Classroom

ESE Mains Achiever's Study Plan

Electronics & Communication Engineering

Signals & Systems
Part-1

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1. A. State and prove time shifting and frequency shifting properties of Fourier transform?

B. If x(t) is a real signal then show that $X^*(\omega) = X(-\omega)$

Sol. A) (i) Time shifting property:

If $x(t) \xleftrightarrow{FT} X(\omega)$

Then $x(t-t_0) \leftrightarrow e^{-j\omega t_0} X(\omega)$

Proof: We know that

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \text{ and let } X'(\omega) = \int_{-\infty}^{\infty} x(t-t_0) e^{-j\omega t} dt \text{ ......(i)}$$

Where $X(\omega)$ is F.T. of x(t) and $X'(\omega)$ is F.T. of $x(t - t_0)$

Let $t - t_0 = \lambda$

$\therefore t = \lambda + t_0$

Equation (i) becomes

$$X'(\omega) = \int_{-\infty}^{\infty} x(\lambda) e^{-j\omega(\lambda+t_0)} d\lambda = \int_{-\infty}^{\infty} x(\lambda) e^{-j\omega\lambda} e^{-j\omega t_0} d\lambda$$

$$X'(\omega) = e^{-j\omega t_0} \int_{-\infty}^{\infty} x(\lambda) e^{-j\omega\lambda} d\lambda$$

$$\therefore X'(\omega) = e^{-j\omega t_0} X(\omega)$$

$$\therefore X(t) \xleftrightarrow{FT} X(\omega)$$

Then $\therefore X(t - to) \xleftrightarrow{FT} e^{-j\omega to} X(\omega)$

(iii) Frequency shifting property:

$$X(t) \xleftrightarrow{FT} X(\omega)$$

Then $e^{j\omega_o t} X(t) \longleftrightarrow X(\omega - \omega_o)$

Proof: we know that,

$$X(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \text{ and let}$$

$$X'(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega - \omega_o) e^{j\omega t} d\omega$$

Let $\omega - \omega_o = \lambda$

$$\therefore X'(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\lambda) e^{j(\lambda+\omega_0)t} d\lambda = e^{j\omega_o t} \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\lambda) e^{j\lambda t} d\lambda$$

$$\therefore x'(t) = e^{j\omega_0 t} x(t)$$

$$\therefore x(t) \xleftrightarrow{FT} X(\omega)$$

Then $e^{j\omega_0 t} x(t) \xleftrightarrow{FT} X(\omega - \omega_0)$

(B) Given that x(t) is real signal

$\therefore x(t) = x^*(t)$

Now $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$\therefore X^*(\omega) = \int_{-\infty}^{\infty} x^*(t) e^{j\omega t} dt$ ....(i)

Now

$X(-\omega) = \int_{-\infty}^{\infty} X(t) e^{j\omega t} dt$

Now

$x(t) = x^*(t)$

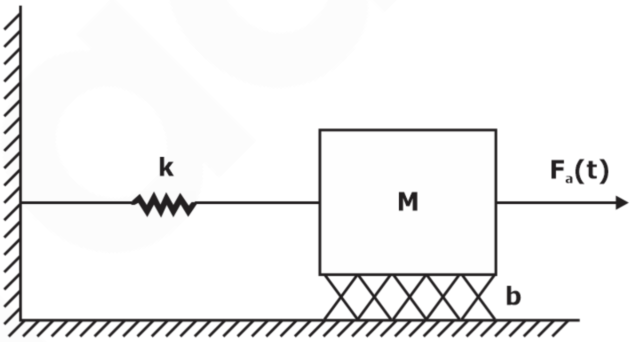
$\therefore X(-\omega) = \int_{-\infty}^{\infty} x^*(t) e^{-j\omega t} dt$ ....(ii)

From (i) and (ii)

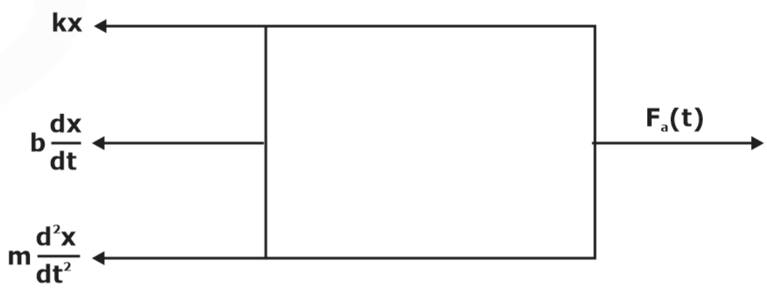
$x(t) = x^*(t)$

$X^*(\omega) = X(-\omega)$

2. Consider the system shown. If m = 3kg, b = 0.6 N-S/m and k = 1.5 N/m. If the position of mass is x(t) and x(t) = 0 at equilibrium. The initial velocity at t = 0^- is –2 m/s and the initial position is $x(0^-)$ = 0 m. The input force on the mass $F_a(t)$ is of 6 Newton towards right for t ≥ 0 then find equation of x(t)?



Sol. The free body diagram can be drawn as,



$$\therefore m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_a(t)$$

Now m = 3, b = 0.6, k = 1.5

$x(0^-) = 0$, $x'(0^-) = -2$, $F_a(t) = 6\,u(t)$

$$\therefore 3\frac{d^2x}{dt^2} + 0.6\frac{dx}{dt} + 1.5x = 6u(t)$$

∴ Laplace transform of the above equation will be,

$$3\left(s^2X(s) - sx(0^-) - x'(0^-)\right) + 0.6\left(sX(s) - x(0^-)\right) + 1.5X(s) = \frac{6}{s}$$

$$\therefore 3\left(s^2X(s) + 2\right) + 0.6sX(s) + 1.5X(s) = \frac{6}{s}$$

$$\therefore \left(3s^2 + 0.6s + 1.5\right)X(s) = \frac{6}{s} - 6$$

$$\therefore X(s) = \frac{-6s + 6}{s\left(3s^2 + 0.6s + 1.5\right)}$$

$$\therefore X(s) = \frac{-2s + 2}{s\left(s^2 + 0.2s + 0.5\right)}$$

$$= \frac{A}{s} + \frac{Bs + C}{(s + 0.1)^2 + 0.49}$$

$$= \frac{4}{s} + \frac{-4s - 2.8}{(s + 0.1)^2 + (0.7)^2}$$

$$\therefore X(s) = \frac{4}{s} + \frac{-4s}{(s + 0.1)^2 + (0.7)^2} - \frac{2.8}{(s + 0.1)^2 + (0.7)^2}$$

$$\therefore x(t) = 4u(t) - 4e^{-0.1t}\cos(0.7t)u(t) - 4e^{-0.1t}\sin(0.7t)u(t)$$

$$\therefore x(t) = 4\left[1 - e^{-0.1t}\left\{\cos(0.7t) + \sin(0.7t)\right\}\right]u(t)$$

3. A. Mention some properties of ROC of z-transform?

B. Using final value theorem, find steady state value of $(0.5^n - 0.5)u(n)$ and verify it?

Sol. A) Properties of ROC of z-transform:

(i) ROC of z-transform is indicated with circle in z-plane.

(ii) ROC does not contain any pole.

(iii) If x(n) is a finite duration causal sequence or right handed sequence then the ROC is entire z-plane except at z = 0.

(iv) If x(n) is a finite duration anti-causal sequence or left sided sequence then the ROC is entire z-plane except z = ∞.

(v) If x(n) is a infinite duration causal sequence, ROC is exterior of the circle with radius a. i.e., |z| > a

(vi) If x(n) is a infinite duration anti-causal sequence, ROC is interior of the circle with radius a. i.e., $|z| < a$.

(vii) If x(n) is a finite duration two-sided sequence, then the ROC is entire z-plane except $z = 0$ and $z = \infty$.

(viii) For a z-transform F(z) the number of ROC's possible will be equal to "Number of distinct pole modulus +1".

B) $f(n) = (0.5^n - 0.5)\, u(n)$

$f(n) = (0.5)^n u(n) - 0.5u(n)$

$$\therefore f(\infty) = \lim_{n\to\infty} (0.5)^n u(n) - 0.5u(n)$$

$$= 0 \times 1 - 0.5 \times 1$$

$\therefore f(\infty) = -0.5$

Now $f(n) = (0.5)^n u(n) - 0.5\, u(n)$

$$\therefore F(z) = \frac{z}{z-0.5} - \frac{0.5z}{z-1}$$

$$\therefore f(\infty) = \lim_{z\to 1} (z-1)F(z)$$

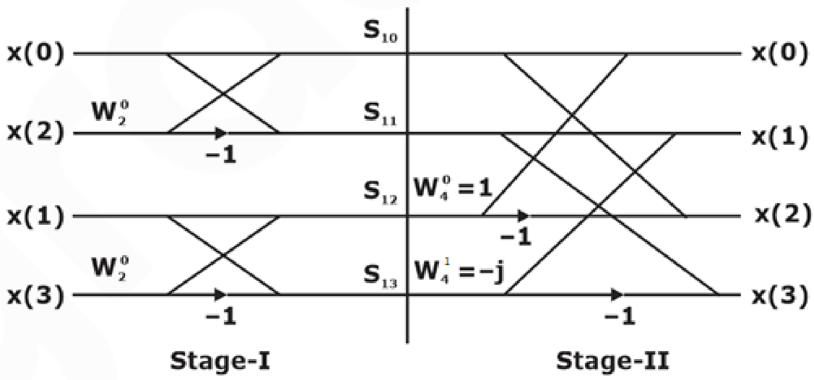
$$= \lim_{z\to 1} (z-1).\frac{z(z-1) - 0.5z(z-0.5)}{(z-0.5)(z-1)} = \frac{-0.5\times 0.5}{0.5}$$

$$\therefore f(\infty) = -0.5$$

4. A. Find the DFT of the sequence using DIT-FFT of x(n) = {1, 2, 3, 4}

B. Using result obtained above again find x(n) using inverse DIF-FFT.

Sol. A) x(n) = {1, 2, 3, 4}



Output of stage-I

$S_{10} = x(0) + x(2) = 4$

$S_{11} = x(0) - x(2) = -2$

$S_{12} = x(1) + x(3) = 6$

$S_{13} = x(1) - x(3) = -2$

∴ output of stage-II

$x(0) = S_{10} + S_{12} = 10$

$x(1) = S_{11} - jS_{13} = -2 + j2$

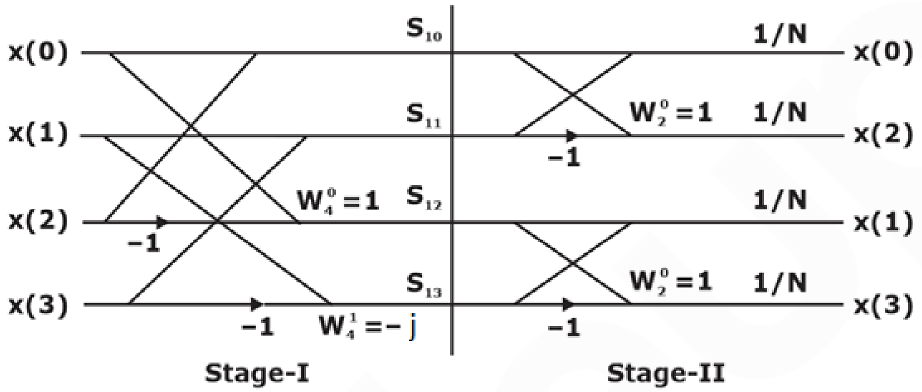
$x(2) = S_{10} - S_{12} = -2$

$x(3) = S_{11} + jS_{13} = -2 - j2$

∴ DFT of the given sequence will be

$X(k) = \{10, -2 + j2, -2, -2 - j2\}$

Now let's find x(n) from X(k) using DIF-FFT



Output of stage-I

$S_{10} = x(0) + x(2) = 8$

$S_{11} = x(1) + x(3) = -4$

$S_{12} = x(0) - x(2) = 12$

$S_{13} = j(x(1) - x(3)) = -4$

Output of stage-II

$$X(0) = \frac{1}{N}(S_{10} + S_{11}) = \frac{1}{4} \times 4 = 1$$

$$X(2) = \frac{1}{N}(S_{10} + S_{11}) = \frac{1}{4} \times 12 = 3$$

$$X(1) = \frac{1}{N}(S_{12} + S_{13}) = \frac{1}{4} \times 8 = 2$$

$$X(3) = \frac{1}{N}(S_{12} + S_{13}) = \frac{1}{4} \times 16 = 4$$

∴ $X(n) = \{1, 2, 3, 4\}$

5. Design a digital filter for using the transfer function $H_a(s) = \frac{s + 0.1}{(s + 0.1)^2 + 16}$. If it has resonant frequency of $\frac{\pi}{2}$ rad / sec ?

Sol. We know that

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

Now general transfer function of filter is,

$$H_a(s) = \frac{s+a}{(s+a)^2 + \Omega_c^2}$$

$$\therefore \Omega_c^2 = 16$$
$$\therefore \Omega_c = 4\text{rad / sec}$$

∴ cut off frequency of analog filter is = 4 rad/sec

We know that,

$$\Omega_c = \frac{2}{T}\tan\left(\frac{\omega_r}{2}\right)$$

$$\therefore T = \frac{2}{\Omega_c}\tan\left(\frac{\omega_r}{2}\right) = \frac{2}{4}\tan\left(\frac{\pi/2}{2}\right)$$

$$\therefore T = \frac{1}{2}\sec$$

$$\therefore H(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$\therefore H(z) = \frac{s+0.1}{(s+0.1)^2+16}\Bigg|_{s=\frac{2}{\frac{1}{2}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = \frac{4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.1}{\left[4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.1\right]^2+16}$$

$$= \frac{\dfrac{4(1-z^{-1})+0.1(1+z^{-1})}{(1+z^{-1})}}{\dfrac{\left[4(1-z^{-1})+0.1(1+z^{-1})\right]^2+16(1+z^{-1})^2}{(1+z^{-1})^2}}$$

$$\therefore H(z) = \frac{\left[4(1-z^{-1})+0.1(1+z^{-1})\right](1+z^{-1})}{\left[4(1-z^{-1})+0.1(1+z^{-1})\right]^2+16(1+z^{-1})^2}$$

$$= \frac{(4-4z^{-1}+0.1+0.1z^{-1})(1+z^{-1})}{(4-4z^{-1}+0.1+0.1z^{-1})^2+16(1+z^{-1})^2}$$

$$= \frac{(4.1-3.9z^{-1})(1+z^{-1})}{(4.1-3.9z^{-1})^2+16(1+2z^{-1}+z^{-2})}$$

$$\therefore H(z) = \frac{0.128z^2+0.006z-0.122}{z^2+0.0006z+0.975}$$

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