

# AE/JE Foundation

Electrical Engineering

Communication  
Systems + Mathematics

Top 100 Most Important  
Questions





$$f_m = 2 \text{ K}$$

$$\text{So, BW} = 2(1 + \beta)f_m = 2(1 + 8)(2\text{K}) = 36 \text{ kHz}$$

5. Which of the following is not an advantage of turned radio frequency receives?
- A. Simple in construction
  - B. High sensitivity
  - C. Performance is good at high frequency.
  - D. No problem of tracking and alignment.

Ans. C

Sol. TRFRs has poor performance at high frequencies.

6. Three message signals, which are bandlimited to 10 kHz are multiplex using TDM. Each sample in encoded transmission bandwidth is:
- A. 300 kHz
  - B. 50 kHz
  - C. 100 kHz
  - D. 150 kHz

Ans. D

Sol. No. of multiplexed signals,  $N = 3$

$$\text{No. of bits/sample} = 5$$

$$\text{Message frequency } f_m = 10 \text{ kHz}$$

$$\text{Sampling frequency, } f_s = 2f_m = 20 \text{ kHz}$$

$$\therefore \text{bit rate, } R_b = N \times n \times f_s = 3 \times 5 \times 20 \text{ kHz} = 300 \text{ kbps}$$

$$\text{Transmission bandwidth} = \frac{R_b}{2} = 150 \text{ kHz}$$

7. The value of SQNR in dB, if message signal is sampled at 8 kHz and transmitted through 512 level PCM system.
- A. 54
  - B. 55
  - C. 55.8
  - D. 56.7

Ans. C

Sol.  $f_s = 8 \text{ kHz}$

$$f_m = 4 \text{ kHz}$$

$$L = 512 = 2^9$$

$$n = 9$$

$$\text{So, } (SQNR)_{dB} = 1.8 + (6n) = 1.8 + 6 \times 9 = 1.8 + 54 = 55.8 \text{ dB}$$

8. Band width requirement for telephone channel is \_\_\_\_\_.
- A. 2 KHz
  - B. 2 MHz
  - C. 4 KHz
  - D. 4 MHz

Ans. C

Sol. Telephone channel carries voice frequency is between 300 Hz to 3500 Hz.

$$\text{So, B.W.} = 3500 - 300$$

$$= 3200 \text{ Hz}$$

$$= 3.2 \text{ KHz}$$

So, most appropriate option is 4 KHz



Sol. The bandwidth of a multitone amplitude modulated signal is two times the maximum frequency present in modulated signal.

13. Consider the following statements regarding PDF and CDF:

1) A probability cumulative distribution function can be non-increasing or monotonically increasing.

2) Maximum value of a probability distribution function is 1

Which of the above statements is/are correct?

A. only 1

B. only 2

C. Both 1 and 2

D. Neither 1 nor 2

Ans. A

Sol. A probability CDF should be monotonically increasing or non-increasing, but it should not decrease as the probabilities are getting added.

The maximum value of probability density function can be greater than 1.

14. A signal is to be digitally encoded with a resolution of 0.02%. The voltage range is 0 – 8V.

What is the analog value of LSB?

A.  $1/2^6$

B.  $1/2^{13}$

C.  $1/2^{10}$

D.  $1/2^9$

Ans. C

Sol. Resolution = 0.02%

$$\text{No of quantization levels} = \frac{100}{.02} = 5000$$

Minimum no of bits required

$$2^N > 5000 \Rightarrow N = 13$$

Analog value of LSB

$$= \frac{1}{2^{13}} \times 8 = \frac{1}{2^{10}}$$

15. For a tuned radio frequency receiver the correct sequence of following devices from input to output is

1) Antenna

2) RF Amplifier

3) Detector

4) Audio amplifier

5) Power amplifier

A. 1, 3, 2, 4, 5

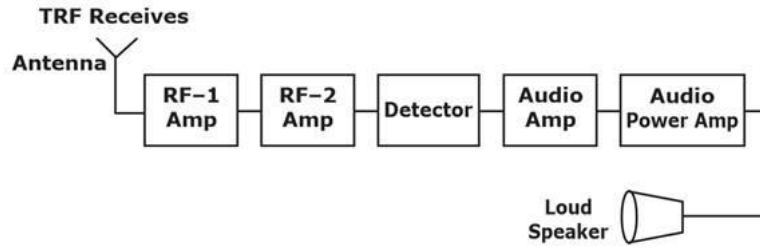
B. 1, 2, 3, 4, 5

C. 1, 2, 3, 5, 4

D. 1, 3, 2, 5, 4

Ans. B

Sol.



16. The PDF of a Cauchy distributed random variable  $X$  is  $f_x(x) = \frac{b/\pi}{x^2 + b^2}, -\infty < x < \infty$ .

The value of  $E[X]$  is

- A.  $\frac{1}{\pi b}$
- B. 0
- C.  $\frac{b}{\pi} \ln b$
- D.  $\frac{b}{2\pi} \ln b$

Ans. B

Sol. Since the function  $f_x(x)$  is symmetric about  $y$  - axis ( $x = 0$ ) so

$$E(x) = \int_{-\infty}^{\infty} xf(x) dx = 0$$

17. Consider the following statement regarding pre-emphasis and De-emphasis:

- 1) Pre-emphasis and De-emphasis techniques are used to improve fidelity of receiver.
- 2) Pre-emphasis is the process of artificial boosting of high frequency component of original signal to increase corresponding SNR.
- 3) De-emphasis will be done after demodulation.

Which of the above statements are correct?

- A. 1 and 2
- B. 2 and 3
- C. 1 and 3
- D. 1, 2 and 3

Ans. D

Sol.

Fidelity is the ability of receives to produce all frequency components of transmitted message signal.

Pre-emphasis and De-emphasis are used to improve fidelity of receives.

Pre-emphasis is the process of boosting original signal high frequency component to increase SNR.

De-emphasis is the reverse of pre-emphasis and will be done after demodulation.

18. For an FM signal, unmodulated carrier frequency in MHz is\_\_\_\_\_, provided,  $f_{max} = 1.5$  MHz,  $\Delta f = 450$  kHz.

- A. 1050
- B. 1.5
- C. 1.05
- D. 105

Ans. C



Sol. Companding is a process of expanding low amplitude signals and compressing high amplitude signals at transmitters and reverse is done at receiver. This can be done before quantization.

22. A message signal is transmitted through 256 level PCM system. This message signal is band limited to 4 kHz. The system BW is \_\_\_\_\_ kHz.

- A. 8
- B. 2
- C. 16
- D. 32

Ans. D

Sol.  $f_m = 4 \text{ kHz}$

$$L = 256 \Rightarrow 2^8 \Rightarrow n = 8$$

$$\text{as } f_s = 2f_m = 8 \text{ kHz}$$

$$\text{So, } BW = \frac{nf_s}{2} = \frac{8 \times 8k}{2} = 32 \text{ kHz}$$

23. **Statement 1:** An 8-bit PCM system perform better than a 6-bit PCM system.

**Statement 2:** 8-bit PCM system requires more channel bandwidth than 6-bit PCM system.

- A. Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)
- B. Both statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
- C. Statement (I) is true but Statement (II) is false
- D. Statement (I) is false but Statement (II) is true

Ans. B

Sol. 8-bit PCM system performs better than 6-bit PCM system as it produces smaller quantization noise.

As no of bits increases, quantization step size decreases hence quantization noise power decreases.

In case of PCM bandwidth requirement is directly proportional to no of bits.

24. Consider the following statements regarding FM and AM

- 1) FM is more immune to noise than AM
- 2) FM requires higher bandwidth compared to AM
- 3) FM requires more modulating power than AM

Which of the above statements is/are not correct?

- A. 1 and 3
- B. 1 and 2
- C. Only 2
- D. only 3

Ans. D

Sol. FM is more immune to noise than AM

$$(BW)_{AM} = 2 f_m$$

$$(BW)_{FM} = 2(\beta+1)f_m$$

In case of FM, modulating power before and after the modulation remains same whereas it increases in case of AM.



25. **Statement 1:** In amplitude modulation, if modulation index is greater than 1 then the signal cannot be detected by envelope detectors.

**Statement 2:** In case of over modulation message signal is not completely stored in positive envelope.

- A. Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)
- B. Both statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
- C. Statement (I) is true but Statement (II) is false
- D. Statement (I) is false but Statement (II) is true

Ans. A

Sol.

For modulation index  $\mu > 1 \Rightarrow A_m > A_c$

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} > 1$$

$$\Rightarrow A_{\min} = \text{negative}$$

Hence it cannot be detected by envelope detectors as it cannot detect the signal less than zero, For detection of such signals synchronous detectors has to be used.

26. Consider an amplitude modulated signal,  $20 \cos(4\pi \times 10^3 t)$  with carrier signal  $100 \cos(4\pi \times 10^3 t)$ , then the modulation index is

- A. 0.2
- B. 0.4
- C. 2
- D. 4

Ans. A

Sol. Modulating signal,  $m(t) = A_m \cos(2\pi f_m t)$

$$= 20 \cos(4\pi \times 10^3 t)$$

$$\text{So, } A_m = 20$$

$$\text{Carrier signal, } c(t) = A_c \cos(2\pi f_c t)$$

$$= 100 \cos(4\pi \times 10^3 t)$$

$$\text{So, } A_c = 100$$

$$\text{So, modulation index, } m = \frac{A_m}{A_c} = \frac{20}{100} = 0.2$$

$$m = 0.2$$

27. The internal state of neuron is called \_\_\_\_\_, is the function of the inputs the neurons receives.

- A. Weight
- B. activation or activity level of neuron
- C. Bias
- D. None of these

Ans. B

28. Which of the following is not an advantage of pulse code modulation?
- A. Private and secured Communication
  - B. Complexity of the system decreases.
  - C. Immunity to noise and interference
  - D. Regeneration of coded signals along the path.

Ans. B

Sol. For a PCM, complexity of the system increases, and transmission bandwidth requirement is also high  
Rest all are advantage of PCM.

29. In AM modulation, the equation of the modulating signal is given by  $f(t) = A_m \cos(\omega_m t)$ . If the amplitude of the carrier wave is A and there is no over-modulation, the modulation efficiency will be
- A. 33.4%
  - B. 38.6%
  - C. 43.3%
  - D. 48.6%

Ans. A

Sol. For under critically modulation,  $\mu \leq 1$   
For critical or maximum modulation  $\mu = 1$

$$\text{modulation efficiency } (\eta) = \frac{\text{Useful Power}}{\text{Total power}}$$

$$= \frac{P_{SB}}{P_T} = \frac{P_c \frac{\mu^2}{2}}{P_c \left(1 + \frac{\mu^2}{2}\right)}$$

$$\eta = \frac{\mu^2}{2 + \mu^2}$$

$$\eta = \frac{1}{2 + 1} = \frac{1}{3} \times 100 = 33.3\%$$

30. **Statement 1:** In case of time division multiplexing, the speed of commutator is equal to nyquist rate of highest frequency is preferred.

**Statement 2:** Channel bandwidth requirement will be minimum when speed of commutator is equal to nyquist rate of lowest frequency.

- A. Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)
- B. Both statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
- C. Statement (I) is true but Statement (II) is false
- D. Statement (I) is false but Statement (II) is true

Ans. D

Sol. When the speed of commutators is equal to nyquist rate of lowest frequency, channel bandwidth requirements are minimum. Hence this case is preferred.

31. In AM signal when the modulation index is 0.5, the maximum power  $P_t$ , (where  $P_c$  is carrier power) is equal to\_\_\_\_\_.

- A.  $P_c$
- B.  $2.5P_c$
- C.  $2P_c$
- D.  $1.125P_c$

Ans. D

Sol.

$$P_t = P_c * \left(1 + \frac{m^2}{2}\right)$$

$$P_t = \left(1 + \frac{0.5^2}{2}\right) * P_c = 1.125P_c$$

32. Consider the following statements:

- A. Both statement (i) and statement (ii) are true, and statement (ii) is the correct explanation of statement (i)
- B. Both statement (i) and statement (ii) are true, but statement (ii) is not the correct explanation of statement (i)
- C. Statement (i) is true, but statement (ii) is false
- D. Statement (i) is false, but statement (ii) is true

**Statement (i):** The process of extracting an original message signal from the modulated wave is known as demodulation.

**Statement (ii):** Envelope Detector is one of the famous demodulators used for demodulating AM wave.

- A. A
- B. B
- C. C
- D. D

Ans. B

Sol. The process of extracting an original message signal from the modulated wave is known as detection or demodulation. The circuit, which demodulates the modulated wave is known as the demodulator. The following demodulators (detectors) are used for demodulating AM wave.

1. Square Law Demodulator
2. Envelope Detector

Square law demodulator is used to demodulate low level AM wave. This demodulator contains a square law device and low pass filter. Envelope detector is used to detect (demodulate) high level AM wave. This envelope detector consists of a diode and low pass filter. Here, the diode is the main detecting element. Hence, the envelope detector is also called as the diode detector. The low pass filter contains a parallel combination of the resistor and the capacitor.

So both statements are correct, but statement (ii) is not correct explanation of statement (i).

33. Match List-I with List-II and select the correct answer using the code given below the lists:-

List-I	List-II
A. Video signal(TV)	1. Square-law detector
B. FM	2. Transmitted power constant
C. AM	3. AM waves

A. A-2 B-1 C-3

B. A-1 B-2 C-3

C. A-3 B-1 C-2

D. A-3 B-2 C-1

Ans. D

Sol. Amplitude modulation detectors are of two types:-

1) Square Law detector

2) Envelope detector

In frequency modulation the transmitted power is constant

Video signal transmits data by AM waves only.

34. Consider the following statements about noise.

1) Solar noise refers to the interference of broad frequency spectrum radiated by the sun with the communication signals.

2) White noise is due to rapid and random motion of the molecules inside the component itself.

3) Thermal noise is also called Gaussian noise as it follows Gaussian probability density function.

Which of the above statements is/are correct?

A. 2 and 3 only

B. 1 and 2 only

C. only 1

D. 1, 2 and 3

Ans. D

Sol. All the statements are correct.

Solar noise is due to broad frequency spectrum radiated by sun.

White noise or thermal noise is due to the rapid and random motion of the molecules inside the component. It is also called Gaussian noise as it follows Gaussian probability density function.

35. The value of image frequency in kHz, provided receiver is tuned to 600 kHz and intermediate receiver is tuned to 600 kHz and intermediate frequency is 500 kHz.

A. 1100

B. 100

C. 1600

D. 1700

Ans. C

Sol.  $f_s = 600 \text{ kHz}$

$f_i = 500 \text{ kHz}$

$f_{si} = f_s + 2f_i = 600 + 2 \times 500 = 1600 \text{ kHz}$

36. **Statement 1:** For values of modulation index less than 0.3 in FM there are only two sidebands.

**Statement 2:** For large values of  $\beta$  in FM the FM wave will contain a carrier and infinite numbers of sidebands.

A. Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)

B. Both statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)

C. Statement (I) is true but Statement (II) is false

D. Statement (I) is false but Statement (II) is true

Ans. A

Sol. In theory for large values of  $\beta$  more than 1 radian, FM contains a carrier and infinite no of sidebands placed symmetrically around the carrier. This is said to be wideband FM.

37. For practical purpose, the signal to noise ratio for acceptable quality transmission of analog signals and digital signals respectively are

A. 10-30 dB and 5-08 dB

B. 40-60 dB and 10-12 dB

C. 60-80 dB and 20-24 dB

D. 70-90 dB and 30-36 dB

Ans. B

Sol. For practical or commercial purposes,

The signal to noise ratio for acceptable quality of transmission of analog signals and digital signals are respectively:

Analog Signals: 40-60 dB

Digital Signals: 10-12 dB

38. Carrier signal is having 10 MHz frequency. The carrier signal is frequency modulated by sinusoidal signal of 500 Hz. Bandwidth using Carson's rule is \_\_\_\_\_ kHz if maximum frequency deviation is 50 KHz.

A. 100

B. 101

C. 102

D. 103

Ans. B

Sol. By Carson's Rule,

$$BW = 2 (\Delta f + f_m)$$

$$= 2(50 + 0.5) \text{ kHz}$$

$$= 101 \text{ kHz}$$

39. In TV, video signals are transmitted through \_\_\_\_\_.

A. frequency modulation

B. pulse modulation

C. amplitude modulation

D. phase modulation

Ans. C

Sol. Amplitude modulation gets more affected by noise in comparison to FM signals. Since Ear can hear the minute distortions in audio signals, hence they are transmitted through FM transmission. Whereas video signals with minute distortions doesn't show any difference when displayed on screen.

40. Consider the following statements:

- A. Both statement (i) and statement (ii) are true, and statement (ii) is the correct explanation of statement (i)
- B. Both statement (i) and statement (ii) are true, but statement (ii) is not the correct explanation of statement (i)
- C. Statement (i) is true, but statement (ii) is false
- D. Statement (i) is false, but statement (ii) is true

**Statement (i):** Balanced modulator consists of four identical AM modulators.

**Statement (ii):** The transmission of a signal, which contains a carrier along with two sidebands is termed as DSBFC.

- A. A
- B. B
- C. C
- D. D

Ans. D

Sol. Balanced modulator consists of two identical AM modulators. These two modulators are arranged in a balanced configuration in order to suppress the carrier signal. Hence, it is called as Balanced modulator. The same carrier signal is applied as one of the inputs to these two AM modulators. The modulating signal is applied as another input to the upper AM modulator. Whereas the modulating signal with opposite polarity is applied as another input to the lower AM modulator. So, statement (i) is incorrect.

The transmission of a signal, which contains a carrier along with two sidebands can be termed as Double Sideband Full Carrier system or simply DSBFC. So, statement (ii) is correct.

41. A FM generator has frequency sensitivity of 40 Hz/volt. Modulating signal having amplitude 5 V and frequency 2000 Hz is applied to this FM generator. Frequency modulation is \_\_\_\_\_ . (Consider modulating signal to be sinusoidal).

- A. 2 Hz
- B. 20 Hz
- C. 200 Hz
- D. 2000 Hz

Ans. C

Sol. Amplitude of modulating signal,  $A_m = 5 \text{ V}$  &  $f_m = 2000 \text{ Hz}$

Frequency sensitivity,  $K_f = 40 \text{ Hz/Volts}$

$\Delta f = \text{Frequency deviation} = K_f A_m$

$= 40 \times 5 = 200 \text{ Hz}$







Sol. In FM frequencies are more affected by the noise compare to low frequencies so preemphasis is done to amplify high frequency and pass the low frequency and pass the low frequency as it is so to improve (S/N) ratio at receiver end at high frequencies.

49.A random communication signal has PDF  $P(x) = ae^{-bx}$ , for  $-1 \leq x \leq 1$  and  $b = 1$  then value of a is

- A.  $\frac{e}{e^2 - 1}$
- B.  $\frac{e}{e - 1}$
- C.  $\frac{1}{2}$
- D. 2

Ans. A

Sol.

$$\text{PDF, } \int_{-\infty}^{\infty} P(x) = 1$$

$$\int_{-1}^1 ae^{-bx} dx = 1$$

$$\frac{-a}{b} [e^{-bx}]_{-1}^1 = 1$$

$$\frac{-a}{b} [e^{-b} - e^b] = 1$$

But  $b = 1$

$$-a [e^{-1} - e] = 1$$

$$a = \frac{1}{e - e^{-1}} = \frac{e}{e^2 - 1}$$

50. Consider the following statements regarding envelope detector:

- 1) Charging time constant of the capacitor should be high for proper envelope detection.
- 2) Discharging time constant should be at an optimum value i.e. neither too high nor too low
- 3) For an AM wave with carrier frequency 2MHz and message frequency 5kHz the discharging time constant = 100  $\mu$ sec will result in proper envelope detection.

Which of the above statements are correct?

- A. 1 and 2
- B. 2 and 3
- C. 1 and 3
- D. 1, 2 and 3

Ans. B

Sol. Charging time constant should be very small so that capacitor charges quickly to the peak value of modulated signal.

Discharge time constant should be

$$\frac{1}{f_c} < R_L C < \frac{1}{f_m}$$

$$0.5 \times 10^{-6} \text{ sec} < R_L C < 0.2 \times 10^{-3} \text{ sec}$$

$R_L C = 100 \mu\text{sec}$  satisfies the criteria

### Mathematics

51. Singular points of  $\frac{z-4}{z^2+2z+5}$  is:

A.  $-1 \pm 2i$

B.  $1 \pm 2i$

C. Both (A) and (B)

D. Neither (A) nor (B)

Ans. A

Sol.

$$F(z) = \frac{z-4}{z^2+2z+5} = \frac{N(z)}{D(z)}$$

For singular point,

$$D(z) = 0$$

$$z^2 + 2z + 5 = 0$$

$$z = -1 \pm 2i$$

52. Maximum value of  $x^{1/x}$  is:

A.  $e^e$

B.  $e^{1/e}$

C.  $e^{-1/e}$

D. 1

Ans. B

Sol.

$$\text{Let } y = x^{\frac{1}{x}}$$

$$\log y = \frac{1}{x} \log x$$

$$\Rightarrow \frac{f}{y} \frac{dy}{dx} = \left( \frac{1}{x^2} - \frac{\log x}{x^2} \right)$$

$$\frac{dy}{dx} = y \left( \frac{1 - \log x}{x^2} \right)$$

$$\frac{dy}{dx} = 0 \Rightarrow x = e \Rightarrow \text{Stationary point}$$

$$\frac{d^2y}{dx^2} = \frac{1}{x^4} \left[ x^2 \left( y \left( 0 - \frac{1}{x} \right) + (1 - \log x) \frac{dy}{dx} - 2xy(1 - \log x) \right) \right]$$

$$= \frac{1}{x^4} \left[ x^2 \left( \frac{-y}{x} + \frac{(1 - \log x)^2 y}{x^2} \right) - 2xy(1 - \log x) \right]$$

$$\text{at } x = e \Rightarrow \frac{d^2y}{dx^2} < 0$$

so maximum value occurs at  $x = e$

i.e.,  $e^{1/e}$ .

53. If  $\left( \frac{1+i}{1-i} \right)^n = 1$ , least positive integer 'n' is:

A. 2

B. 4

C. 6

D. 8

Ans. B

Sol.

$$\begin{aligned} \left(\frac{1+i}{1-i}\right)^n &= \frac{(1+i)^n(1+i)^n}{(1-i)^n(1+i)^n} \\ &= \frac{(1+i)^{2n}}{2^n} \\ &= \left[\frac{(1+i)^2}{2}\right]^n \\ &= \frac{(2i)^n}{2^n} = i^n \\ i^n = 1 &\Rightarrow n = 4 \end{aligned}$$

54. Stokes' theorem is generalization of:

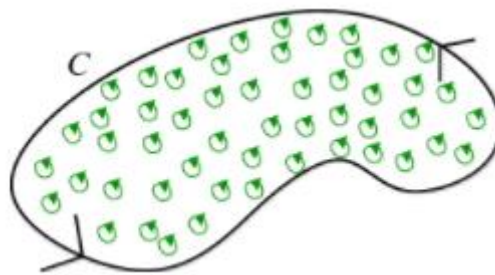
- A. Green's theorem
- B. Lagrange Theorem
- C. Mean Value Theorem
- D. Rolls Theorem

Ans. A

Sol. Stokes' theorem is a generalization of Green's theorem from circulation in a planar region to circulation along a surface.

**Green's theorem states that**, given a continuously differentiable two-dimensional vector field  $F$ , the integral of the "microscopic circulation" of  $F$  over the region  $D$  inside a simple closed curve  $C$  is equal to the total circulation of  $F$  around  $C$ , as suggested by the equation.

$$\int_C \mathbf{F} \cdot d\mathbf{s} = \iint_D \text{"microscopic circulation of } \mathbf{F}\text{" } dA.$$



**Stokes' theorem generalizes Green's theorem to three dimensions.**



Sol.

$$\frac{d^4y}{dx^4} + 13 \frac{d^2y}{dx^2} + 36y = 0$$

i.e.,  $(d^4 + 13D^2 + 36)y = 0$

$$(D^2 + 4)(D^2 + 9)y = 0$$

A.E. has roots  $\pm 2i$  and  $\pm 3i$

$$\therefore y = (c_1 \cos 2x + c_2 \sin 2x) + (c_3 \cos 3x + c_4 \sin 3x)$$

is required solution.

57. The value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  is \_\_\_\_\_. Take:  $u = \tan^{-1} \left( \frac{x^3 + y^3}{x - y} \right)$ .

A.  $\sin(2u)$

B.  $\cos(2u)$

C.  $\tan(2u)$

D. 0

Ans. A

Sol.

$$f(u) = \tan u = \left( \frac{x^3 + y^3}{x - y} \right)$$

Let

$\therefore f(u)$  is a homogeneous function of degree 2

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = n \frac{f(u)}{f'(u)}$$

$$2 \frac{\tan(u)}{\sec^2(u)} = 2 \frac{\sin(u)}{\cos(u)} \cos^2(u) = \sin(2u)$$

58. Consider a partial differential equation (PDE):  $4 \frac{\partial^2 u}{\partial x^2} - 5 \frac{\partial^2 u}{\partial y^2} = 0$ . Its nature is:

A. Hyperbola

B. Straight line

C. Circle

D. Used as any of (A), (B) or (C)

Ans. A

Sol.

Comparing with  $\frac{A\partial^2y}{\partial x^2} + \frac{B\partial^2y}{\partial x\partial y} + \frac{C\partial^2y}{\partial y^2} + \frac{D\partial y}{\partial x} + \frac{E\partial y}{\partial y} + Fu = 0$

$$B^2 - 4AC = 80 > 0 \rightarrow \text{Hyperbola.}$$

59. Residue of  $f(z)$  at  $z = 1$  if  $f(z) = \frac{z^2 + 2}{(z-1)^2(z+2)}$  is

A. 1/3

B. 2/3

C. 1

D. 4/3

Ans. A

Sol.

$$\text{At } z = 1, \text{ Residue} = \frac{1}{1!} \lim_{z \rightarrow 1} \frac{d}{dz} \left( \frac{(z-1)^2 \cdot (z^2 + 2)}{(z-1)^2(z+2)} \right)$$









68. Value of  $2A^{10} - 18A^9 + 40A^8 - 25A^7 + 9A^6 - 20A^5 + 13A^4 - 9A^3 + 20A^2 - 10A$  is

\_\_\_\_\_ , if  $A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 6 & 7 \\ 9 & 0 & 1 \end{bmatrix}$ .

- A. A
- B. 2A
- C. 3A
- D. 4A

Ans. B

Sol.

$$|A - \lambda I| = \begin{vmatrix} 2 - \lambda & 0 & 0 \\ 3 & 6 - \lambda & 0 \\ 3 & 0 & 1 - \lambda \end{vmatrix}$$

$$= (2 - \lambda)(6 - \lambda)(1 - \lambda)$$

$$= (12 - 20\lambda + 9\lambda^2 - \lambda^3)$$

$$\therefore A^3 - 9A^2 + 20A - 12I = 0$$

$$2A^{10} - 18A^9 + 40A^8 - 25A^7 + 9A^6 - 20A^5 + 13A^4 - 9A^3 + 20A^2 - 10A$$

$$= 2A^7(A^3 - 9A^2 + 20A - 12I) - A^7 + 9A^6 - 20A^5 + 13A^4 - 9A^3 + 20A^2 - 10A$$

$$= 0 - A^4(0) + A(A^3 - 9A^2 + 20A - 12I) + 2A$$

$$= A(0) + 2A = 2A$$

69. When Binomial distribution is used in a limiting form is i.e.,  $(n \rightarrow \infty)$

- A. Normal distribution
- B. Poisson distribution
- C. Binomial distribution
- D. None of the above

Ans. A

Sol. For Binomial distribution

$$P(r) = \frac{n!}{r!(n-r)!} \pi^r (1-\pi)^{n-r}$$

If  $n \rightarrow \infty$  it is called normal distribution.

70. The solution of initial value problem;  $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ , where  $u(x, 0) = 6e^{-3x}$  is

- A.  $u = 6e^{-3x + t}$
- B.  $u = 6e^{-(2x + 2t)}$
- C.  $u = 6e^{-(3x + 2t)}$
- D.  $u = 6e^{-(3x + 2t)}$

Ans. C

Sol.

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u \quad \dots(i)$$

$$u(x, 0) = 6e^{-3x}$$

Separation of variables

$$\text{Let, } u(x, t) = X(x) T(t)$$

$$u_x = X'T$$

$$u_t = XT'$$

Put in eq. (i)

$$X'T = 2XT' + XT$$

Divide by XT

$$\frac{X'}{X} = \frac{2T'}{T} + 1 = k$$

$$X' = kX$$

$$(D - k) X = 0$$

$$\text{AE: } m - k = 0$$

$$m = k$$

$$X = C_1 e^{kx}$$

$$2T' = (k - 1)T$$

$$2T' - (k - 1) T = 0$$

$$\text{AE: } 2m - (k - 1) = 0$$

$$m = \frac{(k - 1)}{2}$$

$$T = C_2 e^{\frac{(k-1)}{2}t}$$

$$u(x, t) = XT$$

$$u(x, t) = C_1 C_2 e^{kx + \frac{(k-1)}{2}t}$$

$$u(x, 0) = C_1 C_2 e^{kx} = 6e^{-3x}$$

$$\therefore C_1 C_2 = 6$$

$$k = -3$$

$$u(x, t) = 6e^{-3x-2t}$$

$$u(x, t) = 6e^{-(3x + 2t)}$$

71. For the position vector  $\vec{r}$ ,  $\nabla^2[\log r]$  is:

A.  $1/r$

B.  $-1/r$

C.  $1/r^2$

D.  $-1/r^2$

Ans. C

Sol.

$$\nabla^2[f(r)] = f''(r) + \frac{2}{r} f'(r)$$

$$\nabla^2[\log r] = -\frac{1}{r^2} + \frac{2}{r} \left(\frac{1}{r}\right)$$

$$= \left(\frac{1}{r^2}\right)$$

72. A complex function is defined as  $f(z) = e^{2z}$ . The imaginary part of  $f(z)$  is:

A.  $e^y \sin x$

B.  $e^x \cos y$

C.  $e^{2x} \cos 2y$

D.  $e^{2x} \sin 2y$

Ans. D

Sol.

$$f(z) = e^{2z} = e^{2x+2iy}$$

$$f(z) = e^{2x} \cdot e^{2iy} = e^{2x}[\cos 2y + i \sin 2y]$$

Thus,  $Im [f(z)] = e^{2x} \sin 2y$

73. For the regression equations:

$$y = 0.516x + 33.73 \text{ and } x = 0.512y + 32.52$$

the means of x and y are nearly

- A. 67.6 and 68.6
- B. 68.6 and 68.6
- C. 67.6 and 58.6
- D. 68.6 and 58.6

Ans. A

Sol.  $y = 0.516x + 33.73$

$$y - 0.516x = 33.73 \dots(1)$$

$$- 0.516x + y = 33.73$$

and  $x = 0.512y + 32.52$

$$x - 0.512y = 32.52 \dots(2)$$

On solving eqn. (1) and (2), we get mean of x and y.

$$x = 67.6 \text{ and } y = 68.6$$

74. Find the value of  $\frac{1}{2\pi j} \oint_c \frac{2z^2 + z}{z^2 - 1} dz$  for  $c \equiv |z - 1| = 1$

- A. 0.5
- B. 1
- C. 1.5
- D. 2

Ans. C

Sol.

$$I = \frac{1}{2\pi j} \oint_c \frac{2z^2 + z}{z^2 - 1} dz = \text{Sum of Residues}$$

$$I = \frac{1}{2\pi j} \oint_c \frac{2z^2 + z}{(z - 1)(z + 1)} dz$$

Contour is a circle of radius 1 unit, centred at (1,0),

I has singularity at  $z=1$  and  $z=-1$ ,

But only  $z=1$  lies inside the contour,

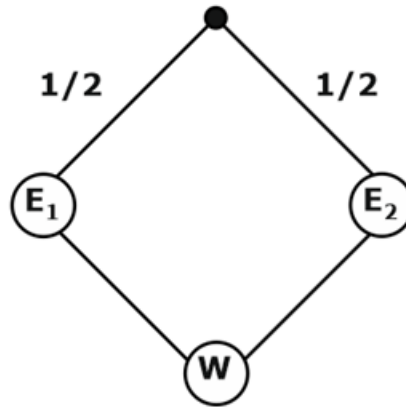
Therefore,  $I = \frac{1}{2\pi j} \oint_c \frac{2z^2 + z}{z^2 - 1} dz = \text{Residue at } z=1$

$$\text{At } z = 1 \Rightarrow \text{Residue} = \lim_{z \rightarrow 1} \frac{2z^2 + z}{z + 1} = \frac{2(1)^2 + 1}{1 + 1} = \frac{3}{2}$$

So,  $I = 3/2 = 1.5$







W → Drawn ball is white

$$P\left(\frac{W}{E_1}\right) = \frac{4}{6} \quad P\left(\frac{W}{E_2}\right) = \frac{3}{6}$$

$$\begin{aligned} P(W) &= P(E_1) \times P\left(\frac{W}{E_1}\right) + P(E_2) \times P\left(\frac{W}{E_2}\right) \\ &= \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{2} \\ &= \frac{7}{12} \end{aligned}$$

80. The coefficient of  $x^3$  in the Taylor series expansion of  $f(x) = 3 \sin x + 2 \cos x$  is:

- A. 3/2
- B. -1/2
- C. 1/2
- D. -3/2

Ans. B

Sol. Taylor series expansion of  $\sin(x)$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$3 \sin x + 2 \cos x = \left[ 3 \left( x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \right) \right] + \left[ 2 \left( 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \right) \right]$$

$$= 2 + 3x - x^2 - \frac{x^3}{2} + \frac{x^4}{12} + \frac{3x^5}{120}$$

Coefficient of  $x^3 = \frac{-1}{2}$ .

81. P, Q and R invested in the ratio 4 : 5 : 6 in a business. If 10% of the total profit goes to charity and Q's share is Rs 4320, find the total profit.

- A. 14000
- B. 15000
- C. 14400
- D. 16400

Ans. C

Sol.: Let the total profit be P remaining profit after 10% donation = 0.9P

$$\begin{aligned}
 \text{Q'share in remaining profit} &= \frac{5}{(4+5+6)} \text{ of remaining profit} \\
 &= \frac{1}{3} \text{ of remaining profit} \\
 &= \frac{0.9P}{3} = 0.3P = 4320 \\
 P &= \frac{4320}{0.3} = 14400
 \end{aligned}$$

82. A and B are two candidates in an election. 10% of the voters in the voter list have not voted and out of the total votes cast, 100 were rejected. If A won from B by 300 votes and gets 46% of the votes listed in the voter list, then find the total number of valid votes cast.

- A. 8900
- B. 9800
- C. 8200
- D. 2800
- E. 10000

Ans. A

Sol.: 10% not vote.

$$\begin{aligned}
 &100 \text{ voters were rejected.} \\
 &\text{A got 46\% and won by 300 vote} \\
 &46\% - (90\% - 46\% - 100) = 300 \\
 &46\% - 44\% + 100 = 300 \\
 &2\% + 100 = 300 \\
 &2\% = 200 \\
 &\text{WE GET,} \\
 &1\% = 100 \\
 &\text{valid voters} = 90 * 100 - 100 = 8900.
 \end{aligned}$$

83. A boat running in downstream covers distance of 40 km in 5hr. While it covers same distance upstream it takes 8hrs. The speed of the stream is what percent of boat.

- A. 20%
- B. 23%
- C.  $23\frac{1}{13}\%$
- D. None

Ans. C

$$\begin{aligned}
 \text{Sol.: } 40 &= (x+y) \times 5 = 8 = x+ y \\
 40 &= (x-y) \times 8 = 5= x-y \\
 x &= \frac{13}{2} = 6.5 \text{ km./hr} \\
 y &= 1.5 \\
 \text{Percent} &= \frac{300}{13} = 23\frac{1}{13}\%
 \end{aligned}$$

84. A number when divided by 49 leaves 32 as remainder. This number when divided by 7 will have the remainder as:

- A. 4
- B. 3
- C. 2
- D. 5

Ans. A

Sol.: Let number be x and divisor be k.

According to the given condition,

$$x = 49k + 32 = 7 \times 7k + 7 \times 4 \times 4 = 7(7k + 4) + 4$$

Hence, remainder is 4.

**Alternate Method:**

Let the number be 81 as it leaves 32 as remainder when divided by 49.

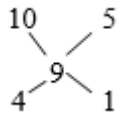
So, 81 leaves 4 as remainder when divided by 7.

85. 90 coins, which contain the coins of 10 Paise and 5 Paise, make Rs. 8.10. Find the number of coins of 5 Paise.

- A. 18
- B. 36
- C. 9
- D. 24

Ans. A

Sol.: The Average value of 90 Coins =  $(8.10/90) \times 100 = 9$  Paise



So, the ratio of 10 Paise and 5 Paise coins = 4 : 1

Total of Ratios = 4 + 1 = 5

$$\text{Number of 5 Paise Coins} = \frac{1}{5} \times 90 = 18$$

86. A man spends 15% of his income. If his expenditure is Rs. 75, his income (in rupees) is:

- A. 400
- B. 300
- C. 750
- D. 500

Ans. D

Sol.: A man spends 15% of his income. If his expenditure is Rs. 75

15% of income = 75

$$\Rightarrow \text{total income} = (75/15) \times 100 = 500$$

87. The ratio of the numbers of boys and girls of a school with 504 students is 13:11. What will be the new ratio if 12 more girls are admitted?

- A. 91:81
- B. 81:91
- C. 9:10
- D. 10:9

Ans. A

Sol.: Total numbers of girls in the school:

$$= 504 \times \frac{11}{13 + 11} = 504 \times \frac{11}{24} = 231$$

Total numbers of boys in the school:

$$= 504 \times \frac{13}{13 + 11} = 504 \times \frac{13}{24} = 273$$



Now, total numbers of girls when 12 more girls are admitted =  $231 + 12 = 243$

∴ New ratio of boys and girls =  $273 : 243 = 91 : 81$

88. The compound interest on Rs. 30,000 at 7% per annum for n years is Rs. 4347. The value of n is

- A. 3
- B. 2
- C. 4
- D. 5

Ans. B

Sol.: Amount = Rs.  $(30000 + 4347) = \text{Rs.}34347$

$$A = P \left( 1 + \frac{R}{100} \right)^T$$

$$\Rightarrow 34347 = 30000 \left( 1 + \frac{7}{100} \right)^n$$

$$\Rightarrow \frac{34347}{30000} = \left( \frac{107}{100} \right)^n$$

$$\Rightarrow \frac{11449}{10000} = \left( \frac{107}{100} \right)^n$$

$$\Rightarrow \frac{11449}{10000} = \left( \frac{107}{100} \right)^n$$

$$\Rightarrow \left( \frac{107}{100} \right)^2 = \left( \frac{107}{100} \right)^n$$

$$\Rightarrow n = 2 \text{ years}$$

89. Two trains 140 m and 160 m long run at the speeds of 60 km/h and 40 km/h respectively in opposite directions on parallel tracks. The time (in seconds) which they take to cross each other, is

- A. 10
- B. 10.8
- C. 9
- D. 9.6

Ans. B

Sol.: Total length of trains =  $140 + 160 = 300\text{m}$

$$\text{Relative speed} = 60 + 40 = 100 \text{ km/h} = 100 \times \frac{5}{18} \text{ m/s} = \frac{250}{9} \text{ m/s}$$

$$\therefore \text{Time taken to cross each other} = \frac{300}{\frac{250}{9}} = \frac{300 \times 9}{250} = 10.8\text{s}$$

90. Parth took a loan of ₹12000 at simple interest. After 5 years he paid ₹3600 as interest. What is the rate of interest per annum?

- A. 6%
- B. 8%
- C. 4%
- D. 10%

Ans. A

Sol.: Principle amount = ₹12000

Time period = 5 years

Interest = ₹ 3600

$$\text{Rate} = \frac{3600 \times 100}{12000 \times 5} = \frac{360000}{60000} = 6\%$$

91. Neelam, Supriti, and Pallavi start running around a circular stadium and complete one round in 14s, 12s and 16s respectively. In how much time, they will meet again at the starting points?

- A. 5 min 30 sec
- B. 6 min 36 sec
- C. 5 min 45 sec
- D. 5 min 36 sec
- E. none of these

Ans. D

Sol.: Required time = LCM of 14,12,16 = 336 s = 5 min 36 sec

92. 9 litre mixture contains spirit and water. In this mixture water is 15%. Now if 9 litre mixture is again added and that have 8% water. Find the total percent of spirit = ?

- A.  $12\frac{1}{2}\%$
- B.  $11\frac{1}{2}\%$
- C.  $98\frac{1}{2}\%$
- D.  $88\frac{1}{2}\%$

Ans. D

Sol.: Water is  $\rightarrow \frac{9 \times 15}{100} = \frac{1.35}{100} = 1.35$

Water is  $\rightarrow \frac{9 \times 8}{100} = 0.72$

Water =  $\frac{1.35 + 0.72}{9 + 9} = 11.5\%$  Spirit = 88.5%

93. An inlet P can fill a tank in 30 minutes. 5 other inlets, each of 25% efficiency of P can fill tank in what time?

- A. 24 minutes
- B. 10 minutes
- C. 30 minutes
- D. 45 minutes

Ans. A

Sol.: Since other inlets are of 25% efficiency of P so they take  $30 \times 4 = 120$  minutes each to fill the tank.

So, in 1 min other inlet will fill =  $1/120$ .

But since there are 5 inlets, they will fill =  $(1/120) \times 5 = 1/24$

So, together they can fill the inlet completely in 24 minutes.

94. The ratio of the present ages of P and Q is 8 : 5. After 6 years their ages will be in the ratio of 3 : 2. Find the ratio of the sum and difference of the present ages of P and Q.

- A. 9:39
- B. 39:9
- C. 39:19
- D. 29:9

Ans. B

Sol.: Accordingly

$$(8x+6)/(5x+6) = 3/2$$

$$15x+18 = 16x+12$$

$$x = 6$$

hence ages will be 48 and 30

$$\text{Ration of sum and difference} = 48 + 30 : 48 - 30 = 78 : 18 = 39:9$$

95. The compound interest on Rs. 24000 at 10% per annum for 3/2 years, interest being compounded semi annually is

A. Rs.3783

B. Rs.3777

C. Rs.3780

D. Rs.3781

Ans. A

Sol.: Formula for CI when it's not compounded annually:

$$A = P \left( 1 + \frac{r}{100 \times n} \right)^{nt}$$

Where A is the amount at the end of time t, P is the principal, t is time, r is rate and n is number of times per year interest is compounded.

According to the question:

The compound interest on Rs. 24000 at 10% per annum for 1 1/2 years, interest being compounded semi annually

$$A = P \left( 1 + \frac{r}{100 \times n} \right)^{nt}$$

$$\Rightarrow A = 24000 \left( 1 + \frac{10}{100 \times 2} \right)^{2 \times 3/2}$$

$$\Rightarrow A = 24000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = 27,783$$

$$\text{Compound interest} = \text{Amount} - \text{Principal} = 27,783 - 24000 = 3783$$

96. **Directions:** Study the information carefully to answer the following questions

A kindergarten consists of 7500 students. The ratio of number of boys to number of girls is 3 : 2. All the students play five different games -Hockey, Cricket, Tennis, Football and Volleyball. 28% of girls play Hockey. 16% of the boys play Tennis. One-fifth of the boys play Volleyball. The ratio of number of girls to number of boys, who play Tennis is 2 : 3 respectively. 25% of the total numbers of students play Cricket. Number of girls, who play Volleyball is 60% of the number of boys, who play Volleyball. 22% of the girls play Football. The rest girls play Cricket. 18% of the boys play Hockey and the rest boys play Football. Number of boys in Hockey forms what per cent of the number of girls in the same game? (rounded off to two digits after decimal)

A. 96.43

B. 113.70

C. 90.36

D. 117.43

E. 128.91

Ans. A

Sol.:

Games	Girls	Boys
Hockey	840	810
Cricket	480	1395
Tennis	480	720
Football	660	675
Volleyball	540	900

$$\text{Required percentage} = \frac{810}{840} \times 100 = 96.43\%$$

97. What is the number of girls in Cricket and Hockey?

- A. 1380
- B. 1220
- C. 1320
- D. 1200
- E. None of these

Ans. C

Sol.: Required number of girls in Cricket and Hockey = (480 + 840) = 1320

98. Number of boys in Cricket forms what per cent of total number of the students in the school?

- A. 16.8%
- B. 9.75%
- C. 12.25%
- D. 16%
- E. 18.6%

Ans. E

Sol.: Required percentage =  $\frac{1395}{7500} \times 100 = 18.6\%$

99. What is the total number of boys in Tennis and Volleyball together?

- A. 1380
- B. 1620
- C. 1400
- D. 1520
- E. None of these

Ans. B

Sol.: Total number of boys in Tennis and Volleyball = (720+900) = 1620

100. What is the number of boys in Football?

- A. 800
- B. 570
- C. 640
- D. 675
- E. None of these

Ans. D

Sol.: Total number of boys in Football = 675

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