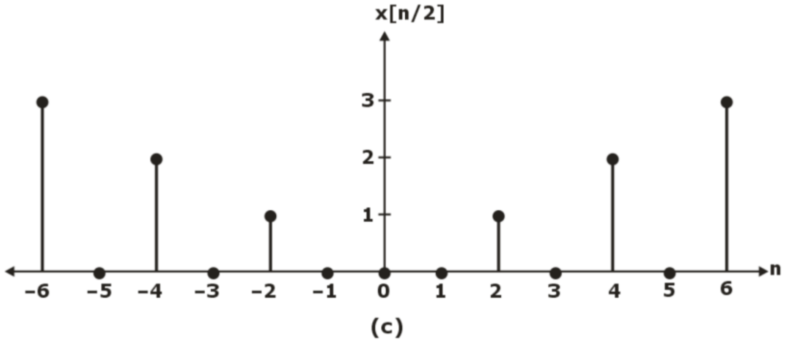


Figure: Time scaling of DT signal, (a) Original DT sequence x[n], (b) Compressed (decimated) version of x[n], (c) Expanded (interpolated) version of x[n]

11.5. Time-Shifting of discrete time Signals

The steps to obtain a time shifted signal from the original signal is given below.

(i) If x[n] is given, then $x[n + n_0]$ is plotted by shifting x[n] to the left by n_0.

(ii) If x[n] is given, then $x[n - n_0]$ is plotted by shifting x[n] to the right by n_0.

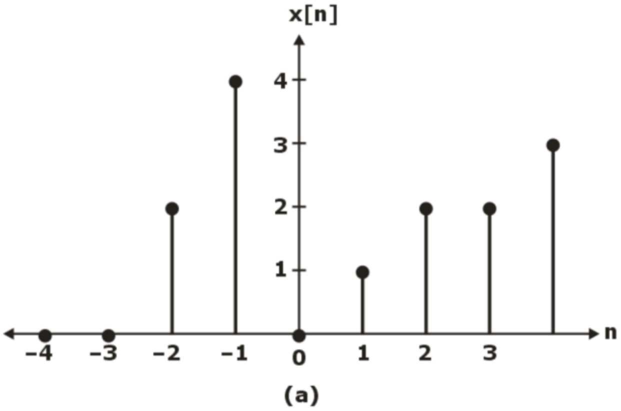
(iii) If x[−n] is given, then $x[-n - n_0]$ is plotted by shifting x[− n] to the left by n_0.

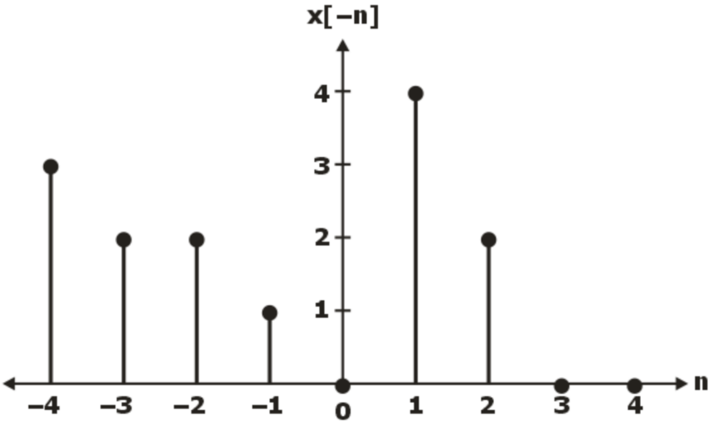
(iv) If x[− n] is given, then $x[-n + n_0]$ is plotted by shifting x[− n] to the right by n_0.

Note: The waveform of $x[n + n_0]$ is identical to that of x[n] except for a shift of n_0 time units towards the left-hand side.

11.6. Time-Reversal (folding) of discrete time signals

The folding operation produces a signal x[−n] which is the mirror image of x[n] about the vertical axis.





(b)

Figure: Time folding of DT Signal, (a) Original DT Sequence x[n], (b) Folded Version of x[n].

12. MULTIPLE OPERATIONS ON DISCRETE TIME SIGNALS

Consider a discrete time signal x[n] with multiple transformation given as

x[n] → A x[bn ± n_0]

Where a and b are assumed to be real numbers. The operations should be performed in the following order.

Methodology

Step 1: First multiply signal by a constant A to obtained amplitude scaled version of x[n] that is A x[n].

Step 2: Shift the signal A x[x] to the left or to right by n_0 time units. This will be produced shifted signal A x[n ± n_0].

Step 3: Scale the signal A x[n ± n_0] by b, the resulting signal represents A x[bn ± n_0].

Step 4: If b is negative, reflect the scaled signal A x[bn ± n_0] about the vertical axis.

The correct sequence for the above transformation is

$$x[n] \xrightarrow{A_{scaling}^{amplitude}} Ax[n] \xrightarrow{n\to n-n_0{}_{shifting}^{time}} Ax[n-n_0] \xrightarrow{n\to b_{scaling}^{time}} Ax[bn-n_0]$$

If time scaling is done before time shifting it will produce incorrect results.

$$x[n] \xrightarrow{A_{scaling}^{amplitude}} Ax[n] \xrightarrow{n\to bn_{shifting}^{time}} Ax[bn] \xrightarrow{n\to n-n_0{}_{scaling}^{time}} Ax[b(n-n_0)] \neq Ax[bn-n_0]$$

13. BASIC DISCRETE TIME SIGNALS

13.1. Discrete Impulse Function

The unit-impulse function in discrete time is defined as

$$\delta[n] = \{ \begin{smallmatrix} 1, & n=0 \\ 0, & n\neq 0 \end{smallmatrix}$$

$$\delta[n] = \left\{\ldots 0,0,0,\underset{\uparrow}{1},0,0,0\ldots\right\}$$

So, δ [n] is referred as the unit sample occurring at n = 0.

Similarly, for the shifted function δ [n − k] the unit sample occurring at n = k

That is,

$$\delta[n-k] = \{ \begin{smallmatrix} 1, & n=k \\ 0, & n\neq k \end{smallmatrix}$$

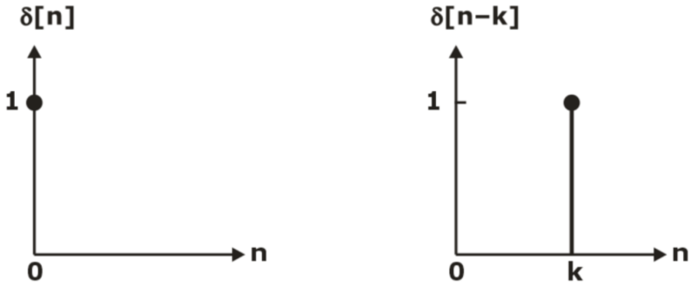


Figure: (a) DT Unit Impulse Function (b) DT Shifted Unit Impulse Function

Properties

Following are some of the important properties of unit impulse function.

(i) Product property

$$x[n]\,\delta[n - n_0] = x[n_0]\,\delta[n - n_0]$$

(ii) Shifting property

$$\sum_{\infty}^{n=-\infty} x[n]\delta[n-n_0] = x[n_0]$$

(iii) Scaling Property

The discrete-time unit impulse does not have a property corresponding to the scaling property of continuous-time unit impulse. Therefore, δ [n] = δ [an] for any nonzero integer value of n.

13.2. Discrete Unit Step Function

The unit-step sequence shown in figure 29 (a) is defined as,

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Or, $u[n] = \left\{\ldots 0,0,0,\underset{\uparrow}{1},1,1,1\ldots\right\}$

Similarly, the shifted unit-step sequence is defined as follows

$$u[n-k] = \begin{cases} 1, & n-k \geq 0 \ or \ n \geq k \\ 0, & n-k < 0 \ or \ n < k \end{cases}$$

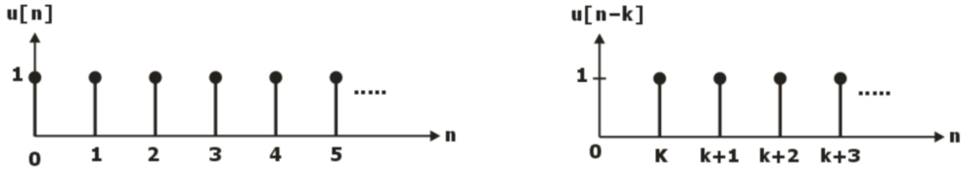


Figure: (a) DT Unit Impulse Function,(b) Shifted DT Unit Impulse Function

Properties

Following are some of the important properties of unit step function.

(i) δ[n] = u[n] – u[n – 1]

(ii) $\overset{\overset{k=-\infty}{\infty}}{\sum} \delta[k] = u[n]$

(iii) $\overset{\overset{k=-\infty}{\infty}}{\sum} \delta[n-k] = u[n]$

13.3. Discrete Unit-ramp Function

The unit ramp sequence is defined as

$$r[n] = \begin{cases} n \; for \; n \geq \; 0 \\ \\ 0 \; for \; n \; < \; 0 \end{cases}$$

Or, $r[n] = nu[n] = \{\underset{\uparrow}{0}, 1,2,3,4,5, \ldots\}$

The graphical representation of r(n) is shown in figure 30.

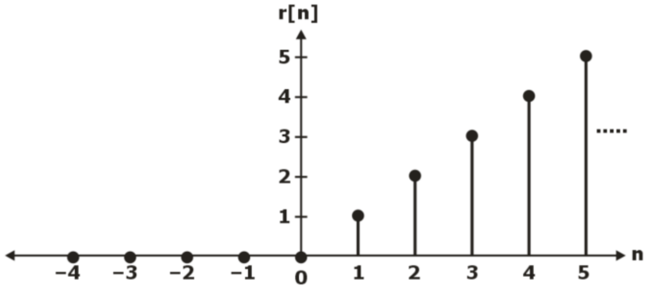


Figure: DT unit Ramp Function

13.4. Unit-Rectangular Function

The discrete-time unit rectangular sequence is shown in figure 31. It is defined as

$$rect\left[\frac{n}{2N}\right] = \begin{cases} 1, \; |n| \; \leq \; N \\ \\ 0, \; |n| \; > \; N \end{cases}$$

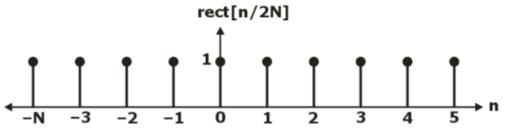


Figure: DT unit rectangular function

The signal rect [n/2N] has (2N + 1) unit samples over the interval – N ≤ n ≤ N.

13.5. Unit-Triangular Function

The discrete-time unit triangular sequence shown in figure 32 is defined as

$$tri\left(\frac{n}{N}\right) = \begin{cases} 1-\frac{|n|}{N} & |n| \le N \\ 0, & |n| > N \end{cases}$$

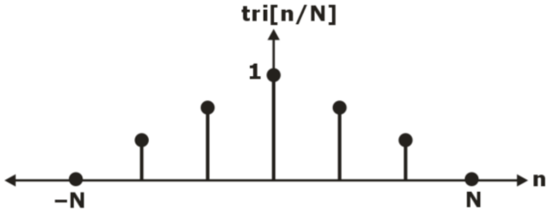


Figure 1: DT unit Triangular Function

The signal tri[n/N] has (2N + 1) unit samples over the interval − N ≤ n ≤ N.

13.6. Unit-Signum Function

The discrete-time function corresponding to the continuous time signum function is defined in figure 33.

$$sgn[n] = \begin{cases} 1, & n > 0 \\ 0, & n = 0 \\ -1, & n < 0 \end{cases}$$

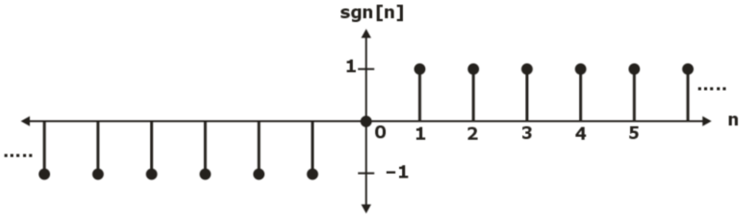


Figure: DT unit Signum Function

CHAPTER 2: CLASSIFICATION OF SIGNALS

1. Periodic and Aperiodic signal

1.1 Condition for continuous-time periodic signal

$x(t + T) = x(t), \quad -\infty < t < \infty \quad \ldots(i)$

Where T is the fundamental period of a signal.

Frequency of the periodic signal is given by

$$f = \frac{1}{T}$$

Angular frequency, measured in radian per second, is defined as

$$\omega = \frac{2\pi}{T}$$

1.2 Condition for discrete-time periodic signal

$x(n + N) = x(n), \quad -\infty < n < \infty \ldots(ii)$

Where N is called the fundamental period of a signal.

The fundamental angular frequency or simply fundamental frequency of x[n] is given by,

$$\Omega = \frac{2\pi}{N} m$$

where, N = fundamental period

m = Smallest integer.

Important point

- The sum of two or more periodic discrete-time sequence is always periodic.
- A constant signal is periodic and its fundamental period is undefined.
- The sum of two or more periodic continuous-time signals need not be periodic. They will be periodic if and only if the ratio of their fundamental periods is rational.

1.3. Steps to determine whether the sum of two or more periodic signals is periodic or not.

Step 1: Determine the fundamental period of the individual signals in the sum signal, say T_1, T_2 ……

Step 2: Find the ratio of the fundamental period of the first signal with the fundamental periods of every other signals.

Step 3: If all the ratios are rational, then the sum signal is also periodic, and its fundamental period is

$$\frac{\text{LCM of Numerator of } \mathrm{T}_1, T_2\ldots\ldots}{HCF \text{ of Denominator of } \mathrm{T}_1, T_2\ldots\ldots}$$

1.4. Steps to determine whether the sum of two or more sequence periodic or not

Step 1: Determine the fundamental period of individual sequence in the sum sequence, say N_1, N_2 ….

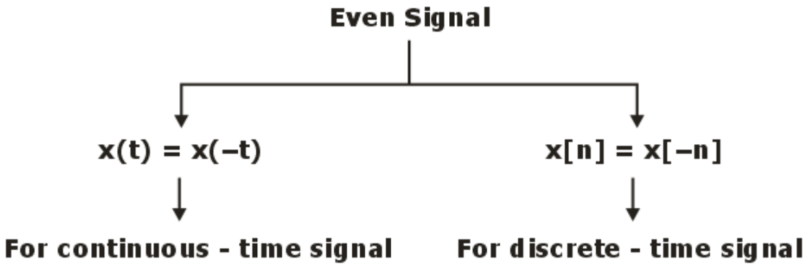
Step 2: If all the individual sequences are periodic then fundamental period is

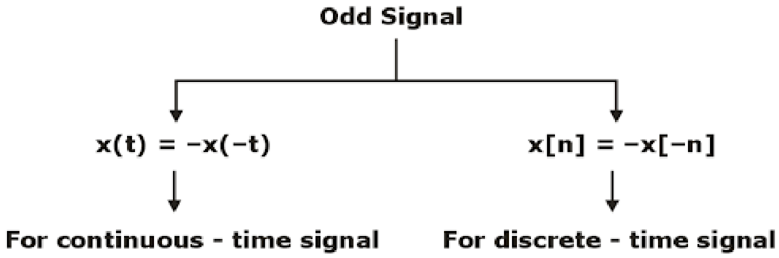
$N = \text{LCM of } N_1, N_2, \ldots..$

Step 3: If any one or more sequence in sum is aperiodic then the resultant sequences are also aperiodic.

2. Even and odd signals

Even signals are symmetric about origin whereas odd signals are antisymmetric about origin.





An arbitrary signal x(t) can always be expressed as a sum of even and odd signals as

$x(t) = x_e(t) + x_o(t)$

Where, $x_e(t)$ is called the even part of x(t) and is given by

$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

and $x_o(t)$ is called the odd part of x(t) and is given by

$$x_0(t) = \frac{1}{2}[x(t) - x(-t)]$$

Basic properties

- The sum of two even function is even and any constant multiple of an even function is even.
- The sum of two odd function is odd, and any constant multiple of an odd function is odd.
- The product of two even functions is an even function
- The product of two odd function is an even function.
- The product of an even function and an odd function is an odd function.
- Due to anti symmetry property, odd signal is always zero at t = 0

therefore, $x_o(0) = y_o(0) = 0$

or, $x_o[0] = y_o[0] = 0$

- Integration of a continuous-time odd signal within the limits [−T, T] results in a zero value ie.

$$\int_{-T}^{T} x_0(t)dt = \int_{-T}^{T} y_0(t)dt = 0$$

- The integral of a continuous-time even signal within the limits [−T, T] can be simplified as follow:

$$\int_{-T}^{T} x_e(t)dt = 2\int_{0}^{T} x_e(t)dt$$

- Adding the samples of discrete-time odd sequence $x_o[n]$ within the range [−N, N] is 0 ie,

$$\sum_{n=-N}^{N} x_o[n] = 0 = \sum_{n=-N}^{N} y_0[n]$$

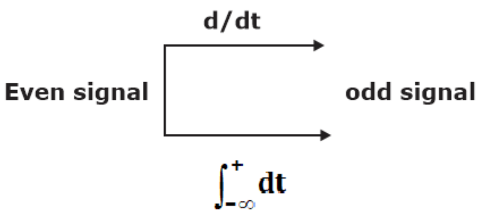
- Adding the samples of discrete-time even sequence $x_e[n]$ within the range [−N, N] simplifies to

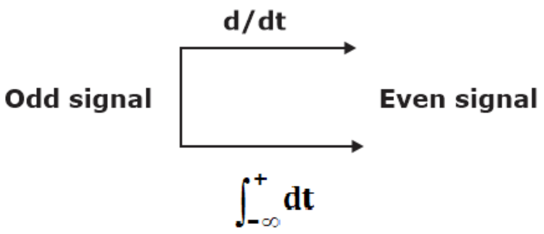
$$\sum_{n=-N}^{N} x_e[n] = x_e[0] + 2\sum_{n=1}^{N} x_e[n]$$

Note: Even and odd signals are mutually exclusive. That is, if a signal is an even signal, it cannot be odd and vice versa. however, there could be certain class of signals that could neither be termed odd nor even signal.

s/g	s/g	s/g	
$x_1(t)x_2(n)$	$x_2(t)x_2(n)$	$x_1 \cdot x_2$	x_1/x_2
E	E	E	E
E	O	O	O
O	E	O	O
O	O	E	E

"only for signals"





3. Energy and power signal

Signals which are classified on the basis of finite energy or finite average power are known as energy or power signal.

Power signal

Periodic signal is called power signal and for power signal, P_∞ = finite & $E_\infty = \infty$.

Signal is referred to as power signal, if and only if the average power of the signal satisfies the condition

$0 < P < \infty$

Energy signal

For energy signal, $P_\infty = 0$ & E_∞ = finite

Signal is referred as energy signal, if and only if the total energy of the signal satisfies the condition,

$0 < E < \infty$

In case of Continuous-time signal

Total energy is given by

$$E = \begin{cases} \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 dt \\ \int_{-\infty}^{\infty} |x(t)|^2 dt \end{cases}$$

Average power is given by,

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

Average power of a periodic signal x(t) of fundamental period T is given by,

$$p = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

In the case of discrete time signal

Energy of signal is given as

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Average power is defined by,

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{n=N} |x[n]|^2$$

Average power in a periodic signal x(n) with fundamental period N is given by

$$p = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$

4. Energy and Power Continuous time Signals

	Energy Signal		Power Signal
1.	The total energy is obtained using $E = \lim_{T\to\infty} \int_{-T}^{T} \lvert x(t)\rvert^2 dt$	1.	The average power is obtained using $P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \lvert x(t)\rvert^2 dt$
2.	For the energy signal 0 < E < ∞, and the average power P = 0.	2.	For the power signal 0 < P < ∞, and the energy E = ∞.
3.	Non-periodic signals are energy signals.	3.	Periodic signals are power signals. However, all power signals need not be periodic.
4.	Energy signals are not time limited.	4.	Power signals exist over infinite time.

Properties of Power signal:

1) Power signal has infinite energy

2) $\boxed{\begin{array}{l} x(t) \longrightarrow P \\ x(-at + b) \longrightarrow P \end{array}}$

$$\begin{cases} A \ \sin t \longrightarrow \frac{A^2}{2} \\ A \ \sin \ (2t) \longrightarrow \frac{A^2}{2} \\ A \sin \ \left(2t \ -\cdot \ \frac{\pi}{6}\right) \longrightarrow \frac{A^2}{2} \end{cases}$$

3) K x$(at + b) \longrightarrow K^2P$

5. Causal and non causal signal

A continuous-time signal x(t) is said to be causal if x(t) = 0 for t < 0, otherwise the signal is non-causal. For an anti-causal signal x(t) = 0 for t > 0.

Similarly, a discrete-time signal x(n) is said to be causal if x(n) = 0 for n < 0; otherwise the signal is non- causal. For an anti-causal discrete-time signal x(n) = 0 for n > 0.

CHAPTER 3: CLASSIFICATION OF SYSTEMS

1. Linear, nonlinear systems

A linear system is one which satisfies the principle of **superposition** and **homogeneity or scaling**.

Consider a linear system characterized by the transformation operator T[]. Let x_1, x_2 are the inputs applied to it and y_1, y_2 are the outputs. Then the following equations hold for a linear system

$$y_1 = T[x_1],\ y2 = T[x_2]$$

Principle of homogeneity: $T\,[a*x_1] = a*y_1,\ T\,[b*x_2] = =b*y_2$

Principle of superposition: $T\,[x_1] + T\,[x_2] = a*y_1+b*y_2$

Linearity: $T\,[a*x_1] + T\,[b*x_2] = a*y_1+b*y_2$

Where a, b are constants.

2. Time variant, time invariant systems

A system is said to be time variant system if its **response varies with time.** If the system response to an input signal does not change with time such system is termed as time invariant system. The behaviour and characteristics of time variant system are fixed over time.

In time invariant systems if input is delayed by time t_0 the output will also gets delayed by t_0. Mathematically it is specified as follows

$$y(t-t_0) = T[x(t-t_0)]$$

For a discrete time invariant system the condition for time invariance can be formulated mathematically by replacing t as n*Ts is given as

$$y(n-n_0) = T[x(n-n_0)]$$

Where n_0 is the time delay.

Methodology

1. Let $y(t, t_0)$ denotes the output corresponding to a delayed input $x(t - t_0)$. This can be obtained by substituting $x(t) \rightarrow x(t - t_0)$ in the given input-output relation

2. Now, obtain the delayed output $y(t - t_0)$, by directly substituting $t \rightarrow t - t_0$ in the given input-output relation

3. If $y(t, t_0) = y(t - t_0)$, then the system is time invariant. Otherwise it is a time-varying system Similarly can be checked for discrete time signals also.

3. Systems with and without memory (Dynamic and Static systems)

A system is said to be static or memory less if its output at any instant depends on the input at that and does not depend on the past or future values of input. Otherwise, if the output at any instant depends on the past or future values of input, then the system is said to be dynamic or with memory.

4. Causal and Non-causal Systems

A system is said to be causal, if the present value of the output signal depends only on the present value or past value or a combination of present and past values of the input signal. A system is said to be non-causal if it is not causal i.e., the present value of output depends on the future values of input. For example: (i) y(t) = x(t) + x(t – 1) is a causal system.

(ii) The system y[t] = x[n] + x[n — 1] is causal

NOTE: All memory less systems are causal systems because the output at any time instant depends only on the input at that time instant. Systems with memory can either be causal or non-causal.

5. Invertible and Non-Invertible Systems

A system is said to be invertible if the input to the system can be uniquely determined from the output. In order to have a system to be invertible, it is necessary that distinct inputs produce distinct outputs i.e., two different inputs cannot produce the same output.

If the system is invertible, there exists an inverse system. If these two systems are cascaded as shown in the figure, then final output is same as the input.

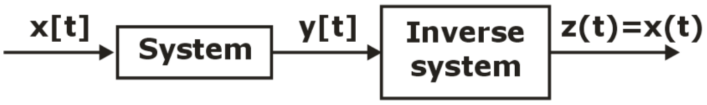


Figure: CT Invertible System

Figure: DT Invertible System

NOTE: Invertible system

A system is invertible if for the given to inputs x1(t) and x2(t) with x1(t) ≠ x2(t), it must be true that $y_1(t) \neq y_2(t)$

6. Stable and Unstable systems

A system is said to be bounded input and bounded output (BIBO) stable if and only if every bounded input produces a bounded output.

The input signal x(t) is said to be bounded if there exists a finite number M_x such that $|x(t)| \leq M_x < \infty$, for all t The system is BIBO stable if for any bounded input x(t) the output signal y(t) is also bounded i.e., $|y(t)| \leq M_y < \infty$, for all t. If the system produces unbounded output for bounded input then it is unstable.

A Discrete Time system is said to be BIBO stable if for any bounded input, it produces a bounded output.

The system is BIBO stable if for any bounded input x[n] the output signal y[n] is also bounded i.e.

$|y[n]| \leq My < \infty$, for all n

If the system produces unbounded output for a bounded input then it is unstable.

7. Properties of LTI systems in Terms of impulse Response

7.1. Memory less LTI system

A CT system is said to be memory less if its output at any time depends only on the value of the input at the same time. A memory less, linear time invariant system has an input output relation that is of the form

y(t) = Kx(t)

where K is any arbitrary constant. By substituting x(t) = δ(t) in equation, memory less LTI system has the impulse response

h(t) = Kδ(t)

NOTE: Memory less LTI continuous system

An LTI continuous system will be memory less if and only if its impulse response h(t) = 0 for t ≠ 0.

7.2. Causal LTI System

An LTI system is said to be causal if the output at any instant depends only on the present and past values of the input. Consider a continuous-time LTI system whose output y(t) can be obtained using convolution integral given by

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

At t = 0 the output can be written as

$$y(0) = \int_{-\infty}^{\infty} h(\tau)x(-\tau)d\tau$$

For a causal system the output depends only on present and past values of input. From equation, we can see that, if τ ≥ 0, the output depends on present and past values of input and the system is causal. But if τ < 0, then output depends on future values of input, therefore the system will be causal if h(τ) = 0 for τ < 0.

NOTE:

An LTI system will be causal if and only if its impulse response h(t) = 0 for t < 0.

7.3. Invertible LTI system

An LTI system is said to be invertible if the input of the system can be recovered from the output. As we discussed earlier. if the inverse system is connected in cascade with the original system, then final output will be same as the input. This can be illustrated in below figure.

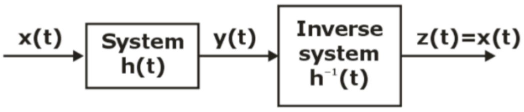


Figure: An LTI inverse system

NOTE:

An LTI system is invertible if its impulse response satisfies $h^{-1}(t) * h(t) = \delta(t)$

NOTE:

An LTI system is BIBO stable if the impulse response is absolutely integrable

$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

8. Table Showing Comparison of Different Signals with Their Properties

S.No.	Relationship between output and input	Linearity	Causality	Static or Dynamic	Time-Variancy
1.	$y(t) = x(t - t_0)$	Linear	Causal	Dynamic	Time-invariant
2.	$y(t) = t.x(t)$	Linear	Causal	Static	Time-variant
3.	$y(t) = x(t) + A$	Non-linear	Causal	Static	Time-invariant
4.	$y(t) = x(\alpha t)$	Linear	Non-causal	Dynamic	Time-variant
5.	$y(t) = =x^2(t)$	Non-linear	Causal	Static	Time-invariant
6.	$y(t) = x(t^2)$	Linear	Non-causal	Dynamic	Time-variant

7.	$y(t) = \frac{dx(t)}{dt}$	Linear	Causal	Dynamic	Time-invariant
8.	$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$	Linear	Causal	Dynamic	Time-invariant
9.	$y(t) = \int_{-\infty}^{at} x(\tau)d\tau$	Linear	Non-causal	Dynamic	Time-variant
10.	y(t) = cos[x(t)]	Non-linear	Causal	Static	Time-invariant
11.	y(t) = x(-t)	Linear	Non-causal	Dynamic	Time-variant
12.	y(t) = $\cos\omega_0 t.x(t)$	Linear	Causal	Static	Time-variant

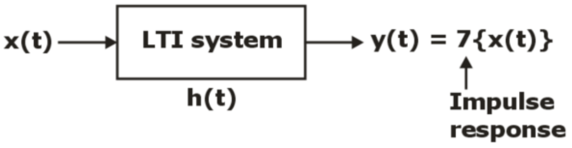
Note:

⇒ All the static systems are causal systems but converse is not true.

⇒ All the non-causal systems are dynamic systems but the converse is not true.

CHAPTER 4: LTI SYSTEM (CONVOLUTION)

1. Convolution Integral



$$\boxed{\boldsymbol{y(t) = \int_{\infty}^{\infty} x(\tau)h(t-\tau)d\tau}}$$

or

$$\boxed{y(t) = x(t) \otimes h(t)}$$

$$\boxed{u(t+\alpha) \otimes u(t+\beta) = r(t+\alpha+\beta)}$$

$$\boxed{\boldsymbol{y(t) = u(t) * r(t) = \frac{t^2}{2}u(t)}} \ \boldsymbol{Parabolic function}$$

$$\boxed{u(t+\alpha)|\otimes r(t+\beta)| = \frac{(t+\alpha+\beta)^2}{2}u(t+\alpha+\beta)}$$

2. Properties of convolution Integral

i. Commutative property

$$\boxed{y(t) = x(t) \otimes h(t) = h(t) \otimes x(t)}$$

ii. Distributive property

$x(t) \otimes [h_1(t) \otimes h_2(t)] = x(t) \otimes h_1(t) + x(t) \otimes h_2(t)$

iii. Associative property

$[x(t) \otimes h_1(t)] \otimes h_2(t) = x(t) \otimes [h_1(t) \otimes h_2(t)]$

iv. Property based on time invariancy

a. $x(t) \otimes h(t) = y(t)$

b. $x(t + \alpha) \otimes h(t) = y(t + \alpha)$

c. $x(t) \otimes h(t + \beta) = y(t + \beta)$

d. $x(t + \alpha) \otimes h(t + \beta) = y(t + \alpha + \beta)$

v) Differentiation property:

If $x(t) * h(t) = y(t)$

Then,

$$\frac{dx(t)}{dt} * h(t) = \frac{dy(t)}{dt}$$

$$x(t) * \frac{dh(t)}{dt} = \frac{dy(t)}{dt}$$

$$\frac{dx(t)}{dt} * \frac{dh(t)}{dt} = \frac{d^2y(t)}{dt^2}$$

vi. When two equal width rectangular pulses of duration 'T' are convoluted, resultant signal is always a triangular pulse of duration "2T".

vii. When two unequal width rectangular pulses of duration T_1 and T_2 are convoluted, then the resultant signal is always a trapezoidal pulse of duration $T_1 + T_2$.

viii. Invertibility of continuous time signals

$$\boxed{h(t) = h_1(t) \otimes h_2(t) = \delta(t)}$$

ix. Scaling property of convolution

$$\boxed{x(at) \otimes h(at) = \frac{1}{a} y(at)}$$

x. Stability

For a continuous time LTI system to be stable, its impulse response should be absolutely integrable. i.e.

$$\boxed{\int_{-\infty}^{\infty} |h(t)| dt < \infty}$$

xi. System in cascade connection

$$\boxed{h_{cascade}(t) = h_1(t) \otimes h(t)}$$

xii. System in parallel connection

$$\boxed{h_{parallel}(t) = h_1(t) + h_2(t)}$$

3. Discrete Time LTI System

$$x[n] \rightarrow \boxed{LTI\ system} \rightarrow y[n] = x[n] \otimes h[n]$$

output response of a LTI system to an input x[n]

$$\boxed{y[n] = x[n] \otimes h[n] = \sum_{K=-\infty}^{\infty} x[K] h[n-K]}$$

convolution sum

4. Properties of convolution sum

i. Commutative property

$x[n] \otimes h[n] = h[n] \otimes x[n]$

ii. Distributive property

$x[n] \otimes (h_1[n] + h_2[n]) = x[n] \otimes h_1[n] + x[n] \otimes h_2[n]$

iii. Associative property

$x[n] \otimes h_1[n] \otimes h_2[n] = x[n] \otimes (h_1[n] \otimes h_2[n])$

iv. Shifting property

If $y[n] = x[n] \otimes h[n]$

then, $x[n - n_0] \otimes h\ n\ [n - n_1] = y[n - n_0 - n_1]$

v. Duration of convolution

Let M be the duration (length) of sequence x[n] and N be the duration (length) of sequence h[n], then the duration of convolution sum.

$y[n] = x[n] \otimes h[n]$ is **M+N-1**

vi. Generalized Results

a. If $x[n] = u(n)$

$h[n] = u(n)$

then $y[n] = x[n] \otimes h[n] = u[n] \otimes u[n] = (n + 1) u[n]$

b. $u[n + \alpha] \otimes u[n + \beta] = [n + \alpha + \beta + 1]\, u\,[n + \alpha + \beta]$

vii. Systems in parallel

$h[n] = h_1[n] + h_2[n]$

viii. System in cascade

$h[n] = h_1[n] \otimes h_2[n]$

CHAPTER 5: CONTINUOUS TIME FOURIER SERIES (CTFS)

1. Existence of Fourier Series

The Fourier series for a periodic signal x(t) exists if it satisfies the following conditions which are knows as Dirichlet conditions:

- The function x(t) has a finite number of maxima and minima in one period.
- The function x(t) has a finite number of discontinuities in one period.
- The function x(t) is absolutely integrable over one period, that is,

$$\int_0^T |x(t)|dt < \infty$$

2. Fourier Series

Fourier Series Form	Mathematical Expression	Coefficients
Trigonometric	$x(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\,\omega_0 t + b_n \sin n\,\omega_0 t)$	$a_0 = \frac{1}{T_0}\int_{T_0} x(t)dt$ $a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos(n\omega_0 t)\,dt$ $b_n = \frac{2}{T_0}\int_{T_0} x(t)(\sin n\,\omega_0 t)dt$
Exponential	$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$	$c_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt$
Polar or Cosine Form	$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \theta_n)$	$A_0 = a_0$ $A_n = \sqrt{a_n^2 + b_n^2}$ $\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$

NOTE: $c_n = \frac{1}{2}(a_n - jb_n)$,

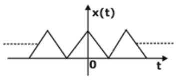
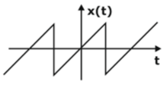
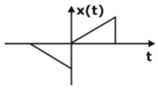
$c_{-n} = \frac{1}{2}(a_n + jb_n)$,.

3. Properties of Fourier Series

S.N.	Property	CTFS Coefficients
1	Linearity	$px_1(t) + qx_2(t) \xleftrightarrow{CTFS} pc_n + qd_n$
2	Time shifting	$x(t - t_0) \xleftrightarrow{CTFS} e^{-jn\omega_0 t_0} c_n$
3	Time reversal	$x(-t) \xleftrightarrow{CTFS} c_{-n}$
4	Time scaling	$x(at) \xleftrightarrow{CTFS} c_n,$ With period aT0
5	Multiplication	$x_1(t)x_2(t) \xleftrightarrow{CTFS} \sum_{l=-\infty}^{\infty} a_l b_{n-l}$

6	Conjugation and conjugate symmetry	$x*(t) \xleftrightarrow{CTFS} c_{-n}^*$ and $c_{-n} = c_n^*$ for x(t) is real
7	Time Differentiation	$\frac{dx(t)}{dt} \xleftrightarrow{CTFS} jn\omega_0 c_n$
8	Time Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{CTFS} \frac{c_n}{jn\omega_0}$
9	Convolution	$x_1(t) * x_2(t) \xleftrightarrow{CTFS} T_0 c_n d_n$
10	Parseval's Theorem If $x_1(t) = x_2(t) = x(t)$	$\int_0^{T_0} x_1(t)x_2^*(t)dt = T_0 \sum_{n=-\infty}^{\infty} c_n d_n^*$ $\int_0^{T_0} \lvert x(t)^2 \rvert dt = T_0 \sum_{n=-\infty}^{\infty} \lvert c_n \rvert^2$
11	Frequency Shifting	$e^{jk\omega_0 t}x(t) \xleftrightarrow{CTFS} c_{n-k}$

4. Condition for periodic signals to be symmetry

Type of symmetry	Condition	Example	a_0	a_n	b_n	Property
Even	x(t) = x(-t)		$a_0 = \frac{2}{T}\int_0^{T/2} x(t)dt$ $\Omega_0 = \frac{2\pi}{T}$ T is fundamental period	$\frac{4}{T}\int_0^{T/2} x(t)\cos(n\Omega_0 t)dt$	0	Cosine terms only
Odd	x(t) = -x(-t)		0	0	$\frac{4}{T}\int_0^{T/2} x(t)\sin(n\Omega_0 t)dt$	Sine terms only
Half wave	$x(t) = -x\left(t \pm \frac{T}{2}\right)$		0	$\frac{4}{T}\int_0^{T/2} x(t)\cos(n\Omega_0 t)dt$	$\frac{4}{T}\int_0^{T/2} x(t)\sin(n\Omega_0 t)dt$	Odd n only

CHAPTER 6: CONTINUOUS TIME FOURIER TRANSFORM (CTFT)

1. Fourier Transform

Fourier transform is a transformation technique which transforms non-periodic signals from the continuous-time domain to the corresponding frequency domain. The Fourier transform of a continuous-time non periodic signal $x(t)$ is defined as

$$X(j\omega) = F[x(t)] = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

If the frequency is represented in terms of cyclic frequency *f* (in Hz), then the above equation is written as

$$X(jf) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$$

2. Existence of Fourier Transform

Dirichlet Conditions

(i) $x(t)$ is absolutely integrable. That is,

$$\int_{-\infty}^{\infty} |x(t)|dt < \infty$$

(ii) $x(t)$ has a finite number of maxima and minima and a finite number of discontinuities within any finite interval.

3.MAGNITUDE AND PHASE SPECTRA

The Fourier transform $X(j\omega)$ of a signal $x(t)$ is in general, complex form can be expressed as

$$X(j\omega) = |X(j\omega)|\boxed{X(j\omega)}$$

The plot of $|X(j\omega)|$ versus $\omega$is called magnitude spectrum of *x(t)* and the plot of $\boxed{X(j\omega)}$versus $\omega$is called phase spectrum. The amplitude (magnitude) and phase spectra are together called Fourier spectrum which is nothing but frequency response of $X(j\omega)$ for the frequency range $-\infty < \omega < \infty$.

4. Inverse Fourier Transform

The inverse Fourier transform of $X(j\omega)$ is given as

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(j\omega)e^{j\omega t}d\omega$$

5. Fourier Transform of Some Basic Signals

S. No.	Time Domain $x(t)$	Fourier Transform $X(j\omega)$
1.	1	$2\pi\delta(\omega)$
2.	$\delta(t)$	1
3.	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
4.	$e^{-at}u(t)$	$\frac{1}{a + j\omega}$
5.	$e^{-a\|t\|}$	$\frac{2a}{a^2 + \omega^2}$
6.	$te^{-at}u(t)$	$\frac{1}{(a + j\omega)^2}$

7.	$t^n e^{-at} u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$
8.	$sgn(t) = \{\begin{matrix} 1\ t > 0 \\ -1 t < 0 \end{matrix}$	$\frac{2}{j\omega}$
9.	$e^{j\omega_o t}$	$2\pi\delta(\omega - \omega_0)$
10.	$cos(\omega_0 t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
11.	$sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
12.	$e^{-at} cos(\omega_0 t)\, u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$
13.	$e^{-at} sin(\omega_0 t)\, u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$
14.	$\text{rect}\left(\frac{t}{\tau}\right) = \{\begin{matrix} 1 \lvert t \rvert \le \tau/2 \\ 0 \lvert t \rvert > \tau/2 \end{matrix}$	$\tau\, sin\, c\left(\frac{\omega\tau}{2\pi}\right)$
15.	$\frac{W}{\pi} sin\, c\left(\frac{Wt}{\pi}\right)$	$\text{rect}\left(\frac{\omega}{2W}\right) = \{\begin{matrix} 1 \lvert\omega\rvert \le W \\ 0 \lvert\omega\rvert > W \end{matrix}$
16.	$\Delta\left(\frac{t}{\tau}\right) = \begin{cases} 1 - \frac{\lvert t \rvert}{\tau} \lvert t \rvert \le \tau \\ 0 Otherwise \end{cases}$	$\tau\, sin\, c^2\left(\frac{\omega\tau}{2\pi}\right)$
17.	$\sum_{k=-\infty}^{\infty} \delta(t - kT_0)$	$\omega_0 \sum_{m=-\infty}^{\infty} \delta(\omega - m\omega_0)$
18.	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$

6. Properties of Fourier Transform

S. No.	Properly	Time Signal x(t)	Fourier Transform X(j ω)
1.	Linearity	$ax_1(t) + bx_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
2.	Time Shifting	$X(t - t_0)$	$e^{-j\omega t_0} X(j\omega)$
3.	Conjugation	$X^*(t)$	$X^*(-j\omega)$
4.	Time Scaling	$X(at)$	$\frac{1}{\lvert a \rvert} X\left(j\frac{\omega}{a}\right)$
5.	Differentiation in time	$\frac{d^n x(t)}{dt^n}$	$(j\omega)^n X(j\omega)$
6.	Differentiation in frequency domain	$t\, x(t)$	$j\frac{dX(j\omega)}{d\omega}$
7.	Time Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{1}{j\omega} X(j\omega) + \pi X(0)\delta(\omega)$
8.	Frequency Shifting	$X(t)\, e^{j\omega t}$	$X[j(\omega - \omega_0)]$
9.	Duality	$X(t)$	$2\pi x(-j\omega)$
10.	Time convolution	$X(t)*h(t)$	$X(j\omega)\, H(j\omega)$
11.	Frequency Convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi}[X_1(j\omega) * X_2(j\omega)]$
12.	Parseval's theorem	$E_x = \int_{-\infty}^{\infty} \lvert x(t) \rvert^2 dt$	$E_x = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega) \rvert^2 d\omega$
13.	Time reversal	$X(-t)$	$X(-j\omega)$

7. Important Points

(i) If $x(t)$ is a real and even symmetric function, then its Fourier transform $X(j\omega)$ is also real and even.

(ii) If $x(t)$ is real and odd symmetric signal, its Fourier transform $X(j\omega)$ is imaginary and odd symmetric.

(iii) If $x(t)$ is an imaginary and even symmetric function, then its Fourier transform $X(j\omega)$ is also imaginary and even symmetric.

(iv) If $x(t)$ is imaginary and odd symmetric signal, its Fourier transform $X(j\omega)$ is real and odd symmetric.

CHAPTER 7: LAPLACE TRANSFORM

1. The Bilateral or Two-Sided Laplace Transform

The bilateral or two-sided Laplace transform of a continuous-time signal x(t) is defined as

$$X(s) = L\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

2. The Unilateral Laplace Transform

The Laplace transform for causal signals and systems is referred to as the unilateral Laplace transform and is defined as follows:

$$X(s) = L\{x(t)\} = \int_{0}^{\infty} x(t)e^{-st}dt$$

Comparison table for unilateral and bilateral Laplace transform:

Bilateral LT	Unilateral LT
1. X(s) $\int_{-\infty}^{\infty} x(t)e^{-st}\,dt = LT[x(t)]$	1. $X(s) = \int_{0^-}^{\infty} x(t)e^{-st}\,dt = ULT[x(t)]\ \backslash$
2. Limits of integration: $-\infty\ to\ +\infty$	2. Limits of integration: $0^-\ to\ \infty$
3. ROC is must	3. No need to specify ROC (ROC must always be RHS of s- plane)
4. BLT is unique if ROC is specified	4.ULT is unique
5. Handles both causal and non-causal systems	5.Handles only causal systems

3. THE EXISTENCE OF LAPLACE TRANSFORM

The bilateral Laplace transform of a signal x(t) exists if the following integral converges (i.e. finite)

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

Substituting s = σ + jω in above equation

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-(\sigma+j\omega)t}dt$$

$= \int_{-\infty}^{\infty}[x(t)e^{-\sigma t}]e^{-j\omega t}dt$

The above integral converges if

$\int_{-\infty}^{\infty}|x(t)e^{-\sigma t}|dt < \infty$

Hence, the Laplace transform of x(t) exists if x(t) $e^{-\sigma t}$ is absolutely integrable.

4. REGION OF CONVERGENCE

Laplace transform of x(t) i.e. X(s) exists if

$$\int_{-\infty}^{\infty} |x(t)e^{-\sigma t}|dt < \infty$$

The range of values of σ (i.e. real part of s) for which the Laplace transform converges is known as Region of Convergence (ROC).

5. Laplace Transform of Some Basic Function

S. No.	CT signal x(t)	Laplace Transform $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$	ROC
1.	δ(t)	1	Entire s-plane
2.	u(t)	$\frac{1}{s}$	Re{s} > 0
3.	u(t) – u(t –a)	$\frac{1}{s}(1-e^{-as})$	Re{s} > 0
4.	$e^{-at}u(t)$	$\frac{1}{a+s}$	Re {s} > – a
5.	t u(t)	$\frac{1}{s^2}$	Re {s} > 0
6.	$t^n u(t)$	$\frac{n!}{s^n+1}$	Re {s} > 0
7.	$te^{-at}u(t)$	$\frac{1}{(a+s)^2}$	Re {s} > – a
8.	$t^n e^{-at}u(t)$	$\frac{n!}{(a+s)^{n+1}}$	Re {s} > – a
9.	$\cos(\omega_0 t)u(t)$	$\frac{s}{\omega_0^2+s^2}$	Re{s} > a
10.	$\sin(\omega_0 t)u(t)$	$\frac{\omega}{\omega_0^2+s^2}$	Re {s} > 0
11.	$x(t) = \cos^2(\omega_0 t)u(t)$	$\frac{(2\omega_0^2+s^2)}{s(4\omega_0^2+s^2)}$	Re {s} > 0
12.	$x(t) = \sin^2(\omega_0 t)\ u(t)$	$\frac{2\omega_0^2}{s(4\omega_0^2+s^2)}$	Re {s} > 0
13.	$x(t) = \exp(-at)\cos(\omega_0 t)\ u(t)$	$\frac{a+s}{(a+s)^2+\omega_0^2}$	Re {s} > – a
14.	$x(t)=\exp(-at)\sin(\omega_0 t)\ u(t)$	$\frac{W_0}{(a+s)^2+w_0^2}$	Re {s} > – a

6. Properties of Laplace Transform

S.N.	**Property**	**Time function x(t)**	**ROC**
1.	Linearity	$ax_1(t) + bx_2(t) \overset{L}{\longleftrightarrow} aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
2.	Time scaling	$x(at) \overset{L}{\longleftrightarrow} \frac{1}{\lvert a\rvert} X\left(\frac{s}{a}\right)$	aR_x
3.	Time shifting	$x(t-t_0) \overset{L}{\longleftrightarrow} e^{-st_0}X(s)$	R_x

4.	Frequency shifting	$e^{s_0 t}x(t)\overset{L}{\longleftrightarrow}X(s-s_0)$	R_x + Re(s_0)
5.	Time differentiation	$\frac{dx(t)}{dt}\overset{L}{\longleftrightarrow}sX(s)-x(0)$	R_x
6.	time integration	$\int_0^t x(\tau)\,d\tau\overset{L}{\longleftrightarrow}\frac{X(s)}{s}$	R ∩ Re(s) >0
7.	s-domain differentiation	$-tx(t)\overset{L}{\longleftrightarrow}\frac{dX(s)}{ds}$	R_x
8.	Conjugation	$x^*(t)\overset{L}{\longleftrightarrow}X^*(s\,*)$	R_x
9.	Time convolution	$x_1(t)*x_2(t)\overset{L}{\longleftrightarrow}X_1(s)X_2(s)$	atleast R_1 ∩ R_2
10.	s-domain convolution	$x_1(t)x_2(t)\overset{L}{\longleftrightarrow}\frac{1}{2\pi j}[X_1(s)*X_2(s)]$	atleast R_1 ∩ R_2
11.	Initial value theorem	$x(0^+)=\lim_{t\to 0^+}x(t)=\lim_{s\to\infty}sX(s)$	
12.	Final value theorem	$x(\infty)=\lim_{t\to\infty}x(t)=\lim_{s\to 0}sX(s)$	
13.	Time Reversal	$x(-t)\overset{L}{\longleftrightarrow}X(-s)$	$-R_x$

7.IMPULSE RESPONSE AND TRANSFER FUNCTION

Let $x(t)\overset{L}{\longleftrightarrow}X(s)$ is the input and $y(t)\overset{L}{\longleftrightarrow}Y(s)$ is the output of an LTI continuous time system having impulse response $h(t)\overset{L}{\longleftrightarrow}H(s)$. The response y(t) of the continuous time system is given by convolution integral of input and impulse response as

$$y(t)=x(t)*h(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)\,d\tau$$

Using convolution property of Laplace transform the above equation can be written as.

Y(s) = X (s) H (s)

Thus $H(s)=\frac{Y(s)}{X(s)}$

Where, H(s) defined as the transfer function of the system. It is the Laplace transform of the impulse response.

Impulse response is

$$h(t)=L^{-1}\{H(s)\}=L^{-1}\left\{\frac{Y(s)}{X(s)}\right\}$$

8.STABILITY AND CAUSALITY

For a causal system the ROC of its rational transfer function H(s) is to the right of the righter most pole.

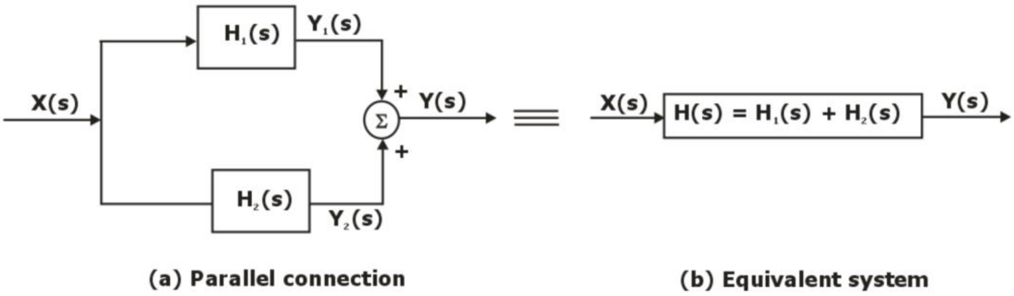
For the system to be stable (i.e. the ROC of its system function H(s) includes the entire jω-axis) the righter most pole of H(s) must be to the left of jω axis.

NOTE: A causal system with rational transfer function H(s) is stable is and only if all its poles lie in the negative half of s-plane.

9. SYSTEM FUNCTION FOR INTERCONNECTED LTI SYSTEMS

1. Parallel Connection

The parallel interconnection of two LTI continuous systems having impulse responses $h_1(t)$ and h_2 (t) is shown in the below figure.



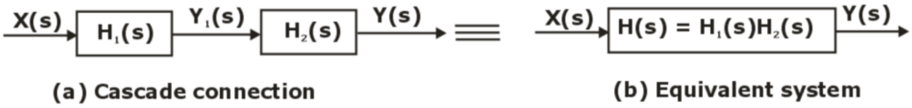
(a) Parallel connection

(b) Equivalent system

Figure: Parallel connection of LTI system in s-domain

2. Cascaded Connection

Two systems with impulse responses $h_1(t)$ and h_2 are connected in cascaded configuration as shown in below figure.



(a) Cascade connection

(b) Equivalent system

Figure: Cascaded connection of LTI system in s-domain

10. ZERO INPUT RESPONSE AND ZERO-STATE RESPONSE

The Laplace transform gives total response which includes zero input response and zero state response components.

Total response = Zero input response + Zero state response

10.1 Zero- input response:

The input is considered as zero and response is due to the initial conditions i.e. initial conditions generates the output.

10.2 Zero state response:

The initial conditions are considered as zero (i.e. zero state) and response is due to applied input. The term zero state signifies the system is initially released.

This is also termed as forced response as we are applying input (i.e., force to the system) with zero initial condition.

CHAPTER 8: Z TRANSFORM

1. The Bilateral or Two-sided Z-transform

The z-transform of a discrete time sequence x[n], is defined as

$$X(z) = Z\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

2. The unilateral or One-sided z-transform

The z-transform for causal signals and systems is referred to as the unilateral z-transform. For a causal sequence

$z[n] = 0$, for $n < 0$

Therefore, the unilateral z-transform is defined as

$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

3. EXISTENCE OF Z-TRANSFORM

For existence of z-transform

$|X(z)| < \infty$

$$\sum_{n=-\infty}^{\infty} x[n]r^{-n} < \infty$$

4. Standard z transforms with their respective ROCs.

S.No.	DT sequence x[n]	z-transform	ROC
1.	$\delta[n]$	1	Entire z-plane
2.	$\delta[n - n_0]$	Z^{-n0}	Entire z-plane except z = 0
3.	$u[n]$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$	$\|z\| > 1$
4.	$a^n u[n]$	$\frac{1}{1-\alpha z^{-1}} = \frac{z}{z-\alpha}$	$\|z\| > \|a\|$
5.	$a^{n-1}u[n - 1]$	$\frac{z^{-1}}{1-\alpha z^{-1}} = \frac{z}{z-\alpha}$	$\|z\| > \|a\|$
6.	$nu[n]$	$\frac{z^{-1}}{(1-z^{-1})^2} = \frac{z}{(z-1)^2}$	$\|z\| > 1$
7.	$na^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2} = \frac{\alpha z}{(z-\alpha)^2}$	$\|z\| > a$
8.	$\cos(\Omega_0 n)\, u[n]$	$\frac{1-z^{-1}\cos\Omega_0}{1-2z^{-1}\cos\Omega_0 + z^{-2}}$ or $\frac{z[z-\cos\Omega_0]}{z^2-2z\cos\Omega_0+1}$	$\|z\| > 1$
9.	$\sin(\Omega_0 n)\, u[n]$	$\frac{z^{-1}\sin\Omega_0}{1-2z^{-1}\cos\Omega_0 + z^{-2}}$ or $\frac{z\sin\Omega_0}{z^2-2z\cos\Omega_0+1}$	$\|z\| > 1$

10.	$a^n \cos(\Omega_0 n)u[n]$	$\frac{1-\alpha z^{-1}\cos\Omega_0}{1-2\alpha z^{-1}\cos\Omega_0+\alpha^2 z^{-2}}$ or $\frac{z[z-\alpha\cos\Omega_0]}{z^2-2\alpha z\cos\Omega_0+\alpha^2}$	$\|z\| > \|a\|$
11.	$a^n \sin(\Omega_0 n)\, u[n]$	$\frac{\alpha z^{-1}\sin\Omega_0}{1-2\alpha z^{-1}\cos\Omega_0+\alpha^2 z^{-2}}$ or $\frac{\alpha z\sin\Omega_0}{z^2-2\alpha z\cos\Omega_0+\alpha^2}$	$\|z\| > a$
12.	$r\, a^n \sin(\Omega_0 n + \theta)$ $u[n]$ with $a \in R$	$\frac{A+Bz^{-1}}{1+2\gamma z^{-1}+\alpha^2 z^{-2}}$ or $\frac{z(Az+B)}{z^2+2\gamma z+\gamma^2}$	$\|z\| \leq \|a\|^{(n)}$

5. Properties of z- transform

Properties	**Time domain**	**z-transform**	**ROC**
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	at least $R_1 \cap R_2$
Time shifting (bilateral or non-causal)	$x[n - n_0]$	$Z^{-n_0}X(z)$	R_x except for the possible deletion or addition of z = 0 or z = ∞
	$x[n + n_0]$	$Z^{n_0}X(z)$	
Time shifting (unilateral or causal)	$x[n - n_0]$	$z^{-n_0}\left(X(z) + \sum_{m=1}^{n_0} x[-m]z^m\right)$	R_x except for the possible deletion or addition of z = 0 or z = ∞
	$x[n + n_0]$	$z^{n_0}\left(X(z) - \sum_{m=1}^{n_0-1} x[m]z^{-m}\right)$	
Time reversal	$x[-n]$	$X\left(\frac{1}{z}\right)$	$1/R_x$
Differentiation in z domain	$nx[n]$	$-z\frac{dX(z)}{dz}$	R_x
Scaling in z domain	$a^n x[n]$	$X\left(\frac{z}{a}\right)$	$\|a\|R_x$
Time scaling (expansion)	$x_k[n] = x[n/k]$	$X(z^k)$	$(R_x)^{1/k}$
Time differencing	$x[n] - x[n - 1]$	$(1 - z^{-1})\, X(z)$	R_x, except for the possible deletion of the origin
Time convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	at least $R_1 \cap R_2$
Conjugations	$x^*[n]$	$X^*(z^*)$	R_x

Initial-value theorem		$x[0] = \lim_{z\to\infty} X(z)$	provided x[n] = 0 for n < 0
Final-value theorem		x[∞] $= \lim_{n\to\infty} x(n)$ $= \lim_{x\to 1}(z-1)X(z)$	provided x[∞] exists

6.Causality

A linear time-invariant discrete time system is said to be causal if the impulse response **h[n] = 0, for n < 0** and it is therefore **right-sided.** The ROC of such a system H(z) is the exterior of a circle. If H(z) is rational then the system is said to be causal if

1. The ROC is the exterior of a circle outside the outermost pole; and
2. The degree of the numerator polynomial of H9z) should be less than or equal to the degree of the denominator polynomial.

NOTE: $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$ condition for the stability.

CHAPTER 9: DISCRETE TIME FOURIER TRANSFORM (DTFT)

1. DTFT

The DTFT of a non-periodic sequence x[n] is given by

$$X(e^{j\Omega}) = F\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

2. Magnitude and Phase Spectra

The Fourier transform X(e$^{j\Omega}$) of x[n] is, in general, complex and can be expressed as

$$X(e^{j\Omega}) = |X(e^{j\Omega})| \angle X(e^{j\Omega})$$

The plot of |X(e$^{j\Omega}$)| versus Ω is called magnitude spectrum of x[n] and the plot of ∠X(e$^{j\Omega}$) versus Ω is called phase spectrum. The amplitude (magnitude) and phase spectra are together called Fourier spectrum of signal x[n].

3. Existence of DTFT

The DTFT X(e$^{j\Omega}$) of a DT sequence x[n] exists, if x[n] is absolutely summable i.e.

$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty$$

4. Inverse DTFT

The inverse discrete time Fourier transform of X(e$^{j\Omega}$) is defined as:

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}d\Omega$$

5. Special forms of DTFT

Discrete Sequence x[n]	**DTFT X(e$^{j\Omega}$)**
Real and Even	Real and Even
Real and Odd	Imaginary and Odd
Imaginary and Even	Imaginary and Even
Imaginary and Odd	Real and Odd

6. DTFT of some basic DT signals

Signal	**Fourier Transform**
1	$2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$
δ[n]	1
u[n]	$\frac{1}{1-e^{-j\Omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\Omega - 2\pi k)$
$a^n u[n], \|a\| < 1$	$\frac{1}{1-ae^{-j\Omega}}$

$\delta[n-n_0]$	$e^{-j\Omega n_0}$
$e^{-j\Omega_0 n}$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega-\Omega_0-2\pi k)$
$cos\, \Omega_0\, n$	$\pi \sum_{k=-\infty}^{\infty} \{\delta(\Omega-\Omega_0-2\pi k)+\delta(\Omega+\Omega_0-2\pi k)\}$
$sin\, \Omega_0\, n$	$\frac{\pi}{j} \sum_{k=-\infty}^{\infty} \{\delta(\Omega-\Omega_0-2\pi k)-\delta(\Omega+\Omega_0-2\pi k)\}$
$\sum_{k=-\infty}^{+\infty} \delta[n-kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\Omega-\frac{2\pi k}{N}\right)$
$(n+1)a^n u[n]$ $\|a\| < 1$	$\frac{1}{(1-ae^{-j\Omega})^2}$

7. Properties of DTFT

S. No.	Property	Sequence	DTFT
1.	Linearity	$a_1x_1[n] + a_2x_2[n]$	$a_1X_1(e^{j\Omega}) + a_2X_2(e^{j\Omega})$
2.	Periodicity	x[n]	$X(e^{j\Omega}) = X(e^{j(\Omega+2\pi)})$
3.	Time shifting	x[n – k]	$e^{-j\Omega k} X(e^{j\Omega})$
4.	Frequency shifting	$e^{j\Omega_0 n}x[n]$	$X\left(e^{j(\Omega-\Omega_0)}\right)$
5.	Time reversal	x[−n]	$X(e^{-\Omega})$
6.	Time expansion	X[n/k], n is a multiple integer of k	$X(e^{jk\Omega})$
7.	Differentiation in the frequency domain	nx[n]	$j\frac{d}{d\Omega}X(e^{j\Omega})$
8.	Conjugation	x*[n]	$X^*(e^{-j\Omega})$
9.	Convolution in time Domain	$x_1[n] * x_2[n]$	$X_1(e^{j\Omega})\, X_2(e^{j\Omega})$
10.	Convolution in the frequency domain	$x_1[n]x_2[n]$	$\frac{1}{2\pi}[X_1(e^{j\Omega}) * X_2(e^{j\Omega})]$
11.	Time differencing	x[n] – x[n – 1]	$(1 - e^{-j\Omega})\, X(e^{j\Omega})$
12.	Time accumulation	$\sum_{n=-\infty}^{\infty} x[n]$	$\frac{1}{(1-e^{-j\Omega})}X(e^{j\Omega})$ $+\pi X(0) \sum_{m=-\infty}^{\infty} \delta(\Omega-2\pi m)$
13.	Parseval's Theorem	energy of the sequence x[n] is given as $\sum_{n=-\infty}^{\infty}\|x[n]\|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\|X(e^{j\Omega})\right\|^2 d\Omega$	

8. Transfer Function & Impulse Response

Let $x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ be the input sequence and $y[n] \xleftrightarrow{DTFT} Y(e^{j\Omega})$ be the output sequence of the system. Then, response (output) of the system is given by following convolution sum.

$$y[n] = x[n] * h[n]$$

by taking DTFT using the convolution property, we have

$Y(e^{j\Omega}) = X(e^{j\Omega})\ H(e^{j\Omega})$

$H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$ (Transfer function)

NOTE: $X(e^{j\Omega})$ exists if the ROC of X(z) includes the unit circle. We know that ROC does not contain any poles. Therefore, it is concluded that DTFT of any sequence x[n] can be obtained from its z-transform X(z) if the poles of X(z) are inside the unit circle.

CHAPTER 10: SAMPLING THEOREM

1. Sampling

It is the process of converting a continuous time-signal x(t) into a discrete time signal x[n] by taking samples of the continuous time signal at discrete intervals of time. That is,

$$x[n] \; = \; x(t)|_{t=\mathrm{nT}_s}$$

Where, n is any integer.

2. SAMPLING THEOREM

The sampling theorem states that, a band-limited signal x(t) having the highest frequency component f_m Hz can be exactly recovered or reconstructed from its samples taken at a rate of $2f_m$ samples per second. Hence, the sampling rate or sampling frequency must satisfy this condition,

$$f_s \geq 2f_m$$

The sampling interval

$$T_s = \frac{1}{f_s} \leq \frac{1}{2f_m}$$

CHAPTER 11: DISCRETE FOURIER TRANSFORM (DFT)

1. Discrete Fourier Transform

The discrete-time Fourier transform of the sequence x[n] is given by

$$X_{DFT}[k] = \sum_{n=0}^{N-1} x[n]e^{\frac{-j2\pi kn}{N}}$$

where k = 0, 1, 2, …., (N – 1)

Discrete Fourier transform $X_{DFT}[k]$ is also denotes as X[k] or DFT{x[n]}

2. INVERSE DISCRETE FOURIER TRANSFORM (IDFT)

The inverse Discrete Fourier transform of $X_{DFT}(k)$ is defined as

$$x[n] = \frac{1}{N}\sum_{k\,=\,0}^{N-1} X[k]e^{j2\pi nk/N}$$

3. Properties of DFT

Discrete Sequence x[n]	**DFT X[k]**
Real and Even	Real and Even
Real and Odd	Imaginary and Odd
Imaginary and Even	Imaginary and Even
Imaginary and Odd	Real and Odd

S. No.	**Properties**	**Discrete time Signal**	**Discrete Fourier Transform**
1	Linearity	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$
2	Periodicity	x[n + N] = x[n]	X[k + N] = X[k]
3	Circular time shift	$x[n - n_0]_N$	$X(k)e^{\frac{-j2\pi kn_0}{N}}$
4	Time reversal	x[N – n]	X(N – k)
5	Conjugation	x*[n]	X*[N – k]
6.	Circular frequency shift	$x[n]e^{\frac{j2\pi k_0 n}{N}}$	$X[(k - k_0)]_N$
7.	Multiplication	$x_1[n]\ x_2[n]$	$\frac{1}{N}(X_1[k] * X_2[k])$
8	Circular Convolution	$x_1[n] \circledast x_2[n]$	$X_1[k]\ X_2[k]$
9	Circular correlation	$\bar{r}_{xy}(m) = \sum_{n=0}^{N-1} x[n]y * [n-m]_N$	$X(k)Y^*(k)$
10	Parseval's relation	$\sum_{n=0}^{N-1} x_1[n]x_2^*[n]$	$\frac{1}{N}\sum_{k=0}^{N-1} X_1[k]X_2^*[k]$

CHAPTER 12: FAST FOURIER TRANSFORM (FFT)

1. Properties of Twiddle factor:

(i) Symmetry property:

$$W_N^{k+\frac{N}{2}} = -W_N^k$$

(ii) Periodicity property:

$$W_N^{k+N} = W_N^k$$

Note: The fast Fourier transform algorithm exploits the two basic properties of the twiddle factor and reduces the number of complex multiplications required to perform DFT from N^2 to $\frac{N}{2} log_2 N$.

2. IDFT USING FFT ALGORITHM

FFT algorithm can be used to compute an inverse DFT without any change in the algorithm. The inverse DFT of an N-point sequence X(k), k = 0, 1, …., N – 1 is defined as

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)W^{-nk}$$

Where, $W = e^{-j2\pi/N}$

Take complex conjugate and multiply by N, we obtain

$$\boldsymbol{Nx * (n) = \frac{1}{N}\sum_{k=0}^{N-1} X * (k)W^{-nk}}$$

The desired output sequence x(n) can then found by complex conjugating the DFT and dividing by N to give

$$x(n) = \frac{1}{N}\left[\sum_{k=0}^{N-1} X * (k)W^{-nk}\right]^*$$

CHAPTER 13: DIGITAL FILTERS

1. There are two class of digital filters, depending on the duration of the impulse response.

1.(i) Finite-duration impulse response (FIR) digital filter, the operation of which governed by linear constant coefficient difference equations of a non-recursive nature.

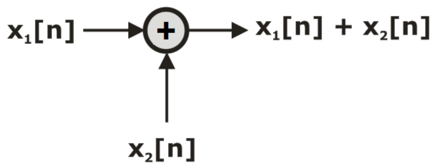
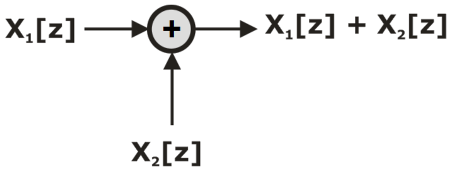
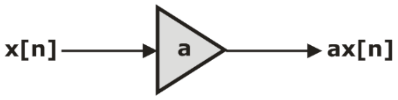
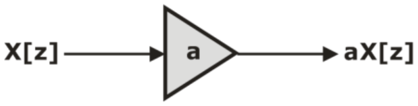
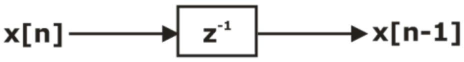
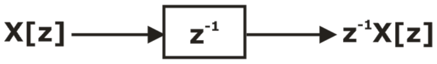
The transfer function of FIR digital filter is a polynomial in z^{-1}.

It has three important properties.

- They have finite memory and therefore, any transient start up is of limited duration.
- They are always BIBO stable.
- They can realize a desired magnetic response with exactly linear phase (i.e. with no phase distortion.)

1.(ii) Infinite-duration impulse response (IIR) digital filter, whose input output characteristics are governed by linear constant coefficient difference equations of recursive nature.

2. Basic Elements of Block Diagram

Elements of Block diagram	**Time Domain Representation**	**s-domain Representation**
Adder		
Constant multiplier		
Unit delay element		
Unit advance element		

3. COMPARISION BETWEEN FILTERS

Table 2: Comparison Between Non-Recursive & Recursive filter

Non-Recursive filters	Recursive filters
$y(n) = \sum\limits_{k=-\infty}^{\infty} a_k x(n-k)$ for causal system $= \sum\limits_{k=0}^{\infty} a_k x(n-k)$ For causal i/p sequence $y(n) = \sum\limits_{k=0}^{N} a_k x(n-k)$ It gives FIR output. All zero filter. It is always stable.	$y(n) = \sum\limits_{k-N_f}^{N_p} a_k x(n-k) - \sum\limits_{k=1}^{M} b_k y(n-k)$ for causal system $y(n) = \sum\limits_{k_0}^{N_p} a_k x(n-k) - \sum\limits_{k=1}^{M} b_k y(n-k)$ It gives IIR output but not always. Ex: y(n) = x(n) – x(n – 3) + y(n – 1) General TF:$H(z) = \frac{\sum_{k=-N_F}^{N_p} a_k z^{-k}}{1-\sum_{k=1}^{M} b_k z^{-k}}$ b_k = 0 for Non-Recursive N_f = 0 for causal system

4. Comparison Between FIR & IIR Filters

FIR filters	IIR filters
1. Linear phase no phase distortion.	Linear phase, phase distortion.
2. Used in speech processing, data transmission & correlation processing.	Graphic equalizers for digital audio, tone generators filters for digital telephone
3. Realized non recursively.	Realized recursively.
4. Stable	Stable or unstable. H(n) = a^n(n), a < 1 stable = 0, a > 1 unstable
5. Filter order is more	Less
6. More co-efficient storage	Less storage
7. Quantization noise due to finite precision arithmetic can be made negligible	Quantization noise
8. Co-efficient accuracy problem is less severe.	More
9. Used in multi rate DSP (variable sampling rate)	