BYJU'S EXAM PREP

AE/JE Foundation

Electrical Engineering

Electromagnetic Fields

Formula Notes

IMPORTANT FORMULAS TO REMEMBER

CHAPTER 1: VECTOR CALCULUS

1. Vector Quantity

A physical quantity which has both magnitude and definite direction is called a vector quantity. The various examples of vector quantity are force, velocity, displacement, electric field intensity, magnetic field intensity, acceleration etc.

1.1. Representation of a Vector

To distinguish between a scalar and a vector it is customary to represent a vector by a letter with an arrow on top of it, such as $\bar{a}$ and $\bar{b}$, or by a letter in boldface type such as $\bar{\mathrm{A}}$ and $\bar{\mathrm{B}}$.

1.2. Unit Vector

A unit vector consists both magnitude and direction. Its direction is same as that of the main vector however, its magnitude is unity. It can be written in various as I_A, i_A, α_A or u_A. A unit vector is defined as the ratio of the main vector itself to its magnitude. For example, the unit vector of A is given as $\alpha_A = \frac{\bar{A}}{|A|}$

Where |A| is the magnitude of the vector and α_A is the unit vector of A.

2. Basic Vector Operations

2.1. Scaling of a Vector

When a vector is multiplied by a scalar it results in a vector quantity.

Consider a vector $\vec{\mathrm{A}}$ and a scalar k. The product $\vec{\mathrm{R}}$ of the two quantities is given as

$\vec{\mathrm{R}} = k\vec{\mathrm{A}}$

Following are some important properties of scaling operation:

Properties of scaling operation

1. Consider the scaling operation $\vec{\mathrm{R}} = k\vec{\mathrm{A}}$. The direction of $\vec{\mathrm{R}}$ is same as that of $\vec{\mathrm{A}}$ if k is positive, and opposite to that of A if k is negative.

2. In rectangular coordinates, assume that the scaling operation is given by

$R_x\bar{a}_x + R_y\bar{a}_y + R_z\bar{a}_Z = k\,(A_x\bar{a}_x + A_y\bar{a}_y + A_z\bar{a}_Z)$

The above equality is satisfied if each component of the LHS is equal to the corresponding component of RHS, i.e.

$R_x = kA_x$, $R_y = kA_y$, $R_z = kA_z$

The magnitude of R is

$|R| = k\sqrt{A_x^2 + A_y^2 + A_z^2} = kA$

3. Let k_1, k_2 be the scalars, and A, B be the vectors then,

$(k_1+k_2)\vec{A} = k_1\vec{A} + k_2\vec{A}$

2.2. Addition of Vectors

Consider the two vectors,

$\vec{A} = A_x\bar{a}_x + A_y\bar{a}_y + A_z\bar{a}_Z$ and

$\vec{B} = B_x\bar{a}_x + B_y\bar{a}_y + B_z\bar{a}_Z$

The addition of these two vectors is given by

$\vec{A} + \vec{B} = (A_x+B_x)\,\bar{a}_x + (A_y+B_y)\,\bar{a}_y + (A_Z+B_Z)\,\bar{a}_Z$

Properties of Vector Addition

1. Vector addition follows the commutative law, i.e.

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$

2. Vector addition follows the associative law, i.e.

$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

3. Similar to the vector addition, the subtraction of the vectors is defined as

$\vec{A} - \vec{B} = (A_x - B_x)\,\bar{a}_x + (A_y - By)\,\bar{a}_y + (A_z-B_Z)\,\bar{a}_Z$

$k_1(\vec{A} + \vec{B}) = k_1\vec{A} + k_1\vec{B}$

2.3. Multiplication of vectors

When two vectors A and B are multiplied, the result is either a scalar or a vector depending on how they are multiplied. There are two types of vector multiplication:

1. Scalar (or dot) product: $\vec{A} \bullet \vec{B}$

2. Vector (or cross) product: $\vec{A} \times \vec{B}$

2.3.1 Scalar Product

The dot product of the vectors A and B is defined as

$\vec{A} \bullet \vec{B} = |A||B| \cos\theta$

Following are some important properties of dot product of two vectors.

Properties of Dot product

1. The dot product of two orthogonal vectors is always zero, i.e.

$\vec{A} \bullet \vec{B} = 0$, if $\theta = 90^0$

2. The dot product of two parallel vectors in equal to the product of their magnitudes, i.e.

$\vec{A} \bullet \vec{B} = AB$, if $\theta = 0^0$

3. In rectangular coordinate systems, the dot products of the unit vectors are given as

$a_x \bullet a_y = a_y \bullet a_z = a_z \bullet a_x = 0$

$a_x \bullet a_x = a_y \bullet a_y = a_z \bullet a_z = 1$

4. if the two vectors are defined in rectangular coordinates as

$\vec{A} = A_xa_x + A_ya_y + A_za_z$

$\vec{B} = B_x a_x + B_y a_y + B_z a_z$

Then, their dot product is evaluated as

$\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z$

5. The dot product follows the commutative law, i.e.

$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$

6. The dot product also follows the distributive law, i.e.

$\vec{A} \bullet (\vec{B} + \vec{C}) = \vec{A} \bullet \vec{B} + \vec{A} \bullet \vec{C}$

2.3.2 Vector of Cross Product

The cross product of two vectors A and B is Defined as

$\vec{A} \times \vec{B} = (AB \sin \theta)\, a_n$

Where a_n is the unit vector normal to the plane containing A and B, θ is the angle between the vector A and B as shown in Figure 1.4 As there are the normal unit vector a_n we use the right- hand rule.

Right hand rule

Let your fingers point in the direction of the first vector and curl around (via the smaller angle) toward second; then your thumb indicates the direction of a_n. the cross-product A× B points upward.

Properties of cross product

1. The cross product of two orthogonal vectors is equal to the product of their magnitudes with the direction perpendicular to the plane, i.e.

$\vec{A} \times \vec{B} = AB a_n$, if, $\theta = 90^0$

2. The cross product of two parallel vectors is always zero, i.e.

$\vec{A} \times \vec{B} = 0$, if $\theta = 0^0$

3. In rectangular coordinate system, the cross product of the unit vectors are given as

$a_x \times a_x = a_y \times a_y = a_z \times a_z = 0$

$a_x \times a_y = a_z$

$a_y \times a_z = a_x$

$a_z \times a_x = a_y$

4. If the two vectors are defined in rectangular coordinates as

$\vec{A} = A_x a_x + A_y a_y + A_z a_z$

$\vec{B} = B_x a_x + B_y a_y + B_z a_z$

Then, their cross product is evaluated as

$$\vec{A} \times \vec{B} = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

5. The cross product is anti-commutative, i.e.

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

6. The cross product follows the distributive law, i.e.

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

3. COORDINATE SYSTEMS

The following three most useful coordinate systems:

1. Cartesian or rectangular coordinates,
2. Circular or cylindrical coordinates, and
3. Spherical coordinates.

3.1. Rectangular or Cartesian Coordinate System

The three coordinate axes are designated as x, y and z which are mutually perpendicular to each other. The variables x, y and z can have any values in the range

$-\infty < x < \infty$, $-\infty, < y < \infty$, $-\infty < z < \infty$

Vector Representation in Rectangular Coordinate System

A vector A in rectangular coordinate system is represented as

$A = A_x a_x + A_y a_y + A_z a_z$

Where a_x, a_y, a_z are the unit vectors along the x, y and z directions.

The magnitude of A is given by

$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

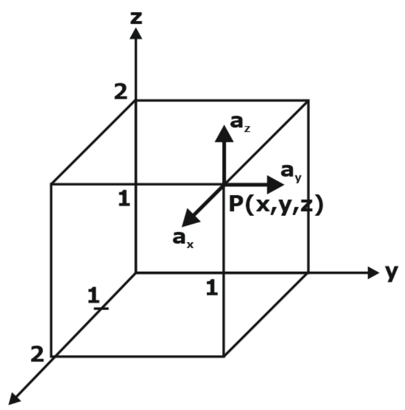


Figure: Representation of cartesian coordinates

3.2. Cylindrical Coordinates

The cylindrical coordinate system is very convenient whenever we are dealing with problems having cylindrical symmetry.

A Point P in cylindrical coordinates is represented as (ρ, ϕ, z) and is as shown in figure below.

The ranges of the variables are:

$0 \le \rho \le \infty$

$0 \le \phi \le 2\pi$

$-\infty \le z \le +\infty$

A vector $\vec{A}$ in cylindrical coordinates can be written as

$\left(A_\rho, A_\phi, A_z\right)$ or $A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$

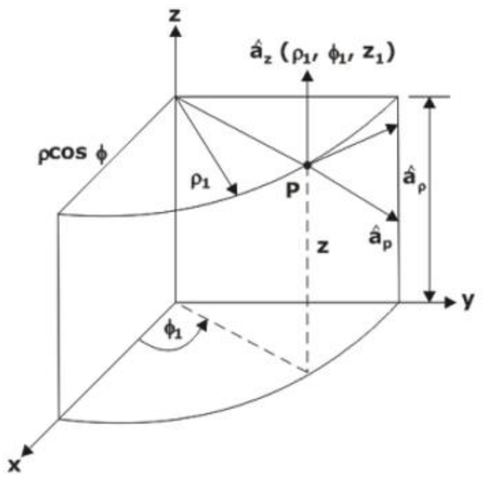


Figure: Representation of cylindrical coordinates

Notice that the unit vectors $\hat{a}_\rho, \hat{a}_\phi$ and $\hat{a}_z$ are mutually perpendicular because our coordinates system is orthogonal.

$\hat{a}_\rho \times \hat{a}_\phi = \hat{a}_\phi \times \hat{a}_z = \hat{a}_z \times \hat{a}_\rho = 0$

$\hat{a}_\rho \times \hat{a}_\rho = \hat{a}_\phi \times \hat{a}_\phi = \hat{a}_z \times \hat{a}_z = 1$

$\hat{a}_\rho \times \hat{a}_\phi = \hat{a}_z$

$\hat{a}_\phi \times \hat{a}_z = \hat{a}_\rho$

$\hat{a}_z \times \hat{a}_\rho = \hat{a}_\phi$

Conversion of cartesian coordinate to cylindrical coordinate and vice-versa

Point transformation,

$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}\frac{y}{x}$$

$$z = z$$

or

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

The relationship between $\hat{a}_x, \hat{a}_y, \hat{a}_z$ and $\hat{a}_\rho, \hat{a}_\phi, \hat{a}_z$ are vector transformation,

$$\hat{a}_x = \cos\phi\,\hat{a}_\rho - \sin\phi\,\hat{a}_\phi$$

$$\hat{a}_y = \sin\phi\,\hat{a}_\rho + \cos\phi\,\hat{a}_\phi$$

$$\hat{a}_z = \hat{a}_z$$

or $\hat{a}_\rho = \cos\phi\hat{a}_x + \sin\phi\hat{a}_y$

$$\hat{a}_\phi = -\sin\phi\hat{a}_x + \cos\phi\hat{a}_y$$

$$\hat{a}_z = \hat{a}_z$$

Finally, the relationship between (A_x, A_y, A_z) and $\left(A_\rho, A_\phi, A_z\right)$ are

$$\begin{vmatrix} A_\rho \\ A_\phi \\ A_z \end{vmatrix} = \begin{vmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix}$$

$$\begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix} = \begin{vmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} A_\rho \\ A_\phi \\ A_z \end{vmatrix}$$

3.3. Spherical Coordinates

A point P can be represented as ($\left(r, \theta, \phi\right)$) and s illustrated in figure below. From figure, we notice that r is defined as the distance from the origin to point P or the radius of a sphere centered at the origin and passing through P; θ (called the colatitudes) is the angle between the z-axis and the position vector of P; and ϕ is measured from the x-axis (the same azimuthal angle in cylindrical coordinates). According to these definitions, the ranges of the variables are

$$0 \le r \le \infty$$

$$0 \le \theta \le \pi$$

$$0 \le \phi \le 2\pi$$

Note: the unit vectors $\hat{a}_r, \hat{a}_\theta$ and $\hat{a}_\phi$ are mutually perpendicular because our coordinate system is orthogonal.

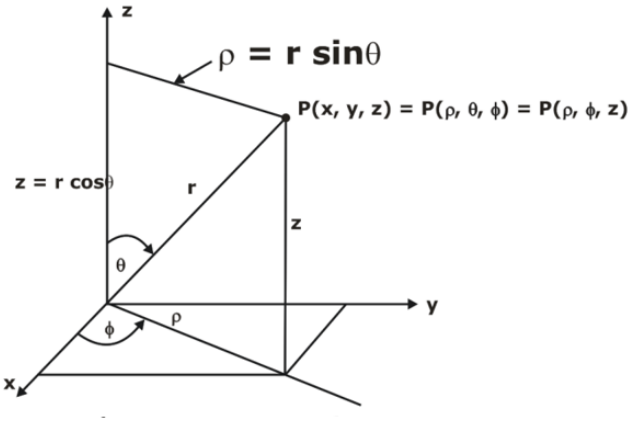
$$\hat{a}_r \times \hat{a}_\theta = \hat{a}_\theta \times \hat{a}_\phi = \hat{a}_\phi \times \hat{a}_r = 0$$

$$\hat{a}_r \times \hat{a}_r = \hat{a}_\theta \times \hat{a}_\theta = \hat{a}_\phi \times \hat{a}_\phi = 1$$

$$\hat{a}_r \times \hat{a}_\theta = \hat{a}_\phi$$

$$\hat{a}\theta \times \hat{a}_\phi = \hat{a}_r$$

$$\hat{a}_\phi \times \hat{a}_r = \hat{a}_\theta$$



Conversion of cartesian coordinate to spherical coordinate and vice-versa

Point transformation,

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2 + z^2}}{z}$$

$$\phi = \tan^{-1} \frac{y}{x}$$

Or

$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta$$

The relationship between $\hat{a}_x, \hat{a}_y \hat{a}_z$ and $\hat{a}_r, \hat{a}_\theta, \hat{a}_\phi$ are

$$\hat{a}_x = \sin\theta \cos\phi \; \hat{a}_r + \cos\theta \cos\phi \; \hat{a}_\theta - \sin\phi \hat{a}_\phi$$

$$\hat{a}_y = \sin\theta \sin\phi \; \hat{a}_r + \cos\theta \sin\phi \; \hat{a}_\theta + \cos\phi \hat{a}_\phi$$

$$\hat{a}_z = \cos\phi \hat{a}_r - \sin\phi \hat{a}_\phi$$

$$\hat{a}_r = \sin\theta \cos\phi \hat{a}_x + \sin\theta \sin\phi \hat{a}_y + \cos\phi \hat{a}_z$$

$$\hat{a}_\theta = \cos\theta \cos\phi \hat{a}_x + \cos\theta \sin\phi \hat{a}_y - \sin\phi \hat{a}_z$$

$$\hat{a}_\phi = -\sin\phi \hat{a}_x + \cos\phi \hat{a}_y$$

Finally, the relationship between (A_x, A_y, A_z) and $\left(A_r, A_\theta, A_\phi\right)$ are

Vector transformation,

$$\begin{vmatrix} A_r \\ A_\theta \\ A\phi \end{vmatrix} = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{vmatrix} \begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix}$$

$$\begin{vmatrix} A_x \\ A_y \\ Az \end{vmatrix} = \begin{vmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\cos\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\phi & -\sin\phi & 0 \end{vmatrix} \begin{vmatrix} A_r \\ A_\theta \\ A_\phi \end{vmatrix}$$

4. DIFFERENTIAL ELEMENTS IN COORDINATE SYSTEMS

4.1. Differential Elements in Rectangular Coordinate System

The differential elements in rectangular coordinate system are defined as follows:

1. Differential length in rectangular coordinate system:

$dL = dx\alpha_x + dy\alpha_y + dz\alpha_z$

2. Differential area in rectangular coordinate system:

$dS = dydz\alpha_x = dxdz\alpha_y = dxdy\alpha_z$

3. Differential volume in rectangular coordinate system:

$dV = dxdydz$

4.2. Differential Elements in Cylindrical Coordinate System

The differential elements in cylindrical coordinate system are defined as follows:

1. Differential length in cylindrical coordinate system:

$dL = d\rho\alpha_\rho + \rho d\varphi\alpha_\varphi + dz\alpha_z$

2. Differential area in cylindrical coordinate system:

$dS = \rho d\varphi dz\alpha_\rho = d\rho dz\alpha_\varphi = \rho d\varphi d\rho\alpha_z$

3. Differential volume in cylindrical coordinate system:

$dV = \rho d\rho d\varphi dz$

4.3. Differential Elements in spherical Coordinate System

The differential elements in spherical coordinate system are defined as follows:

1. Differential length in spherical coordinate system

$dL = dr\alpha_r + rd\theta\alpha_\theta + r\sin\theta d\varphi\alpha_\varphi$

2. Differential area in spherical coordinate system

$dS = r^2\sin\theta d\theta d\varphi\alpha_r = r\sin\theta drd\varphi\alpha_\theta = rdrd\varphi\alpha_\varphi$

3. Differential volume in spherical coordinate system

$dV = r^2\sin\theta drd\theta d\varphi$

5. DIFFERENTIAL CALCULUS

The Del operator (∇), in the different coordinate system, is defined as

$$\nabla = \frac{\partial}{\partial x}\alpha_x + \frac{\partial}{\partial y}\alpha_y + \frac{\partial}{\partial z}\alpha_z \qquad \text{(Rectangular coordinates)}$$

$$\nabla = \frac{\partial}{\partial \rho}\alpha_\rho + \frac{1\partial}{\rho\partial\phi}\alpha_\phi + \frac{\partial}{\partial_z}\alpha_z \qquad \text{(Cylindrical Coordinates)}$$

$$\nabla = \frac{\partial}{\partial r}\alpha_r + \frac{1}{r}\frac{\partial}{\partial\theta}\alpha_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\alpha_\phi \qquad \text{(Spherical coordinates)}$$

6. Gradient of a Scalar

The gradient (or grad) is defined by the operation of the Del operator on a scalar field. For a scalar Field V, we define the gradient in the different coordinates as

$$\nabla V = \frac{\partial V}{\partial x}\alpha_x + \frac{\partial V}{\partial y}\alpha_y + \frac{\partial V}{\partial z}\alpha_z \qquad \text{(Rectangular coordinates)}$$

$$\nabla V = \frac{\partial V}{\partial \rho}\alpha_\rho + \frac{1\partial V}{\rho\partial\phi}\alpha_\phi + \frac{\partial V}{\partial z}\alpha_z \qquad \text{(Cylindrical coordinates)}$$

$$\nabla V = \frac{\partial V}{\partial r}\alpha_r + \frac{1\partial V}{r\partial\theta}\alpha_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\alpha_\phi \qquad \text{(Spherical coordinates)}$$

7. Divergence of a Vector

Divergence of a vector function is a scalar and defined as the net outward flux per unit volume over the elementary closed surface. For a vector function A, we define the divergence in the different coordinates as

$$\nabla\bullet A = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) \qquad \text{(Rectangular)}$$

$$\nabla\bullet A = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z} \qquad \text{(Cylindrical)}$$

$$\nabla\bullet A = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi} \qquad \text{(Spherical)}$$

8. Curl of a Vector

The curl of a vector plays a very important role in electromagnetic theory.

We define the curl of vector A in different coordinate systems as

$$\nabla\times A = \begin{bmatrix} a_x & a_y & a_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix} \qquad \text{(Rectangular coordinates)}$$

$$\nabla \times A = \left(\frac{1}{\rho}\right)\begin{bmatrix} a_x & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A\phi & A_z \end{bmatrix} \qquad \text{(Cylindrical coordinates)}$$

$$\nabla \times A = \left(\frac{1}{r^2 \sin\theta}\right)\begin{vmatrix} a_\rho & r a_\theta & r\sin\theta a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix} \qquad \textbf{(}\text{Spherical coordinates)}$$

9. Laplacian Operator

The Laplacian Operator is the square of the Del operator and written as (∇^2). It can operate both on scalar as well as vector field. The Laplacian of a scalar field is a scalar field whereas the Laplacian of a vector is a vector field.

9.1. Laplacian of a Scalar

The Laplacian of a scalar field V in different coordinate systems is defined as

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \qquad \text{(Rectangular coordinates)}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \qquad \text{(Cylindrical coordinates)}$$

$$\nabla^2 . V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} \qquad \text{(Spherical coordinates)}$$

9.2. Laplacian of a Vector

The Laplacian of a vector is defined as the gradient of divergence of the vector minus the curl of the curl of vector, i.e.

$\nabla^2 A = \nabla (\nabla . A) - \nabla \times \nabla \times A$

10. DIVERGENCE THEOREM

According to divergence theorem, the surface integral of a vector field over a closed surface is equal to the volume integral of the divergence of the vector field over the volume. Mathematically, the divergence theorem is written as

$$\oint_s A \bullet dS = \int_v (\nabla \bullet A) dV$$

Where A is a vector field and V is the volume bounded by the closed surface S.

11. STOKE'S THEOREM

According to Stoke's theorem, the line integral of a vector field around a closed path in equal to the surface integral of the curl of vector field over the open surface bounded by the closed path. Mathematically, the Stoke's theorem is written as

$$\oint_L A \bullet dL = \int_S (\nabla \times A) \bullet dS$$

Where A is a vector field and S is the open surface bounded by the closed path L.

CHAPTER 2: ELECTROSTATICS

1. Electric charge

Electric Charge is a fundamental conserved property of some subatomic particles, which determines their electromagnetic interaction.

1.1 Point Charge

Point charges are very small charges assumed to be of infinitesimally small volume, although they have finite volume considered as a single charge.

1.2. Line Charge

The charge per unit length along the line charge is called line charge density. It is denoted by ρ_L and defined as

$$\rho_L = \lim_{\Delta L \to 0} \frac{\Delta Q}{\Delta L} = \frac{dQ}{dL}$$

where ΔQ is small charge, and ΔL is small length.

1.3. Surface Charge

The charge per unit area over the surface is called the surface charge density. It is denoted by ρ_S and defined as

$$\rho_S = \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta S} = \frac{dQ}{dS}$$

where ΔQ is small charge, and ΔS is small area.

1.4. Volume Charge

The charge per unit volume in the region is called volume charge density. It is denoted by ρ_v and defined as

$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v} = \frac{dQ}{dv}$$

where ΔQ is small charge, and Δv is small volume.

2. Electric flux Density

The electric flux density vector $\vec{D}$ in a medium is defined as the product of the permittivity and the electric field vector

$$\vec{D} = \in E$$

The permittivity of the medium is defined in terms of the free space permittivity and the relative permittivity ($\in'$)as

$$\in = \in' \in_0$$

Electric flux density is independent of the medium properties

For point charge $\vec{E} = \dfrac{Q}{4\pi \in R^2} \hat{a}_R, \vec{D} = \dfrac{Q}{4\pi R^2} \hat{a}_R$

For line charge $\vec{E} = \dfrac{P_L}{2\pi \in_P} \hat{a}_P, \vec{D} = \dfrac{P_L}{2\pi_P} \hat{a}_P$

The unit of electric flux density are

$$\frac{F}{m} \times \frac{F}{m} = \frac{C}{m^2}$$

NOTE: The units of $\vec{D}$ are equivalent to surface density i.e. C/m^2

3. Gauss's Law – Maxwell Equations

The total outward electric flux Ψ through any closed surface is equal to the total charge enclosed by the surface.

In equation form, gauss's law is written as

$$\psi = \oint_s \vec{D}.\, d\vec{S} = Q_{enclosed}$$

Where $= d\vec{S} = \hat{a}_n dS$ and $\hat{a}_n$ is the outward pointing unit normal to closed surface S.

$$\psi = \oint_s \vec{D}.\, d\vec{S} = \text{ total charge enclosed}$$

$$Q = \oint_v P_v dv$$

or $Q = \oint_S \vec{D}.d\vec{S} = \int_V \rho_v dv$

By applying divergence theorem to the middle term, we have

$$\oint_S \vec{D}.d\vec{S} = \int_V \nabla.\vec{D}dv$$

Comparing the two volume integrals

$$\nabla.\vec{D} = \rho_v$$

It states that the volume charge density is the same as the divergence of the electric flux density.

4. Electric field due to a point charge

$$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_\tau$$

$$\vec{E} = \frac{Q}{4\pi \in_0 r^2}\hat{a}_\tau$$

Where, Q is the point charge and r is the distance between point where electric field is calculated and point charge.

5. Electric field due to an Infinite line charge

$$\Rightarrow \vec{D} = \frac{P_L}{2\pi\rho}\hat{a}_P$$

$$and \ \vec{E} = \frac{P_L}{2\pi \in_0 \rho}\hat{a}_P$$

Where, P_L is linear charge density, ρ is distance of the point P (P is the point where electric field is calculated) from line charge and $\hat{a}_P$ is position vector of point P.

6. Electric Field due to an infinite sheet of charge

$$\vec{D} = \frac{\rho_s}{2}\hat{a}_z$$

Or, $\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{\rho_s}{2\epsilon_0}\hat{a}_z$

Where, ρ_s is surface charge density and $\hat{a}_z$ is the unit normal vector from sheet to the pointwhere electric field is calculated.

7. Field due to a uniformly chargedsphere

$$\vec{D} = \begin{cases} \frac{r}{3}\rho_v \hat{a}_\tau; & 0 < r \le a \\ \frac{a^3}{3r^2}\rho_v \hat{a}_\tau; & r \ge a \end{cases}$$ Where, $\rho_v$is volume charge density.

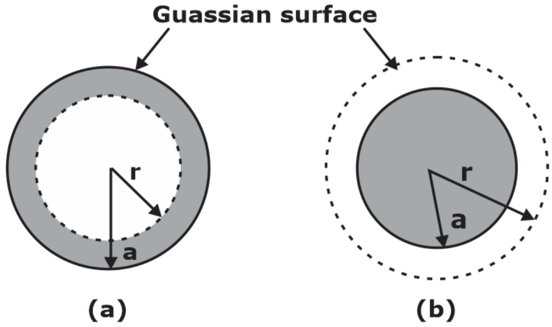


Figure: Gaussian surface for a uniformly charged when (a)r ≥ a and (b) r ≤ a

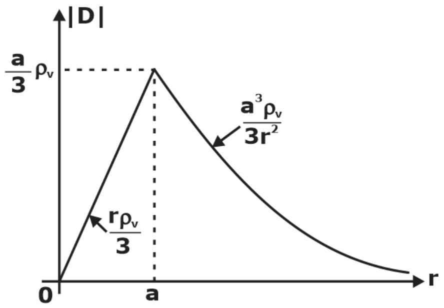


Figure: Sketch of |D| against r for a uniformly charged sphere.

8. Electric field due to multiple point charger

The electric field due to multiple points chargers can be determined using the principle of superposition. for N point charges $Q_1, Q_2, \ldots\ldots Q_N$ located at $\vec{r}_1, \vec{r}_2, \vec{r}_3 \ldots\ldots \vec{r}_N$ the electric fields intensity at point $\vec{r}$ is obtained by equations.

$$\vec{E} = \frac{Q_1(\vec{r} - \vec{r}_1)}{4\pi\epsilon_0 |\vec{r} - \vec{r}_1|^3} + \frac{Q_2(\vec{r} - \vec{r}_2)}{4\pi\epsilon_0 |\vec{r} - \vec{r}_1|^3} \ldots + \frac{Q_N(\vec{r} - \vec{r}_N)}{4\pi\epsilon_0 |\vec{r} - \vec{r}_N|^3}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\sum_{K=1}^{N}\frac{Q_K(\vec{r} - \vec{r}_k)}{|\vec{r}_1 - \vec{r}_k|^3}$$

9. Electric field due to charge distributions

$$dQ = \rho_L dl \Rightarrow Q = \int_L \rho_L dl (\text{line charge})$$

$$dQ = \rho_S dS \Rightarrow Q = \int_S \rho_S dS \ (\text{surface charge})$$

$$dQ = \rho_v dV \Rightarrow Q = \int_V \rho_v dV \ (\text{volume charge})$$

10. Electric field on the axis of a charged ring

Consider a circular ring of radius a with uniform line charge density ρ_L(C/m) and a point P on the axis of ring as shown in figure

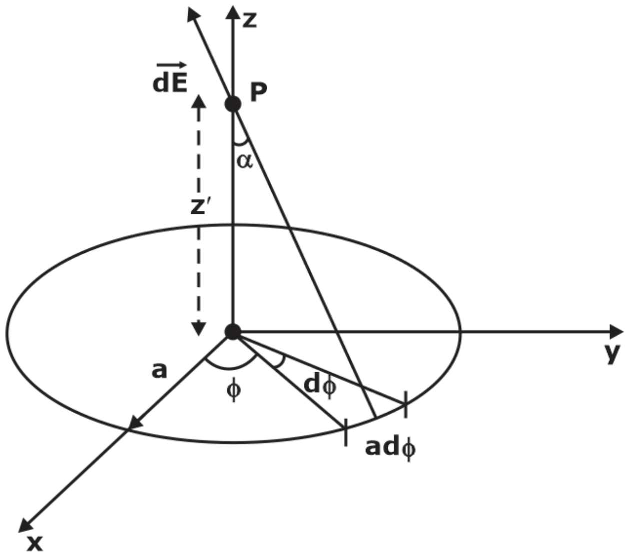


Figure: electric field to circular ring

The total electric field is therefore

$$\vec{E} = \frac{Qz}{4\pi\varepsilon_0(z^2 + a^2)^{3/2}}\hat{a}_z$$

Note: As $z \to \infty$, $\vec{E}$ tends to $\dfrac{Q}{4\pi\varepsilon_0 z^2}$

11. Electric field of a Charged Circular Disk

The electric field due to a uniformly charged circular disk at a point on its axis can be calculated using the result for a ring. Consider a disk of radius a, surface charge density ρ_s*(C/m^2) and point P as shown in the figure

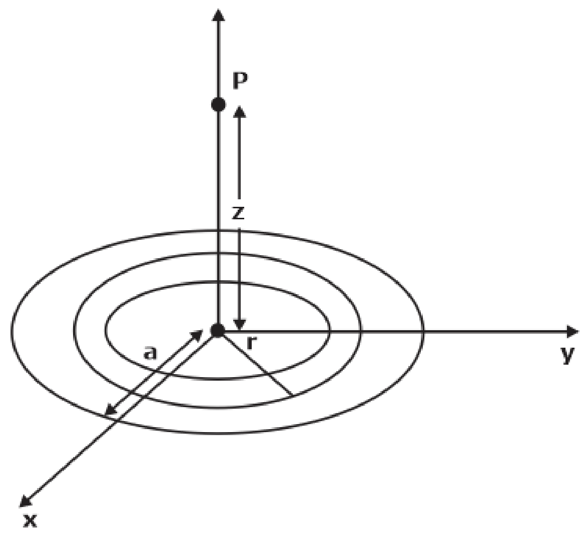


Figure: Electric field due to charged circular

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + a^2}}\right)\hat{a}_z$$

Note: If $z \gg a$ then $\bar{E} = \frac{Q}{4\pi \in_0 z^2}\hat{a}_z$

12. Potential difference in the electric field of a point charge

The potential difference between two points *A* and *B* in the electric fields of the point charge is

$$V_{AB} = -\int_A^B \bar{E}.d\bar{l} = \int_A^B dV = V_B - V_A$$

13. Energy Density in Electrostatic Field

$$W_E = \frac{1}{2}\int_{vol.} (D.E)\, dv = \frac{1}{2}\int_{vol.} \in_0 E^2 dv \text{ (J)}$$

Where, D is electric flux density and E is electric field intensity.

We defined energy density in (J/m^3)

14. Boundary Conditions

Electric field intensity $\vec{E}$ into two orthogonal components

$\vec{E} = \vec{E}_t + \vec{E}_n$

where $\vec{E}_t$ and $\vec{E}_n$ are tangential and normal components of $\vec{E}$ respectively.

14.1. Dielectric-Dielectric Boundary Conditions

Consider the $\vec{E}$ field existing in a region that consist of two different dielectrics characterized by $\epsilon_1 = \epsilon_0\epsilon_n$ and $\epsilon_2 = \epsilon_0\epsilon_a$ as shown in figure.

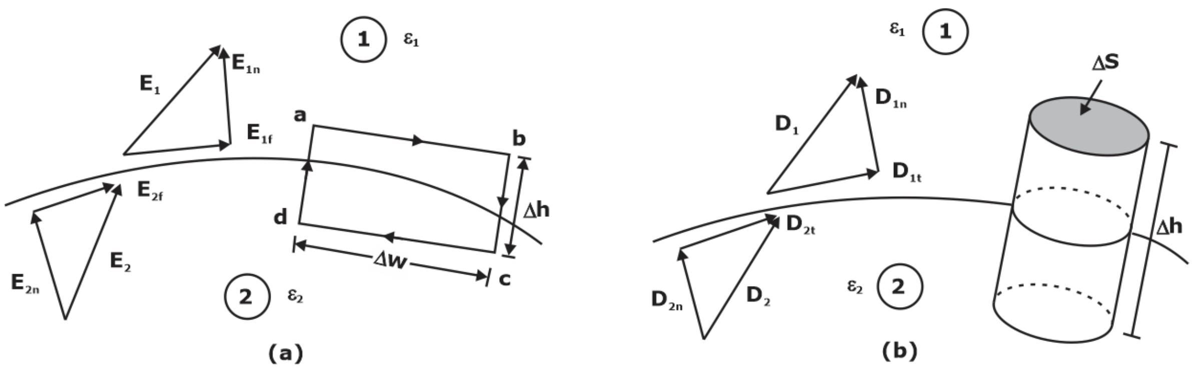


Figure: Dielectric-dielectric boundary: (a) determining $E_{1t} = E_{2t}$ (b) determining $D_{1\sigma} = D_{2\sigma}$.

The fields and $\vec{E}_1$ and $\vec{E}_2$ can be decomposed as

$\vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n}$

$\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n}$

then, $E_{1t} = E_{2t}$

14.2. Conductor-Dielectric Boundary Conditions

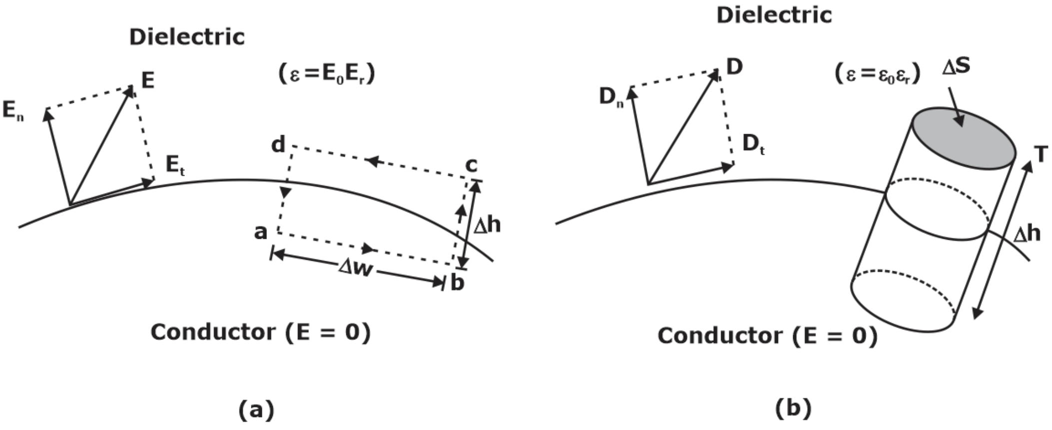


Figure: Conductor-dielectric boundary.

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_S$$

or, $D_n = \rho_S$

14.3. Conductor-Free Space Boundary Conditions

The boundary conditions at the interface of conductor and free space can be obtained from conductor-dielectric boundary conditions with $\epsilon_r = 1$.

Thus the boundary condition are

$E_t = 0$. $D_t = \epsilon_0 E_t = 0$

$$D_n = \frac{\rho_S}{\epsilon_0}$$

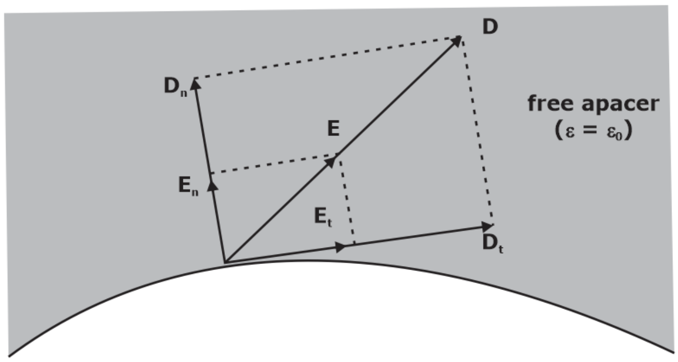


Figure: Conductor-free space boundary

15. Poisson's and Laplace's Equations

$\nabla^2 V = -\frac{\rho_v}{\epsilon}$, Where V is electrostatic potential and ρ_v is volume charge density.

This is known as Poisson's equation.

As special case of this equation occurs when $\rho_v = 0$ (i.e., for a charge free region

$$\nabla^2 V = 0$$

Which is known as Laplace's equation.

16. Coaxial Capacitor

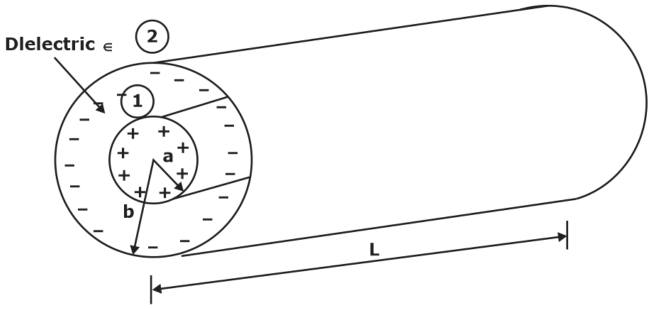


Figure: A coaxial capacitor

The capacitance of a coaxial cylinder is given by

$$C = \frac{Q}{V} = \frac{2\pi \in}{\ln \frac{b}{a}}$$

17. Spherical Capacitor

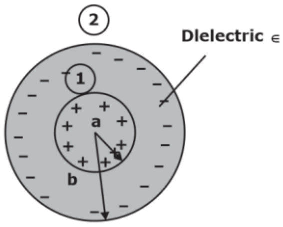


Figure: A spherical capacitor

The capacitance of the spherical capacitor is

$$C = \frac{Q}{V} = \frac{4\pi \in}{\frac{1}{a} - \frac{1}{b}}$$

CHAPTER 3: MAGNETOSTATICS

1. MAGNETIC FLUX DENSITY

Magnetic flux density is the amount of magnetic flux per unit area of a section, perpendicular to the direction of magnetic flux.

It is denoted by B. Mathematically,

$$B = \frac{d\Phi}{dS} a_n$$

Where $d\Phi$ is a small amount of magnetic flux through small area dS of the section perpendicular to magnetic flux a_n is the unit vector normal to the surface area.

also expressed as

$$\Phi = \int_S B \bullet dS$$

2. Relation between Magnetic field Intensity (H) and Magnetic Flux Density (B):

The magnetic field intensity is related to the magnetic flux density as

$$B = \mu H = \mu_0 \mu_r H$$

Where, μ is the permeability of the medium, $\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of free space, and μ_r is the relative permeability of the medium.

3. Biot-Savart's Law

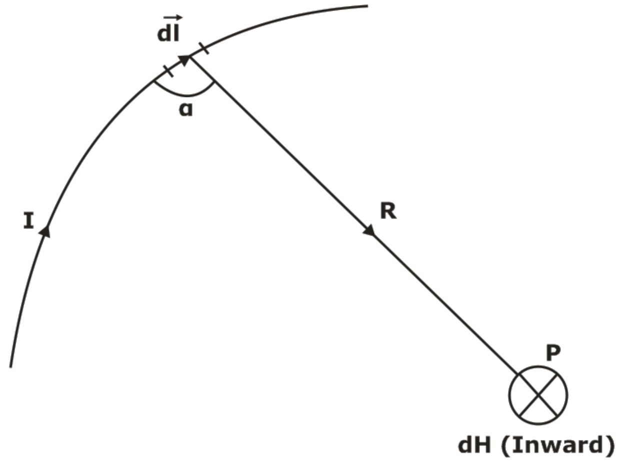


Figure: Magnetic field due to small length at P

Line current $\vec{H} = \int_L \frac{I\vec{dl} \times \vec{a}_R}{4\pi R^2}$

Surface current $\vec{H} = \int_L \frac{\vec{K}dS \times \vec{a}_R}{4\pi R^2}$

Volume current $\vec{H} = \int_V \frac{\vec{J}dv \times \vec{a}_R}{4\pi R^2}$

(H direction = I direction × Radial vector)

4. Ampere's Circuital Law

According to Ampere's circuital law the line integral of magnetic field intensity H around the closed loop L is equal to I, i.e.

$$\oint_L H \cdot dL = I$$

Differential Form of Ampere's Circuital Law

In differential form Ampere's circuital law is defined as

$\nabla \times H = J$

i.e. the curl of the magnetic field intensity (H) is equal to the current density (J) at the point in space.

5. H-field for finite length of current I carrying wire:

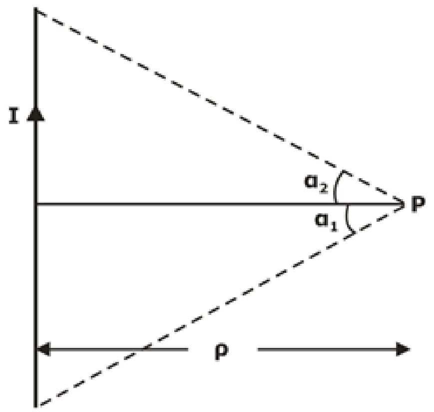


Figure: Field at P due to line conductor

$$\vec{H} = \frac{I}{4\pi\rho}(\sin\alpha_1 + \sin\alpha_2)a_\phi$$

Note: Notice from the above equation that $\vec{H}$ is always along the unit vector $\hat{a}_\phi$ (i.e., along concentric circular paths) irrespective of the length of the wire or the point of interest P.

6. H-field for infinite length of current I carrying wire:

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

The unit vector $\hat{a}_\phi$ must be found carefully. A simple approach is to determine $\hat{a}_\phi$ form

$$\hat{a}_\phi = \hat{a}_l \times \hat{a}_p$$

Where $\hat{a}_l$ is a unit vector along the line current and $\hat{a}_p$ is a unit vector along the perpendicular line from the line current to the filed point.

	Electric force (F_e = QE)	Magnetic Force (F_m = Qu ×B)
1.	It is in the same direction as the field E.	It is perpendicular to both u and B.
2.	It can perform work.	It cannot perform work.
3.	It is independent of the velocity of charge.	It depends upon the velocity of charge.
4.	It can produce change in kinetic energy.	It cannot produce change in kinetic energy.

Table: Comparison between Electric Force and Magnetic Force

7. Force on a Differential Current Element in Magnetic Field

The differential magnetic force experienced by various differential current elements are given below:

$$F_m = \int_L IdL \times B \quad \text{(Line current)}$$

$$F_m = \int_S K \times BdS \quad \text{(Surface current)}$$

$$F_m = \int_V J \times Bdv \quad \text{(Volume current)}$$

Where IdL is the line current element, KdS is surface current element, Jdv is volume current element, and F_m is the magnetic force exerted on the respective elements in presence of magnetic field B

8. Magnetic Force Between Two Current Elements

Consider the two differential current elements I_1dL_1 and I_2dL_2 separated by a distance r. The magnetic force between the two current elements is given by

$$F = \frac{\mu I_1 I_2}{4\pi} \int_{L_1} \int_{L_2} \frac{dL_2 \times (dL_1 \times a_r)}{r^2}$$

This equation is also called Ampere's force law.

9. Magnetic Susceptibility

In a linear material, magnetization is directly proportional to field intensity. i.e.

$$M \propto H$$

or $$M = \chi_m H$$

where χ_m is the magnetic susceptibility of the medium. The magnetic susceptibility of a magnetic material is a measure of the degree of magnetization of a material in response to an applied magnetic field.

10. Relation between Magnetic Field Intensity and Magnetic Flux Density

In a magnetic material, magnetic flux density is expressed in terms of magnetic field intensity as

$$B = \mu_0(H + M) = \mu_0(1 + \chi_m)H$$
$$= \mu_0\mu_r H = \mu H$$

where

$\mu = \mu_0\mu_r$ is called permeability of the medium, expressed in Henry per metre (H/m),

$\mu_0 = 4\pi \times 10^{-7}\ H/m$ is the permeability of free space, known as absolute permeability,

$\mu_r = (1 + \chi_m) = \frac{\mu}{\mu_0}$ is the relative permeability of the medium, it is dimensionless.

11. Energy Density in a Magnetic Field

In a magnetic field with flux density B, the stored magnetic energy density is given by

$$w_m = \frac{1}{2}(B \cdot H)$$

where H is the magnetic field intensity in the region. The total magnetic energy stored in a region is obtained by taking the volume integral of the energy density, i.e.

$$W_m = \int_v w_m dv = \int_v \frac{1}{2}(B \cdot H)\, dv$$

12. Boundary Conditions for Magnetostatic Fields:

$B_{1n} = B_{2n}$ States that Normal component of B is continuous across an interface. $\mu_1 H_{1n} = \mu_2 H_{2n}$

$H_{1t} - H_{2t} = J_{sn}$ States that the Tangential component of H field is discontinuous across an interface where free surface current exist-amount the amount of discontinuity being equal to the surface current density.

When conductivities of both media are finite, current are defined by volume current densities and free surface currents don’t exist on interface hence j equal to zero, and the Tangential component of H field is continuous across the boundary of almost all physical media; it is discontinuous only when an interface with an ideal conductor or a super conductor is assumed.

CHAPTER 4: MAXWELL'S EQUATION

Maxwell Equations

Differential form	integral form	Significance
$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint E.dl = -\iint \frac{\partial B}{\partial t}.ds$	Faraday's Law
$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint H.dl = -\iint (J + \frac{\partial D}{\partial t}).ds$	Ampere's Circuital Law
$\nabla.D = \rho_v$	$\oiint D.ds = Qenclosed$	Gauss Law
$\nabla.B = 0$	$\oiint B.ds = 0$	No isolated magnetic charge

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