

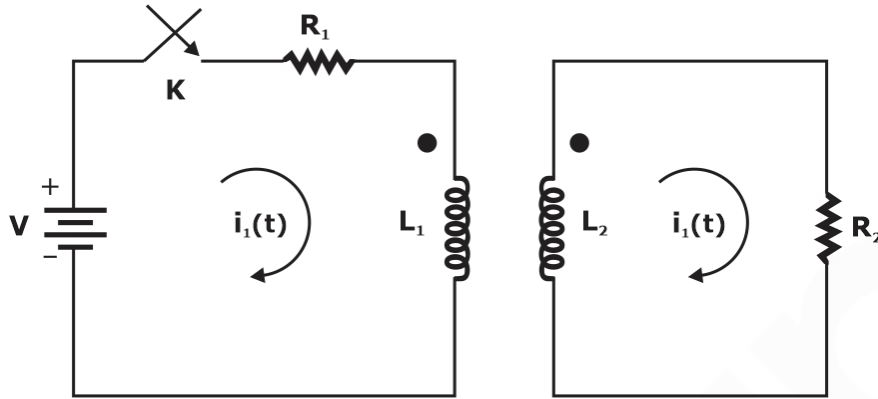
ESE Mains Achiever's Study Plan

Electronics & Communication Engineering

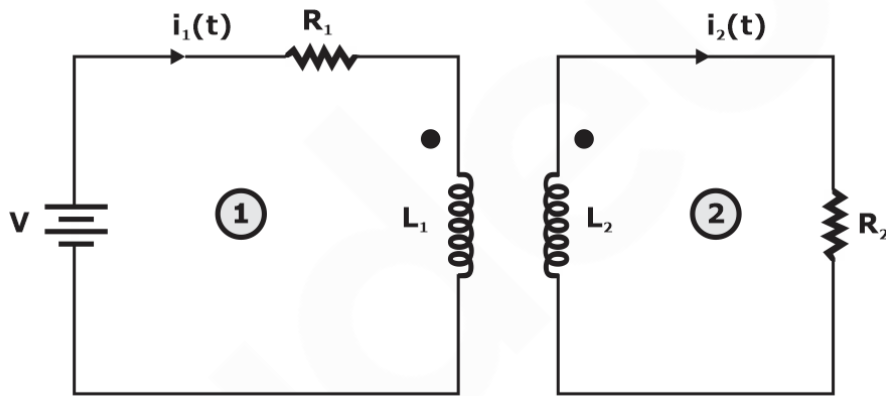
Networks Part-2



1. The circuit shown below is unenergized before closing the switch K at $t = 0$, the circuit parameters are $L_1 = 1\text{H}$, $L_2 = 8\text{H}$, $M = 2\text{H}$, $R_1 = 2\Omega$, $R_2 = 2\Omega$, $V = 10\text{ V}$. Find $i_1(t)$ from the instant switch is closed.



Sol. The circuit for time $t > 0$



Applying KVL in loop (1)

$$V = i_1(t)R_1 + L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt} \dots\dots(i)$$

Applying KVL in loop (2)

$$L_2 \frac{di_2(t)}{dt} + R_2 i_2(t) - M \frac{di_1(t)}{dt} = 0 \dots\dots(ii)$$

Taking Laplace transform of equation (i) & (ii)

$$\frac{V}{s} = R_1 I_1(s) + sL_1 I_1(s) - sM I_2(s) \dots\dots(iii)$$

$$sL_2 I_2(s) + R_2 I_2(s) - sM I_1(s) = 0 \dots\dots(iv)$$

$$(sL_2 + R_2) I_2(s) = sM I_1(s)$$

$$I_2(s) = \frac{sM}{(sL_2 + R_2)} I_1(s) \dots\dots(v)$$

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By (iii) & (v)

$$\frac{V}{s} = (R_1 + sL_1)I_1(s) - \frac{sM \cdot sM}{(sL_2 + R_2)} I_1(s)$$

$$\frac{V}{s} = \left[R_1 + sL_1 - \frac{s^2 M^2}{(sL_2 + R_2)} \right] I_1(s)$$

Putting values of parameters:

$$\frac{10}{s} = \left[2 + s - \frac{4s^2}{8s + 2} \right] I_1(s)$$

$$\frac{10}{s} = \frac{[(s + 2)(8s + 2) - 4s^2]}{(8s + 2)} I_1(s)$$

$$= \left[\frac{8s^2 + 16s + 2s + 4 - 4s^2}{(8s + 2)} \right] I_1(s)$$

$$I_1(s) = \frac{10(8s + 2)}{4s^2 + 18s + 4}$$

$$= \frac{2.5(8s + 2)}{(s^2 + 4.5s + 1)}$$

$$= \frac{A}{s + 0.234} + \frac{B}{s + 4.26}$$

Solving partial differentiation:

$$As + 4.26A + Bs + 0.264B = 2.5 \times 8s + 2.5 \times 2$$

$$A + B = 20$$

$$4.26A + 0.234B = 5$$

$$\text{So, } A = 0.0795$$

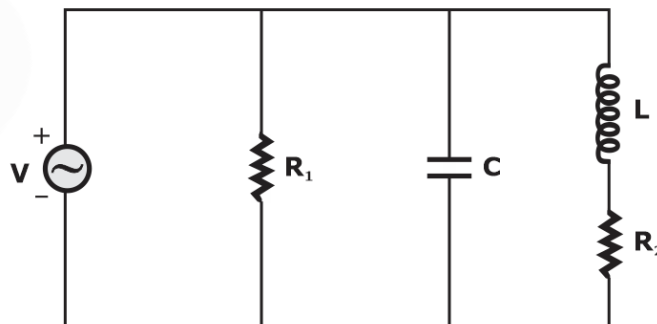
$$B = 19.920$$

$$I_2(s) = \frac{0.0795}{s + 0.234} + \frac{19.920}{s + 4.26}$$

$$i_1(t) = 0.0795e^{-0.234t} + 19.920e^{-4.26t} \text{ A}$$

2. Obtain resonant frequency of the circuit shown in figure below.

Given value of parameters as $L = 1\text{H}$, $R_1 = 1\Omega$, $R_2 = 10\Omega$, $C = 1\text{F}$



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Sol. To find resonant frequency we know that imaginary part of admittance is zero in case of parallel circuit.

$$Y(j\omega) = \text{admittance} = \frac{1}{R_2 + j\omega L} + j\omega C + \frac{1}{R_1}$$

$$Y(j\omega) = \frac{1}{10 + j\omega(1)} + j\omega(1) + \frac{1}{1}$$

$$Y(j\omega) = \frac{10 - j\omega}{100 - \omega^2} + j\omega + 1$$

$$\text{Im} [Y(j\omega)] = 0$$

$$\frac{-\omega}{100 - \omega^2} + \omega = 0$$

$$\omega = \frac{\omega}{100 - \omega^2}$$

$$100 - \omega^2 = 1$$

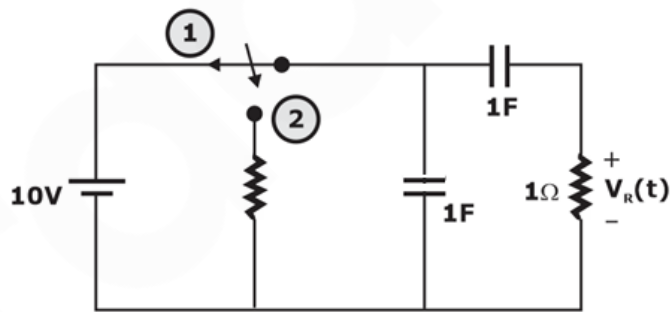
$$100 - 1 = \omega^2$$

$$\omega = \pm\sqrt{99}$$

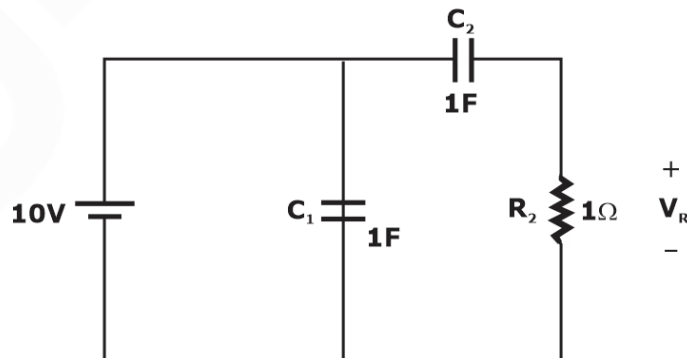
$$\omega = \pm 9.95 \text{ rad / sec}$$

The resonant frequency for this parallel combination is 9.95 rad / sec.

3. The network shown in figure below, switch 'K' is connected at position (1) for long time. At $t = 0$, the switch 'K' is transferred to position (2) Determine the voltage $V_R(t)$



Sol. For $t < 0^-$



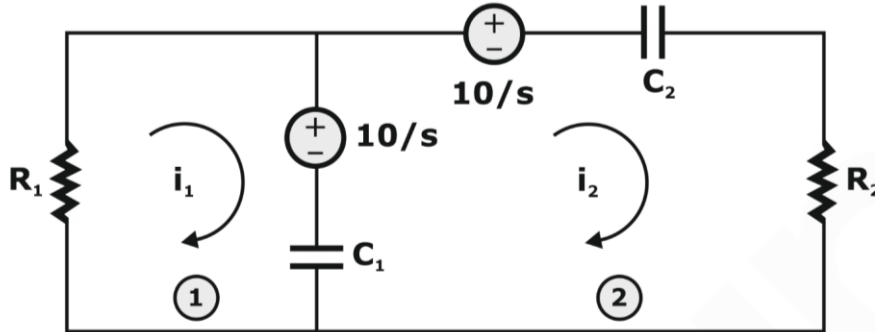
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$$V_{C1}(0^-) = 10 \text{ V}$$

$$V_{C2}(0^-) = 10 \text{ V}$$

[voltage across capacitor in parallel to voltage source is same as voltage source at steady state]

At $t = 0^+$ switch moved to position (2) for $t = 0^+$, the circuit will be



Applying KVL in loop (1)

$$\frac{-10}{s} + I_1(s)R_1 + \frac{1}{C_1s} [I_1(s) - I_2(s)] = 0$$

$$I_1(s) \left[R_1 + \frac{1}{C_1s} \right] - I_2(s) \frac{1}{C_1s} = \frac{10}{s} \dots\dots(i)$$

KVL in loop (2)

$$\frac{-10}{s} + \frac{10}{s} + \frac{1}{C_1s} [I_2(s) - I_1(s)] + \left[\frac{1}{C_2s} + R_2 \right] I_2(s) = 0$$

$$-\frac{1}{C_1s} I_1(s) + \left[R_2 + \frac{1}{C_1s} + \frac{1}{C_2s} \right] I_2(s) = 0 \dots\dots(ii)$$

Putting values of parameter in equation (i) and (ii)

$$I_1(s) \left[1 + \frac{1}{s} \right] + \frac{1}{s} [-I_2(s)] = \frac{10}{s} \dots\dots(iii)$$

$$-\frac{1}{s} I_1(s) + \left[1 + \frac{1}{s} + \frac{1}{s} \right] I_2(s) \dots\dots(iv)$$

By (iv)

$$I_1(s) = (s + 2)I_2(s)$$

$$I_2(s) = \frac{1}{s+2} I_1(s) \dots\dots(v)$$

By (iii) & (v)

$$I_1(s) \left[\frac{s+1}{s} \right] + \frac{1}{s} \left(\frac{1}{s+2} \right) I_1(s) = \frac{10}{s}$$

$$I_1(s) \left[\frac{s+1}{s} + \frac{1}{s(s+2)} \right] = \frac{10}{s}$$

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$$I_1(s) = \frac{10 \times s \times s(s+2)}{s[s(s+1)(s+2) + s]}$$

$$I_1(s) = \frac{10(s+2)}{s^2 + 3s + 3}$$

$$I_2(s) = \frac{I_1(s)}{s+2} = \frac{10}{s^2 + 3s + 3}$$

(By V)

$$\text{So, } I_2(s) = \frac{10}{s^2 + 3s + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 3}$$

$$= \frac{10}{\left(s + \frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$I_2(t) = L^{-1} [I_2(s)]$$

$$= L^{-1} \left[\frac{10 \times \sqrt{3} / 2}{\sqrt{3} / 2 \left[\left(s + \frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 \right]} \right]$$

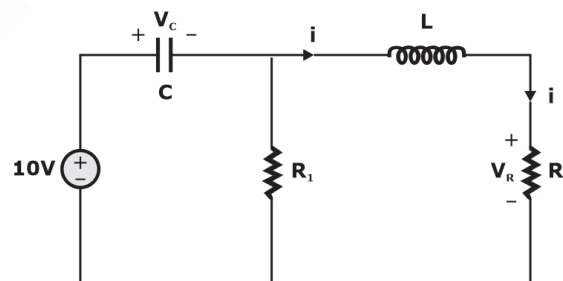
$$= 10 \frac{2}{\sqrt{3}} e^{-(3/2)t} \sin\left(\frac{\sqrt{3}}{2} t\right)$$

$$i_2(t) = \frac{20}{\sqrt{3}} e^{-(3/2)t} \sin\left(\frac{\sqrt{3}}{2} t\right)$$

$V_R(t) = R_2 i_2(t)$

$$V_R(t) = \frac{20}{\sqrt{3}} e^{-1.5t} \sin\left(\frac{\sqrt{3}}{2} t\right)$$

4. Determine the voltage V_R in the circuit given below

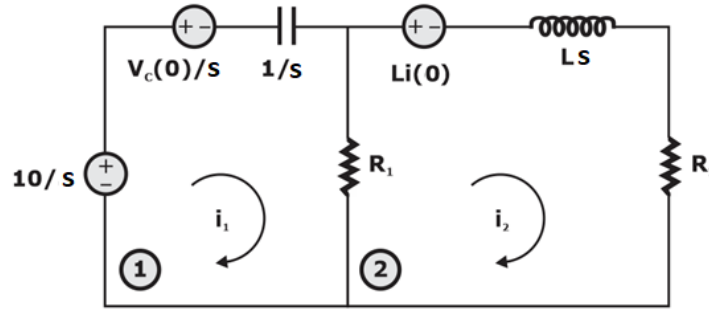


The values of parameters are $C = 1 \text{ F}$, $R_1 = R_2 = 1 \Omega$

$L = 1 \text{ H}$, $V_c(0) = 8 \text{ V}$, $i(0) = 1 \text{ A}$

Find the value of $i(t)$ for time $t > 0$

Sol. Drawing circuit considering the initial conditions



Applying KVL in loop (1)

$$-\frac{10}{s} + \frac{V_c(0)}{s} + \left[R_1 + \frac{1}{Cs} \right] I_1(s) - R_1 I_2(s) = 0$$

$$\left(1 + \frac{1}{s} \right) I_1(s) - I_2(s) = \frac{10}{s} - \frac{8}{s} = \frac{2}{s} \dots(i)$$

Applying KVL in loop (2)

$$- I_1(s) R_1 + (R_2 + Ls + R_1) I_2(s) - Li(0^+) = 0$$

$$- I_1(s) + (2 + s) I_2(s) = 1 \dots\dots\dots(ii)$$

$$Eq^n(ii) \times \left(\frac{s+1}{s} \right) + Eq^n(i)$$

$$\Rightarrow -\left(\frac{s+1}{s} \right) i_1(s) + (2+s) \times \frac{(s+1)}{s} I_2(s)$$

$$= \left(\frac{s+1}{s} \right) + \frac{2}{s} + \left(\frac{s+1}{s} \right) I_1(s) - I_2(s)$$

$$\Rightarrow \frac{(s+1)(s+2)}{s} I_2(s) - I_2(s) = \left(\frac{s+3}{s} \right)$$

$$I_2(s) = \frac{s+3}{s^2 + 2s + 2}$$

$$I_2(s) = \frac{s+1}{(s+1)^2 + 1} + \frac{2}{(s+1)^2 + 1}$$

$$i_2(t) = e^{-t} \cos t + 2e^{-t} \sin t$$

$$\therefore I_2(t) = e^{-t} \cos t + 2e^{-t} \sin t \text{ for } t > 0$$

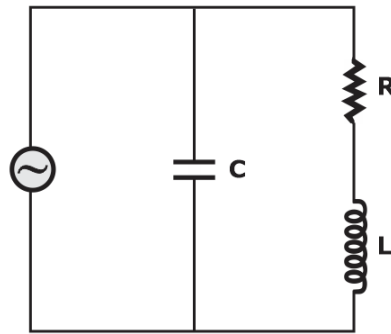
5. For the circuit shown below, derive the condition & expression of resonant frequency. Also calculate resonant frequency if parameters are $L = 1$, $C = 1nF$, $R = 1 K\Omega$, $V=1.5V$

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Sol. The equivalent impedance is given by

$$Z = (R + X_L) \parallel (X_C)$$

$$= (R + j\omega L) \parallel \frac{1}{j\omega C}$$

$$= \frac{(R + j\omega L) \times \frac{1}{j\omega C}}{\left(R + j\omega L + \frac{1}{j\omega C}\right)}$$

$$= \frac{(R + j\omega L)}{Rj\omega C - \omega^2 LC + 1}$$

$$= \frac{R + j\omega L}{(1 - \omega^2 LC) + j\omega RC}$$

$$= \frac{(R + j\omega L)[(1 - \omega^2 LC) - j\omega RC]}{(1 - \omega^2 LC)^2 + \omega^2 R^2 C^2}$$

At resonance imaginary part is zero as current and voltage are in phase at resonance

So,

$$-j\omega R^2 C + j\omega L(1 - \omega^2 LC) = 0$$

$$R^2 C/L = 1 - \omega^2 LC$$

$$\omega^2 LC = 1 - R^2 C/L$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2 C}{L^2 C}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

The expression for resonant frequency is derived as

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

So, putting values R, L and C the value of resonant frequency will be

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{1 \times 10^{-9}} - \frac{(10 \times 10^3)^2}{(1)^2}}$$
$$= \frac{1}{2\pi} \sqrt{10^9 - 10^8}$$

$$f = 4774.6 \text{ Hz}$$

The resonant frequency will be 4774.6 Hz or 4.77 KHz.



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