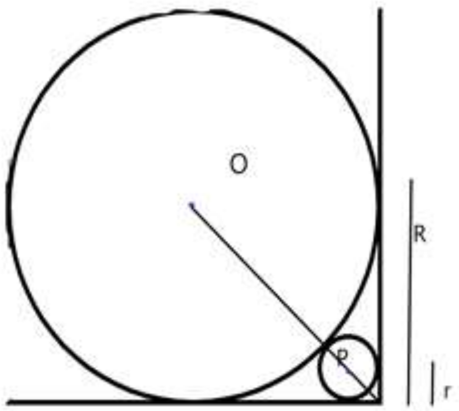
CDS I 2020 Mathematics Solution

1. Ans. D.



Let a circle of radius R = 8 cm and centre O toches the axis and a small circle of radius r and centre at P are in the gap.

Let the distance between the small circle and the origin is x.

Then in small circle

$$r^2+r^2=(r+x)^2$$
$$x=(\sqrt{2}-1)r$$

And in big circle

$$R^2+R^2=(R+2r+x)^2$$
$$\sqrt{2}R=R+2r+x$$
$$(\sqrt{2}-1)8=2r+(\sqrt{2}-1)r$$
$$(\sqrt{2}-1)8=(1+\sqrt{2})r$$
$$r=\frac{8(\sqrt{2}-1)}{(1+\sqrt{2})}\frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}$$
$$r=\frac{8(2+1-2\sqrt{2})}{2-1}$$
$$r=8(3-2\sqrt{2})cm$$

2. Ans. C.

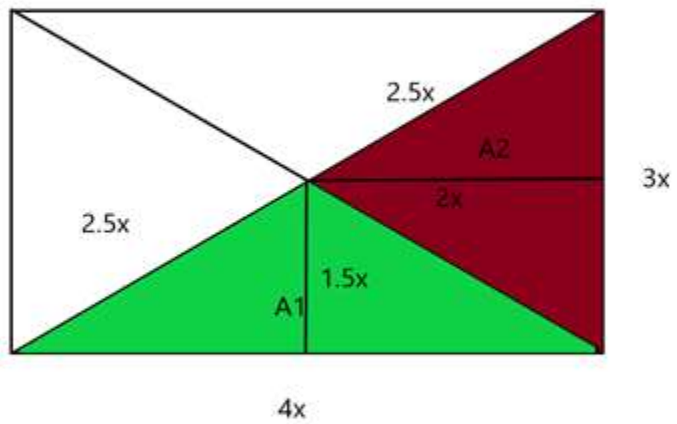
Let the side of the square is a, then one side of the rectangle is a^2, and another side is b.

According to the question, area of square = area of the rectangle

$$a^2 = a^2 \times b$$
$$b = 1$$

Another side of the rectangle is of unit length.

3. Ans. D.



If the length and breadth of a rectangle are in the ratio 4: 3.

Let the sides of the rectangle are 4x and 3x.

Now the diagonal of the rectangle $= \sqrt{(4x)^2 + (3x)^2} = 5x$

The sides of the big triangle are 2.5x,2.5x and 4x

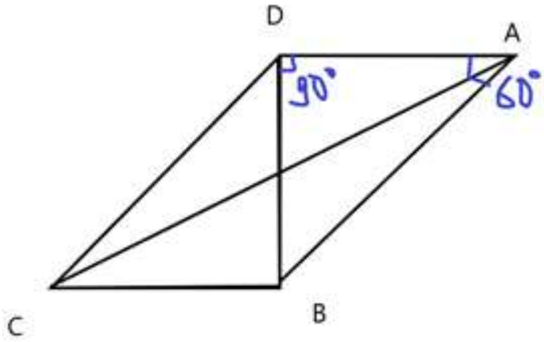
And area = $\frac{1}{2} \times 1.5x \times 4x = 3x^2$

The sides of the small triangle are 2.5x,2.5x and 3x

And area $= \frac{1}{2} \times 2x \times 3x = 3x^2$

So the required ratio is 1:1

4. Ans. B.



In the right-angled triangle ABD

$$\sin 60^\circ = \frac{BD}{AB}$$
$$\frac{\sqrt{3}}{2} = \frac{BD}{AB}$$
$$BD = \frac{\sqrt{3}}{2}AB$$
$$BD^2 = \frac{3}{4}AB^2$$

5. Ans. A.

Let a parallelogram ABCD where line AQ meets at the middle point of CD at P.

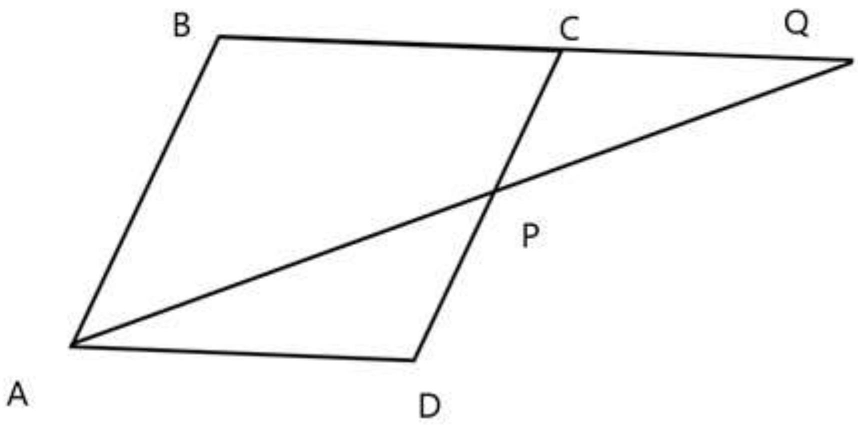
i. Since BC and AD is Parallel then $\angle PAD = \angle CQP$

And CP = PD, both the triangle APD and CPQ are congruent. So their areas are equal.

ii. In the triangle AQB and CPQ

Angle BQA = Angle CQP = same angle

Since the triangle PCQ is a part of the triangle BQA. So their areas can’t equal.



6. Ans. C.

The lengths of sides of a triangle are 3x, $4\sqrt{y}$, $5\sqrt[3]{z}$, where 3x< $4\sqrt{y}$ <5 $\sqrt[3]{z}$. Since it makes a right-angle triangle. Then,

$$\left(5\sqrt[3]{z}\right)^2 = (3x)^2 + \left(4\sqrt{y}\right)^2$$

$$25z^{\frac{2}{3}} = 9x^2 + 16y$$

If the value of x,y and z are 1 then it satisfies the condition of the right-angle triangle.

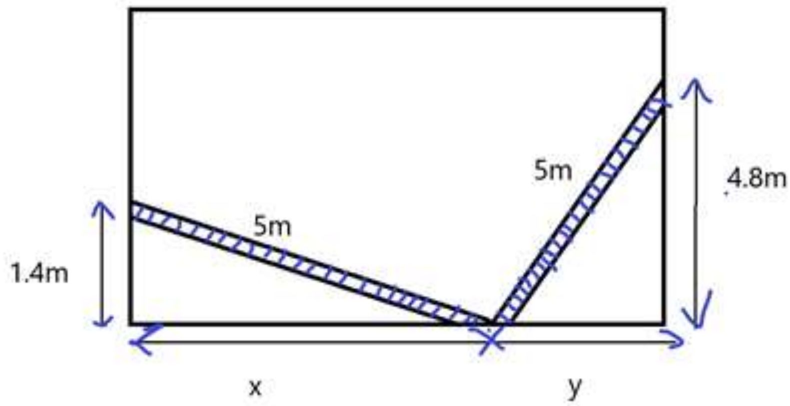
7. Ans. A.

A triangle has a maximum of 1 circum-circle that touches all the vertex of the triangle.

8. Ans. A.

The length of the arc $= r\theta = 6\times(30^\circ \times \frac{\pi}{180^\circ}) = \pi\, cm$

9. Ans. C.



As we can see in the above figure, the ladder makes a right-angle triangle, and the sum of the base of the triangle is the breath of the room.

$$x=\sqrt{5^2-1.4^2} \quad and \quad y=\sqrt{5^2-4.8^2}$$
$$x=4.8 \quad and \quad y=1.4$$

So the breath of the room = x + y = 4.8 + 1.4 = 6.2m

10. Ans. D.

If a square of maximum area is cut from the circular disk, then the vertex of the square touch the circle.

Let the side of the square is a, and the radius of circle is 1 unit, then

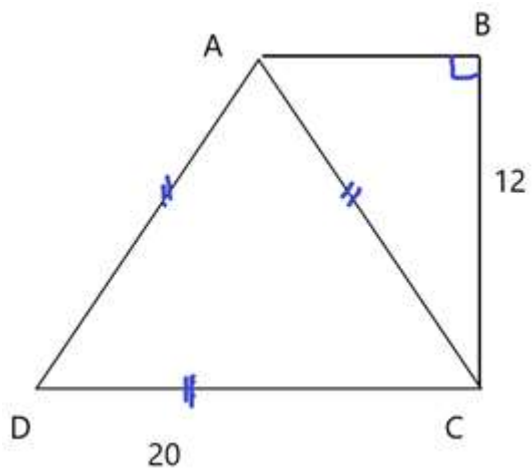
The diagonal of the square = diameter of the circle

$$\sqrt{2}a=2\times1$$
$$a=\sqrt{2}$$

So the area of the largest square plate $=a^2=\left(\sqrt{2}\right)^2=2\ unit^2$

11. Ans. A.

Let ABCD is a quadrilateral such that AD = DC = CA = 20 units, BC = 12 units and $\angle ABC$ = 90°



Since we can see tha triangle ABC are a triangle and ABC is a right angle triangle. In right angle triangle ABC

$$AB^2 + BC^2 = AC^2$$
$$AB^2 + 12^2 = 20^2$$
$$AB = \sqrt{400-144} = \sqrt{256} = 16\,units$$

Now the total area

$$= \triangle ABC + \triangle ACD$$
$$= \frac{1}{2} \times 12 \times 16 + \frac{\sqrt{3}}{4} \times 20^2$$
$$= 96 + 173$$
$$= 269\,unit^2$$

12. Ans. A.

The radius of the circle is 2 cm. when we join the centre of each circle it makes equilateral triangle

The area of the circle which is inside the circle = $= \frac{60^\circ}{360^\circ} \times \pi \times 2^2 = \frac{2}{3}\pi$

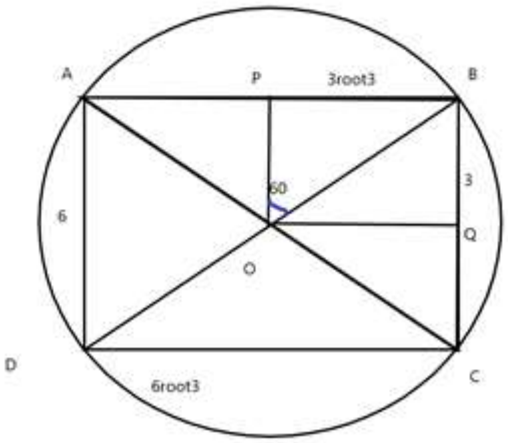
The ares of the shaded region = area of triangle – area of the part of the circle

$$= \frac{\sqrt{3}}{4} 4^2 - 3\left(\frac{2}{3}\pi\right)$$
$$= 4\sqrt{3} - 2\pi\ cm^2$$

13. Ans. B.

The sides of the rectangle are $6\sqrt{3}$ and 6 unit

Radius of the circle $= \frac{1}{2}(\text{diagonal of rectangle}) = \frac{1}{2}\left(\sqrt{\left(6\sqrt{3}\right)^2 + 6^2}\right) = 6\,unit$



The angle at the centre by long arc

$$\tan\theta = \frac{PB}{PO} = \frac{3\sqrt{3}}{3} = \sqrt{3} = \tan 60^\circ$$
$$\theta = 60^\circ$$

So $\angle AOB = 2\times 60^\circ = 120^\circ$

The shaded area = Area of part of circle OAB – Area of triangle AOB

$$= \frac{120^\circ}{360^\circ}\times\left(\pi\times 6^2\right) - \frac{1}{2}\times 3\times 6\sqrt{3}$$
$$= 12\pi - 9\sqrt{3}$$
$$= 3\left(4\pi - 3\sqrt{3}\right)unit^2$$

14. Ans. C.

if $\frac{y}{x}$ = 6 and $\frac{z}{x}$ = 5, then y = 6x and z = 5x,

As we can see that, x + y + z = 180°

or x + 6x + 5x =180°

or 12x = 180°

or x = 15°

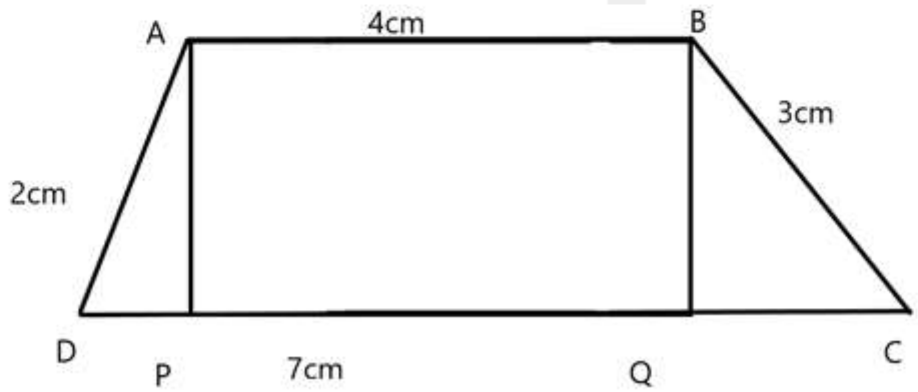
15. Ans. D.

In the trapezium AB = PQ = 4 cm

Let AP = BQ = h and DP = x, then QC = CD – DP - PQ = 7 – 4 – x = 3 – x

Since APD and BCQ is right angle triangle,so

$$h^2 = 2^2 - x^2 = 3^2 - (3-x)^2$$
$$4 - x^2 = 9 - (9 - 6x + x^2)$$
$$4 - x^2 = 6x - x^2$$
$$6x = 4$$
$$or \quad x = \frac{2}{3}$$
$$hence,\ h = \sqrt{4 - x^2} = \sqrt{4 - \frac{4}{9}} = \sqrt{\frac{32}{9}} = \frac{4\sqrt{2}}{3}$$



Now, the area of the trapezium

$$= \frac{1}{2}(sum\ of\ the\ parallel\ sides) \times height$$
$$= \frac{1}{2} \times (4+7) \times \frac{4\sqrt{2}}{3}$$
$$= \frac{22\sqrt{2}}{3} cm^2$$

16. Ans. B.

In each triangle, 3rd angle = 180° - sum of other two angle

Since opposite angle are equal then,

$$\text{Sum of the angle of a quadrilateral} = 360^\circ$$
$$(180^\circ - \angle 1 - \angle 2) + (180^\circ - \angle 3 - \angle 4) + (180^\circ - \angle 5 - \angle 6) + (180^\circ - \angle 7 - \angle 8) = 360^\circ$$
$$720^\circ - (\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8) = 360^\circ$$
$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 720^\circ - 360^\circ = 360^\circ$$

17. Ans. B.

Since PQ and RS are parallel then Angle AEF = Angle EGH $=95^\circ$

Angle BGH = $180^\circ - 95^\circ = 85^\circ$

Angle BHG = $180^\circ - 110^\circ = 70^\circ$

Now in triangle BGH, Angle BGH +Angle GHB + Angle GBH = 180°

$85^\circ + 70^\circ + x = 180^\circ$

$x = 180^\circ - 155^\circ$

$x = 25^\circ$

18. Ans. D.

Given that AC = 25 units

Since F is the mid point of AC , so Af = FC = 12.5 units

Radius of semicircle ABC = 25/2 = 12.5 unit

Radius of semicircle AEF and CDF = 12.5/2 = 6.25 cm

Total area of the figure

$$= \frac{1}{2}\left(\pi \times (12.5)^2 + \pi \times (6.25)^2 + \pi \times (6.25)^2\right)$$
$$= \frac{1}{2}\pi(234.375)$$
$$= 368.15 \ unit^2$$

19. Ans. D.

The sides of the rectangle are 8cm and 6cm, the the diagonal of the rectangle

$= \sqrt{8^2 + 6^2} = 10$

And the radius of the circle = $\frac{1}{2}\text{diagonal of the rectangle} = \frac{1}{2}(10) = 5cm$

Area of the shaded region = Area of the circle – Area of the rectangle

$= \pi \times 5^2 - 8 \times 6 = 30.53 cm^2$

20. Ans. A.

We know that if we add one to the multiplication of prime numbers then result obtained is also a prime number.

Consider 2x3x5x7x11+1

Note : 2,3,5,7 and 11 all are prime numbers and 1 is also added to their product.

So, 2x3x5x7x11+1 is a prime number.

21. Ans. B.

Let x = $(54)^{10}$

Taking logarithm both sides

$\log x = \log(54)^{10}$

$\Rightarrow \log x = 10 \times \log(2 \times 3^3)$

$\Rightarrow \log x = 10 \times \{\log 2 + 3\log 3\}$

$\Rightarrow \log x = 10 \times \{0.301 + 3(0.477)\}$

$\Rightarrow \log x = 17.32$

Since, the characteristic of log x is 17

So number of digits in $(54)^{10}$ = 17 + 1 = 18

22. Ans. D.

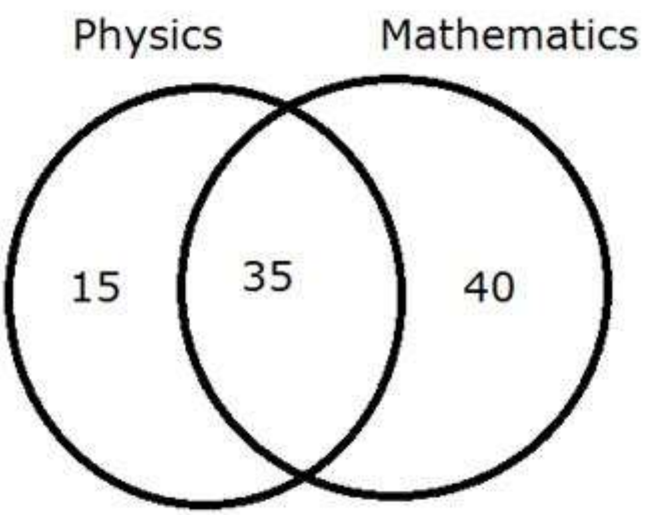
Total students registered = 250 students

Students registered for Maths = 75

Students registered for Physics = 50

Students registered for both subjects = 35

Venn Diagram:



Number of Students registered for at least one subject = 15+35 +40 = 90

Hence , Number of students who have registered neither for Physics nor for Mathematics = 250-90 = 160

23. Ans. A.

Consider 2222^2, 222^{22}, 22^{222}, 2^{2222}

Take logarithm

$\Rightarrow \log(2222)^2, \log(222)^{22}, \log(22)^{222}, \log(2)^{2222}$

$\Rightarrow 2\log(2222), 22\log(222), 222\log(22), 2222\log(2)$

$\Rightarrow 2(\log 2\times 1111), 22\log(2\times 111), 222\log(2\times 11), 2222\log 2$

$\Rightarrow 2\{\log 2+\log 1111\}, 22\{\log 2+\log 111\}, 222\{\log 2+\log 11\}, 2222\log 2$

Now, log 2 = 0.301

log (1111) = 3(approx)

log(111) = 2 (approx)

log(11) = 1 (approx)

If we put these values , we can clearly see that $2222\log 2$ is largest.

⇒ 2^{2222} is largest.

24. Ans. A.

Let unit digit of number = y

Ten's digit of a number = x

Number = 10x + y............(1)

If digits are interchanged then

unit digit of number = x

Ten's digit of a number = y

Interchanged Number = 10y + x............(2)

According to question

10x + y = 4(10y + x)+9

⇒ 10x +y = 40 y + 4x +9

⇒ 6x-39y = 9

⇒ 2x – 13y = 3................(3)

Also, product of digits in the two-digit number is 8

⇒ xy = 8

⇒ $y=\frac{8}{x}$ ....(4)

Put value of y from (4) to equation (3)

$2x-13\left(\frac{8}{x}\right)=3$

⇒ $2x^2 - 3x - 104 = 0$

Solve quadratic equation usong quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

⇒ $$x == \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2 \times -104)}}{2 \times 2}$$

⇒ x = 8, $-\frac{13}{2}$

As x is a digit so only possible value of x = 8.

Put value of x in equation (4)

y = 1

Required Number = $10(8) + 1 = 81$

25. Ans. A.

Given α and ß are the roots of the quadratic equation $x^2+kx-15=0$

Sum of roots = $-\frac{b}{a} = -k$

⇒ α +ß = -k ……………….(1)

Product of roots = $\frac{c}{a} = -15$

⇒ (α.ß) = - 15……………(2)

Also , α-ß=8 (Given)

We know that $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

⇒ $64 = k^2 - 4(-15)$

⇒ $k^2 = 64 - 60 = 4$

⇒ $k = \pm 2$

According to question , k =2

26. Ans. B.

If (x+k) is the HCF of x^2+5x+6 and $x^2+8x+15$ then

x = -k will satisfy both equations x^2+5x+6 and $x^2+8x+15$

Put x = -k in equation x^2+5x+6

⇒ $k^2-5k+6=0$……………(1)

Put x = -k in equation $x^2+8x+15$

⇒ $k^2-8k+15=0$………..(2)

Subtract (1) from (2)

⇒ -3k +9 = 0

⇒ -3k = -9

⇒ k = 3

27. Ans. A.

Consider $x^2+ 9y^2=6xy$

⇒ $x^2+9y^2-6xy=0$

⇒ $(x-3y)^2=0$

⇒ $x=3y$

⇒ $\frac{y}{x}=\frac{1}{3}$

Hence , y:x = 1:3

28. Ans. B.

Consider $\frac{1}{a^{m-n}-1}+\frac{1}{a^{n-m}-1}$

$\Rightarrow \frac{1}{\frac{a^m}{a^n}-1}+\frac{1}{\frac{a^n}{a^m}-1}$

$\Rightarrow \frac{a^n}{a^m-a^n}+\frac{a^m}{a^n-a^m}$

$\Rightarrow \frac{a^n}{a^m-a^n}-\frac{a^m}{a^m-a^n}$

$\Rightarrow \frac{a^n-a^m}{a^m-a^n}=-\left(\frac{a^m-a^n}{a^m-a^n}\right)=-1$

29. Ans. B.

Consider x=$\sqrt{2}$, y=$\sqrt[3]{3}$ and z=$\sqrt[6]{6}$

$x=(2)^{\frac{1}{2}}, y=(3)^{\frac{1}{3}}$ & $z=(6)^{\frac{1}{6}}$

Now we will take LCM of 2,3,6

⇒ LCM(2,3,6) = 6

$x^6=(2)^{\frac{6}{2}}, y^6=(3)^{\frac{6}{3}}$ & $z^6=(6)^{\frac{6}{6}}$

$x^6=(2)^3, y^6=(3)^2$ & $z^6=(6)^1$

$x^6=8, y^6=9$ & $z^6=6$

Clearly, $z^6<x^6<y^6$ …………(1)

Also x , y and z all are positive numbers.

So from equation (1) we can conclude that z < x < y

30. Ans. C.

Given y=$x^{\log x}$

$\log y=\log\left(x^{\log x}\right)=\log x.\log x=(\log x)^2$

But log x=1.2500

Hence, log y= $(1.2500)^2$ = 1.5625

31. Ans. C.

Area of a square = $2401x^4+196x^2+4$ = $\left(49x^2+2\right)^2$

We know that , Area of square = $(side)^2$

$\Rightarrow (side)^2 = \left(49x^2+2\right)^2$

$\Rightarrow (side) = \left(49x^2+2\right)$

32. Ans. C.

Given, x varies as yz

$x \propto yz$

$\Rightarrow y \propto \frac{x}{z}$

$\Rightarrow y \propto \frac{1}{\frac{z}{x}}$

y varies inversely as $\frac{z}{x}$

33. Ans. B.

5x+2y=7xy ……………(1)

10x+3y=8xy………………(2)

Multiply equation (1) by 2

10x+4y=14xy ……………(3)

Subtract equation (3) from equation (2)

⇒ - y = - 6 xy

⇒ y = 6xy

⇒ x = $\frac{1}{6}$

Put value of x in equation (1)

⇒ $5\left(\frac{1}{6}\right)+2y=\frac{7}{6}y$

⇒ $2y-\frac{7}{6}y=-\frac{5}{6}$

⇒ $\frac{5}{6}y=-\frac{5}{6}$

⇒ $y=-1$

34. Ans. D.

Let number of rows= x

Number of students in each row = y

Hence, Number of students = xy

If number of students is increased by 2 in each row, then the number of rows decreases by 3

⇒ (x-3)(y+2)=xy...........(1)

⇒ $xy+2x-3y-6=xy$

⇒ $2x-3y-6=0$................(2)

If number of students is increased by 4 in each row, then the number of rows decreases by 5

⇒ (x-5)(y+4)=xy

⇒ $xy+4x-5y-20=xy$

⇒ $4x-5y-20=0$............(3)

Multiply equation (2) by 2 and then subtract from equation (3)

⇒ y-8=0

⇒ y=8

Put the value of y in (2)

⇒ x = 15

Hence number of students in the class = xy = $(15).(8)=120$

35. Ans. A.

Consider 3, 5, 9, 4, 6, 11, 18

Arrange the data in ascending order

3,4,5,6,9,11,18

Here, Total number of observation (n) = 7

As n is odd , median = $\left(\frac{n+1}{2}\right)^{th}$

$\left(\frac{n+1}{2}\right)^{th} = \left(\frac{7+1}{2}\right)^{th} = 4^{th}$

4^{th} term is 6.

Hence, median will be 6.

36. Ans. B.

Ratio of the angles of the sectors is 1 : 2 : 3

Let angle of the first sector = x

Angle of the second sector = 2x

Angle of the third sector = 3x

We know that complete angle of a pi-diagram is 360°

⇒ x + 2x + 3x = 360°

⇒ 6x = 360°

⇒ x = 60°

Angle of the largest sector = 3x = $3\times 60^\circ = 180^\circ$

37. Ans. C.

Given the maximum marks in a Test are converted from 250 to 50 for the purpose of an Internal Assessment.

$\Rightarrow \frac{50}{250}=\frac{1}{5}$

This means marks are reduced to $\frac{1}{5}$th for the purpose of an Internal Assessment.

The highest marks scored were 170 and lowest marks were 70.

For internal assessment:

Equivalent of 170 = $\frac{170}{5}=34$

Equivalent of 70= $\frac{70}{5}=14$

Required Difference = 34 – 14 = 20

38. Ans. B.

Total released convicts from Jail A = 86 + 45 +25 = 156

% of trained convicts of jail A = $\frac{45}{156}\times 100=28.84\%$

Total released convicts from Jail B = 1305 + 903 +461 = 2669

% of trained convicts of jail B = $\frac{903}{2669}\times 100=33.83\%$

Total released convicts from Jail C = 2019 +940+474=3433

% of trained convicts of jail C = $\frac{940}{3433}\times 100=27.38$

Total released convicts from Jail D =1166 +869+416 =2451

% of trained convicts of jail D = $\frac{869}{2451}\times100=35.45\%$

Total released convicts from Jail E =954+544+254=1752

% of trained convicts of jail E = $\frac{544}{1752}\times100=31.05\%$

Total released convicts from Jail F = 1198+465+174=1837

% of trained convicts of jail F = $\frac{465}{1837}\times100=25.31\%$

Jails with highest and smallest percentage of trained convicts are respectively are D and F.

39. Ans. D.

Placement rate of trained convicts of Jail A= $\frac{25}{45}\times100=$ 55.55

Placement rate of trained convicts of Jail B= $\frac{461}{903}\times100$ =51.05

Placement rate of trained convicts of Jail C= $\frac{474}{940}\times100$ =50.43

Placement rate of trained convicts of Jail D= $\frac{416}{869}\times100$ =47.87

Placement rate of trained convicts of Jail E= $\frac{254}{544}\times100$ =46.69

Placement rate of trained convicts of Jail F= $\frac{174}{465}\times100$ =37.41

Clearly, Jail A have highest placement rate of trained convicts

40. Ans. A.

Trained convicts of Jail A = 45

Half of the trained convicts = 22.5

Convicts who got job from Jail A = 25

Trained convicts of Jail B = 903

Half of the trained convicts = 451.5

Convicts who got job from Jail B = 461

Trained convicts of Jail C = 940

Half of the trained convicts = 470

Convicts who got job from Jail C = 474

Trained convicts of Jail D = 869

Half of the trained convicts = 434.5

Convicts who got job from Jail D = 416

Trained convicts of Jail E = 544

Half of the trained convicts = 272

Convicts who got job from Jail E = 254

Trained convicts of Jail E = 465

Half of the trained convicts = 232.5

Convicts who got job from Jail E = 174

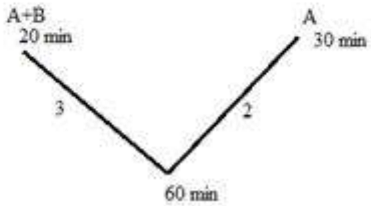
Jails from which more than half of the trained convicts are offered jobs, are A , B , C

41. Ans. D.

1^{st} outlet :A & 2^{nd} outlet: B

A & B both opened together takes 20 min. to empty the tank

While when A opens, it takes 30 min. to empty the tank.



B takes : 1 unit à60 min

Hence, option D is correct.

2^{nd} method:

By using the formula

y is the time taken when both outlets are open

And x is the time taken by 1^{st} outlet.

Then B takes = $\frac{xy}{x-y} = \frac{30\times 20}{30-20} = 60$ min.

42. Ans. C.

Given: $(x^8 + \frac{1}{x^8}) = 47$

$(x^8 + \frac{1}{x^8}) + 2 = 47 + 2$

Make perfect square

Then $(x^4 + \frac{1}{x^4}) = 7$

Again add 2 both side and try to make a perfect square

$(x^4 + \frac{1}{x^4}) + 2 = 7 + 2$

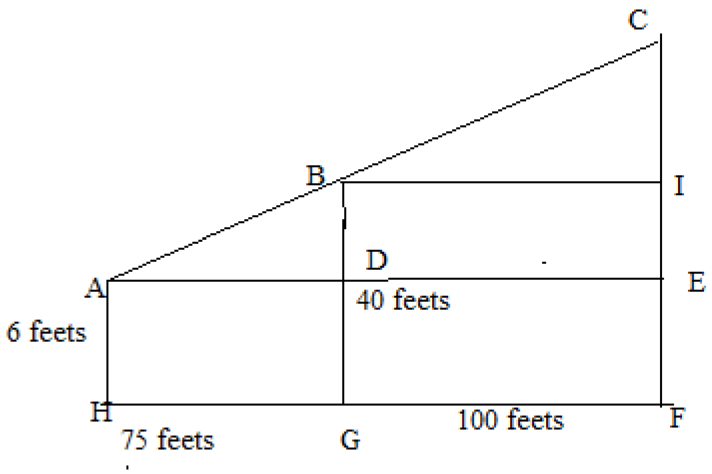
Then you will get, $(x^2 + \frac{1}{x^2}) = 3 = k$

Then you know that $(x^6 + \frac{1}{x^6})^3 = k^3 - 3k = 3^3 - 3x3$

$= 27 - 9 = 18$

Hence, option C is correct.

43. Ans. A.



Here, angle CBI = angle BAD

Then $\frac{BD}{AD} = \frac{CI}{BI}$, BD = 40 – 6 = 34 cm

$$\frac{34}{75} = \frac{CI}{100}$$
$$CI = \frac{100}{75} \times 34$$
$$CI = 45.33$$

Then, IF = 40 feets

Now, CF = IF + CI

CF = 40 + 45.33

CF = 85 feets (approximately)

Hence option A is correct.

44. Ans. D.

Given: $\sin^2 6° + \sin^2 12° + \sin^2 18° + ... + \sin^2 84° + \sin^2 90°$

$\sin^2 90° = 1$,

$\sin^2 6° + \sin^2 12° + \sin^2 18° + ... + \sin^2 84° + 1$

obseve angles are in A.P : 6, 12, 18, 24, 30, 36, 42 ………., 84

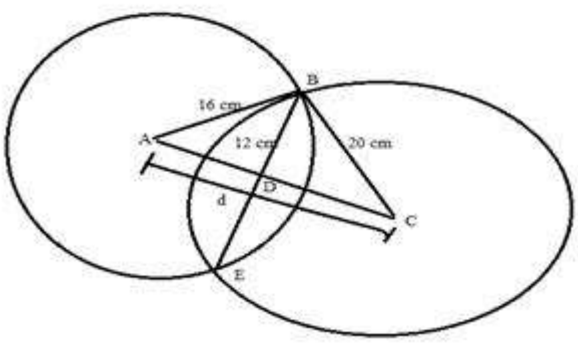
convert last half of sin term in to cos term, i.e,

$(\sin^2 6^\circ + \cos^2 6^\circ) + (\sin^2 12^\circ + \cos^2 12^\circ) + ... + (\sin^2 42 + \cos^2 42)$ +1

1+1+1....+1 = 8

Hence option D is correct.

45. Ans. B.



We have AD = 12 cm, BD ⊥ AC (property)

Now just apply the Pythagoras theorem in triangles ABD and BDC to get AD and DC respectively.

In triangle ADB: AD = $\sqrt{AB^2 - BD^2} = \sqrt{16^2 - 12^2} = \sqrt{112}$

$10 < \sqrt{112} < 11$

Similarly, DC = 16 cm (use Pythagoras in ADB)

AC (D) = AD + DC = $\sqrt{112}$ + 16 cm

26 cm < d < 27 cm

Hence, option B is correct.

46. Ans. A.

Length = 48 cm

And breadth = 14 cm. it is given that the diagonal makes an angle θ with the longer side

Then, $\tan\theta = \frac{perp.}{base} = \frac{14}{48} = \frac{7}{24}$

Perpendicular = 14 cm, base = 48 cm then we can find hypotenuse by Pythagoras theorem

Hypotenuse (H) = $\sqrt{perp.^2 + base^2}$

$\sqrt{14^2 + 48^2} = 50$

H = 50 cm

$\sqrt{perp.^2 + base^2}$

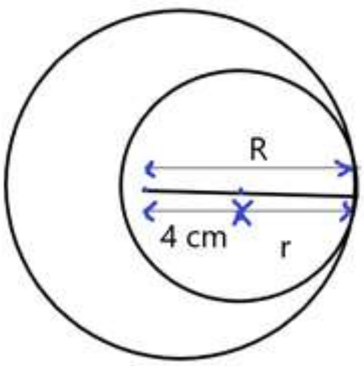
$\sec\theta = \frac{H}{Base}$ & $\operatorname{cosec}\theta = \frac{H}{perp.}$

$\sec\theta = \frac{50}{48}$ & $\operatorname{cosec}\theta = \frac{50}{14}$

$\sec\theta + \operatorname{cosec}\theta = \frac{50}{48} + \frac{50}{14} = \frac{775}{168}$

So, option A is correct.

47. Ans. B.



Let a circle of radii R and a small circle of radii r toches internally.

Hence R= r+4 (i)

Acc. to question, sum of their areas is 136πcm^2

$$\pi R^2 + \pi r^2 = 136\pi$$
$$R^2 + r^2 = 136$$
$$from\ \ eqn\ (i)$$
$$(r+4)^2 + r^2 = 136$$
$$r^2 + 4r - 60 = 0$$
$$(r+10)(r-6) = 0$$

(r-6) = 0 or r = 6

Hence the radii of the small circle is 6 cm, and the big circle is 10 cm.

48. Ans. C.

Let the radius of the circle is r and the sides of the square is a.

Then according to the question

Area of circle = Area of the square

$$\pi r^2 = a^2$$
$$\frac{r^2}{a^2} = \frac{1}{\pi}$$
$$\frac{r}{a} = \sqrt{\frac{1}{\pi}}$$

Hence the ratio of their perimeter is

$$\frac{2\pi r}{4a} = \frac{\pi}{2}\frac{r}{a} = \frac{\pi}{2}\sqrt{\frac{1}{\pi}} = \frac{\sqrt{\pi}}{2}$$

49. Ans. C.

If the thickness of cylinder is 1 foot and the inner radius of the cylinder is r1 = 3 feet, then the outer radius is r2 = 4 feet. And height is h = 7 feet.

The inner surface of the circle is

$$= 2\pi r_1 l = 2\pi \times 3 \times 7 = 42\pi\, feet^2 \cong 1\ liter\ paint$$
$$1\, feet^2 \cong \frac{1}{42\pi}\ liter\ paint$$

The otal surface of the cylinder is

$$=\text{innersurface}+\text{outer surface}+\text{base surface}+\text{upper surfce}$$
$$=2\pi r_1h+2\pi r_2h+2\pi\left(r_2^{\,2}-r_1^{\,2}\right)$$
$$=2\pi(r_1h+r_2h+r_2^{\,2}-r_1^{\,2})$$
$$=2\pi(3\times7+4\times7+4^2-3^2)$$
$$=2\pi\times56$$
$$=112\pi\ feet^2$$

So total required point is

$$=112\pi\times\frac{1}{42\pi}=\frac{8}{3}\ litre$$

50. Ans. D.

The radius of the circle is 30 feet.

Then the area of the circle $=\pi(30)^2=900\pi\ feet^2$

If a sector of 20° is removed, then the only sector of 340° area remains.

$$\text{If the area for the } 360^\circ \text{ is } 900\pi\text{feet}^2$$
$$\text{then the area for the sector of } 340^\circ=\frac{900\pi}{360^\circ}\times340^\circ\text{feet}^2$$
$$=850\pi\text{feet}^2$$

51. Ans. D.

When we dug in such a way, then it makes a cylinder

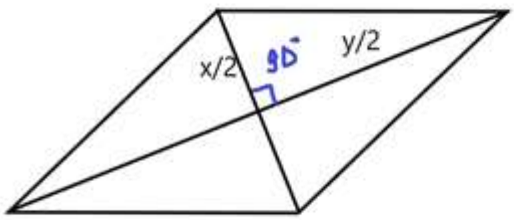
The cubic metre volume of earth to be dig is

$$=\pi r^2h=\pi(1.4)^2\times5=30.8\ cubic\ metre$$

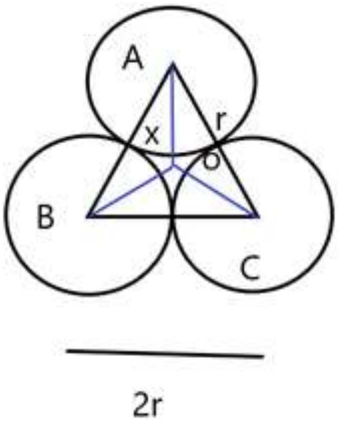
52. Ans. A.

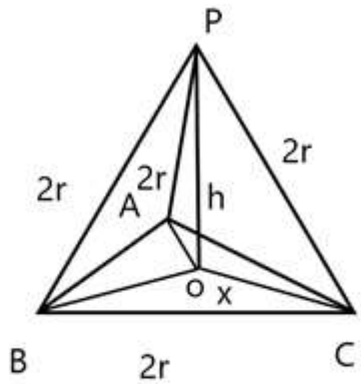
Let a rhombus which diagonals are x and y. Since the diagonal of rhombus cut each other at right angle and at the midpoint. Then area of the rhombus

$$=4(Area\ of\ a\ part)$$
$$=4\left(\frac{1}{2}\times\frac{x}{2}\times\frac{y}{2}\right)=\frac{xy}{2}$$



53. Ans. A.





let A, B and C are 3 balls of radius r are placed in the plane, and 4th ball P is placed on the top of them.

Let h is the distance of centre of ball P from the ground.

In the triangle ABC, let a point Q at the mid point of BC. Since triangle ABC is a equilateral then

Angle ACB = 60° and $\angle$OCB = 30° so in the $\triangle$OCQ

$$\cos 30^\circ = \frac{CQ}{CO}$$

$$\frac{\sqrt{3}}{2} = \frac{r}{x}$$

$$x = \frac{2r}{\sqrt{3}}$$

Again in the triangle POC which also a right angle triangle

$$OP^2 = CP^2 - OC^2$$
$$h^2 = (2r)^2 - x^2$$
$$h^2 = 4r^2 - \left(\frac{2r}{\sqrt{3}}\right)^2$$
$$h^2 = 4r^2 - \frac{4r^2}{3}$$
$$h = \sqrt{4r^2\left(1-\frac{1}{3}\right)}$$
$$h = 2r\sqrt{\frac{2}{3}}$$

54. Ans. A.

Let h is the hight of the cylinder and r is the radius.

If a right circular cylinder just encloses a sphere. Then

height of cylinder = diameter of the sphere = diameter of the cylinder

h=2r

now,

p = the surface area of the sphere = $4\pi r^2$

q = the curved surface area of the cylinder = $2\pi rl = 2\pi r(2r) = 4\pi r^2$

here we can see tha, p = q

55. Ans. B.

The diameter PS is divided in three equal parts

PS = 2(9) = 18 cm

PQ = QR = RS = PS/3 = 6 cm

The area of the shaded region

$$= \frac{1}{2}(area\ of\ big\ circle - area\ of\ small\ circle)$$
$$= \frac{1}{2}(\pi \times 9^2 - \pi \times 6^2)$$
$$= \frac{45}{2}\pi$$

The area of the unshaded region

$$= \frac{1}{2}(area\ of\ big\ circle + area\ of\ small\ circle)$$
$$= \frac{1}{2}(\pi \times 9^2 + \pi \times 6^2)$$
$$= \frac{117}{2}\pi$$

So the required ratio

$$= \frac{45}{2}\pi \Big/ \frac{117}{2}\pi$$
$$= \frac{45}{117} = \frac{5}{13}$$

56. Ans. A.

Since ABCD is a rhombus, then

angle BAC = angle FDG = 55°

now, at point A

angle HAB + Angle BAC + angle CAE = 180°

x + 55° + 40° = 180°

x = 180° - 95°

x = 85°

57. Ans. B.

ABCD is a trapezoid where AD is parallel to BC and perpendicular to the line CD. Hence $\angle$ADC and $\angle BCD$ is right angle.

II. At point O $\angle$AOD and $\angle BOC$ are opposisite angle.

Hence, $\angle$AOD = $\angle BOC$

III. In ΔADC, AD + DC > AC ...(i)

and In ΔBDC, BC + CD > BD ...(ii)

By adding both equation

AD + DC + BC + CD > AC + BD

58. Ans. B.

Given that the product of the two numbers in each pair is 2160 and the HCF of the two numbers in each pair is 12.

Let those numbers are ‘a’ and ‘b’.

HCF(a,b) = 12

We can write, a = 12m and b = 12n where m and n are co-prime.

Also , ab = 2160

⇒ (12m)(12n)=2160

⇒ 144mn = 2160

⇒ $mn = 15$

Now there are two cases

Either m = 5 and n = 3

Or m = 15 and n =1

So , Either a = 60 and b = 36

Or a = 180 and b = 12

Hence possible pairs of unequal number satisfying both the conditions are (60,36) and (180,12)

Now, mean of 60 and 36 (x)= $\frac{60+36}{2} = \frac{96}{2} = 48$

Mean of 180 and 12 (y)= $\frac{180+12}{2}=\frac{192}{2}=96$

Mean of x and y = $\frac{96+48}{2}=\frac{144}{2}=72$

59. Ans. B.

m is the number of prime numbers between 0 and 50

Prime numbers between 0 and 50 =2,3,5,7,11,13,17,19,23,29,31,37,41,43,47

Number of prime numbers between 0 and 50 = 15

⇒ m = 15

Prime numbers between 50 and 100 = 53 , 59 , 61 , 67 , 71 , 73 , 79 , 83 , 89 , 97

Number of prime numbers between 50 and 100 = 10

⇒ n = 10

Hence , m – n = 15 – 10 = 5

60. Ans. C.

$3^x+3^{x+1}+3^{x+2}$ = $3^x(1+3+9)=13(3)^x$

Here, x is a natural number.

Put x = 1

$13(3)^x=13\times3=39$

Put x = 2

$13(3)^x=13(3)^2=13\times9=117$, which is also divisible by 39

Hence, largest divisor of $3^x+3^{x+1}+3^{x+2}$, if x is any natural number will be 39.

61. Ans. A.

If (x-1) and (x+2) divide the polynomial x^3+4x^2+px+q

Then x = 1 and x = -2 both will satisfy equation x^3+4x^2+px+q

Put x= 1 in equation x^3+4x^2+px+q

⇒ $1+4+p+q=0$

⇒ $p+q=-5$ …………(1)

Put x= -2 in equation x^3+4x^2+px+q

⇒ $-8+16-2p+q=0$

⇒ $-2p+q=-8$ …………..(2)

Subtract (2) from (1)

⇒ 3p=3

⇒ p = 1

Put the value of p in equation (1)

q = - 6

62. Ans. D.

Given $5^{x+1}-5^{x-1}=600$………..(1)

Also, $5^4-5^2=625-25=600$…………(2)

From (1) and (2)

⇒ $x+1=4$

⇒ x = 3

Hence , 10^{2x} = $(10)^6=1000000$

63. Ans. C.

f(x) is divided by (x-α) (x-ß) where α≠ß

Then f(x) = (x-α) (x-ß) g(x) + (ax+b) ………(1)

where g(x) is quotient and (ax+b) is linear remainder

Put $x=\alpha$ in (1)

$f(\alpha)=a\alpha+b$.....(2)

Put $x=\beta$ in (1)

$f(\beta)=a\beta+b$.........(3)

Subtract (3) from (2)

$$f(\alpha)-f(\beta)=a\alpha+b-(a\beta+b)$$

$$a=\frac{f(\alpha)-f(\beta)}{\alpha-\beta}$$ ....(4)

Put the value of 'a' in equation (2)

$$f(\alpha)=\left[\frac{f(\alpha)-f(\beta)}{\alpha-\beta}\right]\alpha+b$$

$$\alpha.f(\alpha)-\beta f(\alpha)=\alpha.f(\alpha)-\alpha f(\beta)+b(\alpha-\beta)$$

$$b=\frac{\alpha f(\beta)-\beta f(\alpha)}{\alpha-\beta}$$ ....(5

Remainder = (ax+b) = $\frac{f(\alpha)x-f(\beta)x}{\alpha-\beta}+\frac{\alpha f(\beta)-\beta f(\alpha)}{\alpha-\beta}$ = $\frac{(x-\beta)f(\alpha)-(x-\alpha)f(\beta)}{\alpha-\beta}$

64. Ans. A.

Points P and Q represent real numbers $0.7\overline{3}$ and $0.5\overline{6}$ on the number line.

Let $0.7\overline{3}$ =x

Multiply by 10 both side

7.333333..... = 10x............(1)

Multiply by 10 both side again

⇒ 73.3333333..... = 100x.............(2)

Subtract (1) from (2)

⇒ 90x = 66

⇒ $x = \frac{66}{90} = \frac{11}{15}$

Let $0.5\overline{6}$ = y

Multiply by 10 both side

5.666666….. = 10y…………(1)

Multiply by 10 both side again

⇒ 56.666666….. = 100y…………..(2)

Subtract (1) from (2)

⇒ 90y = 51

⇒ $y = \frac{51}{90} = \frac{17}{30}$

Distance between P and Q = $\frac{11}{15} - \frac{17}{30} = \frac{22}{30} - \frac{17}{30} = \frac{5}{30} = \frac{1}{6}$

65. Ans. C.

Ratio of speeds of X and Y is 5 : 6.

Let speed of X = 5x

Speed of y =6x

Distance travelled by X = (1200-70) = 1130 m

Distance travelled by Y = 1200 m

Time taken by X to travel 1130 m = $\frac{1130}{5x} = \frac{226}{x}$

Time taken by Y to travel 1200 m= $\frac{1200}{6x} = \frac{200}{x}$

As $\frac{226}{x} > \frac{200}{x}$

Hence , Y wins the race.

Let Y completes the race 't' time

And X covers a distance of l metre in 't' time.

If time is constant then Ratio of speed = Ratio of distance

$\Rightarrow \frac{5}{6} = \frac{l}{1200}$

$\Rightarrow l = 1200 \times \frac{5}{6} = 1000$

Hence , Distance by which Y wins the race = 1200 – 1000-70 = 130 m

66. Ans. D.

Let usual speed of train = x km/hr

Distance travelled = 300 km

Time taken by train = $\frac{300}{x}$ hours

It is given that train takes two hours less for a journey of 300 km if its speed is increased by 5 km/hr from is usual speed.

New Speed = (x+5) km/hr

So, time taken by train = $\frac{300}{x+5}$ hours

According to question

$$\frac{300}{x} - \frac{300}{x+5} = 2$$

$$300(x+5) - 300(x) = 2(x)(x+5)$$

$$300x + 1500 - 300x = 2x^2 + 10x$$

$\Rightarrow 2x^2+10x-1500=0$

$\Rightarrow x^2+5x-750=0$

$\Rightarrow x^2+30x-25x-750=0$

$\Rightarrow x(x+30)-25(x+30)=0$

$\Rightarrow (x+30)(x-25)=0$

⇒ x = - 30 or x = 25

But speed can not be negative.

Hence , x = 25 km/hr

Usual speed = 25 km/hr

67. Ans. A.

Let a man’s one day work = x unit

Woman’s one day work = y unit

6 men and 8 women can do a piece of work in 10 days

Work done by 6 men and 8 women in one day = $\frac{1}{10}$

⇒ 6x + 8y = $\frac{1}{10}$ ……………..(1)

Now, 13 men and 24 women can do the same work in 4 days.

⇒ 13x + 24 y = $\frac{1}{4}$ …………….(2)

Multiply equation (1) by 3 and subtract from (2)

⇒ -5x = $\frac{1}{4}-\frac{3}{10}=-\frac{1}{20}$

⇒ x = $\frac{1}{100}$

Put the value of x in (1)

$$\frac{6}{100}+8y=\frac{1}{10}$$

$\Rightarrow$ $8y=\frac{1}{25}$

$\Rightarrow$ $y=\frac{1}{200}$

Hence , Ratio of x and y = 2:1.

68. Ans. C.

In a class room the ratio of number of girls to that of boys is 3 : 4.

Number of boys = 4x

Number of girls = 3x

Total number of students =7x

Average height of students in the class is 4.6 feet.

Sum of heights of all the students = $4.6\times 7x$ = $32.2x$

Average height of the boys in the class is 4.8 feet

Sum of heights of all the boys = $4x\times 4.8=19.2x$

Sum of heights of all the girls = $32.2x-19.2x=13x$

Average height of the girls in the class = $\frac{13x}{3x}=4.33$

Hence, Average height of the girls in the class More than 4.3 feet but less than 4.4 feet

69. Ans. C.

Let the three digits number = abc

Then, the reverse of it will be cba

Both the numbers are divisible by 7 then both the number should be multiple of 7

We can write both the number as

7X = 100a + 10b + c ....1st eq.

7Y = 100c + 10b + a ....2nd eq.

Subtract 2nd from 1st, we get

7(x-y) = 99(a – c)

Here, 99 is not divisible by 7 then, it is compulsory that (a-c) should be a factor of 7.

Then the possible value of (a – c) are a = 2 & c = 9, a = 1 & c = 8

Then possible numbers are 168, 861, 259 &952

Note: 343 will not be the required number as the three digits should be different

Hence C will be correct option.

70. Ans. A.

$$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}-\frac{\sin\theta+1}{\cos\theta}$$

Divide neumrater and denominator by cosθ, then we get

$$\frac{\tan\theta-1+\sec\theta}{\tan\theta-\sec\theta+1}-\tan\theta-\sec\theta$$

$$\frac{(\tan\theta+\sec\theta)-1}{(\tan\theta-\sec\theta)+1}-\tan\theta-\sec\theta$$

$$\frac{(\tan\theta+\sec\theta)-1)(\tan\theta+\sec\theta)}{((\tan\theta-\sec\theta)+1)(\tan\theta+\sec\theta)}-\tan\theta-\sec\theta$$

$$\frac{(\tan\theta+\sec\theta)-1)(\tan\theta+\sec\theta)}{(\tan^2\theta-\sec^2\theta)+(\tan\theta+\sec\theta)}-\tan\theta-\sec\theta$$

$$\frac{(\tan\theta+\sec\theta)-1)(\tan\theta+\sec\theta)}{-1+(\tan\theta+\sec\theta)}-\tan\theta-\sec\theta$$

$$\tan\theta+\sec\theta-\tan\theta-\sec\theta$$

$$=0$$

So, option A is correct.

71. Ans. A.

$(tan\,x + tan\,y)(1 - cot\,x\,cot\,y) + (cot\,x + cot\,y)(1 - tan\,x\,tan\,y)$

Convert every term in tan form

Then we get

$$\frac{(\tan x+\tan y)(\tan x\tan y-1)}{\tan x\tan y}+\frac{(\tan x+\tan y)(1-\tan x\tan y)}{\tan x\tan y}$$
$$\frac{(\tan x+\tan y)(\tan x\tan y-1)}{\tan x\tan y}-\frac{(\tan x+\tan y)(\tan x\tan y-1)}{\tan x\tan y}$$
$$=0$$

Hence, option A is correct.

72. Ans. C.

$$\sqrt{\frac{\sec x-\tan x}{\sec x+\tan x}}$$

Multiply numerator and denominator by $\sqrt{\sec x+\tan x}$

Then,

$$\sqrt{\frac{\sec x-\tan x}{\sec x+\tan x}}\,\frac{\sqrt{\sec x+\tan x}}{\sqrt{\sec x+\tan x}}$$
$$\sqrt{\frac{\sec^2 x-\tan^2 x}{(\sec x+\tan x)^2}}=\sqrt{\frac{1}{(\sec x+\tan x)^2}}=\frac{1}{(\sec x+\tan x)}$$

Hence, option C is correct.

73. Ans. C.

$$\cot\theta=\frac{base}{perpendicular}=\frac{63}{16}$$

Now, here we have base = 63, perpendicular = 16, then with the help of Pythagoras theorem, we can find the hypotenuse as 65.

$hypo = \sqrt{perp.^2 + base^2}$

$hypo = \sqrt{16^2 + 63^2}$

Hypo = 65

Then $\sin\theta = \frac{16}{65}$ & $\cos\theta = \frac{63}{65}$

Then $\sin\theta + \cos\theta = \frac{16}{65} + \frac{63}{65} = \frac{79}{65}$

Hence, option C is correct.

74. Ans. D.

Either you can directly check it by just putting some values of θ for ex: as θ = 0 or 90^0, you will get 5 as your answer.

Or go through the conventional method

Put 1 as $\left(\sin^2\theta + \cos^2\theta\right)^2$ and expand it.

$$\frac{1 - 2\sin^2\theta\cos^2\theta}{\sin^4\theta + \cos^4\theta}$$

$$\frac{\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta}{\sin^4\theta + \cos^4\theta}$$

$$\frac{\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta}{\sin^4\theta + \cos^4\theta}$$

$$\frac{\sin^4\theta + \cos^4\theta}{\sin^4\theta + \cos^4\theta} = 1$$

Then $\frac{1 - 2\sin^2\theta\cos^2\theta}{\sin^4\theta + \cos^4\theta}$ +4 = 1 + 4 = 5

Hence, option D is correct.

75. Ans. C.

triangle having side lengths $\frac{y}{z} + \frac{z}{x}, \frac{z}{x} + \frac{x}{y}, \frac{x}{y} + \frac{y}{z}$

As we can see from here all sides are of different length, then the triangle will be a scalene triangle,

$$\alpha = \cot\theta.\tan\theta.\cot 2\theta.\tan 2\theta.\cot 3\theta.\tan 3\theta..........$$

Then, the area of triangle = where a, b, c are the sides of the triangle.

$$a = \frac{y}{z} + \frac{z}{x}, b = \frac{z}{x} + \frac{x}{y}, c = \frac{x}{y} + \frac{y}{z}$$

Here, s = $\dfrac{\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+\frac{x}{y}+\frac{y}{z}+\frac{z}{x}}{2} = \frac{x}{y}+\frac{y}{z}+\frac{z}{x}$

S = $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$

Put the value of each term, then you will get,

Area = $\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{x}{y}+\frac{y}{z}+\frac{z}{x}}$

Hence, option C is correct.

76. Ans. C.

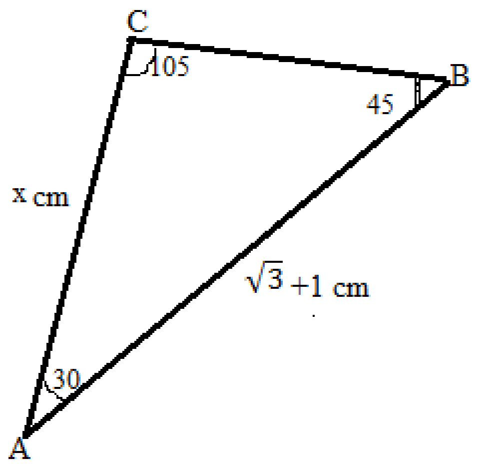
The angles of a triangle are 30° and 45° and the included side is ($\sqrt{3}$+1) cm

1^{st} angle = 30°

2^{nd} angle = 45°

3^{rd} angle = 180 –(30 + 45) = 105^0

Then angle 105^0 will be opposite to ($\sqrt{3}$+1) cm side



Similarly x cm is the side opposite to 45° angle.

We can use here sin rule.

$$\frac{\sqrt{3}+1}{\sin 105^0}=\frac{x}{\sin 45^o}$$

After solving you will get X = 2 cm

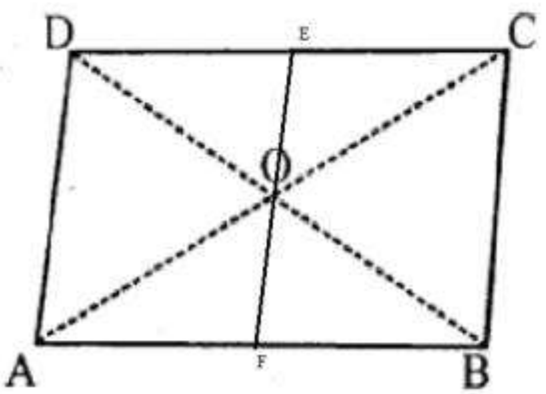
Area = $\frac{1}{2}(\text{product of adjecent sides})\sin(\text{angle b/w them})$

Area = $\frac{1}{2}(2\times(\sqrt{3}+1))\sin 30^o$

Area = $\frac{1}{2}(\sqrt{3}+1)$ cm^2

Hence, option C is correct.

77. Ans. D.



Now, as we know that the diagonal of a parallelogram cut the parallelogram in to four equal parts i.e, the area of the triangles formed by the diagonals should always be equal in a parallelogram(property of a parallelogram)

i.e, Ar(ABO) = Ar(BCO) = Ar(DOC) = Ar(DOA)

we can also easily prove that the triangles DOE and EOC are congruent to each other

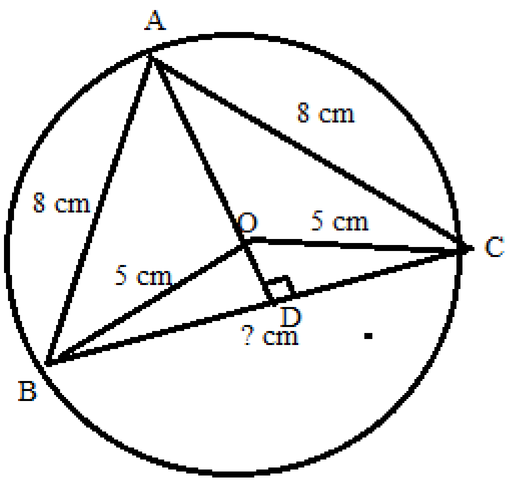
i.e Ar(DOE) = Ar(EOC) = $\frac{1}{2}$ Ar(DOC) = $\frac{1}{8}$ Ar(ABCD)

then Ar(remaining portion after removal of the triangle DOE) = $\frac{7}{8}$ Ar(ABCD)

, now , we have to find the ratio of area of remaining portion of the plate to the whole = $\frac{7}{8}$

Hence, option D is correct.

78. Ans. C.



AD $\perp$ BC & AO = 5 cm (Radius)

Assume OD = x cm & DC = BD = y cm, AD = 5 + x

Now, in triangle ADC & triangle ODC : DC is common.

$8^2 - (5 + x)^2 = 5^2 - x^2$

From here after simplification, we get x = $\frac{7}{5}$

Then y = $\sqrt{5^2 - x^2} = \sqrt{25 - \frac{49}{25}} = \sqrt{\frac{625-49}{25}} = 4.8$

Then, BC = 2y = 2 x 4.8 = 9.6 cm

Hence, option C is correct.

79. Ans. A.

Given $x^{\sqrt{x}} = \sqrt[n]{x^x}$ ....(1)

$\Rightarrow x^{\sqrt{x}} = (x^x)^{\frac{1}{n}}$

$\Rightarrow x^{\sqrt{x}} = (x)^{\frac{x}{n}}$...................(1)

Note : If $x^a = x^b$ then a = b

From (1)

$\Rightarrow \sqrt{x} = \frac{x}{n}$

$\Rightarrow \sqrt{x} = n$

Squaring both sides

⇒ x = n^2

If x = n^2 then correct option will be either A or D

Now we will check whether x = 1 or x = n satisfy equation (1)

Put x = 1 in equation (1)

$\Rightarrow (1)^{\sqrt{1}} = (1)^{\frac{1}{1}}$

⇒ 1= 1

This means x = 1 satisfies equation (1)

Put x = n in equation (1)

$\Rightarrow (n)^{\sqrt{n}} \neq (n)$

This means x = n does not satisfy equation (1)

Hence correct option will be A.

80. Ans. B.

Number under consideration : $(10)^n - 1$

Put n = 1 , $(10)^1 - 1 = 9$

Put n = 2 , $(10)^2 - 1 = 99$

Put n = 3 , $(10)^3 - 1 = 999$

Clearly, Sum of digits of $(10)^n - 1$ =9n , where n is a natural number.

⇒ 9n = 3798

⇒ n = 421

81. Ans. C.

Given that a number divides 12288, 28200 and 44333 so as to leave the same remainder in each case.

Let when 12288, 28200 and 44333 are divided by x and r is obtained as a remainder.

$12288 = q_1x + r$, where q_1 is the quotient..........(1)
$28200 = q_2x + r$, where q_2 is the quotient..........(2)
$44333 = q_3x + r$, where q_3 is the quotient..........(3)

Subtract (1) from (2)

$(28200\text{-}12288) = (q_2 - q_1)x$

$\Rightarrow 15912 = (q_2 - q_1)x$ ....(4)

Subtract (1) from (3)

$(44333\text{-}12288) = (q_3 - q_1)x$

$32045 = (q_3 - q_1)x$..............(5)

Subtract (2) from (3)

$(44333\text{-}28200) = (q_3 - q_2)x$

$16133 = (q_3 - q_2)x$..............(6)

From (4) , (5) & (6) we can say that 15912 , 32045 and 16133 all are divisible by x

HCF (15912 , 16133, 32045) = HCF {(16133-15912) , (32045 – 16133) , (32045 – 15912)}

= HCF(221 , 15912 , 16133) = 221

Hence , x = 221

221 divides 12288, 28200 and 44333 so as to leave the same remainder in each case.

82. Ans. B.

Given m and n are positive integers such that $m^n=1331$.

We know that $(11)^3 = 1331$

Hence, m = 11 and n = 3

⇒ $(m-1)^{n-1}$ = $(10)^2 = 100$

83. Ans. B.

Price of article X was ₹ 5000 and price of article Y was ₹ 2000 in the year 2010.

Difference = 5000 – 2000 = 3000 Rs.

In 2012 , Price of article X = $5000\left(1+\frac{20}{100}\right)^2 = 5000\times\frac{6}{5}\times\frac{6}{5} = Rs.7200$

Price of article Y = $2000\left(1+\frac{10}{100}\right)^2 = 2000\times\frac{11}{10}\times\frac{11}{10} = Rs.2420$

Difference = 7200 – 2420 = Rs. 4780 which is less than 5000

In 2013 , Price of article X = $5000\left(1+\frac{20}{100}\right)^3 = 5000\times\frac{6}{5}\times\frac{6}{5}\times\frac{6}{5} = Rs.8640$

Price of article Y = $2000\left(1+\frac{10}{100}\right)^3 = 2000\times\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10} = Rs.2662$

Difference = 8640 – 2662 = Rs. 5978 which is more than 5000

Hence , Required year = 2013

84. Ans. D.

Let a sum of Rs. x was put at simple interest at rate of r % per annum for 2 years.

$\text{Simple interest} = \frac{\text{Principal}\times\text{Rate}\times\text{Time}}{100}$

$\Rightarrow \text{S.I.} = \frac{x\times r\times 2}{100}$ ....(1)

Had it been put at 1% higher rate of interest, it would have fetched ₹24 more.

$\Rightarrow (\text{S.I.}+24) = \frac{x\times(r+1)\times 2}{100}$ ....(2)

Subtract (1) from (2)

$\Rightarrow 24 = \frac{2x}{100}$

$\Rightarrow 24 = \frac{x}{50}$

$\Rightarrow x = 24\times 50 = 1200$

Hence, Required sum = Rs. 1200

85. Ans. D.

Population of first village = 1525

Ratio of male to female population in the first village is 27 : 34

If male population in the first village is 27x then number of female in the first village will be 34x

⇒ 61x = 1525

⇒ x = 25

Hence, population of male in the first village = $27 \times 25 = 675$

population of female in the first village = $34 \times 25 = 850$

Population of second village =2600

Ratio of male to female population in the second village is 6:

If male population in the second village is 6y then number of female in the second village will be 7y

⇒ 13y = 2600

⇒ y = 200

Hence, population of male in the second village = $6 \times 200 = 1200$

population of female in the second village = $7 \times 200 = 1400$

Total number of male in both villages = 675 +1200 = 1875

Total number of Female in both villages = 850 +1400 = 2250

Required Ratio = 1875 : 2250 = 5:6

86. Ans. B.

Here, the given equation is 5x + 9y = 7

Under the condition: x & y should be integer b/w -500 to 500.

Now, we have to check the value by hit and trial method and try to observe the relation

Here, 1st convert the given equation as y = $\frac{7-5x}{9}$

Try to put some value of x (integer) to get y as integer

x	y
-1	12/9
-4	3
-13	8
-22	13
-31	18

Here, we can see that when we put x = -4, -13, -22, -31…………..(common difference b/w each term = 9). Then y = integer value.

As x $\in$[-500, 500]

X may be -495, -486, ……………..-4, -13 , -22, ………….., 495 (common difference = 9)

It is nothing but an A.P having 1st term = -495 & last term = 495 and d (common difference) = 9

Then, no. of possible value (n) = no. of terms of the A.P

n = $\frac{l-a}{d}+1$

$=\frac{495-(-495)}{9}+1$

$\frac{990}{9}+1$

= $=110+1=111$

Hence, option B is correct.

87. Ans. D.

It is given that, S = XYZ + YZX + ZXY

Then it can be written as:S = 100x + 10y +z + 100y + 10z + x + 100z + 10x + y

S = 111(x + y + z)

= 3 $\times$ 37(x + y + z)

Now, check the given condition,

I. S is always divisible by 3 and (X + Y + Z) : true

II. S is always divisible by 9 : not necessary (if x+y+z = multiple of 3. Only then s is divisible by 9.)

III. S is always divisible by 37 : true

Hence, option D is correct.

88. Ans. B.

It is given that: For two observations, the sum is S and product is P

Assume two observation are x & y then x + y = S and x.y = P

And we know that H.M = $\frac{\frac{1}{x}+\frac{1}{y}}{2} = \frac{x+y}{2xy} = \frac{S}{2P}$

Hence, option B is correct.

89. Ans. C.

We have to find the least perfect square which is divisible by 3, 4, 5, 6 and 7

A number which is divisible by 3,4,5,6 and 7 should be a multiple of 3,4,5,6 and 7

N = $3 \times 4 \times 5 \times 6 \times 7 \times A$

Now, the number should also be a least perfect square, then to make 'N' a perfect square, we have to take A = $3 \times 4 \times 5 \times 6 \times 7$

Then N become $3^2 \times 4^2 \times 5^2 \times 6^2 \times 7^2 = 44100$

Hence, option C is correct.

90. Ans. D.

Given: $(x^2 - 1)$ is a factor of $ax^4+bx^3+cx^2+dx+e$

i.e, $(x-1)(x+1)$ is a factor of $ax^4+bx^3+cx^2+dx+e$

then x = 1 & -1 will be the rots of the equation and will satisfy the given equation i.e remainder = 0

f(-1) = 0

$f(-1) = a(-1)^4+b(-1)^3+c(-1)^2+d(-1)+e = 0$

a – b + c – d +e = 0

a + c + e = b + d

hence, option D is correct.

91. Ans. C.

A wheel makes 360 revolutions in one minute

Means 6 revolution per second

And we know that 1 revolution = 2 π radians

Then in 1 sec the number of radians it turns = 6 x 2 π radians = 12 π

Hence, option C is correct.

92. Ans. B.

Given: $(25 \operatorname{cosec}^2 x + \sec^2 x)$

We have, $a \operatorname{cosec}^2\theta + b\sec^2\theta$

Then, minimum value of the given function = $\left(\sqrt{a}+\sqrt{b}\right)^2$ (remember it)

i.e,

$\left(\sqrt{25}+\sqrt{1}\right)^2$

$(5+1)^2$

$6^2 = 36$

Hence, option B is correct.

Note: Other method is to go through the conventional method i.e, the concept of maxima minima, first of all, find the angle at which dy/dx = 0 then check the condition for maxima or minima. But for the exam point of view, this method is not suitable.(too much lengthy)

Hence, it is convenient to remember the result for the given particular type of function.

93. Ans. A.

Given: α $= \Pi_{n=1}^{99}$ cotnθ

$$\frac{34}{75} = \frac{CI}{100}$$

It means $CI = \frac{4}{3} \times 34 =$ ....1st eq.

An it is given that 100 θ = 90° it implies that $90^0 = 100^{\theta}$

Then,

$$\alpha = \cot\theta.\cot 2\theta.\cot 3\theta............\cot 99\theta$$

$$\alpha = \cot\theta.\cot 2\theta.\cot 3\theta............\tan(90^0 - 99\theta)$$

$$\alpha = \cot\theta.\cot 2\theta.\cot 3\theta............\tan(100\theta - 99\theta)$$

$$\alpha = \cot\theta.\cot 2\theta.\cot 3\theta............\tan\theta$$

Now, rearrange the terms

$$\alpha = \cot\theta.\tan\theta.\cot 2\theta \tan 2\theta.\cot 3\theta.\tan 3\theta............$$

Then α = 1

Hence, option A is correct.

94. Ans. A.

If tan 6θ = cot2θ

It is only possible when '$6^{\theta} + 2^{\theta} = 90^o$'

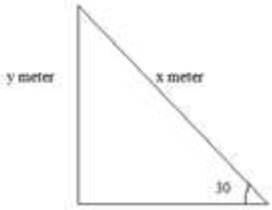
i.e, $8^{\theta} = 90^0$

$4^{\theta} = 45^o$

Then $\sec 4^{\theta} = \sec 45^o = \sqrt{2}$

Hence, option A is correct.

95. Ans. C.



x + y = 15 meter ………….1st eq.

$$\sin 30 = \frac{perp.}{hypo.} = \frac{y}{x}$$

$$\frac{1}{2} = \frac{y}{x}$$

$$x = 2y$$

From 1st , we get : 2y + y = 15

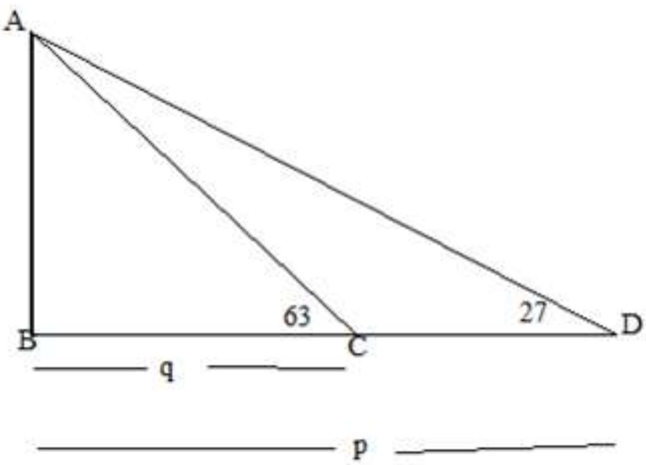
3y = 15

y = 5 meter

hence, height = 5 meter.

So, option C is correct.

96. Ans. B.



Assume H is the height of the tower.

Then, AB = H, $\tan 63^\circ = \frac{AB}{BC}$ ……1st eq. & $\tan 27^\circ = \frac{AB}{BD}$ ……2nd eq.

Multiply e. 1st and 2nd ,

Tan63°.tan27° = $\frac{AB}{BC} \cdot \frac{AB}{BD}$

Here, tan(90° – 63) = cot27°

Now, AB^2 = BC.BD

$AB = \sqrt{pq}$

Hence, option B is correct.

97. Ans. B.

$$\frac{cos\theta}{1+sin\theta}+\frac{1}{cot\theta}$$
$$\frac{cos\theta}{1+sin\theta}+\frac{\sin\theta}{\cos\theta}$$
$$\frac{\cos^2\theta+\sin^2\theta+\sin\theta}{\cos\theta(1+\sin\theta)}$$
$$\frac{1+\sin\theta}{\cos\theta(1+\sin\theta)}$$
$$=\frac{1}{\cos\theta}=\sec\theta$$

Hence, option B is correct.

98. Ans. C.

Given $\frac{a}{b}=\frac{1}{3}$..................(1)

$\frac{b}{c}=2$.............(2)

$\frac{c}{d}=\frac{1}{2}$..................(3)

$\frac{d}{e}=3$ ....(4)

$\frac{e}{f}=\frac{1}{4}$..................(5)

Multiply (1) , (2) and (3)

$$\frac{a}{b}.\frac{b}{c}.\frac{c}{d}=\frac{1}{3}\times 2\times \frac{1}{2}$$

$\Rightarrow \frac{a}{d}=\frac{1}{3}$ ……………….(6)

Multiply (2) , (3) and (4)

$$\frac{b}{c}.\frac{c}{d}.\frac{d}{e}=2\times \frac{1}{2}\times 3$$

$\Rightarrow \frac{b}{e}=3$ ……………….(7)

Multiply (3) , (4) and (5)

$$\frac{c}{d}.\frac{d}{e}.\frac{e}{f}=\frac{1}{2}\times 3\times \frac{1}{4}$$

$\Rightarrow \frac{c}{f}=\frac{3}{8}$ ……………….(8)

Multiply (6) , (7) and (8)

$$\frac{abc}{def}=\frac{1}{3}\times 3\times \frac{3}{8}=\frac{3}{8}$$

99. Ans. C.

We know that when the distance is same, then speed is inversely proportional to time i.e,

$$\frac{s_x}{s_y}=\frac{t_y}{t_x}=\frac{4}{5}$$

$t_y : t_x = 4:5$

=> Then 1 unit difference ≅ 45 minutes

Time taken by y = 4 units = 4 x 45 = 180 minutes

Hence, option C is correct.

100. Ans. B.

Let Y’s income = 100 units

Then X’s income = 100 + 20 = 120 units (X’s income is 20% more than that of Y)

Now, Y’s income less than that of X’s income

$= \frac{120-100}{120}\times 100 = \frac{20}{120}\times 100 = 16\frac{2}{3}$

Hence, option B is correct.