Classroom

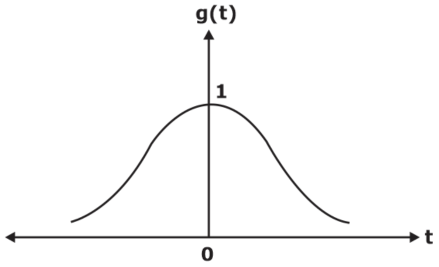
ESE Mains Achiever's Study Plan

Electronics & Communication Engineering

Signals & Systems
Part-2

1. Find Fourier transform of the Gaussian signal $g(t) = e^{-\pi t^2}$

Sol. As the given gaussian signal g(t) can be drawn as,



$$\therefore G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

$$\therefore G(f) = \int_{-\infty}^{\infty} e^{-\pi t^2} e^{-2\pi jft} dt$$

We know that

$$f(t) \xleftrightarrow{FT} F(f)$$

$$(-2\pi jt) f(t) \xleftrightarrow{FT} \frac{d}{df} F(f)$$

$$\therefore \frac{dG(f)}{df} = \int_{-\infty}^{\infty} e^{-\pi t^2} (-2\pi jt) e^{-2\pi jft} dt \text{ ....(i)}$$

We know that

$$\int uV = u\int V - \iint V \, du$$

$$\therefore \int u \, dV = uV - \int V \, du$$

So, let $u = e^{-2\pi jft}$

$\therefore du = (-2\pi jf) e^{-2\pi jft} dt$ & let $dV = (-2\pi jt) e^{-\pi t^2} dt$

$\therefore V = je^{-\pi t^2}$

So equation (i) becomes

$$\frac{dG(f)}{df} = -\int_{-\infty}^{\infty} je^{-\pi t^2} (-2\pi jf) e^{-2\pi jft} dt$$

Here the turn u. v. becomes zero as the limits are evaluated from - ∞ to ∞

$$\therefore \frac{dG(f)}{df} = -2\pi f \int_{-\infty}^{\infty} e^{-\pi t^2} e^{-2\pi jft} dt$$

$$\therefore \frac{dG(f)}{df} = -2\pi f G(f)$$

After solving this differential equation, we will get $G(f) = G(0) e^{-\pi f^2}$

Now $G(0) = \int_{-\infty}^{\infty} e^{-\pi t^2} dt = 1$

$$G(f) = e^{-\pi f^2}$$

$$\therefore e^{-\pi t^2} \xleftrightarrow{FT} e^{-\pi f^2}$$

2. (a) Prove that Laplace transform of t^n is $\frac{\eta!}{S^{n+1}}$

(b) Using properties of Laplace transform find value of $\int_0^\infty te^{-3t}\cos t\ dt$

Sol. (a) Laplace transform of a function f(t) can be calculated as,

$$F(s)=\int_0^\infty f(t)e^{-st}dt$$

$$\therefore F(s)=\int_0^\infty t^n e^{-st}dt\ ....(I)$$

Let st = u

When t = 0, u = 0

When t = ∞, u = ∞ and sdt = du

$$\therefore dt=\frac{du}{s}$$

∴ eqn. (I) becomes

$$F(s)=\int_0^\infty\left(\frac{u}{s}\right)^n e^{-u}\frac{du}{s}=\frac{1}{S^{n+1}}\int_0^\infty e^{-u}u^n du\ ....(II)$$

By the definition of gamma function

$$\overline{|n}=\int_0^\infty e^{-x}x^{n-1}dx,\ n>0$$

$$\overline{|n+1}=\int_0^\infty e^{-x}x^{n+1}dx\ ....(III)$$

From (II) and (III)

$$F(s)=\frac{\overline{|n+1}}{S^{n+1}},n>-1$$

$$\therefore L(t^n)=\frac{n!}{S^{n+1}}$$

(b) We know that

$$F(s)=L(f(t))=\int_0^\infty e^{-st}f(t)dt$$

Comparing this with the required quantity s = 3 and f(t) = t cost

$$\text{Now }\cos t\xleftrightarrow{LT}\frac{s}{s^2+1}$$

$$t\cos t\xleftrightarrow{LT}(-1)\frac{d}{ds}\left(\frac{s}{s^2+1}\right)=-1\left(\frac{(s^2+1)-2s^2}{(s^2+1)^2}\right)$$

$$L(t\cos t)=\frac{s^2-1}{(s^2+1)^2}$$

$$\therefore \int_0^\infty e^{-st} t\cos t\, dt = \frac{s^2-1}{\left(s^2+1\right)^2}$$

$$\therefore \int_0^\infty e^{-3t} t\cos t\ dt = \frac{9-1}{(9+1)^2} = \frac{8}{100}$$

$$\therefore \int_0^\infty te^{-3t} t\cos t\ dt = \frac{2}{25}$$

3. Given the following difference equation, find H(z)

y(n – 2) + 3y(n – 1) + 2y(n) = x(n – 1)

Also for y(−1) = 1, y(−2) = 0 and x(n) = u(n) find the ZIR and ZSR.

Sol.

For finding the transfer function we take initial conditions as zero.

∴ y(n – 2) + 3y(n – 1) + 2y(n) = x(n – 1)

∴ $z^{-2}Y(z) + 3z^{-1}Y(z) + 2Y(z) = z^{-1}\, x(z)$

$$\therefore \frac{Y(z)}{x(z)} = \frac{z^{-1}}{z^{-2}+3z^{-1}+2} \quad \text{........(i)}$$

(i) To find ZSR

X(n) = u(n)

$$\therefore X(z) = \frac{1}{1-z^{-1}}$$

Therefore equation (i) becomes,

$$\therefore Y(z) = \frac{z^{-1}}{z^{-2}+3z^{-1}+2} \times \frac{1}{1-z^{-1}} = \frac{A}{1-z^{-1}} + \frac{B}{z^{-1}+2} = \frac{C}{z^{-1}+1}$$

$$\therefore Y(z) = \frac{\frac{1}{6}}{1-z^{-1}} + \frac{\frac{2}{3}}{z^{-1}+2} + \frac{-\frac{1}{2}}{z^{-1}+1}$$

$$\therefore Y(z) = \frac{1}{6}u(n) + \frac{1}{3}\left(-\frac{1}{2}\right)^n u(n) - \frac{1}{2}(-1)^n u(n)$$

$$\therefore Y(n) = \left[\frac{1}{6} + \left(\frac{1}{3}\right)\left(-\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)(-1)^n\right]u(n)$$

(ii) To find ZIR

We consider initial conditions and input to be zero.

∴ $z^{-2}Y(z) + z^{-1}y(-1) + y(-2) + 3z^{-1}Y(z) + 3y(-1) + 2Y(z) = 0$

∴ $[z^{-2} + 3z^{-1} + 2]\ Y(z) = -z^{-1} - 3$

$$\therefore Y(z) = \frac{-z^{-1}-3}{z^{-2}+3z^{-1}+2} = \frac{-z^{-1}-3}{\left(z^{-1}+1\right)\left(z^{-1}+2\right)} = \frac{A}{z^{-1}+1} + \frac{B}{z^{-1}+2}$$

$$\therefore Y(z) = \frac{-2}{z^{-1}+1} + \frac{1}{z^{-1}+2}$$

$$\therefore y(n) = -2(-1)^n u(n) + \frac{1}{2}\left(-\frac{1}{2}\right)^n u(n)$$

$$\therefore y(n) = \left[-2(-1)^n + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)^n\right] u(n)$$

4. If the DFT of x(n) is X(k) = {3, 2 + j, 1, 2 – j} then which properties of DFT find following quantities.

(a) DFT of x(−n)

(b) DFT of x(n – 2)

(c) DFT of x*(n)

(d) DFT of $x^2(n)$

(e) signal energy

(f) DFT of [x(n) * x(n)]

Sol. (a) $x(n) \xleftrightarrow{DFT} X(k)$

Then $x(-n) \xleftrightarrow{DFT} X(-k) = X(-k + iN)$

Where i is any integer and N is total number of similar.

$\therefore x(-n) \xleftrightarrow{DFT} X(N - k)$

∴ DFT of x(-n) = {3, 2 – j, 1, 2 + j}

(b) $x(n) \xleftrightarrow{DFT} X(k)$

$x(n-2) \xleftrightarrow{DFT} X(k)e^{-j\frac{2\pi}{N}2k} = X(k)e^{-j\frac{2\pi}{2}2k} = X(k)(-1)^k$

∴ DFT of x(n – 2) = {3, -2 –j, 1, -2 + j}

(c) $x(n) \xleftrightarrow{DFT} X(k)$

$x^*(n) \xleftrightarrow{DFT} X^*(-k) = x^*(-k + N)$

∴ DFT of x*(n) = {3, 2 + j, 1, 2 –j}

(d) $\text{DFT}\{x^2(n)\} = \frac{1}{N}(X(k) \odot$

Where $\odot$ is circular convolution

Now linear convolution is calculated as

	3	2 + j	1	2 – j
3	9	6 + 3j	3	6 – 3j
2 + j	6 + 3j	3 + 4j	2 + j	5
1	3	2 + j	1	2 – j
2 – j	6 – 3j	5	2 + j	3 – 4j

= {9, 12 + 6j, 9 + 4j, 16 – 4j, 11, 4 – 2j, 3 – 4j}

So circular convolution is given as,

{9, 12 + 6j, 9 + 4j, 16 – 4j, 11, 4 – 2j, 3 – 4j}

= {20, 16 + 4j, 12, 16 – 4j}

∴ DFT of $x^2(n)$ = {20, 16 + 4j, 12, 16 – 4j}

(e) Signal energy = $\frac{1}{N}\sum_{k=0}^{N-1}|X(k)|^2 = \frac{1}{4}[9+5+1+5]$

∴ Signal energy = 5 joules

(f) DFT of $[X(n)*X(n)] = \frac{1}{N}[X(k).X(k)] = \frac{1}{4}\{9, 3+4i, 3-4i\}$

DFT of $[X(n)*X(n)] = \{2.25, 0.75+i, 0.25, 0.75-i\}$

5. Design a infinite impulse response (IIR) low pass filter for following specifications:

Pass band ripple: < 1 dB

Pass band edge: < 1 kHz

Stopband attenuation: > 40 dB

Stopband edge: 1.5 kHz

Sample rate: 4 kHz

Determine required filter order for

(a) Butterworth filter

(b) TChebyshev filter

$\propto_P$ = 1 dB, f_P = 1 kHz

$\propto_S$ = 40 dB, f_S = 1.5 kHz

F_S = 4 kHz

Sol. (a) Butterworth filter design

$$\omega_p = \frac{2\pi f_p}{F_s} = \frac{2\pi \times 1}{4} = \frac{\pi}{2}$$

$$\omega_s = \frac{2\pi f_s}{F_s} = \frac{2\pi \times 1.5}{4} = \frac{3\pi}{4}$$

$$\text{Then, } N \geq \frac{\log\left[\sqrt{\frac{10^{0.1\propto s}-1}{10^{0.1\propto p}-1}}\right]}{\log\left[\frac{\Omega_s}{\Omega_p}\right]}$$

Now $\frac{\Omega_s}{\Omega_p} = \frac{\tan\left(\frac{\omega_s}{2}\right)}{\tan\left(\frac{\omega_p}{2}\right)} = \frac{\tan\left(\frac{3\pi}{8}\right)}{\tan\left(\frac{\pi}{4}\right)}$

$\frac{\Omega_s}{\Omega_p} = 2.4142$ .... (i)

& $\sqrt{\frac{10^{0.1\propto S}-1}{10^{0.1\propto p}-1}} = \sqrt{\frac{10^{0.1\times 40}-1}{10^{0.1\times 1}-1}}$

$\sqrt{\frac{10^{0.1\propto S}-1}{10^{0.1\propto p}-1}} = 196.51$ ....(ii)

∴ From (i) and (ii)

$N \geq \frac{\log(196.51)}{\log(2.4142)}$

$N \geq 5.99$

∴ N = 6

∴ Order = 6 for butter worth filter

(b) Tchebyshev filter design

$$N \geq \frac{\cosh^{-1}\left[\sqrt{\frac{10^{0.1\propto s}-1}{10^{0.1\propto p}-1}}\right]}{\cosh^{-1}\left[\frac{\Omega_s}{\Omega_p}\right]}$$

∴ From (i) and (ii)

$N \geq \frac{\cosh^{-1}[196.51]}{\cosh^{-1}[2.4142]}$

$\therefore N \geq 3.90$

∴ N = 4

∴ Order = 4 for Chebyshev filter

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