AE/JE Foundation

Electrical Engineering

Communication Systems

Formula Notes

COMMUNICATION SYSTEM (FORMULA NOTES)

Chapter-1

Basics of Communication System

1. INTRODUCTION

Communication is the process of establishing connection or link between two points for information exchange.

OR

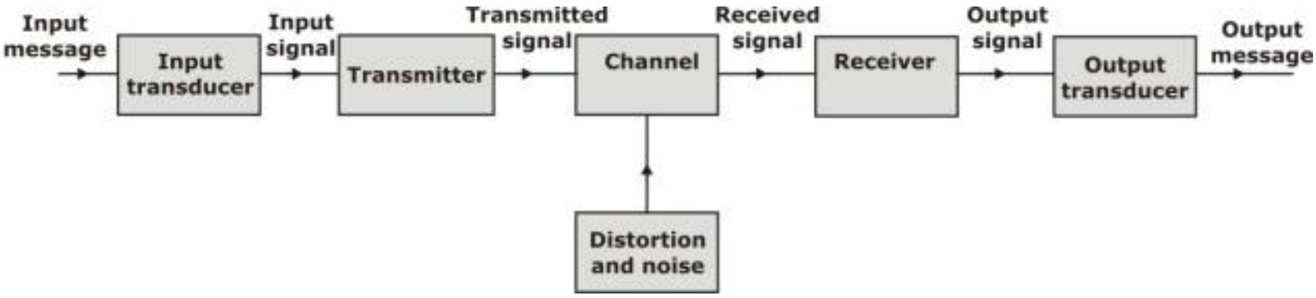
Communication is simply the basic process of exchanging information.

communication system.

Typical examples of communication system are line telephony and line telegraphy, radio telephony and radio telegraphy, radio broadcasting, point to point communication and mobile communication, computer communication, radar communication, television broadcasting, radio telemetry, radio aids to navigation, radio aids to aircraft landing etc.

2. THE COMMUNICATION PROCESS: ELEMENTS OF A COMMUNICATION SYSTEM

The whole idea of presenting the model of communication is to analysis the key concepts used in communication in isolated parts and them combining them to form the complete picture.



3. CONCEPT OF BANDWIDTH & FREQUENCY SPECTRUM

Bandwidth: Different types of passband signals such as voice signal, music signal, TV signal, etc. Each of these signals will have its own frequency range. This frequency range of a signal is known as its bandwidth.

Thus, we write $\boxed{BW = f_2 - f_1}$

The bandwidth of different signals has been listed in table 1.

TABLE 1

S. No.	Type of the signal	Range of frequency in Hz	Bandwidth in Hz
1.	Voice signal (speech) for telephony	300 – 3400	3,100
2.	Music signal	20 – 15000	14, 980
3.	TV signals (picture)	0 – 5 MHz	5 MHz
4.	Digital data	300 – 3400 (If it is using the telephone line for its transmission)	3,100

Frequency Spectrum: Frequency spectrum may be defined as the presentation of a signal in the frequency domain. It can be obtained by using either Fourier series or Fourier transform. It consists of the amplitude and phase spectrums of the signal.

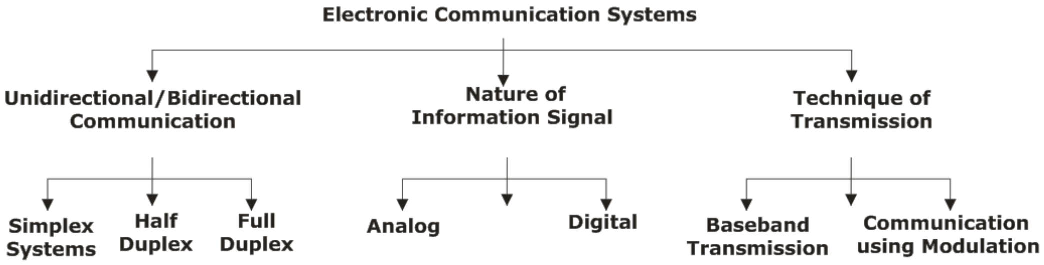
4. CLASSIFICATION OF COMMUNICATION SYSTEM

It shows that the electronic communication system may be basically categorised into three groups based on:

(i) Whether the system is unidirectional or bidirectional.

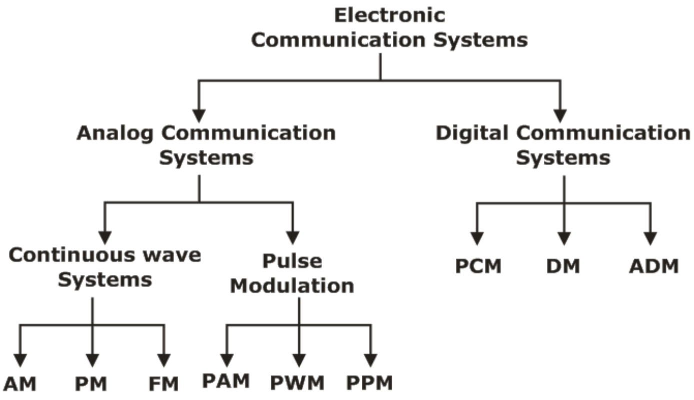
(ii) Whether it uses an analog or digital information signal.

(iii) Whether the system uses baseband transmission or uses some kind of modulation.



Classification of Electronic Communication Systems

5. CLASSIFICATION BASED ON THE NATURE OF INFORMATION SIGNAL



Classification based on analog or digital communication

6. ANALOG COMMUNICATION

The modulation system or techniques in which one of the characteristics of the carrier is varied in proportion with the instantaneous value of modulating signal is called as analog modulation system.

Advantages of analog communication

Some of the advantages of analog communication are as under:

(i) Transmitters and receivers are simple.

(ii) Low bandwidth requirement

(iii) FDM (frequency division multiplexing) can be used.

Drawbacks of analog communication

Some of the drawbacks are as under:

(i) Noise affects the signal quality

(ii) It is not possible to separate noise and signal.

(iii) Repeaters cannot be used between transmitters and receivers.

7. DIGITAL COMMUNICATION

The modulation system or technique in which the transmitted signal is in the form of digital pulses of constant amplitude, constant frequency and phase is called as digital modulation system.

Advantages of digital communication

Some of the advantages of digital communication are as under:

(i) Due to the digital nature of the transmitted signal, the interference of additive noise does not introduce many errors. Hence, digital communication has a better noise immunity.

(ii) Due to the channel coding techniques used in digital communication, it, is possible to detect and correct the errors introduced during the data transmission.

(iii) Repeaters can be used between transmitter and receiver to regenerate the digital signal. This improves the noise immunity further.

Drawbacks of digital communication

Some of the important drawbacks of digital communication arc as under:

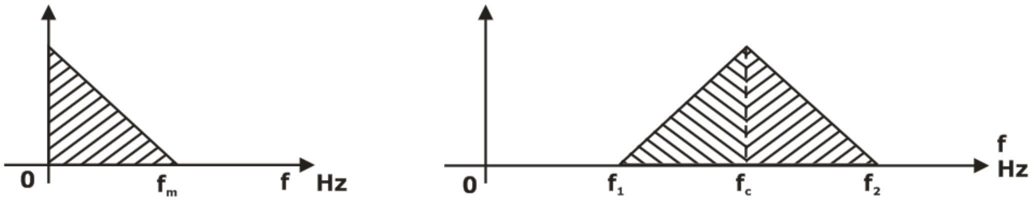
(i) The bit rates of digital systems are high. Therefore, they require a larger channel bandwidth as compared to analog systems.

(ii) Digital modulation needs synchronization in case of synchronous modulation.

8. BASEBAND AND BANDPASS SIGNALS

Baseband Signal

The information or the input signal to a communication system can be analog i.e. sound, picture or it can be digital e.g. the computer data. The electrical equivalent of this original information signal is known as the baseband signal.



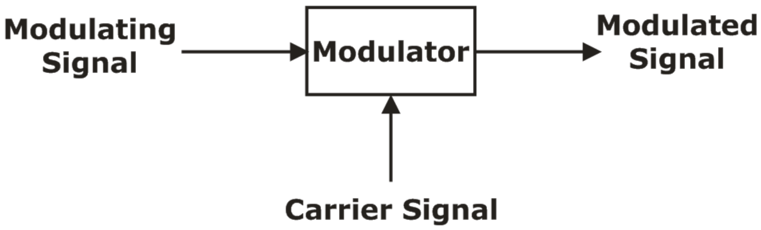
(a) Spectrum of a baseband signal **(b) spectrum of a bandpass signal**

Bandpass signal

It may be defined as a signal which has a non-zero lowest frequency in its spectrum. This means that the frequency spectrum of a bandpass signal extends from f_1 to f_2 Hz. The modulated signal is called as the bandpass signal.

9. MODULATION

In the modulation process, two signals are used namely the **modulating signal** and the **carrier signal**. The modulating signal is nothing but the baseband signal or information signal while carrier is a high frequency sinusoidal signal.



Need of Modulation

i. To avoid the mixing of signals

ii. To decrease the length of transmitting and receiving antenna

iii. To allow the multiplexing of signals

10. COMPARISION BETWEEN ANALOG & DIGITAL COMMUNICATION

Table 2

S. No.	Analog modulation	Digital modulation
(i)	Transmitted modulated signal is analog in nature.	Transmitted signal is digital, i.e. train of digital pulses.
(ii)	Amplitude, frequency or phase variations in the transmitted signal represent the information or message.	Amplitude, width or position of the transmitted pulses is constant. The message is transmitted in the form of code words.
(iii)	Noise immunity is poor for AM, but improved for FM and PM.	Noise immunity is excellent.

(iv)	It is not possible to separate out noise and signal. Therefore, repeaters cannot be used.	It is possible to separate signal from noise. Therefore, repeaters can be used.
(v)	Coding is not possible.	Coding techniques can be used to detect and correct the errors.
(vi)	Bandwidth required is lower than that for the digital modulation methods.	Due to higher bit rates, higher channel bandwidth is required.
(vii)	FDM is used for multiplexing.	TDM is used for multiplexing.
(viii)	Not suitable for transmission of secret information in military applications.	Due to coding techniques, it is suitable for military applications.
(ix)	Analog modulation systems are AM,FM, PM, PAM, PWM etc.	Digital modulation systems are PCM, DM, ADM, DPCM, etc.

Chapter-2

Amplitude Modulation

1. AMPLITUDE MODULATION

Amplitude modulation is defined as a process in which the amplitude of the carrier wave c(t) is varied linearly with the message signal m(t) keeping other parameters constant.

Time-Domain Description

The standard form of an amplitude-modulated (AM) wave is defined by

$x(t) = A_C [1 + k_a m(t)] \cos(2\pi f_c t)$

Where k_a is a constant called the **amplitude sensitivity** of the modulator. The modulated wave so defined is said to be a "standard" AM wave, because its frequency content is fully representative of amplitude modulation.

- The amplitude of the time function multiplying $\cos(2\pi f_c t)$ is called the envelope of the AM wave s(t). Using a(t) to denote this envelope, we may thus write

  $a(t) = A_c |1 + k_a m(t)|$
- Two cases arise, depending on the magnitude of $k_a m(t)$, compared to unity.

Case 1:

$|k_a m(t)| \le 1$, for all t

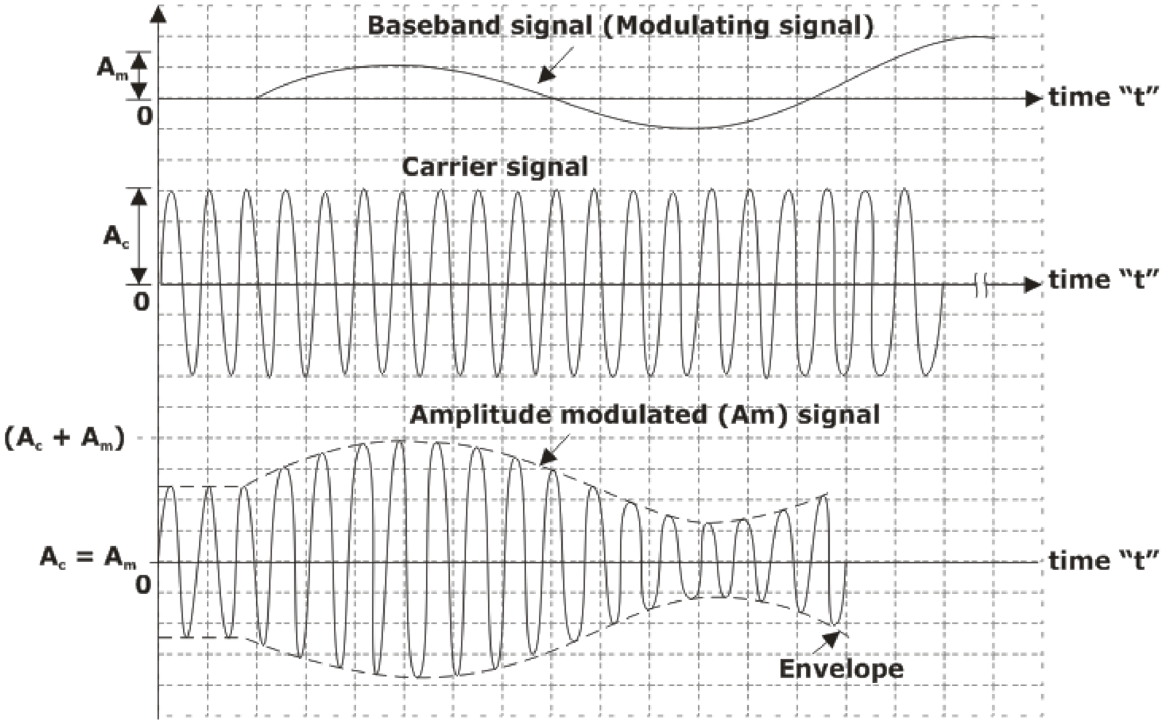
Under this condition, the term $1 + k_a m(t)$, is always non-negative. We may therefore simplify the expression for the envelope of the AM wave by writing

$a(t) = A_c(1 + k_a m(t))$, for all t

Case 2:

$|k_a m(t)| > 1$, for all t

The maximum absolute value of $k_a m(t)$ multiplied by 100 is referred to as the **percentage modulation**. Accordingly, case 1 corresponds to a percentage modulation less than or equal to 100%, whereas case 2 corresponds to a percentage modulation in excess of 100%.

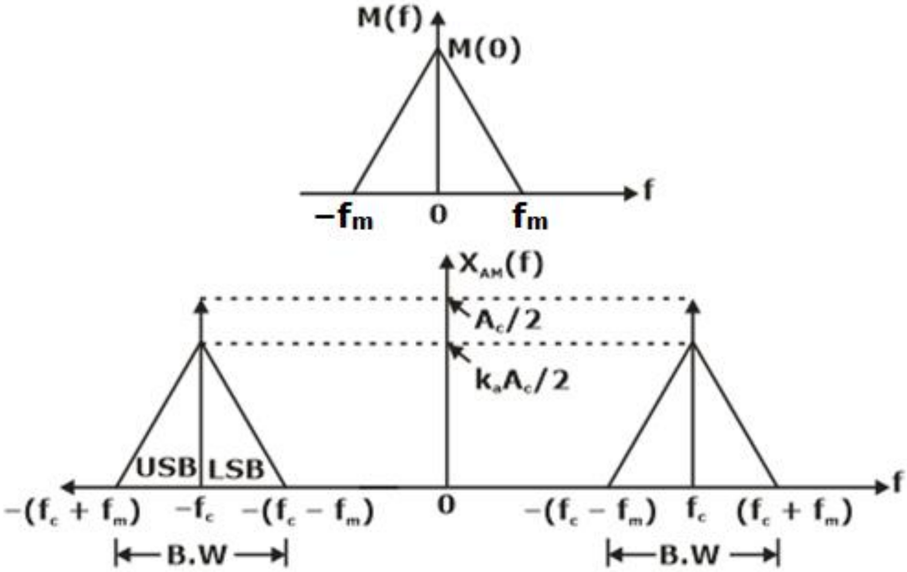


AM waveform for sinusoidal modulating signal

2. FREQUENCY DOMAIN DESCRIPTION

To develop the frequency description of the AM wave, we take the Fourier transform of both sides. Let S(f) denote the Fourier transform of s(t), and M(f) denote the Fourier transform of the message signal m(t); we refer to M(f) as the message spectrum. Accordingly, using the Fourier transform of the cosine function $A_C \cos(2\pi t_c t)$ and the frequency-shifting property of the Fourier transform. we may write

$$S(t) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$



B.W = $(f_c + f_m) - (f_c - f_m)$

B.W = $2f_m$ Hz or kHz

B.W = $2\omega_m$ rad/sec

3. SINGLE TONE AMPLITUDE MODULATION

Let carrier signal,

$x(t) = A_C \cos \omega_c t$

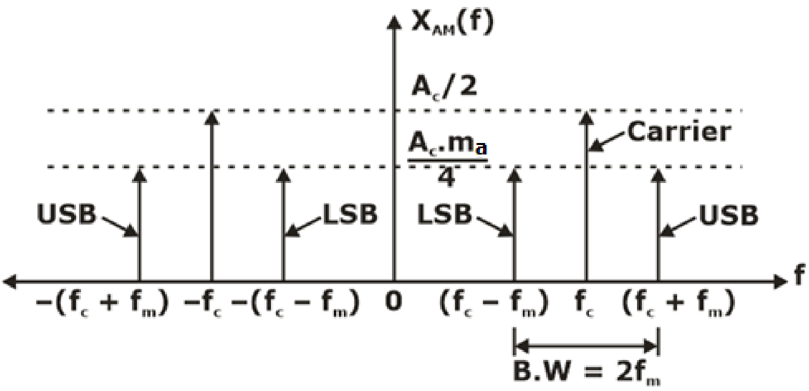
And the message signal,

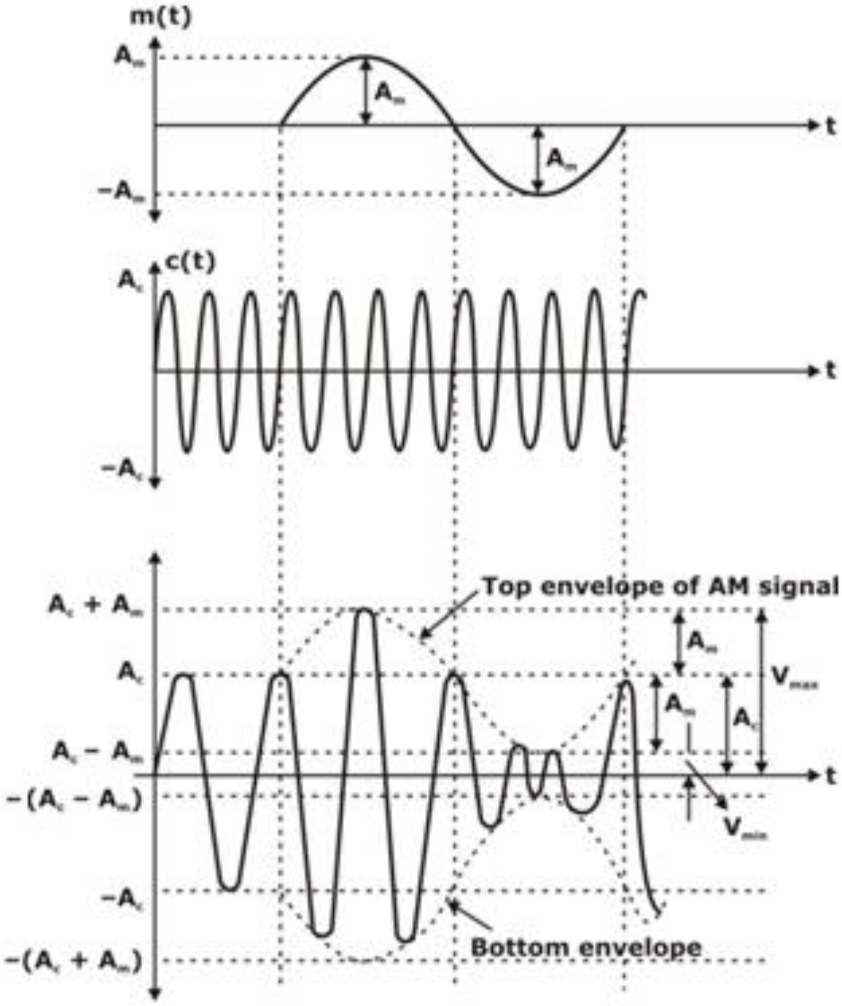
$m(t) = A_m \cos \omega_m t$

then after modulation, we get

$$X_{AM}(t) = \underbrace{A_C \cos \omega_c t}_{\text{Full carrier}} + \frac{1}{2} m_a A_c \underbrace{\cos(\omega_c + \omega_m)t}_{\text{USB}} + \frac{1}{2} m_a A_C \underbrace{\cos(\omega_c - \omega_m)t}_{\text{LSB}}$$

4. SPECTRUM OF SINUSOIDAL AM SIGNAL





$2A_m = V_{max} - V_{min}$

$$\Rightarrow A_m = \frac{V_{max} - V_{min}}{2}$$

$$A_C = \frac{V_{max} + V_{min}}{2}$$

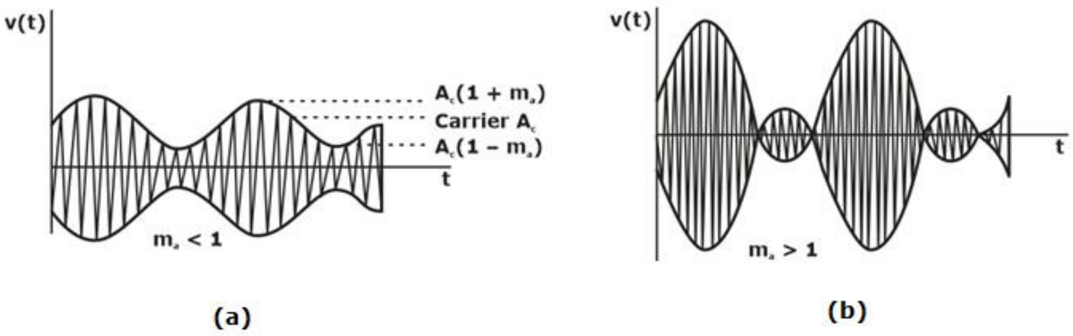
Finally, we get,

$$\boxed{m_a = \frac{A_m}{A_C} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}} \rightarrow \text{modulation index}$$

- % modulation = $m_a \times 100$

5. OVER MODULATION

When $m_a > 1$ i.e. $A_m > A_C$, over modulation takes place and the signal gets distorted. Because, the negative part of waveform gets cut from the waveform leaving behind a "square wave type" of signal, which generates infinite number of harmonics. This type of distortion is known as "**Non-linear distortion**" or "**Envelope distortion**"



(a) Under modulated AM wave (b) Over modulated AM wave

6. POWER RELATIONS IN AM

The Total Power in AM

The total power in an AM wave is given by,

P_t = [Carrier Power] + [Power in USB] + [Power in LSB]

$$\therefore P_t = \frac{E^2}{R} + \frac{E^2_{USB}}{R} + \frac{E^2_{LSB}}{R}$$

Where E, E_{USB} and E_{LSB} are the RMS values of the carrier and sideband amplitudes and R is the characteristic resistance of antenna in which the total power is dissipated.

Carrier Power (P_c)

The carrier power is given by

$$P_c = \frac{E^2}{R} = \frac{[E_c/\sqrt{2}]^2}{R} = \frac{E_C^2}{2R}$$

Power in the sidebands

- The power in the two sidebands is given as

$$P_{USB} = P_{LSB} = \frac{E_{SB}^2}{R}$$

- As we know the peak amplitude of each sideband is $\frac{m_a E_c}{2}$

$$P_{USB} = P_{LSB} = \frac{[m_a E_c/2\sqrt{2}]^2}{R} = \frac{m_a^2 E_c^2}{8R}$$

$$P_{USB} = P_{LSB} = \frac{m_a^2}{4} \times \frac{E_c^2}{2R}$$

$$P_{USB} = P_{LSB} = \frac{m_a^2}{4} P_c$$

Total Power: The total power is given by

$$P_t = P_c + P_{USB} + P_{LSB} = P_c + \frac{m_a^2}{4} P_c + \frac{m_a^2}{4} P_c$$

$$\therefore\ P_t = \left[1 + \frac{m_a^2}{2}\right] P_c$$

$$\text{or, } \frac{P_t}{P_C} = 1 + \frac{m_a^2}{2}$$

7. TRANSMISSION EFFICIENCY

- Transmission efficiency of an AM wave is the ratio of the transmitted power which contains the information (i.e. the total sideband power) to the total transmitted power.

$$\therefore\ \eta = \frac{P_{LSB} + P_{USB}}{P_t} = \frac{\left[\frac{m_a^2}{4} P_c + \frac{m_a^2}{4}\right]}{\left[1 + \frac{m_a^2}{2}\right] P_C} = \frac{m_a^2/2}{1 + \frac{m_a^2}{2}} = \frac{m_a^2}{2 + m_a^2}$$

- The percentage transmission efficiency is given by

$$\eta\% = \frac{m_a^2}{2 + m_a^2} \times 100\%$$

8. AM POWER IN TERMS OF CURRENT

Assume I_C to be the RMS current corresponding to the unmodulated carrier and I_t to be the RMS current AM wave.

- AM wave.

$$P_c = I_c^2 R \text{ and } P_t = I_t^2 R$$

$$\therefore\ \frac{P_t}{P_c} = \frac{I_t^2}{I_c^2} \times \frac{R}{R} = \left[\frac{I_t}{I_c}\right]^2$$

$$\frac{P_t}{P_c} = \left[1 + \frac{m_a^2}{2}\right]$$

$$\left[\frac{I_t}{I_c}\right]^2 = \left[1 + \frac{m_a^2}{2}\right]$$

$$I_t = I_c \left[1 + \frac{m_a^2}{2}\right]^{1/2}$$

9. MULTIPLE SINGLE TONE AMPLITUDE MODULATION

Let us assume that there are two modulating signals.

$x_1(t) = E_{m1} \cos\omega_{m1} t$

and $x_2(t) = E_{m2} \cos\omega_{m2} t$

$$e_{AM} = E_c\left[1 + \frac{E_{m1}}{E_c}\cos\omega_{m1}t + \frac{E_{m2}}{E_c}\cos\omega_{m2}t\right]\cos\omega_c t$$

Where, $\frac{E_{m1}}{E_c} = m_1$ and $\frac{E_{m2}}{E_c} = m_2$

Use the following identity to simplify equation

$$\cos A\cos B = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$$

$$e_{AM} = E_c\cos\omega_c t + \frac{m_1E_c}{2}\cos(\omega_c + \omega_{m1})t + \frac{m_1E_c}{2}\cos(\omega_c - \omega_{m1})t$$

$$+\frac{m_2E_c}{2}\cos(\omega_c + \omega_{m2})t + \frac{m_2E_c}{2}\cos(\omega_c - \omega_{m2})t$$

Total Power in AM Wave

The total power is given as,

$P_t = P_c + P_{USB1} + P_{LSB1} + P_{USB2} + P_{LSB2}$

Extending the concept to the AM wave with n number of modulating signals with modulating indices m_1, m_2…m_n the total power is given by,

$$P_t = P_c\left[1 + \frac{m_1^2}{2} + \frac{m_2^2}{2} + \ldots + \frac{m_n^2}{2}\right]$$

Effective Modulation Index (m_t)

We know that $P_t = P_c\left[1 + \frac{m_t^2}{2}\right]$

$$m_t = \left[m_1^2 + m_2^2 + \ldots m_n^2\right]^{1/2}$$

10. GENERATION OF AM WAVES USING NONLINEAR PROPERTY

The circuit that generates the AM waves is called as amplitude modulator and modulators are named as,

i. Square law modulator

ii. Switching modulator

11. DISADVANTAGES OF AM (DSBFC)

The AM signal is also called as "**Double Sideband Full Carrier (DSBFC) signal**. The main disadvantage of this technique is:

- Power wastage takes place.
- AM needs larger bandwidth.
- AM wave gets affected due to noise.

  These are explained as follows
- The carrier signal in the DSBFC system does not convey any information.

12. DETECTION OF AM WAVES

Square-law detector

A square-law detector is essentially obtained by using a square-law modulator for the purpose of detection. Consider the transfer characteristic equation of a nonlinear device, which is reproduced here for convenience

$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$

where $v_1(t)$ and $v_2(t)$ are the input and output voltages, respectively and a_1 and a_2 are constants.

Envelope detector

Charging time constant = $RC >> \frac{1}{f_c}$

Discharging time constant = $RC << \frac{1}{f_m}$

As the varying voltage across R follows the envelope.

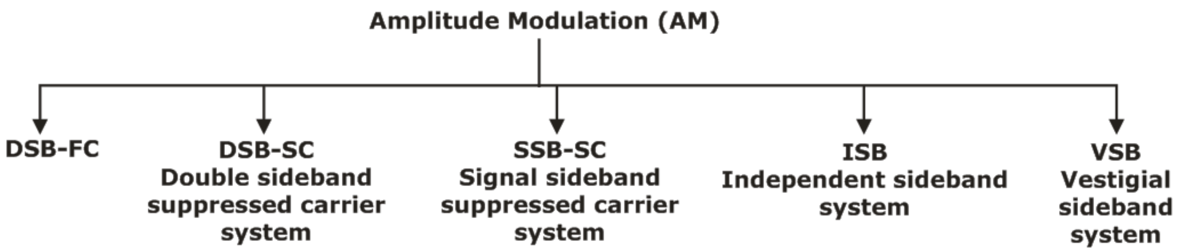
So that, $\frac{1}{f_c} << RC << \frac{1}{f_m}$

If RC is very small or RC is very large, then in both the cases we can't get the envelope of message signal waveform.

For getting envelope of m(t), exact value of RC is given as,

$$RC \le \frac{1}{\omega_m} \cdot \frac{\sqrt{1 - m_a^2}}{m_a}$$

13. TYPES OF AM



14. DOUBLE-SIDEBAND SUPPRESSED-CARRIER MODULATION

Time-Domain Description

To describe a double-sideband suppressed-carrier (DSBSC) modulated wave as a function of time, we write

$s(t) = c(t)m(t) = A_c \cos(2\pi f_c t)\, m(t)$

Frequency-Domain Description

The suppression of the carrier from the modulated wave is well-appreciated by examining its spectrum. Specifically, by taking the Fourier transform

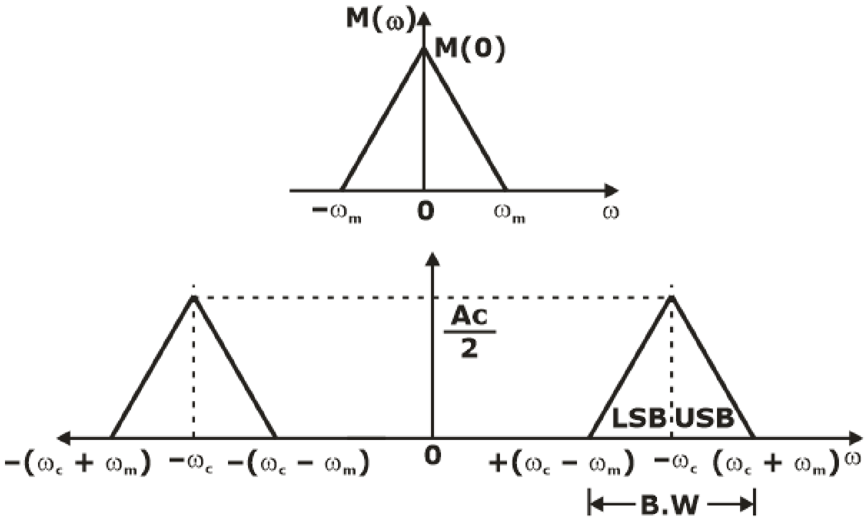
$$S(f) = \frac{1}{2} A_c [M(f - f_c) + M(f + f_c)]$$

where, as before, S(f) is the Fourier transform of the modulated wave s(t) and M(f) is the Fourier transform of the message signal m(t).

15. GENERATION OF DSBSC WAVES

A double-sideband suppressed-carrier modulated wave consists simply of the product of the message signal and the carrier wave. A device for achieving this requirement is called a product modulator.

Spectrum of DSB-SC Signal



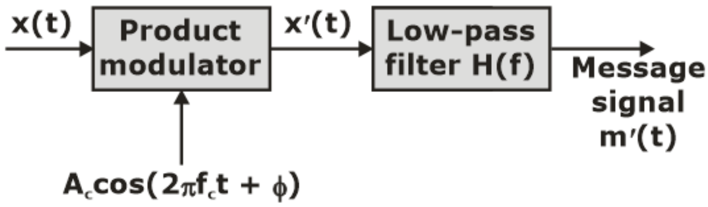
Modulated DSBSC signal

Transmission B.W = $2\omega_m$

16. COHERENT (SYNCHRONOUS) DETECTION OF DSB-SC WAVES

Let x(t) be the DSB-SC signal at the input of the product modulator and the local oscillator having frequency $A_c \cos(2\pi f_c t + \varphi)$. The signal x(t) can be represented as

$$x(t) = m(t) \times A_c \cos(2\pi f_c t)$$



Hence the output of the product modulator is given by

$x'(t) = m(t).\ A_c \cos(2\pi f_c t) \cos(2\pi f_c t + \varphi)$

$x'(t) = m(t).\ A_c \cos(2\pi f_c t + \varphi) \cos(2\pi f_c t)$

But $\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)$

Therefore, $x'(t) = \frac{1}{2}m(t)A_c[\cos(4\pi f_c t + \phi) + \cos\phi]$

$$x'(t) = \frac{1}{2}A_c \cos\phi m(t) + \frac{1}{2}m(t)A_c \cos(4\pi f_c t + \phi)$$

Signal x'(t) is them passed through a low pass filter. Which allows only the first term to pass through and will reject the second term. Hence the filter output is given by,

$$m'(t) = \frac{1}{2}A_c \cos\phi m(t)$$

17. HILBERT TRANSFORM

Hilbert transform of x(t) is represented with $\hat{x}(t)$,and it is given by

$$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(k)}{t-k}dk$$

SINGLE SIDE-BAND

Let m(t) is modulating signal and $\hat{m}(t)$ is Hilbert transform of m(t) then,

$X_{SSB\text{-}SC}(t) = m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t$	$\Rightarrow$ LSB
$X_{SSB\text{-}SC}(t) = m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t$	$\Rightarrow$ USB

Also,

B.W = $\omega_c + \omega_m - \omega_c$

B.W = ω_m

Power Saving

In DSB-SC:

Power saved in DSBSC $= \frac{P_c}{P_t} \times 100$

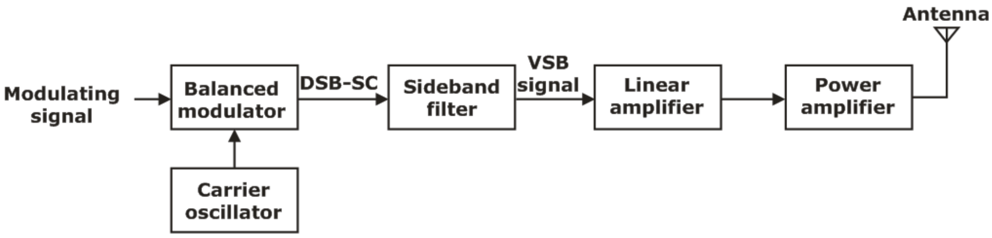
$$P_{save} = \frac{2}{2 + m_a^2} \times 100\%$$

In SSB-SC:

$$\text{Power saved in SSB} = \frac{P_c + P_{USB} \text{or } P_{LSB}}{P_t} \times 100$$

$$P_{save} = \frac{4 + m_a^2}{4 + 2m_a^2} \times 100\%$$

18. VESTIGIAL SIDE-BAND MODULATION (VSB)



VSB Transmitter

Transmission bandwidth

The transmission bandwidth of the VSB modulated wave is given by,

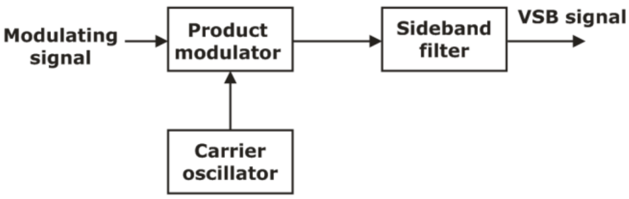
$B = (f_m + f_v)$ Hz

where f_m = Message bandwidth

And f_v = Width of the vestigial sideband.

Generation of VSB Modulated Wave

The modulating signal x(t) is applied to a product modulator. The output of the carrier oscillator is also applied to the other input of the product modulator. The output of product modulator is given by



Generation of VSB signal

$$m(t) = x(t).c(t) = x(t).V_c \cos(2\pi f_c t)$$

The spectrum of the VSB modulated signal is given by,

$$S(f) = \frac{V_c}{2}[X(f - f_c) + X(f + f_c)]H(f)$$

Chapter-3

Angle Modulation

1. ANGLE MODULATION

An angle modulated wave can be expressed mathematically as

$s(t) = A_c \cos[\omega_c t + \theta(t)]$

Where A_c is the peak carrier amplitude,

ω_c is carrier frequency and

$\theta(t)$ is the instantaneous phase deviation.

In angle modulation $\theta(t)$ is a function of modulating signal.

2. PHASE MODULATION (PM)

Mathematical Expression for PM

A phase modulated can be mathematically expressed in time domain as follows:

$$s(t) = \underbrace{A_c}_{\text{Constant amplitude}} \cos [2\pi f_c t + \underbrace{k_p m(t)}_{\text{Varying phase angle}}]$$

Where k_p is the phase sensitivity in radians per volt and m(t) is the message waveform.

Mathematical Expression for FM

We can use θ(t) to write the expression for FM in time domain as under:

$s(t) = A_c \cos \theta(t)$

FM wave:

$$s(t) = \underbrace{A_c}_{\text{Constant amplitude}} \cos [\underbrace{2\pi f_c t}_{\text{Unmodulated carrier}} + 2\pi K_f \int_0^t m(t)\, dt]$$

Where k_f is the frequency sensitivity in radians per volt and m(t) is the message waveform.

3. SINGLE TONE FREQUENCY MODULATION

Changing the frequency of the carrier according to the-message signal is called Frequency Modulation.

$f_i(t) = f_c + K_f m(t)$

K_f = Frequency sensitivity (Hz/Volt)

$f_i(t) = f_c + K_f A_m \cos 2\pi f_m t$

$f_{i,\ max} = f_c + K_f A_m$

$f_{i,\ min} = f_c - K_f A_m$

$\Delta f = K_f A_m$ = frequency deviation

Maximum Frequency of FM Wave:

The maximum frequency of FM wave is given by

$f_{max} = f_c \pm \Delta f$

For a single tone modulation:

$m(t) = A_m \cos 2\pi f_m t$

$$s(t) = A_c\left(\cos\left(2\pi f_c t + \frac{2\pi K_f A_m}{2\pi f_m}\sin 2\pi f_m t\right)\right) = A_c\left(\cos\left(2\pi f_c t + \frac{K_f A_m}{f_m}\sin 2\pi f_m t\right)\right)$$

Modulation Index: The modulation index of FM wave is defined as under:

$$\beta_f = \frac{\text{Frequency deviation}}{\text{Modulating frequency}}$$

$$\beta = \frac{K_f A_m}{f_m} = \frac{\Delta f}{f_m}$$

Deviation Ratio: The modulation index corresponding to the maximum deviation and maximum modulating frequency is called as the deviation ratio.

$$\text{Deviation ratio} = \frac{\text{Maximum deviation}}{\text{Maximum modulating frequency}}$$

Percentage Modulating of FM Wave:

$$\text{\% Modulation} = \frac{\text{Actual frequncy deviation}}{\text{Maximum allowable deviation}}$$

4. TYPES OF FM

The FM systems are basically classified into following two types:

i. Narrow band FM ($\beta << 1$)

ii. Wide band FM ($\beta >> 1$)

BW of NBFM = $2f_m$

The spectrum of AM and FM are identical except that the spectral component at $f_c - f_m$ is 180° out of phase.

Generation of NBFM signal

Figure 6 shows the generation of narrow band FM using balanced modulator.

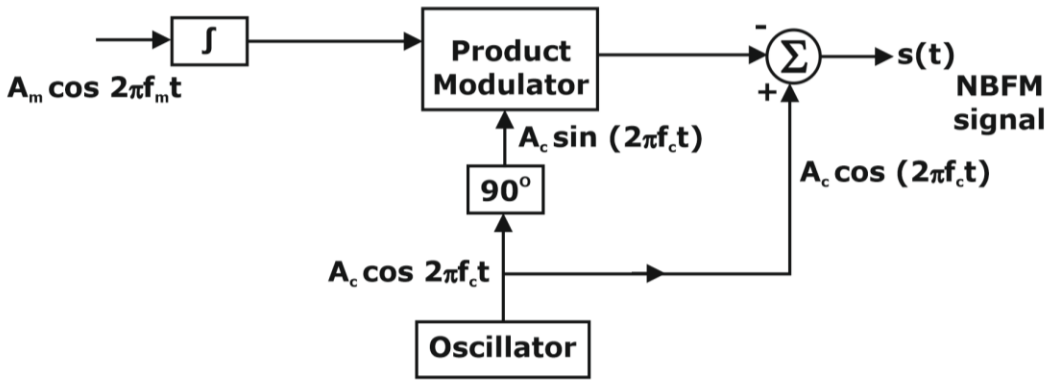


Figure 6: Generation of Narrow band FM

5. WIDEBAND FM

Bessel function of order 'n' is given by

$$J_n(x) = (1/2\pi)\int e^{j(x\sin\theta - n\theta)}\, d\theta$$

S(t) is wideband FM

$$s(t) = A_c\, J_0(\beta)\cos 2\pi f_c t + A_c J_1(\beta)[\cos 2\pi(f_c + f_m)\, t - \cos 2\pi(t_c - f_m)t]$$
$$+ A_c\, J_2(\beta)[\cos 2\pi(f_c + 2f_m)t + \cos 2\pi(f_c + f_m)t]$$

POWER CALCULATION

Total Power

$$P_t = \frac{A_C^{\,2}}{2R}\sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

According to property of Bessel function

$$P_t = \frac{A_C^2}{2R}.1$$

$$\boxed{\text{Total power} = \frac{A_C^2}{2R}}$$

Same as unmodulated carrier power

i.e. $\boxed{P_t = P_C}$

The total power is independent of modulation index. AM takes more power compared to FM for the same message and carrier.

6. CALCULATION OF PRACTICAL B.W OF WBFM USING CARSON'S RULE

Carson has proved that the number of sidebands having significant amplitudes containing 99% of the total power is β + 1.

B.W. = $2(\beta + 1)f_m$

$$= 2\left(\frac{\Delta f}{f_m} + 1\right) f_m$$

$$= 2\Delta f + 2f_m$$

7. PHASE MODULATION

In phase modulation, phase of the carrier is varied according to message signal. Time domain equation of PM modulated signal can be written as,

$$\boxed{\begin{array}{l} S(t) = A_c \cos[2\pi f_c t + K_p(t)] \\ \phi = K_p m(t) \end{array}} \text{ multitone modulation}$$

Where, K_p =phase sensitivity (units =rad/volt)

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + K_p A_m \cos 2\pi f_m t\right]} \quad \text{single ton modulation}$$

where $\boxed{K_p A_m = A\phi}$ called phase deviation

$$\boxed{s(t) = A_c \cos[2\pi f_c t + \beta \cos 2\pi f_m t]}$$

$\boxed{\Delta\phi = \beta}$ = modulation index

8. GENERATION OF WBFM SIGNALS

WBFM Signal can be generated by two methods

i. Direct Method or parameter variation method

ii. Indirect Method or Armstrong Method

Direct Method

This method is most widely used for generation of WBFM signal.

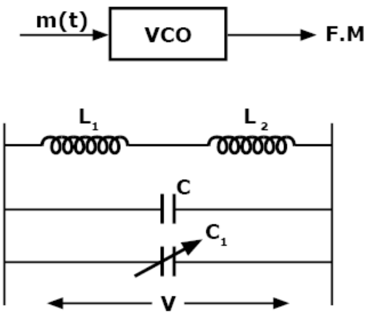
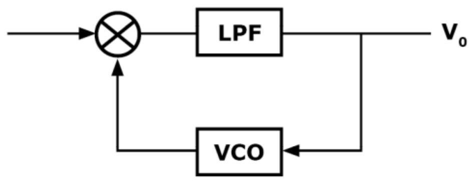


Figure 14(a): Voltage control oscillator

Frequency of oscillation, $f = \dfrac{1}{2\pi\sqrt{(L_1 + L_2)(C + C_1)}}$

9. FM DEMODULATION USING PLL



First order PLL

When the input to the PLL is of the form cos[2пft + φ] the output voltage is

$\boxed{V_0 \propto \dfrac{d}{dt}[\phi]}$.

When the input to the PLL is an FM signal, $A_c \cos[2\pi f_c t + 2\pi K_f \int m(t)\,dt]$, the output voltage is

$$V_0 \propto \frac{d}{dt} 2\pi K_f \int m(t)\,dt$$

$$V_0 \propto 2\pi K_f m(t)$$

$$V_0 = \frac{1}{2\pi K_V}\left[2\pi K_f m(t)\right]$$

Where, $\dfrac{1}{2\pi K_V}$ =proportionality constant

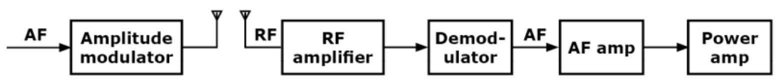
$\boxed{V_0 = \dfrac{K_f}{K_v} m(t)}$ K_f – frequency sensitivity of VCO at transmitter
K_v – frequency sensitivity of VCO at Receiver

Chapter-5

Receivers and Noise

1. TRF RECEIVER

Block diagram for TRF Receiver is as follows



Tuned radio frequency receiver

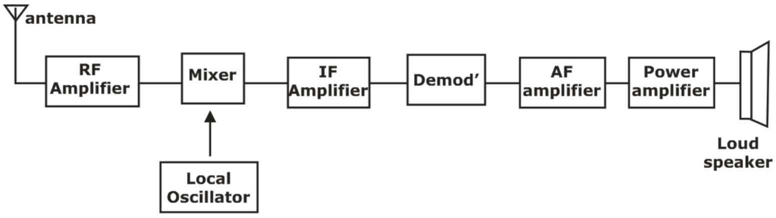
Carrier frequencies allotted from FM = (88– 108) MHz

Carrier frequencies allotted from AM =(550 – 1650) KHz

BW allotted to each AM broadcasting station = 10kHz

2. SUPERHETERODYNE RECEIVER

Block diagram for TRF Receiver is as follows



Superheterodyne Receiver

Mixer will change the carrier frequency from f_s to f_{IF}.

Intermediate frequency for MW is 455 KHz.

Image frequency:

$F_{si} = f_s + 2\ IF$

Where IF is image frequency

The resonant frequency of IF tuned amplifier is constant i.e., IF.

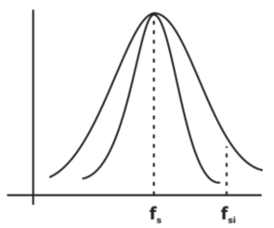
$f_l - f_s = IF$

Where f_l is local oscillator frequency

Image (Frequency) Rejection Ratio:

$$IRR = \frac{\text{Gain at } f_s}{\text{Gain at } f_{si}}$$

Gain at $f_{si} \ll 1$



By increasing the Intermediate frequency, IRR can be increased. By increasing the bandwidth, the gain at f_{si} can be decreased so that IRR increases.

$$IRR \propto \frac{1}{B.W}$$

$$IRR \propto Q$$

$$IRR = \sqrt{1 + Q^2\rho^2}$$

where, $\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$

Since, the frequency of local oscillator is given by

$$f_{LO} = \frac{1}{2\pi\sqrt{LC}}$$

where L is inductance and C is capacitance. Now, for a fix value of L, we have $1/f_{LO}$

$$\sqrt{C} = \frac{1}{2\pi\sqrt{L}}$$

$$\text{or} \quad C = \frac{1}{2\pi L}\frac{1}{f_{LO}^2}$$

So, maximum value of capacitance exists for minimum value of f_{LO}, i.e

$$C_{max} = \frac{1}{2\pi L}\frac{1}{f_{LO,min}^2}$$

Similarly, we get

$$C_{min} = \frac{1}{2\pi L}\frac{1}{f_{LO,\,max}^2}$$

3. CLASSIFICATION OF NOISE

The fundamental noise sources produce different types of noise. They may be listed as under:

(i) Thermal noise

(ii) Partition noise

(iii) Shot noise

(iv) Low frequency or flicker noise

(v) High frequency or transit time noise

Shot Noise

The mean square shot noise current for a diode is given as

$I_n^2 = 2(I + 2I_0)q\,B$ amperes^2

Where I = direct current across the junction (in amp.)

I_0 = reverse saturation current (in amp.)

q = electronic charge = 1.6×10^{-19}C

B = effective noise bandwidth in Hz.

Thermal Noise or Johnson Noise

The average thermal noise power is given by,

$P_n = kTB$ Watts

Where k = Boltzmann's constant = 1.38×10^{-23} Joules/Kelvin

B = Bandwidth of the noise spectrum (Hz)

T = Temperature of the conductor, °Kelvin

White Gaussian Noise

The power spectral density (psd) of a white noise is given by,

$$S_n(f) = \frac{N_0}{2}$$

N_0 is defined, as under:

$N_0 = kT_e$

where K = Boltzmann's constant and

T_e = Equivalent noise temperature or the system

Signal To Noise Ratio

It is defined as the ratio of signal power to the noise power at the same point.

Therefore, $\frac{S}{N} = \frac{P_s}{P_n}$

where, P_s = Signal power

P_n = Noise power at the same point.

S/N (dB) = 10 $\log_{10}$ (P_s/P_n)

4. SINAD

This is another variation of signal to noise ratio. SINAD stands for signal noise and distortion and it is defined as,

$$\text{SINAD} = \frac{S + N + D}{N + D}$$

Where, S = Signal, N = Noise and D = Distortion

SINAD is generally used in the specifications of FM receiver.

5. NOISE FACTOR

It is defined as,

$$F = \frac{\text{S/N ratioat the input}}{\text{S/N ratio at the output}}$$

$$F = \frac{P_{si}}{P_{ni}} \times \frac{P_{n0}}{P_{s0}}$$

Where P_{si} and P_{ni} = Signal and noise power at the input

and P_{so} and P_{no} = Signal and noise power at the output

6. NOISE FIGURE

Sometimes, the noise factor is expressed in decibels. When noise factor is expressed in decibels, it is known as noise figure.

Noise figure F_{dB} = 10 $\log_{10}$ F

Substituting the expression for the noise factor, we get

$$\text{Noise figure} = 10 \log_{10}\left[\frac{\text{S/N at the input}}{\text{S/N at the output}}\right] = 10 \log_{10} (S/N)_i - 10 \log_{10} (S/N)_0$$

Hence, Noise figure $F_{dB} = (S/N)_i$ dB – $(S/N)_0$ dB

The ideal value of noise figure is 0 dB.

7. NOISE TEMPERATURE

The equivalent noise temperature of the amplifier is given by,

$T_{eq} = (F - 1)\ T_0$

Where F is noise factor and T_0 is temperature

8. FIGURE OF MERIT

Figure 7

$(S/N)_i = (S_i/N_i) = \dfrac{\text{Powerof the modulated signal}}{\text{Power of noise in message bandwidth}}$

$(S/N)_0 = (S_0/N_0) = \dfrac{\text{Powerof the demodulated signal}}{\text{Power of noise in message bandwidth}}$

Figure of Merit = $\dfrac{(S/N)_0}{(S/N)_i} = \dfrac{1}{\text{Noise Figure}}$

Noise Figure = $\dfrac{(S/N)_i}{(S/N)_0}$

$(S/N)_0$ depends mainly on modulation scheme and receiver characteristics.

Figure of Merit of a DSB system

∴ Figure of merit = $\dfrac{A_c^2 P/2WN_0}{A_c^2 P/2WN_0} = 1$

Figure of Merit of a SSB system:

∴ Figure of merit = $\dfrac{A_c^2 P/4WN_0}{A_c^2\ P/4WN_0} = 1$

Figure of Merit of AM system:

Figure of Merit $\cong \dfrac{K_a P}{1 + K_a^2\ P}\ \dfrac{K_a^2\ A_m^2}{2 + K_a^2\ A_m^2} = \dfrac{\mu^2}{2 + \mu^2} = \eta$ [modulation efficiency]

Chapter-5
Mathematical Tool

1. PROBABILITY

Probability Axioms

1. (Nonnegativity) P(A) ≥ 0, for every event A.

2. (Additivity) If A and B are two disjoint events, then the probability of their union satisfies $P(A \cup B) = P(A) + P(B)$.

   Furthermore, if the sample space has an infinite number of elements and A_1, A_2, ... is a sequence of disjoint events, then the probability of their union satisfies

   $P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$

3. (Normalization) The probability of the entire sample space $=\Omega$ is equal to 1, that is, $P(\Omega) = 1$.

Properties of Probability Laws

Consider a probability law, and let A, B, and C be events.

(a) If $A \subset B$, then $P(A) \leq P(B)$.

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(c) $P(A \cup B) \leq P(A) + P(B)$.

(d) $P(A \cup B \cup C) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C)$.

conditional probability for event A and B:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

2. TOTAL PROBABILITY THEOREM AND BAYES' RULE

Total Probability Theorem

Let $A_1, \dots, A_n$ be disjoint events that form a partition of the sample space (each possible outcome is included in one and only one of the events $A_1, \dots, A_n$) and assume that $P(A_i) > 0$, for all $i = 1, \dots, n$. Then, for any event B, we have

$P(B) = P(A_1 \cap B) + \dots + P(A_n \cap B) = P(A_1)P(B \mid A_1) + \dots + P(A_n)P(B \mid A_n)$.

Bayes' Rule

Let $A_1, A_2, \dots, A_n$ be disjoint events that form a partition of the sample space, and assume that $P(A_i) > 0$, for all i. Then, for any event B such that $P(B) > 0$, we have

$$P(A_i \mid B) = \frac{P(A_i)P(B / A_i)}{P(B)} = \frac{P(A_i)P(B / A_i)}{P(A_1)P(B \mid A_1) + \dots + P(A_n)P(B \mid A_n)}$$

Independent Events:

We say that A is independent of B

$P(A \mid B) = P(A)$.

$P(A \mid B) = P(A \cap B)/P(B)$, this is equivalent to

$P(A \cap B) = P(A)P(B)$.

3. INTRODUCTION TO RANDOM VARIABLES

A random variable is a rule or relationship, denoted by X, that assigns a real number X(S) to every point in the sample space S. The random variables can be distinguished as

1. Discrete Random Variable

2. Continuous Random Variable

3 Cumulative Distribution Function

4 Mean, Variance and Standard Deviation of a Random Variable

5 Different types of Random Variable

4. DISCRETE RANDOM VARIABLE

When the random variable takes only a discrete set of values, then it is called a discrete random variable. For example, we flip a coin, the possible outcomes are head (H), and tail (T), so S contains two points labelled H and T. Suppose, we define a function X(S) such that

$$X(S) = \begin{cases} 1 \text{ for} & S = H \\ -1 \text{ for} & S = T \end{cases}$$

Thus, we have mapped the two outcomes into the two points on the real line. So, this is called a discrete random variable.

5. PROBABILITY DENSITY FUNCTION OF DISCRETE RANDOM VARIABLE

Let a discrete random variable X having the possible outcomes, $X = \{X_1, X_2, \ldots X_n\}$

So, the probability density function (PDF) of the discrete random variable is defined as

$f_x(x_1) = P(X = x_1)$ i = 1, 2, ...n

6. PROBABILITY MASS FUNCTION OF DISCRETE RANDOM VARIABLE

Suppose that the jumps in $F_x(x)$ of a discrete random variable X occur at the points x_1, x_2 , ..., where the sequence may be either finite or countably infinite, and we assume $x_i < x_j$ if $i < j$.

Then $F_x(x_i) - F_x(x_{i-1},) = P(X \leq x_i) - P(X \leq x_{i-1}) = P(X = x_i)$

Let $P_x(x) = P(X = x)$

The function $p_x(x)$ is called the probability mass function (pmf) of the discrete random variable X.

Properties of $p_x(x)$:

I. $0 \leq p_x(x_k) \leq 1 \quad k = 1, 2, \ldots$

2. $p_x(x) = 0$ if $x \# x_k$ $(k = 1, 2, \ldots)$

3. $\sum_k p_x(x_k) = 1$

The cdf $F_x(x)$ of a discrete random variable X can be obtained by

$F_x(x) = P(X \leq x) = \sum_{xk \leq x} p_x(x_k)$

7. CONTINUOUS RANDOM VARIABLE

If the random variable X takes any value in a whole observation interval, X is called a continuous random variable. For example, if we define a function X(θ) such that

$X(\theta) = \tan^2 \theta$

Then, every value in the range $0 \le x < \infty$ is a possible outcome of this experiment. Thus, we can say that $X(\theta)$ is a continuous random variable.

8. CONTINUOUS RANDOM VARIABLES AND PDFS

A random variable X is called continuous if its probability law can be described in terms of a nonnegative function f_x , called the probability density function of X, or PDF for short, which satisfies

$$P(X \in B) = \int_B fx(x)dx,$$

for every subset B of the real line. In particular, the probability that the value of X falls within an interval is

$$P(a \le X \le b) = \int_a^b f_x(x)dx,$$

excluding the endpoints of an interval has no effect on its probability:

$P(a \le X \le b) = P(a < X < b) = P(a \le X < b) = P(a < X \le b)$.

Note that to qualify as a PDF, a function f x must be nonnegative, i.e., $f_x(x) \ge 0$ for every x, and must also satisfy the normalization equation

$$\int_{-\infty}^{\infty} fx(x)dx = P(-\infty < X < \infty) = 1 .$$

9. CUMULATIVE DISTRIBUTION FUNCTION OF CONTINUOUS RANDOM VARIABLE

The cumulative distribution function (CDF) of the continuous random variable X is given by $F_x(x) = P(X \le x)$

Some important properties of CDF of continuous random variable are given below.

Properties of CDF of Continuous Random Variable:

1. $F_x(-\infty) = 0$
2. $F_x(\infty) = 1$
3. $P(a < x \le b) = F_x(b) - F_x(a)$

Probability Density Function of Continuous Random Variable

The probability density function (PDF) of a continuous random variable is defined as

$$f_x(x) = \frac{dF_x(x)}{dx}$$

Some important properties of PDF of continuous random variable are given below.

10. PROPERTIES OF PDF OF CONTINUOUS RANDOM VARIABLE

1. $f_x(x) \ge 0$
2. $\int_{-\infty}^{\infty} f_x(x)dx = 1$

3. $P(X \le x) = F_x(x) = \int_{-\infty}^{x} f_x(\lambda)d\lambda$

4. $P(a < x \le b) = \int_{a}^{b} f_x(X)dx$

11. EXPECTED VALUE OR MEAN OF A CONTINUOUS RANDOM VARIABLE

The expected value or mean of a continuous random variable X is defined by

$E[X] = \int_{-\infty}^{\infty} xfx(x)dx$.

- The variance of g(X) is defined by $Var(X) = E[(X - E[X])^2] = \int_{-\infty}^{\infty} (x - E[X])^2 fx(x)dx$

12. CUMULATIVE DISTRIBUTION FUNCTIONS

The CDF of a random variable X is denoted by F_x and provides the probability $P(X \le x)$. In particular, for every x we have

$$Fx(x) = P(X \le x) = \begin{cases} \sum_{k \le x} px(k) & X: \text{discrete,} \\ \int_{-\infty}^{x} fx(t)dt & X: \text{continuous} \end{cases}$$

Properties of a CDF

The CDF F_x of a random variable X is defined by $F_x(x) = P(X \le x)$, for all x,

and has the following properties.

- F_x is monotonically nondecreasing:
  if $x \le y$, then $F_x(x) \le F_x(Y)$.
- $Fx(x)$ tends to 0 as $x \to \infty$, and to 1 as $x \to \infty$.
- If X is discrete, then F_x has a piecewise constant and staircase-like form.
- If X is continuous, then F_x has a continuously varying form.
- If X is discrete and takes integer values, the PMF and the CDF can be obtained from each other by summing or differencing:

  $Fx(k) = \sum_{i=-\infty}^{k} Px(i)$,

  $p_x(k) = P(X \le k) - P(X \le k - 1) = F_x(k) - F_x(k - 1)$,

  for all integers k.
- If X is continuous, the PDF and the CDF can be obtained from each other by integration or differentiation:

  $F_x(x) = \int_{-\infty}^{x} fx(t)dt$,

13. STATISTICAL AVERAGE OF RANDOM VARIABLE

Mean or Expected Value:

Let a random variable X characterized by its PDF $f_x(x)$. The mean or expected value of X is defined as

$$E(X) = \overline{X} = \int_{-\infty}^{\infty} x f_x(x) dx$$

Similarly, we obtain the expected value of a function g(X) as $E[g(X)] = \overline{g(X)} = \int_{-\infty}^{\infty} g(x) f_x(x) dx$

If X is a discretely distributed random variable, then the expected value of X is given by

$$E[X] = \overline{X} = \mu_x = \sum_{i=1}^{n} x_i f_x(x_i)$$

Variance

The variance σ_x^2 of a random variable X is the second moment taken about its mean. i.e.

$$\text{Var}[X] = \sigma_x^2 = E[(X - \mu_x)^2]$$

$$= \int_{-\infty}^{\infty} (x - \mu_x)^2 f_x(x) dx$$

Expanding the above equation, we can write

$$\sigma_x^2 = E[X^2] - \{E[X]\}^2 = \overline{X^2} - \mu_x^2$$

Standard Deviation

The standard deviation σ_x of a random variable is the square root of its variance, i.e.,

$$\sigma_x = \sqrt{\text{var}[x]} = \sqrt{\overline{X^2} = \mu_x^2}$$

Covariance

The covariance of the random variables X and Y is defined as:

$$\text{cov}[XY] = \sigma_{xy} = E[(X - \mu_x)(Y - \mu_y)] = \overline{(X - \mu_x)(Y - \mu_y)}$$

where μ_x and μ_y are the mean of random variables X and Y, respectively. We may expand the above result as

$$\text{cov}[XY] = \sigma_{xy} = E[XY] - \mu_x\mu_y = \overline{XY} - \mu_x\mu_y$$

Correlation Coefficient

The correlation coefficient of random variables X and Y can he defined as

$$\rho_{xy} = \frac{\text{cov}[xy]}{\sigma_x \sigma_y}$$

where cov [XY] is the covariance of X and Y, and σ_x, σ_y are the standard deviations of random variables.

NOTE:

1. The random variables X and Y are uncorrelated if and only if their covariance is zero, i.e cov[XY] = 0

2. The random variables X and Y are orthogonal if and only if their correlation is zero, i.e. $E[XY] = 0$

14. DIFFERENT TYPES OF RANDOM VARIABLE

Exponential Random Variable

An exponential random variable has a PDF on the form

$$fx(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

Where λ is a positive parameter characterizing the PDF

$$E[X] = \int_0^\infty x\lambda e^{-\lambda x}dx = (-xe^{-\lambda x}\Big|_0^\infty + \int_0^\infty e^{-\lambda x}dx = 0 - \frac{e^{-\lambda x}}{\lambda}\Big|_0^\infty = \frac{1}{\lambda}.$$

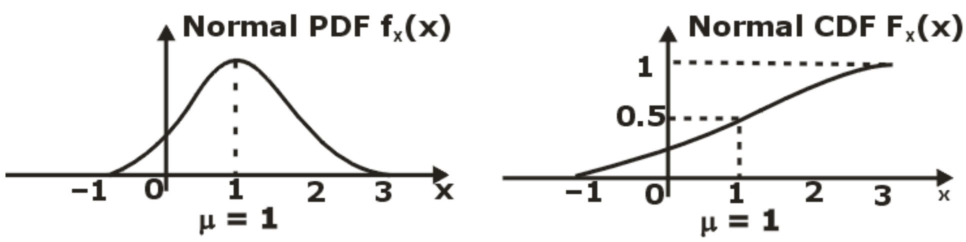
$$var(X) = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}.$$

Normal or Gaussian Random Variable

A continuous random variable X is said to be normal or Gaussian if it has a PDF of the form (see below figure)

$$F_x(x) = \frac{1}{\sqrt{2\pi}\sigma} e - (x-\mu)^{2/2\sigma^2},$$

Where μ and σ are two scalar parameters characterizing the PDF



The mean and the variance can be calculated to be

$E[X] = \mu$, $\quad var(X) = \sigma^2$.

CDF calculation of the Normal Random variable

The CDF of a normal variable X with mean μ and variance σ^2 is obtained using the standard normal table as

$$P(X \leq x) = P\left(\frac{X-\mu}{\sigma} \leq \frac{X-\mu}{\sigma}\right)$$

$$= P\left(Y \leq \frac{x-\mu}{\sigma}\right) = \phi\left(\frac{x=\mu}{\sigma}\right),$$

Where Y is a standard random variable.

Conditional PDF and Expectation Given an Event

- The conditional PDF $f_{x|A}$ of a continuous variable X given an event A with $P(A) > 0$, satisfies

$$P(X \in B \mid A) = \int_B f_{x|A}(x)dx$$

- If A be a subset of the real line with $P(X \in A) > 0$, then

$$P_{X|A}(x) = \begin{cases} \dfrac{f_x(x)}{P(X \varepsilon A)} & \text{if } x \varepsilon A, \\ 0 & \text{otherwise,} \end{cases}$$

and $P(X \in B \mid X \in A) = \int_B f_{x|A}(x)dx,$

For any set B.

- The corresponding conditional expectation is defined by]

$$E[X \mid A] = \int_{-\infty}^{\infty} x f_{x|A}(x)dx$$

- The expected value rule remains valid:

$$E[g(X) \mid A] = \int_{-\infty}^{\infty} g(x) f_{x|A}(x)dx$$

- If $A_1, A_2, \ldots A_n$ are disjoint events with $P(A_i) > 0$ foreach I, that form a partition of the sample space, then

$$f_x(x) = \sum_{i=1}^{n} P(A_i) f_{x|A_i}(x)$$

(a version of the total theorem), and

$$E[X] = \sum_{i=1}^{n} P(A_i) E[X \mid A_i]$$

(the total expectation theorem). Similarly,

$$E[g(X)] = \sum_{i=1}^{n} P(A_i) E[g(X) \mid A_i]$$

15. MULTIPLE CONTINUOUS RANDOM VARIABLES

A joint PDF fx,y, if fx,y is a nonnegative function that satisfies

$$P((X,Y) \in B) = \int \int_{(x,y) \in B} f_{x,y}(x,y)dxdy$$

for every subset B of the two-dimensional plane. The notation above means that the integration is carried over the set B. In the particular case where B is a rectangle of the form $B = [a, b] \times [c, d]$, we have

$$P(a \le X \le b, c \le Y \le d) = \int_c^d \int_a^b f_{x,y}(x,y)dxdy$$

Furthermore, by letting B be the entire two-dimensional plane, we obtain the normalization property

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y)dxdy = 1$$

Expectation

If X and Y are jointly continuous random variables, and g is some function, then Z = g(X, Y) is also a random variable. For now, let us note that the expected value rule is still applicable and

$$E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)f_{x,y}(x,y)dxdy$$

As an important special case, fur any scalars a, is, we have

E(aX + by] = aE[X] + bE[Y].

16. INDEPENDENCE OF CONTINUOUS RANDOM VARIABLES

Suppose that X and Y are independent, that is,

$f_{x,y}(x,Y)= f_x(x)fy(y)$, for all x,y.

17. BERNOULLI RANDOM VARIABLE

Consider the toss of a biased coin, which comes up a head with probability p, and a tail with probability 1-p. The Bernoulli random variable takes the two values 1 and 0, depending on whether the outcome is a head or a tail:

$$X = \begin{cases} 1 & \text{if a head,} \\ 0 & \text{if a tail} \end{cases}$$

Its PMF is

$$P_x(x) = \begin{cases} P & \text{if } x = 1, \\ 1-p & \text{if } x = 0. \end{cases}$$

18. BINOMIAL RANDOM VARIABLE

A biased coin is tossed n times. At each toss, the coin comes up a head with probability p, and a tail with probability 1-p, independently of prior tosses. Let X be the number of heads in the n-toss sequence. We refer to X as a binomial random variable with parameters n and p. The PMF of X consists of the binomial probabilities

$$px(k) = P(X = k) = \binom{n}{k}p^k(1-p)^{n-k}, \; k = 0,1,...,n.$$

(Note that here and elsewhere, we simplify notation and use k, instead of x, to denote the experimental values of integer-valued random variables.) The normalization property $\sum_x Px(x) = 1$, specialized to the binomial random variable, is written as

$$\sum_{k=0}^{n}\binom{n}{k}p^k(1-p)^{n-k} = 1$$

19. POISSON DISTRIBUTION

A random variable X is called a Poisson random variable with parameter $\lambda(> 0)$ if its pmf is given by

$$p_x(k) = P(x = k) = e^{-\lambda}\frac{\lambda^k}{k!} \qquad k = 0, 1....$$

The corresponding cdf of X is

The mean and variance of the Poisson random variable X are:

$$\mu_x = E(X) = \lambda$$

$$\sigma_x^{\ 2} = Var(X) = \lambda$$

20. UNIFORM DISTRIBUTION

A random variable X is called a uniform random variable over (a, b) if its pdf is given by

$$f_x(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

The corresponding cdf of X is

$$f_x(x) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x \ge b \end{cases}$$

The mean and variance of the uniform random variable X are:

$$\mu_x = E(x) = \frac{a+b}{2}$$

$$\sigma_x^{\ 2} = Var(X) = \frac{(b-a)^2}{12}$$

21. RAYLEIGH DISTRIBUTION

The Rayleigh distribution describes a continuous random variable obtained from two Gaussian random variables. If X and Y are independent Gaussian random variables with zero mean and the same variance σ^2, then the corresponding Rayleigh random variable is defined by

$$R = \sqrt{x^2 + y^2}$$

The probability density function of the Rayleigh random variable is given by

$$f_R(r) = \frac{r}{\sigma^2}e^{-r^2/2a^2}$$

The corresponding CDF of Rayleigh random variable

$$f_R(r) = 1 - e^{-r^2/2\sigma^2}$$

The resulting mean of R is

$$\overline{R} = \sqrt{\frac{\pi}{2}}\sigma$$

The resulting second moment of R is $\overline{R^2} = 2\sigma^2$

Properties of $F_{XY}(x, y)$:

The joint cdf of two random variable's has many properties analogous to those of the cdf of a single random variable

1. $0 \le F_{XY}(x, y) \le 1$
2. If $x_1 \le x_2$, and $y_1 \le y_2$, the
   $F_{XY}(x_1, y_1) \le F_{XY}(x_2, y_1) \le F_{XY}(x_2, y_2)$
   $F_{XT}(x_1, y_1) \le F_{XY}(y_1, y_2 \le F_{XY}(x_2, y_2)$
3. $\lim\limits_{\substack{x\to\infty \\ y\to\infty}} F_{XY}(x, y) = F_{XY}(\infty, \infty) = 1$
4. $\lim\limits_{x\to-\infty} F_{XY}(x, y) = F_{XY}(-\infty, y) = 0$
5. $\lim\limits_{x\to a^+} F_{XY}(x, y) = F_{XY}(-a^+, y) = F_{XY}(a, y)$
6. $P(X_1 < X \le x_2, Y \le y) = F_{XY}(x_2, y) - F_{XY}(x_1, y)$
   $(X \le x, y_1 < Y \le y_2) = F_{XY}(x, y_2) - F_{XY}(x, y_1)$
7. If $x_1 \le x_2$ and $y_1 \le y_2$, then
   $F_{XY}(x_2, y_2) - F_{XY}(x_1, y_2) - F_{XY}(x_2, y_1) + F_{XY}(x_1, y_1) \ge 0$

22. MARGINAL DISTRIBUTION FUNCTIONS

Now, $\lim\limits_{x\to\infty}(X \le x, Y \le y) = (X \le x, Y \le \infty) = (X \le x)$

Since the condition $y \le \infty$ is always satisfied. Then

$\lim\limits_{x\to\infty} X_{XY}(x, y) = F_{XY}(x, \infty) = F_X(x)$

Similarly, $\lim\limits_{x\to\infty} X_{XY}(x, y) = F_{XY}(\infty, x) = F_Y(y)$

The cdf's $F_X(x)$ and $F_Y(y)$ are referred to as the marginal cdf's of X and Y, respectively.

23. INDEPENDENT RANDOM VARIABLES

If X and Y are independent random variable's, then

$p_{XY}(x_i, y_j) = p_X(x_i)\, p_Y(y_j)$

24. JOINT PROBABILITY DENSITY FUNCTIONS

Let (X, Y) be a continuous bivariate random variable with cdf $F_{XY}(x, y)$ and let

$$f_{XY}(x, y) = \frac{\partial^2 F_{xy}(x, y)}{\partial x\, \partial y}$$

The function $f_{XY}(x, y)$ is called the joint probability density (joint pdf) of (X, Y).

Properties of $f_{XY}(x,)$:

1. $f_{XY}(x, y) \ge 0$

2. $\iint_{R_A} f_{XY}(x, y)\, dx\, dy = 1$

3. $f_{XY}(x, y)$ is continuous for all values of x or concept possible a finite set.

4. $P[(X, Y)] \in A] = \iint_{R_A} f_{XY}(x, y)\, dx\, dy$

5. $P(a < X \le b, c \le d) = \int_c^d \int_a^b f_{XY}(x, y)\, dx\, dy$

Since P[(X = a)] = 0 = P(Y = c)

it follows that

$P(a < X \le b, c < Y \le d) = P(a \le X \le b, c \le Y \le d) = P(a \le X < b, c \le Y < d)$

$$= P(a < X \le b, c < Y < d) = \int_c^d \int_a^b f_{XY}(x, y)\, dx\, dy$$

25. MARGINAL PROBABILITY DENSITY FUNCTIONS

$$F_X(x) = F_{XY}(x, \infty) = \int_{-\infty}^{x} \int_{-\infty}^{\infty} f_{XY}(\xi, \eta) d\eta\, d\xi$$

Hence $f_x(x) = \dfrac{dF_X(x)}{dx0} = \int_{-\infty}^{\infty} f_{xy}(x, \eta) d\eta$

or $f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x, y) d\eta$

or $f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x, y) dy$

Similarly, $f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x, y)\, dx$

The pdf's $f_X(x)$ and $f_Y(y$ are referred to as the marginal pdf's of X and Y, respectively.

26. CLASSIFICATION OF RANDOM PROCESS

Random processes may be classified as continuous or discrete.

Continuous Random Process: A continuous random process consists of a random process with associated continuously distributed random variables

$X(t, s_i)$. The Gaussian random process is an example of the continuous random process.

Discrete Random Process: A discrete random process consists of the random variables with discrete distributions. For example, the output of an ideal (hard) limiter in a binary (discrete with two levels) random process.

27. PROBABILITY DENSITY FUNCTION OF RANDOM PROCESS

A complete description of a random process {X(t, s)} is given by the N-fold joint pdf that probabilistically describes the possible values assumed by a typical sample function at time $t_N > t_{N-1} > \ldots > t_1$, when N is arbitrary.

For N = 1, we can interpret this joint pdf as

$$f_{X_1}(x_1, t_1)dx_1 = P(x_1 - dx_1 < X_1 \le x_1 \text{ at time } t_1)$$

Where $X_1 = X(t_1, s)$. Similarly, for N = 2, we can interpret the joint pdf as $f_{X_1X_2}(x_1, t_1 : x_2, t_2)$ $dx_1dx_2 = P(x_1 - dx_1 < X_1 \le x_1$ and $x_2 - dx_2 < X_2 \le x_2)$ where $X_2 = X(t_2, s)$. In general, we denote the N-dimensional PDF of a random process as

$f_{X(t)}(x) = f_{X(t)}(x(t_1), x(t_2), \dots, x(t_N))$

28. STATIONARY RANDOM PROCESS

A random process X(t) is said to be stationary to the order N if, for any $t_1, t_2 \dots, t_N$

$$f_{X(t)}(x(t_1), x(t_2), \dots x(t_N)) = f_{X(t)}(x(t_1 + t_0), x(t_2 + t_0), \dots x(t_N + t_0))$$

where t_0 is any arbitrary real constant. Furthermore, the process is said to be strictly stationary if it is stationary to the order $N \to \infty$.

29. AVERAGES OF RANDOM PROCESS

We may define the time average and ensemble average of a random process in the following ways:

Time Average of a Random Process

Consider a stationary random process X(t) with the sample function x(t). The time average of this random process is defined as

$$< X(t) > = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)dt$$

30. ENSEMBLE AVERAGE OF A RANDOM PROCESS

For a given random process X(t), the ensemble average is defined as the mean value of X(t) at arbitrary time t, i.e.

$$E[X(t)] = \overline{X(t)} = \mu_X(t) = \int_{-\infty}^{\infty} x f_{x(t)}(x)dx .$$

An important property of ensemble average is given below.

Property of Ensemble average:

The mean (ensemble average) of a stationary process is constant, i.e.

$\mu_X(t) = \mu_X$ for all t

31. AUTOCORRELATION FUNCTION

The autocorrelation function of a random process X(t) is defined as the expectation of the product of two random variables $X(t_1)$ and $X(t_2)$, i.e.

$$R_X(t_1, t_2) = E[X(t_1)\, X(t_2)] = \overline{X(t_1)\, X(t_2)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1x_2\, f_{x(t)}(x_1, x_2)dx_1dx_2$$

where $x_1 = x(t_1)$ and $x_2 = x(t_2)$. Some important properties of the autocorrelation function of a stationary random process are given below.

Properties of Auto Correlation Function:

1. The autocorrelation function of strictly stationary random process is a function only of the time difference $\tau = t_2 - t_1$, i.e.

$$R_X(\tau) = E[X(t)\,X(t+\tau)] = \overline{X(t)\,X(t+\tau)}$$

2. The mean square value of the process may be obtained from $R_x(\tau)$ simply by putting $\tau = 0$ in above equation, i.e.

$$R_X(0) = E[X^2(t)] = \overline{X^2(t)}$$

3. If the autocorrelation function of a random process has no periodic component then, we may define

$$\overline{X(t)}^2 = R_X(\pm\infty) \text{ or } \overline{X(t)} = \sqrt{R_X(\pm\infty)}$$

4. The autocorrelation function is the even function of τ, i.e.

$$R_X(\tau) = R_X(-\tau)$$

5. The autocorrelation has its maximum magnitude at $\tau = 0$, i.e.

$$R_X(0) \geq R_X(\tau)$$

32. CROSS-CORRELATION FUNCTION

The cross-correlation function for two random processes X(t) and Y(t) is defined as:

$$R_{XY}(t_1, t_2) = E[X(t_1)\,Y(t_2)] = \overline{X(t_1)\,Y(t_2)}$$

where t_1 and t_2 denote the two values of time at which the processes are observed. Following are some important properties of the cross-correlation function of two jointly stationary random processes X(t) and Y(t):

Properties of Cross Correlation Function:

1. The cross-correlation function of jointly random processes X(t) and Y(t) is a function only of the time difference $\tau = t_2 - t_1$, i.e.

$$R_{XY}(t_1, t_2) = R_{XY}(\tau)$$

2. The cross-correlation function is the even function of τ, i.e.

$$R_{XY}(-\tau) = R_{YX}(\tau)$$

3. The random processes X(t) and Y(t) are said to be uncorrelated if, for all t_1 and t_2,

$$R_{XY}(t_1, t_2) = \overline{X(t_1)} \times \overline{Y(t_2)}$$

4. $|R_{XY}(\tau)| \leq \sqrt{R_X(0)\,R_Y(0)}$

5. $|R_{XY}(\tau)| \leq \frac{1}{2}[R_X(0) + R_Y(0)]$

33. ERGODIC PROCESS

A random process is said to be ergodic if all time averages of any sample function are equal to the corresponding ensemble averages (expectations). As the ergodic process has its ensemble average equal to its time average, we may deduce the following properties for the ergodic process:

Properties of Ergodic Process:

1. The dc value of an ergodic process can be defined in terms of ensemble average as

$$X_{dc} = \langle X(t) \rangle = E[X(t)] = \overline{X(t)}$$

2. The dc power of an ergodic process can be defined as

$$P_{dc} = \langle X(t) \rangle^2 = \{E[X(t)]\}^2 = \{\overline{X(t)}\}^2$$

3. The power in the ac (time-varying) component is given by

$$P_{ac} = \sigma_x^2 = \langle X^2(t) \rangle = \langle X(t) \rangle^2 = \overline{X^2(t)} - \overline{X(t)}^2$$

4. The rms power (total power) of an ergodic process is defined as

$$P_{total} = \langle X^2(t) \rangle = E[X^2(t)] = \overline{X^2(t)} = \sigma^2 x + \overline{X(t)}^2$$

5. The rms value of an ergodic process can be defined as

$$X_{rms} = \sqrt{\langle X^2(t) \rangle} = \sqrt{E[X^2(t)]} = \sqrt{\overline{X^2(t)}} = \sqrt{\sigma_x^2 + \overline{X(t)}^2}$$

34. WIDE SENSE STATIONARY PROCESS

A random process is said to be wide-sense stationary (WSS) if its mean is independent of time and autocorrelation function depends on the time difference, i.e.

$E[X(t)] = \mu_X$ where μ_X is a constant

and $R_X(t_1, t_2) = R_X(\tau)$, where $\tau = t_2 - t_1$

35. POWER SPECTRAL DENSITY

Following are some important properties of the power spectral density of a stationary process.

Properties of Power Spectral Density:

1. The power spectral density $S_X(f)$ is always real and nonnegative, i.e.

   $S_X(f) \geq 0$

2. The power spectral density of a real-valued random process is an even function of frequency, i.e.

   $S_X(-f) = S_X(f)$

3. The total normalized power of a random process is defined in terms of power spectral density as

$$P_{total} = \int_{-\infty}^{\infty} S_X(f)\,df$$

or $E[X^2(t)] = \int_{-\infty}^{\infty} S_X(f)\,df$

or $R_X(0) = \int_{-\infty}^{\infty} S_X(f)\,df$

4. The zero-frequency value of the power spectral density of a stationary process equals the total area under the graph of the autocorrelation function, i.e.

$$S_X(0) = \int_{-\infty}^{\infty} R_X(\tau)\,d\tau$$

36. CROSS SPECTRAL DENSITY

Let X(t) and Y(t) be two jointly stationary processes with their cross-correlation functions denoted by $R_{XY}(\tau)$ and $R_{YX}(\tau)$. We then define the cross-spectral densities for the random processes as

$$S_{XY}(f) = \int_{-\infty}^{\infty} R_{XY}(\tau)e^{-j2\pi f\tau}\,d\tau$$

and $$S_{YX}(f) = \int_{-\infty}^{\infty} R_{YX}(\tau)e^{-j2\pi f\tau}\,d\tau$$

Accordingly, using the formula for inverse Fourier transformation, we may also write

$$R_{XY}(\tau) = \int_{-\infty}^{\infty} S_{XY}(f)e^{j2\pi f\tau}\,df$$

and $$R_{YX}(\tau) = \int_{-\infty}^{\infty} S_{YX}(f)e^{j2\pi f\tau}\,df$$

Properties of Cross Spectral Density:

The cross-spectral densities $S_{XY}(f)$ and $S_{YX}(f)$ are not necessarily the real function or f. However, using the property of autocorrelation function, we may deduce that

$$S_{XY}(f) = S_{YX}(-f) = S_{YX}^*(f)$$

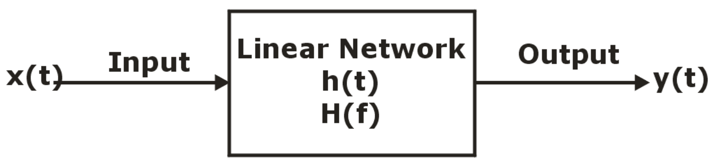
37. LINEAR SYSTEM

Consider a linens system shown in Figure 10. The input-output relationship for the system is

y(t) = h(t) * x(t)

The corresponding Fourier transform relationship is

Y(f) = H(f) X(f)



X(f) Y(f)

$R_X(\tau)$ $R_Y(\tau)$

$S_X(f)$ $S_Y(f)$

Input-Output Relationship of a Linear System

If x(t) is a wide-sense stationary random process, then the output autocorrelation is defined as

$R_Y(\tau) = h(-\tau) * h(\tau) * R_X(\tau)$

Correspondingly, the output power spectral density is given by

$S_Y(f) = |H(f)|^2 S_X(f)$

Thus, the power transfer function of the network is

$$G(f) = \frac{S_Y(f)}{S_X(f)} = |H(f)|^2$$

Chapter-6
Baseband Modulation

1. SAMPLING PROCESS

Sampling Period: The time interval between two consecutive samples is referred as sampling period.

Sampling Rate: The reciprocal of sampling period is referred as sampling rate, i.e.

$\mathbf{f_S = 1/T_S}$

2. EXPLANATION OF SAMPLING THEOREM

Consider a message signal m(t) bandlimited to W, i.e.

$M(f) = 0 \qquad \text{For } |f| \geq W$

Then, the sampling Frequency f_S, required to reconstruct the bandlimited waveform without any error, is given by

$F_s \geq 2\,W$

3. NYQUIST RATE

Nyquist rate is defined as the minimum sampling frequency allowed to reconstruct a bandlimited waveform without error, i.e.

$f_N = \min\{f_S\} = 2W$

Where W is the message signal bandwidth, and f_S is the sampling frequency.

4. NYQUIST INTERVAL

The reciprocal of Nyquist rate is called the Nyquist interval (measured in seconds), i.e.

$$T_N = \frac{1}{f_N} = \frac{1}{2W}$$

Where f_N is the Nyquist rate, and W is the message signal bandwidth.

5. SAMPLING OF BANDPASS SIGNALS

The sampling theorem for bandpass signals may be expressed as under:

The bandpass signal x(t) whose maximum bandwidth is $2f_m$ can be completely represented into and recovered from its samples if it is sampled at the minimum rate of twice the bandwidth.

Hence if the bandwidth is $2f_m$, then the minimum sampling rate for bandpass signal must be $4f_m$ samples per second.

This bandpass signal is first represented in terms of its inphase and quadrature components

Let $x_I(t)$ = Inphase component of x(t)

And $X_Q(t)$ = Quadrature component of x(t)

Thus, the signal x(t) in terms of inphase and quadrature components will be expressed as

$x(t) = x_I(t) \cos(2pf_ct) - X_Q(t) \sin(2pf_ct)$

Thus, if $4f_m$ samples per second are taken, then the bandpass signal of bandwidth $2f_m$ can be completely recovered from its samples. Hence, for bandpass signals of bandwidth $2f_m$.

Minimum sampling rate = Twice of bandwidth = $4f_m$ samples per second.

6. SAMPLING TECHNIQUE

The sampling of a signal is done in several ways.

Basically, there are three types of sampling technique as under:

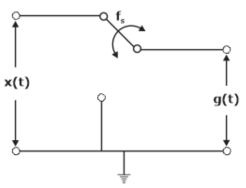
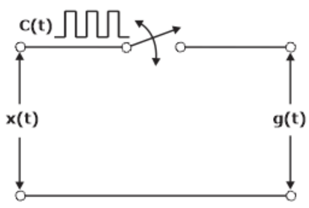
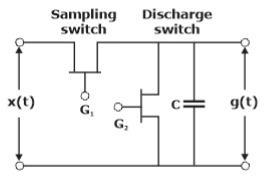
i. Instantaneous sampling

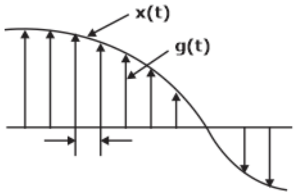
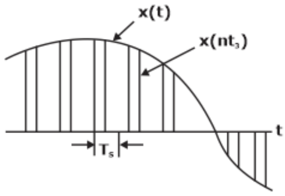
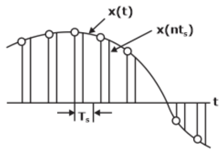
ii. Natural sampling

iii. Flat top sampling

Out of these three, instantaneous sampling is called **ideal sampling** whereas natural sampling and flat-top sampling are called **practical sampling** methods.

Table 1: Performance Comparison of three Sampling Technique

S. No	Parameter of comparison	Ideal or instantaneous sampling	Natural sampling	Flat top sampling
1.	Sampling principle	It uses multiplication	It uses chopping principle	It uses sample and hold circuit
2.	Generation circuit			

3.	Waveforms involved			
4.	Feasibility	This is not a practically possible method	This method is used practically	This method is also used practically
5.	Sampling rate	Sampling rate tends to infinity	Sampling rate satisfies Nyquist criteria	Sampling rate satisfies Nyquist criteria
6.	Noise interference	Noise interference is maximum	Noise interference is minimum noise	Interference is maximum
7.	Time domain representation	$g(t)=\sum_{n=-\infty}^{\infty} x(nT_s)\delta(t-nT_s)$	$g(t)=\frac{\tau A}{T_s}\sum_{n=-\infty}^{\infty} x(t)\,\mathrm{sinc}(nf_s\tau)e^{j2\pi nf_s t}$	$g(t)=\sum_{n=-\infty}^{\infty} x(nT_s)h(t-nT_s)$
8.	Frequency domain representation	$G(f)=f_s\sum_{n=-\infty}^{\infty} X(f-nf_s)$	$G(f)=\frac{\tau A}{T_s}\sum_{n=-\infty}^{\infty} \mathrm{sinc}(nf_s\tau)X(f-nf_s)$	$G(f)=f_s\sum_{n=-\infty}^{\infty} X(f-nf_s)H(f)$

7. PULSE MODULATION

Pulse modulation is the process of changing a binary pulse signal to represent the information to be transmitted. Pulse modulation can be either analog or digital.

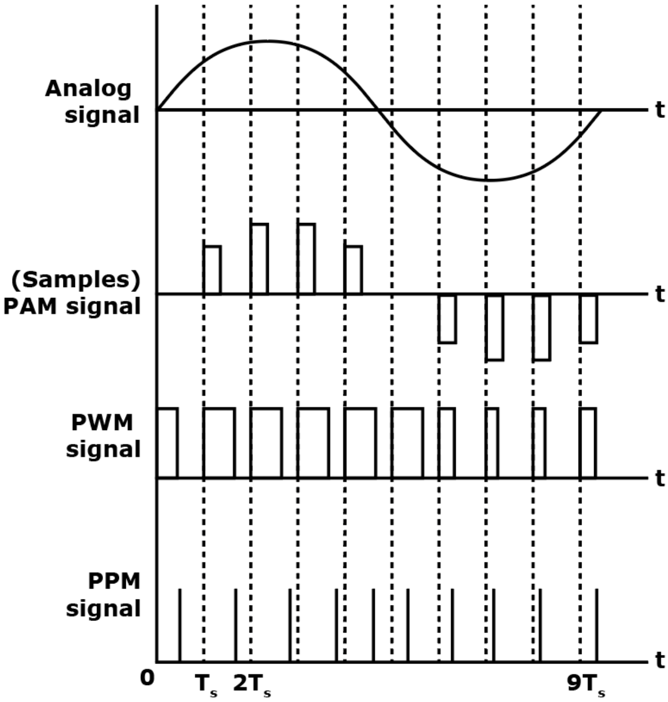
Analog Pulse Modulation

Analog pulse modulation results when some attribute of a pulse varies continuously in one-to-one correspondence with a sample value. In analog pulse modulation systems, the amplitude, width, or position of a pulse can vary over a continuous range in accordance with the message amplitude at the sampling instant.

i. Pulse Amplitude Modulation (PAM)

ii. Pulse Width Modulation (PWM)

iii. Pulse Position Modulation (PPM)



Representation of Various Analog Pulse Modulation

Digital Pulse Modulation

In systems utilizing digital pulse modulation, the transmitted sample take on only discrete values. Two important types of digital pulse modulation are:

i. Delta Modulation (DM)

ii. Pulse Code Modulation (PCM)

iii. Differential Pulse Code Modulation (DPCM)

iv. Adaptive Delta Modulation (ADM)

8. PULSE CODE MODULATION

The PCM signal is generated by carrying out the following three basic operations:

i. Sampling

ii. Quantizing

iii. Encoding

Block Diagram Representation of PCM System

9. Quantization

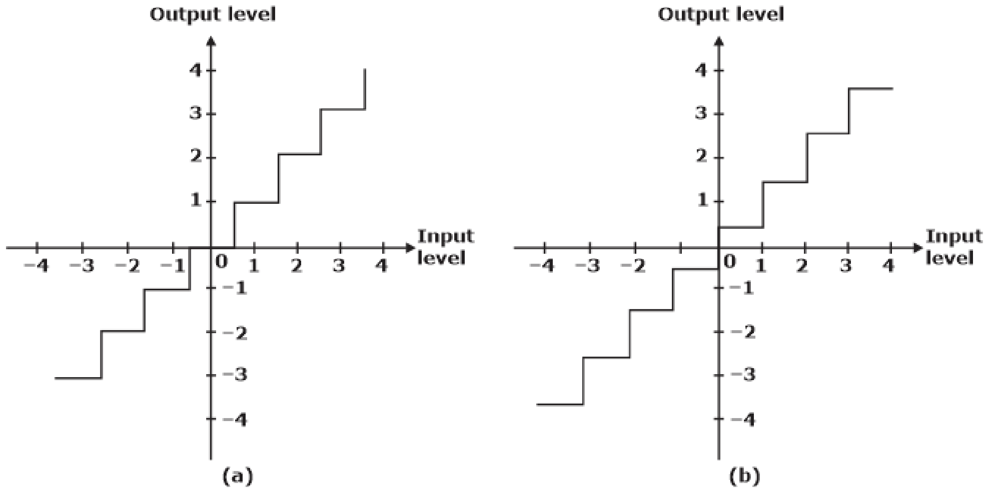
Uniform Quantizer

A quantizer is called as a uniform quantizer if the step size remains constant throughout the input range.

There are two types of uniform quantizer as under:

i. Symmetric quantizer of the midtread type

ii. Symmetric quantizer of the midrise type



Two types of Uniform Quantization (a) Midtread, and (b) Midrise

Nonuniform Quantizer

Nonuniform quantization is required to be implemented to improve the signal to quantization noise ratio of weak signals. It is equivalent to passing the baseband signal through a compressor and then applying the compressed signal to a uniform quantizer. A particular form of compression law that is used in practice is called as μ-law, which is defined by

$$|m_q| = \frac{1n(1+\mu|m|)}{1n(1+\mu)}$$

Where m and m_q are the normalized input and output voltages, and μ is a positive constant.

Encoding

An encoder translates the quantized samples into digital code words. The encoder works with M-ary digits and produces for each sample a code word consisting of n digits in parallel. Since, there are M^n possible M-ary codewords with n digits per word, unique encoding of the q different quantum levels requires that

$M^n \geq q$

The parameters M, n, and q should be chosen to satisfy the equality, so that

$q = M^n$ or $n = \log_M q$

Encoding in Binary PCM

For binary PCM, each digit may be either of two distinct values 0 or 1, i.e.

M = 2

If the code word of binary PCM consists of n digits, then number of quantization levels is defined as

$q = 2^n$

or $n = \log_2 q$

In general, we must remember the following characteristics of a PCM system:

10. CHARACTERISTICS OF PCM SYSTEM

- A sampled waveform is quantized into q quantization levels; where q is an integer.
- If the message signal is defined in the range ($-m_p$, m_p), then the step size of quantizer is
  $$\delta = \frac{2m_p}{q}$$
- For a binary PCM system with n digit codes, the number of quantization level is defined as
  $q = 2^n$
- If the message signal is sampled at the sampling rate f_S, and encoded to n number of bits per sample; then bit rate (bits/sec) of the PCM is defined as
  $R_b = nf_S$

11. METHODOLOGY TO EVALUATE BIT RATE FOR PCM SYSTEM

If the number of quantization levels q and message signal frequency f_m for a PCM signal is given, then bit rate for the PCM system is obtained in the following steps:

Step 1: Obtain the sampling frequency for the PCM signal. According to Nyquist criterion, the minimum sampling frequency is given by

$f_S = 2f_m$

Step 2: Deduce the number of bits per sample using the expression

$n = \log_2 q$

Step 3: Evaluate bit rate (bits/sec) for the PCM system by substituting the obtained values in the expression

$R_b = nf_S$

12. TRANSMISSION BANDWIDTH IN A PCM SYSTEM

$$B_{PCM} \geq \frac{1}{2}R_b = \frac{1}{2}nf_s$$

Where R_b is the bit rate, n is the number of bits in PCM word, and f_S is the sampling rate. Since, the required sampling rate for no aliasing is

$f_S \geq 2\ W$

Where W is the bandwidth of the message signal (that is to be converted to the PCM signal). Thus, the bandwidth of the PCM signal has a lower bound given by

$B_{PCM} \geq nW$

13. NOISE CONSIDERATION IN PCM

In PCM (pulse code modulation), there are two error sources:

i. Quantization noise

ii. Channel noise

Quantization Noise

For a PCM system, the kth sample of quantized message signal is represented by

$M_q(kT_S) = m(kT_S) + \varepsilon(kT_S)$

Where $m(kT_S)$ is the sampled waveform, and $\varepsilon(kT_S)$ is the quantization error. Let the quantization levels having uniform step size δ. Then, we have

$$-\frac{\delta}{2} \le \varepsilon \le \frac{\delta}{2}$$

So, the mean-square error due to quantization is

$$\overline{\varepsilon^2} = \frac{1}{\delta}\int_{-\delta/2}^{\delta/2} \varepsilon^2 d\varepsilon = \frac{\delta^2}{12} \quad ....(i)$$

14. METHODOLOGY TO EVALUATE BIT RATE FOR PCM SYSTEM

Step 1: Obtain the sampling frequency for the PCM signal. According to Nyquist criterion, the minimum sampling frequency is given by

$f_S = 2f_m$

Step 2: Obtain the maximum quantization error for the PCM system using the expression

$$|\text{ error }| = \left|\frac{\delta}{2}\right| = \left|\frac{2m_p}{2q}\right| = \left|\frac{m_p}{q}\right| = \left|\frac{m_p}{2^n}\right|$$

Step 3: Apply the given condition of accuracy as

$|\text{error}| \le x$ % of full-scale value

Step 4: Solve the above condition for the minimum value of number of bits per second (n).

Step 5: Obtain the bit rate by substituting the approximated integer value of n in the expression

$R_b = nf_S$

15. SIGNAL TO QUANTIZATION NOISE RATIO

For PCM system, we have the message signal m(t), and quantization error ε. So, we define the signal to quantization noise ratio as

$$(SNR)_Q = \frac{\overline{m^2(t)}}{\overline{\varepsilon^2}} = \frac{\overline{m^2(t)}}{\delta^2 / 12} \quad ....(ii)$$

Where δ is the step size of the quantized signal defined as

$$\delta = \frac{2m_p}{q} \quad ....(iii)$$

Substituting equation (iii) in equation (ii), we get the expression for signal to quantization noise ratio as

$$(SNR)_Q = 12\frac{\overline{m^2(t)}}{(2m_p / q)^2}$$

$$(SNR)_Q = 3q^2 \frac{\overline{m^2(t)}}{m_p^2} \text{..............(iv)}$$

Where m_p is the peak amplitude of message signal m(t), and q is the number of quantization level.

16. CHANNEL NOISE

If a PCM signal is composed of the data that are transmitted over the channel having bit error rate P_e, then peak signal to average quantization noise ratio is defined as

$$(SNR)_{peak} = \frac{3q^2}{1 + 4(q^2 - 1)P_e}$$

Similarly, for the channel with bit error probability P_e, the average signal to average quantization noise ratio is defined as

$$(SNR)_{avg} = \frac{q^2}{1 + 4(q^2 - 1)P_e}$$

17. COMPANDING

The signal to quantization noise ratio for μ-law companding is approximated as

$$(SNR)_Q = \frac{3q^2}{[\ln(1 + \mu)]^2}$$

Where q is the number of quantization level, and μ is a positive constant.

18. NOISE CONSIDERATION IN DELTA MODULATION

The quantizing noise error in delta modulation can be classified into two types of noise:

i. Slope Overload Noise

ii. Granular Noise

Slope Overload Noise

The maximum slope that can be generated by the accumulator output is

$$\frac{\delta}{T_s} = \delta f_s$$

We have the required condition to avoid slope overload as,

$$\max\left|\frac{d\,m(t)}{dt}\right| \le \delta\, f_s$$

Where m(t) is the message signal, δ is the step size of quantized signal, and f_s is the sampling rate.

Granular Noise

The granular noise in a DM system is similar to the granular noise in a PCM system. Form equation (i), we have the total quantizing noise for PCM system,

$$(\overline{\varepsilon^2})_{PCM} = \frac{1}{\delta}\int_{-\delta/2}^{\delta/2} \varepsilon^2 d\varepsilon = \frac{\delta^2}{12} = \frac{(\delta/2)^2}{3}$$

Replacing δ/2 of PCM by δ for DM, we obtain the total granular quantizing noise as

$$(\overline{\varepsilon^2})_{DM} = \frac{\delta^2}{3}$$

Thus, the power spectral density for granular noise in delta modulation system is obtained as

$$S_N(f) = \frac{\delta^2/3}{2f_s} = \frac{\delta^2}{6f_s}$$

Where δ is the step size, and f_S is the sampling frequency.

19. METHODOLOGY FOR FINDING MINIMUM STEP SIZE IN DELTA MODULATION

Following are the steps involved in determination of minimum step size to avoid slope overload in delta modulation:

Step 1: Obtain the sampling frequency for the modulation. According to Nyquist criterion, the minimum sampling frequency is given by

$f_S = 2f_m$

Step 2: Obtain the maximum slope of message signal using the expression

$$\max\left|\frac{dm(t)}{dt}\right| = 2\pi f_m A_m$$

Where f_m is the message signal frequency and A_m is amplitude of the message signal.

Step 3: Apply the required condition to avoid slope overload as

$$\delta f_s \geq \max\left|\frac{dm(t)}{dt}\right|$$

Step 4: Evaluate the minimum value of step size δ by solving the above condition.

20. MULTILEVEL SIGNALING

Baud

Let a multilevel signalling scheme having the symbol duration T_S seconds. So, we define the symbols per second transmitted for the system as

$$D = \frac{1}{T_s}$$

Where D is the symbol rate which is also called baud.

Bits per Symbol

For a multilevel signalling scheme with M number of symbols (levels), we define the bits per symbol as

$K = \log_2 M$

Relation Between Baud and Bit Rate

For a multilevel signalling scheme, the bit rate and baud (symbols per second) are related as

$R_b = kD = D\log_2 M$ ……………..(v)

Where R_b is the bit rate, $k = \log_2 M$ is the bits per symbol, and D is the baud (symbols per second).

21. RELATION BETWEEN BIT DURATION AND SYMBOL DURATION

For a multilevel signalling scheme, the bit duration is given by

$$T_b = \frac{1}{R_b}$$

Where R_b is the bit rate. Also, we have the symbol duration

$$T_s = \frac{1}{D}$$

Where D is the symbol rate. Thus, by substituting this expression in equation (v), we get the relation

$T_S = kT_b = T_b \log_2 M$

Where $k = \log_2 M$ is the bits per symbol.

22. TRANSMISSION BANDWIDTH

The null to null transmission bandwidth of the rectangular pulse multilevel waveform is defined as

B_T = D symbols/sec

The absolute transmission bandwidth for $\frac{\sin x}{z}$ pulse multilevel waveform is defined as

$$B_T = \frac{D}{2} \text{ symbols / sec}$$

Chapter-7

Bandpass Modulation

1. DIGITAL BANDPASS MODULATION

There are three basic modulation schemes:

i. Amplitude shift keying (ASK)

ii. Frequency shift keying (FSK)

iii. Phase shift keying (PSK)

A. Amplitude-Shift Keying (ASK):

In ASK, the modulated signal can be expressed as

$$x_c(t) = \begin{cases} A\cos\omega_c t & \text{symbol 1} \\ 0 & \text{symbol 2} \end{cases}$$

Note that the modulated signal is still an on-off signal. Thus, ASK is also known as on-off keying (OOK).

B. Frequency-Shift Keying (FSK):

In FSK, the modulated signal can be expressed as

$$x_c(t) = \begin{cases} A\cos\omega_1 t & \text{symbol 1} \\ A\cos\omega_2 t & \text{symbol 0} \end{cases}$$

C. Phase shift keying (PSK):

In PSK, the modulated signal can be expressed as

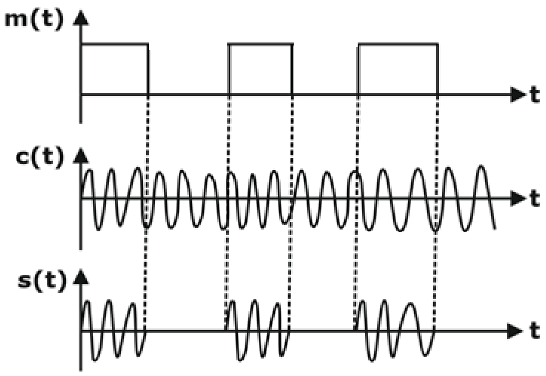
$$x_c(t) = \begin{cases} A\cos\omega_c t & \text{symbol 1} \\ A\cos\omega_c t + \pi & \text{symbol 0} \end{cases}$$

2. AMPLITUDE SHIFT KEYING

ASK is often referred to as on-off keying (OOK). The ASK signal is represented by

$s(t) = A_c\, m(t) \cos\omega_c t$

where m(t) is a unipolar baseband data signal.

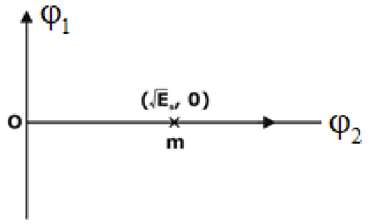


$$s(t) = A_c \cos\omega_c t = \begin{cases} A_c \cos 2\pi_c t & \rightarrow 1 \\ 0 & \rightarrow 0 \end{cases}$$

$$P_S = \frac{A_C^2}{2}$$

$$s(t) = \begin{cases} \sqrt{2p_s}\cos 2\pi f_c t \rightarrow 1 & 0 \le t \le T_2 \\ 0 \qquad\qquad\qquad \rightarrow 0 & \text{elsewhere} \end{cases}$$

Because of this it is called **"ON OFF KEYING"**



$$s(t) = \sqrt{\frac{2E_b}{T_b}}\cos \pi f_c t \rightarrow 1 \qquad 0 \le t \le T_b$$

$$= \sqrt{E_b}\,\phi_1(t)$$

Ask is one dimensional from constellation diagram

3. TRANSMISSION BANDWIDTH OF ASK SIGNAL

For ASK signal, the transmission bandwidth is given by

$B_T = 2R_b$

If raised cosine-roll off is used (to conserve bandwidth), the absolute transmission bandwidth (for rectangular pulse waveform) of AKS signal is obtained as

$B_T = (1+\alpha)R_b$

Where α is the roll-off factor of the filter.

4. BIT ERROR PROBABILITY OF ASK SIGNAL

The probability of bit error for coherent ASK system is given by

$$P_e = Q\sqrt{(E_b / N_0)} = Q\left(\sqrt{\gamma_b}\right)$$

Where E_b is the bit energy, N_0 is the noise power density, and γ_b is the bit energy to noise density ratio.

5. BINARY PHASE SHIFT KEYING

Binary phase shift keying (BPSK) system consists of shifting the phase of a sinusoidal carrier 0° or 180° with a unipolar binary signal, as shown in Figure 2(d). The BPSK signal is represented by

$S(t) = A_c \cos [\omega_c t + k_p m(t)]$

Where m(t) is the polar baseband data signal, as shown in Figure 2(b). Let us obtain the transmission bandwidth, and bit error probability for BPSK system

6. TRANSMISSION BANDWIDTH OF BPSK SIGNAL

The null-to-null transmission bandwidth for BPSK system is same as that found for amplitude shift keying (ASK). The null-to-null transmission bandwidth for BPSK system is given by

$B_T = 2R_b$

Where R_b is the bit rate of the digital signal.

7. Calculation of probability of error

$$P_e(\min) = \frac{1}{2}\text{erfc}\sqrt{\frac{r_{max}^{\;2}}{8}}$$

$$\gamma^2_{max} = \frac{2}{N_0}\int_0^{T_b}[s_1(t) - s_2(t)]^2 dt$$

NOTE: Probability of error in term of distance (d)

$$P_e = \frac{1}{2}\text{erfc}\left(\frac{d}{2\sqrt{N_0}}\right) = Q\left(\frac{d}{\sqrt{2}\sqrt{N_o}}\right) = Q\left(\frac{d}{\sqrt{2N_o}}\right)$$

$$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\frac{2\sqrt{E_b}}{\sqrt{2N_b}}\right) = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

If distance between two manage points is decreased, then probability of error will increase.

8. PROBABILITY OF ERROR IN CASE OF NON-SYNCHRONIZED BPSK

If ϕ is the error between local oscillation and modulated signal, then P_e will become

$$P_e = \frac{1}{2}\text{erfc}\sqrt{\frac{E_b\cos^2\phi}{N_0}} = Q\left(\frac{2E_b\cos^2\phi}{N_0}\right)^{1/2}$$

$$\boxed{SNR = \frac{E_b\cos^2\phi}{N_0}}$$

9. BIT ERROR PROBABILITY OF BPSK SIGNAL

If we consider phase error φ in demodulation, then the bit error probability is expressed as

$$P_e = Q\left(\sqrt{2\gamma_b\cos^2\phi}\right)$$

10. TRANSMISSION BANDWIDTH OF COHERENT BINARY FSK SIGNAL

The transmission bandwidth for FSK signal may be expressed as

$B_T = 2(\Delta f + R_b)$

Where R_b is the bit rate of the modulating signal and Δf is the peak frequency deviation.

The above expression can be more generalised for the following cases:

Case I: Narrowband FSK

For narrowband FSK signal, $\Delta f \ll R_b$. So, the transmission bandwidth of narrowband FSK is given by

$B_T = 2R_b$

Case II: Wideband FSK

For wideband GSK signal, $\Delta f \gg R_b$. So, the transmission bandwidth of wideband FSK given by

$B_T = 2\Delta f$

Case III: FSK with Raised Cosine Roll-off Factor

If a raised cosine roll-off factor α is used, equation (ii) becomes

$B_T = 2\Delta f + (1 + \alpha) R_b$

11. BIT ERROR PROBABILITY OF COHERENT BINARY FSK SIGNAL

For coherent binary FSK signal, we define the bit error probability as

$$P_e = Q\left(\sqrt{E_b / N_0}\right) = Q\left(\sqrt{\gamma_b}\right)$$

Where E_b is the bit energy, N_0 is the noise power density, γ_b is the bit energy to noise density ratio.

Note:

For larger value of z, the Q(z) function can be approximated as

$$Q(z) \approx \frac{1}{\sqrt{2\pi}z} e^{-z^2/2}, \quad z \gg 1$$

Q(z) function can be expressed in terms of complementary error function as

$$Q(z) = \frac{1}{2}\text{erfc}\left(\frac{z}{\sqrt{2}}\right)$$

11. NONCOHERENT BINARY SYSTEMS

The most common noncoherent bandpass modulation techniques are:

i. Differential phase shift keying (DPSK)

ii. Noncoherent frequency shift keying

12. METHOD OF DIFFERENTIAL ENCODING

Differential encoding of a message sequence is illustrated in Table 1. The steps for differential encoding are as follows.

Following are the steps involved in differential encoding of a message sequence:

Step 1: An arbitrary reference binary digit is assumed for the initial digit of the encoded sequence. In the example shown in Table 1, a 1 has been chosen.

Step 2: For each digit of the encoded sequence, the present digit is used as a reference for the following digit in the sequence.

Step 3: A 0 in the message sequence is encoded as a transition from the state of the reference digit to the opposite state in the encoded message sequence; a 1 is encoded as no change of state. In the example shown, the first digit in the message sequence is a 1, so no change in state is made in the encoded sequence, and a 1 appears as the next digit.

Step 4: This serves as the reference for the next digit to be encoded. Since the next digit appearing in the message sequence is a 0, the next encoded digit is the opposite of the reference digit, or a 0.

Step 5: The encoded message sequence then phase-shift keys a carrier with the phases 0 and π as shown in the table.

Table 1: Differential Encoding Example

	Reference Digit									
Message Sequence		1	0	0	1	1	1	0	0	0
Encoded Sequence	1	1	0	1	1	1	1	0	1	0
Transmitted Phase	0	0	π	0	0	0	0	Π	0	π

14. PROBABILITY OF BIT ERROR FOR BINARY DPSK (NON COHERENT PSK)

$$P_{e\ (bit-error)} = \frac{1}{2}\exp\left(\frac{-E_b}{N_0}\right)$$

Probability of bit error for binary DPSK is higher than binary BPSK.

P_e (DPSK) > P_e (BPSK)

15. BIT ERROR PROBABILITY FOR NONCOHERENT FREQUENCY SHIFT KEYING

The bit error probability for noncoherent frequency shift keying is defined

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right) = \frac{1}{2}\exp\left(-\frac{\gamma_b}{2}\right)$$

where E_b is the bit energy, N_a is the noise power density, and γ_b is the bit energy to noise density ratio.

16. CONTINUOUS PHASE FREQUENCY SHIFT KEYING (CPFSK)

$$s(t) = \sqrt{\frac{2\,E_b}{T_b}}\cos\omega_i t$$

$\omega_i t = \omega_c t + \theta(t)$

Here $\theta(t) = \theta(0) \pm \frac{n\pi ht}{T_b}$ where h = Deviation Ratio

$$\omega_1 t = \omega_c t + \theta(0) + \frac{n\pi ht}{T_b}$$

$$\omega_2 t = \omega_c t + \theta(0) - \frac{n\pi ht}{T_b}$$

Since θ(t) is changing with time, hence it is known as CPFSK.

$(\omega_1 - \omega_2)\ t = \frac{2n\pi ht}{T_b}$

$$\boxed{(f_1 - f_2) = \frac{nh}{T_b}}$$

$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_1 t) \rightarrow +1$

$s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_2 t) \rightarrow -1/0$

n → integer taken as '1' for smallest value.

17. MSK (MINIMUM SHIFT KEYING)

Here, $h = \frac{1}{2}$ (Deviation ratio)

Type of CPFSK in which $\left(h = \frac{1}{2}\right)$

In case of MSK, both S_1 and S_2 will be orthogonal to each other. This type of FSK is also known as fast FSK.

For MSK:

$(f_1 - f_2) = \frac{h}{T_b} = \frac{1}{2T_b}$ (for n =1) $= \frac{\text{Bit rate}}{2}$

18. RELATION BETWEEN BIT RATE AND SYMBOL RATE

Since, $k = \log_2 M$ bits per symbol are transmitted, so symbol rate for MPSK system can be defined in terms of bit rate R_b as

$$R_s = \frac{R_b}{k} = \frac{R_b}{\log_2 M}$$

19. RELATION BETWEEN BIT ENERGY AND SYMBOL ENERGY

For a multilevel Signaling scheme, assume that the signal energy per bit is E_b, and signal energy per symbol is E_s. We express the relationship between these two quantities as

$E_s = E_b(\log_2 M)$

20. RELATION BETWEEN PROBABILITY OF BIT ERROR AND PROBABILITY OF SYMBOL ERROR FOR ORTHOGONAL SIGNALS

Let P_E be the average probability of symbol error, and P_e be the average probability of bit error (bit error rate) for an M-ary orthogonal system (such as MFSK).

$$\frac{P_e}{P_E} = \frac{2^{k-1}}{2^k - 1} = \frac{M/2}{M-1} \text{......(v)}$$

In the limit as k increases, we get

$$\lim_{x\to\infty} \frac{P_e}{P_E} = \frac{1}{2}$$

21. RELATION BETWEEN PROBABILITY OF BIT ERROR AND PROBABILITY OF SYMBOL ERROR FOR MULTIPLE PHASE SIGNALS

For a multiple phase system (such as MPSK), the probability of bit error (P_e) can be expressed in terms of probability error (P_E) as

$$P_e = \frac{P_E}{\log_2 M}$$

22. M-ARY PHASE SHIFT KEYING (MPSK)

Transmission Bandwidth

For an M-ary PSK signal, we define the transmission bandwidth as

$B_T = 2R_s$

where R_s is the symbol rate. Substituting equation (iii) in above expression, We get transmission bandwidth of MPSK system as

$$B_T = \frac{2R_b}{\log_2 M} \text{..................(vii)}$$

Where R_b is the bit rate for the system. Also, we have overall absolute transmission bandwidth with raised cosine filtered pulses as

$$B_T = \frac{(1+\alpha)R_s}{\log_2 M}$$

were α is the roll off factor.

23. PROBABILITY OF SYMBOL ERROR

The probability of symbol error for MPSK system is defined as

$$P_E \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}} \sin\frac{\pi}{M}\right)$$

Where $M = 2^k$ is the size of the symbol set, and E_s is the energy per symbol. Since, the symbol energy E_s is given by

$E_s = E_b(\log_2 M) = kE_b$

Where $k = \log_2 M$ is the number of bits transmitted per symbol. So, we can express the probability of symbol error in terms of E_b/N_0 as

$$P_E = 2Q\left(\sqrt{\frac{2kE_b}{N_0}}\sin\frac{\pi}{M}\right)$$

24. PROBABILITY OF BIT ERROR

Using equation (vi), we express the bit error probability in terms of symbol error probability for an M-ary PSK system as

$$P_e = \frac{P_E}{\log_2 M} = \frac{P_E}{k}$$

Thus, by substituting equation (ix) in above expression we get the probability of bit error for M-ary PSK system as

$$P_e = \frac{2}{k}Q\left(\sqrt{\frac{2kE_b}{N_0}}\sin\frac{\pi}{M}\right)$$

$$= \frac{2}{k}Q\left(\sqrt{2k\gamma_b\sin^2\frac{\pi}{M}}\right)$$

25. QUADRATURE PHASE SHIFT KEYING (QPSK)

Transmission Bandwidth

Substituting M = 4 in equation (vii), we get the transmission bandwidth for QPSK system as

$$B_T = \frac{2R_b}{\log_2 4} = R_b$$

26. PROBABILITY OF SYMBOL ERROR

Substituting M = 4 in equation (viii), we get the probability of symbol error for QPSK system as

$$P_E \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{4}\right)$$

or $P_E = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right)$

Since, the symbol energy E_s is given by

$E_s = E_b(\log_2 M) = E_b(\log_2 4) = 2E_b$

So, we can express the probability of symbol error in terms of E_b/N_0 as

$$P_E = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

27. PROBABILITY OF SYMBOL ERROR

The probability of symbol error for an M-ary QAM system is given by

$$P_E \approx \left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3k}{M-1}\frac{E_b}{N_0}}\right)$$

$$P_E \approx 4\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3k}{M-1}\gamma_b}\right)$$

Where $k = \log_2 M$ is the number of bits transmitted per symbol, E_b is the bit energy, N_0 is the noise power density, and γ_b is the bit energy to noise density ratio.

28. Probability of Bit Error

Using equations (vi) and (xi), we obtain the bit error probability for an M-ary QAM system as

$$P_e = \frac{P_E}{\log_2 M} = \frac{P_E}{k}$$

$$= \frac{4}{k}\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3k}{M-1}\gamma_b}\right)$$

29. M-ARY FREQUENCY SHIFT KEYING (MFSK)

Transmission Bandwidth

The transmission bandwidth for an M-ary FSK system is defined as

$$B_T = \frac{R_b M}{2\log_2 M}$$

Where R_b is the bit rate, and $M = 2^k$ is the size of the symbol.

30. PROBABILITY OF SYMBOL ERROR

The probability of symbol error for an M-ry FSK system is given by

$$P_E \le (M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right) = (M-1)Q\left(\sqrt{\frac{E_b \log_2 M}{N_0}}\right)$$

or $$P_E \le (M-1)Q\left(\sqrt{\gamma_b \log_2 M}\right)$$

31. PROBABILITY OF BIT ERROR

Using equation (v) and (xii), we obtain the bit error probability for an M-ary FSK system as

$$P_e = \frac{\frac{M}{2}}{M-1}P_E$$

or $$P_e \le \frac{M}{2}Q\left(\sqrt{\gamma_b \log_2 M}\right)$$

33. OVERALL COMPARISON

S. No	Parameter	ASK	BPSK	QPSK	QAM	M-ary	BFSK	M-ary FSK	MSK
1.	Information is transmitted by change in	Amplitude	Phase	Phase	Amplitude and phase	Phase	Frequency	Frequency	Frequency
2.	Expression for transmitted signal	$A_c \cos\omega_c t$ for symbol 1 0 for symbol 0	$A_c b(t) \cos\omega_c t$ b(t) = 1 for 1 And b(t)=−1 for 0	$A_c \cos\left[\omega_c t + (2i-1)\frac{\pi}{4}\right]$ Where, m = 0, 1, 2, 2	$k_1\sqrt{0.2P_s}$ $\cos\omega_c t$ Where, m= 0, 1,2, 3				
3.	Number of bits per symbol	N = 1	N = 1	N =2	N	N	N = 1	N	N = 2
4.	Number of possible symbol $M = 2^N$	Two	Two	Four	$M = 2^N$	$M = 2^N$	Two	$M = 2^N$	Four
5.	Detection method	Coherent	Coherent	Coherent	Coherent	Coherent	Non- coherent	Non- coherent	coherent
6.	Minimum Euclidean distance	$\sqrt{E_b}$	$2\sqrt{E_b}$	$2\sqrt{E_b}$	$\sqrt{0.2E_s}$ for M = 16	$2\sqrt{E_s}$	$2\sqrt{E_b}$	$2\sqrt{2N E_b}$	$2\sqrt{E_b}$
7.	Minimum bandwidth	$2f_b$	$2f_b$	f_b	$2\frac{f_b}{N}$	$2\frac{f_b}{N}$	$4f_b$	$\frac{2^{N+1}f_b}{N}$	$1.5f_b$
8.	Symbol duration T_s	T_b	T_b	$2T_b$	NT_b	NT_b	T_b	NT_b	$2T_b$

33. OVERALL CONCLUSION OF FORMULAE

(i) Probability of error of ASK, FSK, PSK and QPSK using constellation diagram

$$P_e = Q\left(\frac{d}{2\sigma}\right) = Q\left(\frac{d}{\sqrt{2N_0}}\right) \qquad (d = d_{min})$$

For ASK : $d_{min} = \sqrt{E_b}$ $\quad \left(E_b = \text{Bit energy} = \frac{A_c^2 T_b}{2}\right)$

$$P_e = Q\left(\sqrt{\frac{E_b}{2N_0}}\right) = Q\left(\sqrt{\frac{A_c^2 T_b}{4N_0}}\right)$$

For PSK : $d_{min} = 2\sqrt{E_b}$

$$P_e = Q\left(2\sqrt{\frac{E_b}{2N_0}}\right) = Q\left(\sqrt{\frac{A_c^2 T_b}{N_0}}\right)$$

For FSK : $d_{min} = \sqrt{2E_b}$

$$P_e = Q\left(\sqrt{\frac{2E_b}{2N_0}}\right) = Q\left(\sqrt{\frac{A_c^2 T_b}{2N_0}}\right)$$

For QPSK : $d_{min} = \sqrt{2E_s} = \sqrt{4E_b}$ $\quad (E_s = 2E_b)$

$$P_e = Q\left(\sqrt{\frac{4E_b}{2N_0}}\right) = Q\left(\sqrt{\frac{A_c^2 T_b}{N_0}}\right)$$ (P_e= Bitt error probability)

(ii) Probability of error for various signalling scheme:

QPSK : $P_{e(symbol)} \simeq 2Q\left(\sqrt{\frac{E}{N_0}}\right)$; $P_{e(bit)} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left(E_b = \frac{A_c^2 T_b}{2}\right)$

DPSK : $P_e = \frac{1}{2} e^{-Es/N_0}$ $\left(E_s = \frac{A_c^2 T}{2}\right)$

16-QAM : $P_e = 3Q\left(\sqrt{\frac{E_s}{N_0}}\right) - 2.25\left[Q\left(\sqrt{\frac{E_s}{5N_0}}\right)\right]^2$

MSK : $P_e = Q\left[\sqrt{\frac{d^2}{2\eta}}\right] = Q\left[\sqrt{\frac{2E_b}{\eta}}\right]$

34. MATCHED FILTER

Impulse response of matched filter, h(t) is

- $h(t) = s_i(T - t)$ $s_i(t) \to$ Real
- $h(t) = s_i(T - t)$ $s_i(t) \to$ Imaginary
- $h(t) = s_i^*(T - t)$ $s_i(t) \to$ Complex

Chapter-8
Information Theory

1. INTRODUCTION TO INFORMATION THEORY

The amount of the information associated with x_i is defined as

$$I(x_i) = \log_a \frac{1}{P(x_i)}$$

or $$I_i = \log_a \frac{1}{p_i}$$

2. PROPERTIES OF INFORMATION

a) If we are absolutely certain of the outcome of an event, even before it occurs, there is no information gained, i.e.

$I_i = 0$ for $p_i = 1$

b) The occurrence of an event either provides some or no information, but never brings about a loss of information, i.e.

$I_i \geq 0$ for $0 \leq p_i \leq 1$

c) The less probable an event is, the more information we gain when it occurs.

$I_j > I_i$ for $p_j < p_i$

d) If two events x_i and x_j are statistically independent, then

$I(x_i x_j) = I(x_i) + I(x_j)$

3. ENTROPY

If each symbol x_i occurs with probability p_i and conveys the information I_i, then the average information per symbol is obtained as

$$H(X) = E\left[I(x_i)\right] = \sum_{i=1}^{n} p_i I_i$$

$$H(X) = \sum_{i=1}^{n} p_i \log_2 \frac{1}{p_i}$$

4. PROPERTIES OF ENTROPY

Following are some important properties of source entropy.

a) In a set of symbol X, if the probability p_i = 1 for some i, and the remaining probabilities in the set are all zero; then the entropy of the source is zero, i.e

H(X) = 0

b) If all the n symbols emitted from a source are equiprobable, then the entropy of the source is

H(X) = $\log_2 n$

c) From above two results, we can easily conclude that the source entropy is bounded as

$$0 \le H(X) < \log_2 n$$

5. INFORMATION RATE

The information rate for a source having entropy H is given by

$$R = \frac{H}{T} \text{bits / sec}$$

where T is the time required to send a message.

If the message source generates messages at the rate of r messages per second, then we have

$$T = \frac{1}{r}$$

The information rate of the source as

R = rH bits/sec

6. METHODOLOGY TO EVALUATE SOURCE INFORMATION RATE

For a given set of source symbol, we evaluate the information rate in the following steps:

Step 1: Obtain the probability p_i of each symbol emitted by source.

Step 2: Deduce the amount of information conveyed in each symbol using expression,

$$I_i = \log_2 \frac{1}{p_i} \text{bits}$$

Step 3: Obtain the source entropy by substituting the above results in the expression

$$H = \sum_{i=1}^{n} p_i I_i = \sum_{i=1}^{n} p_i \log_2\left(\frac{1}{p_i}\right)$$

Step 4: Obtain the average message transmission rate using the expression

$$r = \frac{1}{T}$$

where T is the time required to send a message

Step 5: Evaluate information rate of the source by substituting the above results in the expression

R = rH bits / sec

7. AVERAGE CODE – WORD LENGTH

Let the binary code – word assigned to symbol x_i by the encoder have length I_i measured in bits. Then, the average code – word length is defined as

$$\bar{L} = \sum_{i=1}^{n} l_i p_i$$

8. SOURCE CODING THEOREM

According to source encoding theorem, the minimum average code – word length for any distortion less source encoding scheme is defined as

$$\bar{L}_{min} = \frac{H(X)}{\log_2 k}$$

Where H(X) is the entropy of the source, and k is the number of symbols in encoding alphabet.

Thus, for the binary alphabet (k = 2), we get the minimum average code – word length as

$$\bar{L}_{min} = H(X)$$

9. CODING EFFICIENCY

The coding efficiency of a source encoder is defined as

$$\eta = \frac{\bar{L}_{min}}{\bar{L}}$$

$$\eta = \frac{H(X)}{\bar{L}\log_2 k}$$

10. Shannon-Fano Coding:

Methodology: Shannon – Fano encoding algorithm:

Step 1: The source symbols are first ranked in order of decreasing probability.

Step 2: The set is then partitioned into two sets that are as close to equiprobable as possible

Step 3: 0's are assigned to the upper set and 1's to the lower set.

Step 4: The above process is continued, each time partitioning the sets with as nearly equal probabilities as possible, until further partitioning is not possible.

Or we can represent this coding as:

1. List the source symbols in order of decreasing probability.
2. Partition the set into two sets that are as close to equiprobable as possible, and assign 0 to the upper set and 1 to the lower set.
3. Continue this process, each time partitioning the sets with as nearly equal probabilities as possible until further partitioning is not possible.

11. HUFFMAN CODING

Methodology: Huffman encoding algorithm:

Following are the steps involved in Huffman encoding coding of a source symbol:

Step 1: The source symbols are listed in order of decreasing probability.

Step 2: The two source symbols of lowest probability are assigned a 0 an a 1.

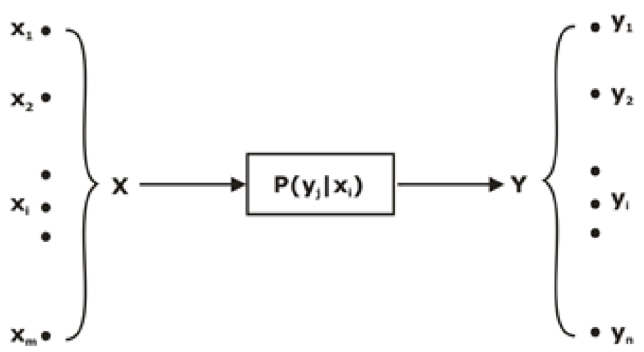
Step 3: These two source symbols are regarded as being combined into a new source symbol with probability equal to the sum of the two original probabilities. (the list source symbols, and therefore source statistics, is thereby reduced in size by one.)

Step 4: The probability of the new symbol is placed in the list in accordance with its value.

Step 5: The above procedure is repeated until we are left with a final list of source statistics (symbols) of only two for which a 0 and a 1 are assigned.

Step 6: The code for each (original) source symbol is found by working backward and tracing the sequence of 0s and 1s assigned to that symbol as well as its successors.

12. DISCRETE MEMORYLESS CHANNELS



Discrete memoryless channel

The matrix of transition probabilities [P(Y|X)], given by:

$$[P(Y|X)] = \begin{bmatrix} P(y_1|x_1) & P(y_2|x_1) & \cdots & P(y_n|x_1) \\ P(y_1|x_2) & P(y_2|x_2) & \cdots & P(y_n|x_2) \\ \cdots & \cdots & \cdots & \cdots \\ P(y_1|x_m) & P(y_2|x_m) & \cdots & P(y_n|x_m) \end{bmatrix}$$

if the input probabilities P(X) are represented by the row matrix

$[P(X)] = [P(x_1) \quad P(x_2) \quad \ldots \quad P(X_m)]$

and the output probabilities P(Y) arc represented by the row matrix

$[P(X)] = [P(y_1) \quad P(y_2) \quad \ldots \quad P(y_m)]$

then $[P(Y)] = [P(X)][P(Y|X)]$

If **P(X) is represented as a diagonal matrix**

$$[P(X)]_d = \begin{bmatrix} P(x_1) & 0 & \ldots & 0 \\ 0 & P(x_2) & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & P(x_m) \end{bmatrix}$$

then $[P(X, Y)] = [P(X)_d[P(Y|X)]$

13. ENTROPY FUNCTIONS FOR DISCRETE MEMORYLESS CHANNEL

If the channel has n inputs and m outputs, then we can define several entropy functions for input and output as

$$H(X) = -\sum_{i=1}^{n} P(x_1)\log_2 P(x_i)$$

$$H(Y) = -\sum_{j=1}^{m} P(y_j)\log_2 P(y_j)$$

a) Joint Entropy

The joint entropy of the system is obtained as

$$H(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2 P(x_i, x_j)$$

b) Conditional Entropy

The several conditional entropy functions for the discrete memoryless channel is defined as

$$H(Y|x_i) = -\sum_{j=1}^{m} p(y_j | x_i)\log_2 P(y_j | x_i)$$

$$H(X|y_j) = -\sum_{i=1}^{n} P(x_i | y_j)\log_2 (x_i | y_j)$$

$$H(Y|X) = -\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2 P(y_j | x_i)$$

$$H(X|Y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2 P(x_i | y_j)$$

14. MUTUAL INFORMATION

The mutual information I(X; Y) of a channel is defined by

$I(X; Y) = H(X) - H(X|Y)$ b/symbol

Also, we can define the mutual information as

$I(X; Y) = H(Y) - H(Y \mid X)$

15. Channel Capacity

The channel capacity is defined as the maximum of mutual information, i.e.

$C = \max \{I(X;Y)\}$

Substituting above equation, we get the channel capacity as

$C = \max \{H(X) - H(X|Y)\}$

This result can be more generalized for the Gaussian channel. The information capacity of a continuous channel of bandwidth B hertz is defined as

$C = B\log_2 (1 + S/N)$

where S/N is the signal to noise ratio. This relationship is known as the **Hartley – Shannon law** that sets an upper limit on the performance of a communication system.

16. CHANNEL EFFICIENCY

The channel efficiency is defined as the ratio of actual transformation to the maximum transformation, i.e.

$$\eta = \frac{I(X;Y)}{\max\{I(X;Y)\}}$$

or $$\eta = \frac{I(X;Y)}{C}$$

17. CAPACITIES OF SPECIAL CHANNELS

Lossless Channel:

For a lossless channel, $H(X|Y) = 0$ and

$I(X; Y)= H(X)$

Thus, the mutual information (information transfer) is equal to the input (source) entropy, and no source information is lost in transmission. Consequently, the channel capacity per symbol is

$$C_s = \max_{\{P(x_i)\}} H(X) = \log_2 m$$

where m is the number of symbols in X.

Deterministic Channel:

For a deterministic channel, $H(Y|X) = 0$ for all input distributions $P(x_i)$, and

$I(X; Y) = H(Y)$

Thus, the information transfer is equal to the output entropy. The channel capacity per symbol is

$$C_s = \max_{\{P(x_i)\}} H(Y) = \log_2 n$$

where n is the number of symbols in Y.

Noiseless Channel:

Since a noiseless channel is both lossless and deterministic, we have

$I(X; Y) = H(X) = H(Y)$

and the channel capacity per symbol is

$C_s = \log_2 m = \log_2 n$

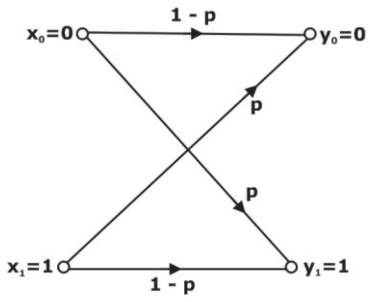
18. Binary Symmetric Channel:

For the BSC of Figure below, the mutual information is:

$I(X; Y) = H(Y) + p\log_2 p + (1 - p)\log_2(1 - p)$

and the channel capacity per symbol is

$C_s = 1 + p\log_2 p + (1 - p)\log_2(1 - p)$



Binary Symmetric Channel

19. DIFFERENTIAL ENTROPY

The average amount of information per sample value of x(t) is measured by

$$H(X) = -\int_{-\infty}^{\infty} f_X(x)\log_2 f_X(x)\,dx \ \text{ b/sample}$$

The entropy H(X) defined by above is known as the differential entropy of X.

$$H(Y) = -\int_{-\infty}^{\infty} f_Y(y)\log_2 f_Y(y)\,dy$$

$$H(X|Y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\log_2 f_X(x \mid y)\,dxdy$$

$$H(X|Y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\log_2 f_Y(y \mid x)\,dxdy$$

20. ERROR DETECTION AND CORRECTION CAPABILITIES

The minimum distance d_{min} of a linear code C is an important parameter of C. It determines the error detection and correction capabilities of C. This is stated in the following theorems.

Theorem 1:

"A linear code C of minimum distance d_{min} can detect up to r errors if and only if

$d_{min} \geq t + 1$"

Theorem 2:

"A linear code C of minimum distance d_{min} can correct up to t errors if and only if

d_{min} ≥ 2t + 1 ", there exists a received word r such that $d(c_i, r) \le t$, and yet r is as close to c_j as it is to c_i. Thus, the decoder may choose c_j, which is incorrect.

21. PARITY-CHECK MATRIX

Let H denote an m X n matrix defined by

$H = [P \quad I_m]$

where m = n – k and I_m is the mth-order identity matrix. Then

$$H^T = \begin{bmatrix} P^T \\ I_m \end{bmatrix}$$

Using above equations, we have

$$GH^T = \begin{bmatrix} I_k & P^T \end{bmatrix} \begin{bmatrix} P^T \\ I_m \end{bmatrix} = P^T \oplus P^T = 0$$

where 0 denotes the k × m zero matrix. Now we have,

$cH^T = dGH^T = 0(11.13)$

where 0 denotes the 1 × m zero vector.

The matrix **H is called the parity-check matrix** of C. Note that the rank of H is m = n – k and the rows of H are linearly independent. The minimum distance d_{min} of a linear block code C is closely related to the structure of the parity-check matrix H of C.

22. SYNDROME DECODING

With syndrome decoding, an (n, k) linear block code can correct up to t errors per codeword if n and k satisfy the following Hamming bound.

$$2^{n-k} \ge \sum_{i=0}^{t} \binom{n}{i}$$

Where $\binom{n}{i} = \frac{n!}{(n-1)!i!}$

A block code for which the equality holds is known as the perfect code. Single error-correcting perfect codes are called Hamming codes.

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