Classroom

ESE Mains Achiever's Study Plan

Electronics & Communication Engineering

Communication System Part-2

1. For a PCM system with the following parameters, determine

(i) Minimum sample rate

(ii) Minimum no. of bits used in the PCM code

(iii) Resolution

(iv) Maximum Quantization error

(v) Coding efficiency

(vi) Actual Dynamic Range in dB

Maximum analog input frequency = 6 kHz

Maximum decoded voltage at the receiver = ± 5.55 V.

Minimum dynamic range = 56 dB

Sol. (i) The minimum sample rate is

$f_s = 2 \times f_m = 2 \times 6$ kHz $= 12$ kHz

(ii) To find the value for Dynamic Range

$$56\text{dB} = 20\log\frac{V_{max}}{V_{min}}$$

$$\Rightarrow 2.8 = \log\frac{V_{max}}{V_{min}}$$

$$\Rightarrow 10^{2.8} = \frac{V_{max}}{V_{min}} = DR$$

$\Rightarrow$ DR = 630.957

In PCM code $2^n - 1 \geq$ DR

For a minimum number of bits

$2^n - 1 \geq$ DR

$\Rightarrow$ 2n = DR + 1

$\Rightarrow \log 2^n = \log(DR + 1)$

$\Rightarrow n \log 2 = \log(DR + 1)$

$$n = \frac{\log(DR+1)}{\log 2}$$

$$= \frac{\log(630.957+1)}{\log 2} = 9.3036$$

The closest whole no. greater than 9.3036 is 10 i.e. 10 bits must be used for the magnitude.

(iii) Resolution $= \dfrac{V_{max}}{2^n - 1} = \dfrac{5.55}{2^{10} - 1} = \dfrac{2.55}{1024 - 1} = 0.0025\text{V}$

(iv) Maximum Quantization error is

$$Q_e = \frac{\text{Resolution}}{2} = \frac{0.0025\text{V}}{2} = 0.00125\text{V}$$

(v) Coding efficiency

$$= \frac{\text{Minimum no. of bits (including sign bit)}}{\text{Actual no. of bits (including sign bit)}} \times 100$$

$$= \frac{9.3036+1}{10+1} \times 100 = \frac{10.3036}{11} \times 100 = 93.67\%$$

(vi) Actual Dynamic Range

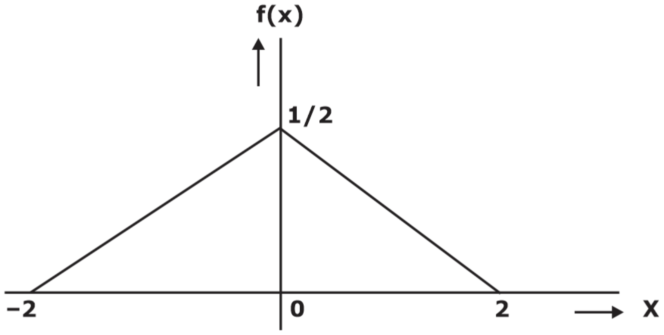
DR (dB) = 20 log (2^n -1)

= 20 log (1024 – 1)

20 log 1023

= 60.1975 dB

2. A signal can be modelled as a low pass stationary process X(t) whose probability Density function (Pdf) is given below:



The Bandwidth (BW) of the above process is 10 (kHz) and it is desired to transmit it using a PCM system.

If the sampling is done at the twice of Nyquist Rate and a uniform quantizer with 32 levels employed, the resulting Bit rate and SQNR are respectively ______

Sol. Since L = 32 (levels)

N = 5 (Bits/sample)

$f_s = f_q = 2 * 2f_m = 2 \times 2 \times 10 \times 10^3$

$= 40 \times 10^3$ (sample/sec)

$\therefore$ Bit Rate (R_b) = nf_s

$= 5 \times 40 \times 10^3 = 200$ (kbps)

$$E\left[x^2\right] = \int x^2 f(x)dx$$

$$f(x) = \frac{x+2}{4}, -2 \le x \le 0$$

$$f(x) = \frac{2-x}{4}, 0 \le x \le 2$$

$$\therefore E\left[x^2\right] = \int_{-2}^{0} \frac{(x+2)}{4} x^2 dx = \frac{1}{4}$$

$$\left[\int_{-2}^{0} 2x^2dx + \int_{-2}^{0} x^3dx\right]$$

When $-2 \le x \le 0$

$$E\left[x^2\right] = \frac{1}{4}\left[\frac{2}{3}(8) + \frac{1}{4}(-16)\right] = \frac{1}{4}\left[\frac{16}{3} - 4\right] = \frac{1}{3}W$$

$$E\left[x^2\right] = \int_{0}^{2} \frac{x^2(2-x)}{4}dx = \frac{1}{4}\left[\int_{0}^{2} 2x^2dx - \int_{0}^{2} x^3dx\right]$$

When $0 \le x \le 2$

$$E\left[x^2\right] = \frac{1}{4}\left[\frac{2}{3}\times 8 - \frac{1}{4}\times 6\right] = \frac{1}{4}\left[\frac{16}{3} - 4\right] = \frac{1}{3}W$$

$$\therefore E\left[x^2\right] = \frac{2}{3}W$$

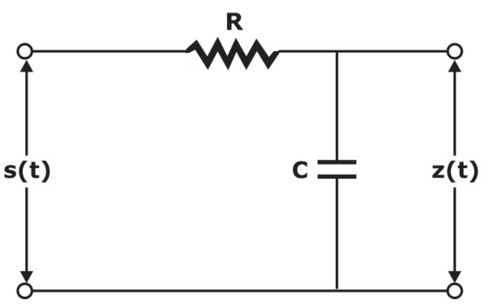
Total power of RP x(t)

$$N_q = \frac{\Delta^2}{12} = \frac{\left(V_{max} - V_{min}\right)^2}{L^2 \times 12} = \frac{\left(2-(-2)\right)^2}{32\times 32\times 12} = \frac{1}{32\times 24}$$

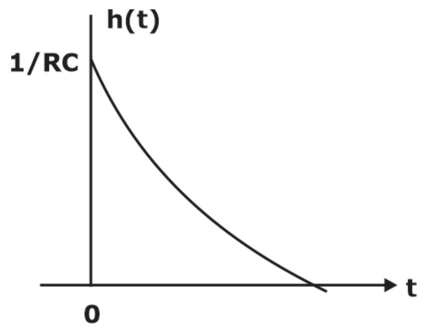
$$\frac{S}{N_q} = \frac{2}{3}\times 32\times 24 = 16\times 32 = 512$$

$\therefore$ (S/N_q) = 27.09 (dB)

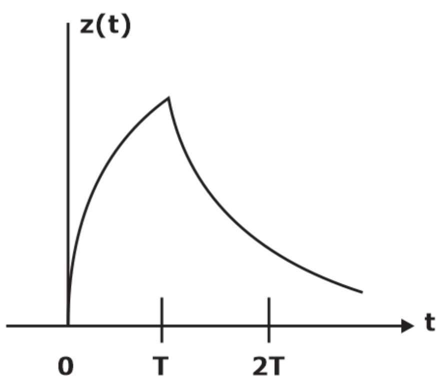
3. If a simple RC filter as shown in Figure. (a) is used instead of the matched filter, find its corresponding output and determine by what factor the maximum output SNR will get reduced



(a)



(b)



Sol. We know that

$$h(t) = \frac{1}{RC} e^{-t/(RC)} u(t)$$

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

Then the output z(t) is given by

Z(t) = s(t)* h(t)

$$= \begin{cases} 0 & t < 0 \\ A\left(1 - e^{-t/(RC)}\right) & 0 \le t \le T \\ A\left(1 - e^{-T/(RC)}\right)e^{-(t-T)/RC} & t > T \end{cases}$$

The maximum value of z(t) reached at t = T is given by z(T) = A $(1 - e^{-T/(RC)})$

The average output noise power is

$$N_o = E\left[n_o^2(t)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\eta}{2} \frac{d\omega}{1 + (\omega RC)^2} = \frac{\eta}{4RC}$$

Thus, $$\left(\frac{S}{N}\right)_o = \frac{z^2(T)}{N_o} = \frac{4A^2T}{\eta} \frac{\left(1 - e^{-T/(RC)}\right)^2}{T/(RC)}$$

We now maximum (S/No) with respect to RC. Letting x = T/(RC) and $g(x) = \frac{\left(1 - e^{-x}\right)^2}{x}$

And setting $$g'(x) = \frac{2xe^{-x}\left(1 - e^{-x}\right) - \left(1 - e^{-x}\right)2}{x^2} = 0$$

We obtain 2 $x3^{-x}$ = 1 $-e^{-x}$ or 1 + 2 x = e^x

Solving for x, we obtain

$$x = \frac{T}{RC} \approx 1.257$$

Substituting this value in the equation for output SNR

$$\left(\frac{S}{N}\right)_{o_{max}} = (0.815)\frac{2A^2T}{\eta}$$

Thus, by using an RC filter the maximum output SNR is reduced by a factor of 0.815 or 0.89 dB from that of the matched filter.

4. A binary channel matrix is given by

outputs

$$\text{Inputs} \quad \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & \frac{2}{3} & \frac{1}{3} \\ x_2 & \frac{1}{10} & \frac{9}{10} \end{array}$$

This means $P_{y/x}(y_1|x_1) = 2/3$, $P_{y/x}(y_2/x_1) = 1/3$, etc.

It is also given that $P_x(x_1) = 1/3$ and $P_x(x_2) = 2/3$. Determine the values of

(i) $P(y_1)$ & $P(y_2)$,

(ii) $H(x)$,

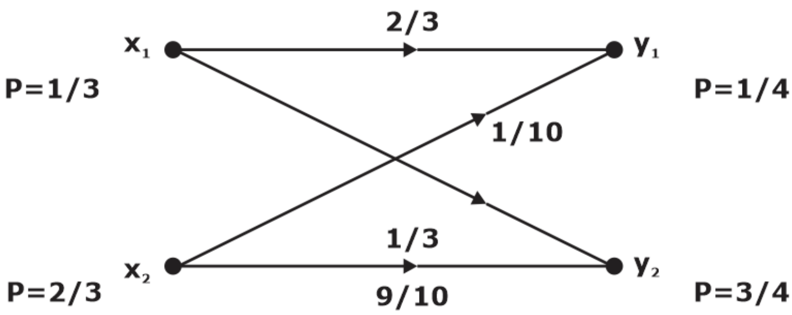
(iii) $H(x|y)$,

(iv) $H(y)$,

(v) $H(y|x)$, and

(vi) $I(x; y)$.?

Sol. (i) The channel matrix can be represented as shown in Fig.



$P(y_1) = P(y_1 |X_1)\, P(x_1) + P(y_1|x_2)\, P(x_2)$

$$= \frac{2}{3}\cdot\frac{1}{3} + \frac{1}{10}\cdot\frac{2}{3} = \frac{13}{45}$$

$$P(y_2) = 1 - P(y_1) = \frac{32}{45}$$

(ii) $$H(x) = P(x_1)\log\frac{1}{P(x_1)} + P(x_2)\log\frac{1}{P(x_2)}$$

$$= \frac{1}{3}\log_2 3 + \frac{2}{3}\log_2\frac{3}{2} = 0.918 \text{ bits}$$

(iii) To compute $H(x/y)$, we find

$$P(x_1 \mid y_1) = \frac{P(y_1 \mid x_1)P(x_1)}{P(y_1)} = \frac{10}{13}$$

$$P(x_1 \mid y_2) = \frac{P(y_2 \mid x_1)P(x_1)}{P(y_1)} = \frac{5}{32}$$

$$P(x_2 \mid y_1) = \frac{P(y_1 \mid x_2)P(x_2)}{P(y_1)} = \frac{3}{13}$$

$$P(x_2 \mid y_1) = \frac{P(y_1 \mid x_2)P(x_2)}{P(y_2)} = \frac{54}{64} = \frac{27}{32}$$

$$H(x \mid y_1) = P(x_1 \mid y_2)\log\frac{1}{P(x_1 \mid y_1)} + P(x_2 \mid y_1)\log\frac{1}{P(x_2 \mid y_1)}$$

$$= \frac{10}{13}\log\frac{13}{10} + \frac{3}{13}$$

$$\log_2\frac{13}{3} = 0.779$$

$$H(x \mid y_2) = P(x_1 \mid y_1)\log\frac{1}{P(x_1 \mid y_1)} + P(x_2 \mid y_2)\log\frac{1}{P(x_2 \mid y_2)}$$

$$= \frac{5}{32}\log\frac{32}{64} + \frac{54}{64}$$

$$\log\frac{64}{54} = 0.624$$

And

$H(x|y) = P(y_1)\ H(x|y) + P(y_2)\ H(x|y)$

$$= \frac{13}{45}(0.779) + \frac{32}{45}(0.624) = 0.6687$$

(iv) $H(y) = \sum_i P(y_i)\log\frac{1}{P(y_i)} = \frac{13}{45}\log\frac{45}{13} + \frac{32}{45}$

$$\log\frac{45}{32} = 0.8673 \text{ bits / symbol}$$

(v) $H(y|x) = H(y) - I(x|y) = 0.8673 - 0.2493$

$= 0.618$ bits/symbol

(vi) Thus, $I(x, y) = H(x) - H(x|y) = 0.918 - 0.6687$

$= 0.24893$ bits/symbol

5. Write a short notes on entropy coding/source coding with examples? ENTROPY CODING

Sol. The design of a variable – length code such that its average code word code length approaches the entropy of the DMS is often referred to as entropy coding. In the section we present two examples of entropy coding.

(a) Shannon-Fano Coding

And efficient code can be obtained by the following simple procedure, know as Shannon fano algorithm:

1. List the source symbols in order of decreasing probability
2. Partition the set into two sets that are as close to equiprobable as possible, and assign 0 to the upper set and 1 to the lower set.
3. Continue this process, each time partitioning the sets with as nearly equal probabilities as possible until further partitioning not possible. An example of Shannon-fano encoding is shown in table.

Table Shannon-Fano encoding

X_i	$P(x_i)$	Step 1	Step 2	Step 3	Step 4	Code
X_1	0.30	0	0			00
X_2	0.25	0	1			01
X_3	0.20	1	0			10
X_4	0.12	1	1	0		110
X_5	0.08	1	1	1	0	1110
X_6	0.05	1	1	1	1	1111

$H(X) = 2.36$ b/symbol, $L = 2.38$ b/symbol, $\eta = H(X)/L = 0.99$

(b) Huffman encoding

In general, Huffman encoding results in an optimum code. Thus, it is the code that has the highest efficiency. The problem Huffman encoding procedure is a follows:

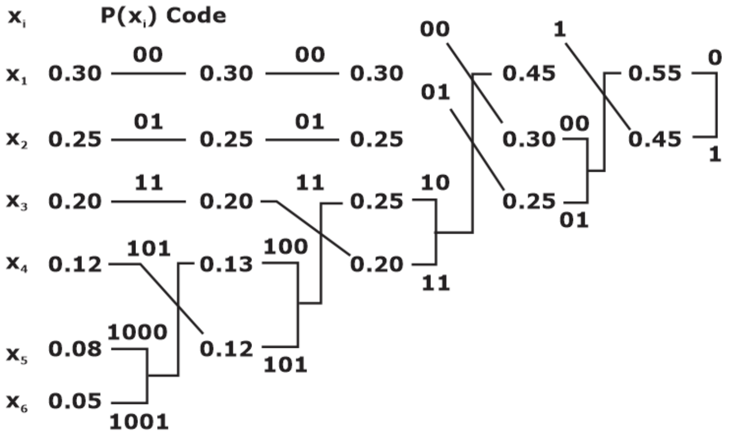
1. List the source symbols in order of decreasing probability
2. Combine the probabilities of the two symbols having the lowest probabilities, and reorder the resultant probabilities; this step is called reduction 1. The same procedure is repeated until there are two ordered probabilities remaining.
3. Start encoding with last reduction, which consist of exactly two ordered probabilities. Assign 0 as the first digit in the code words for all the source symbols associated with first probability; assign 1 to the second probability.
4. Now go back and assign 0 and 1 to the second digit for the two probabilities that were combined in the previous reduction step, retaining all assignments made in step 3.
5. Keep regressing this way until the first column is reached.

An example of Huffman encoding is shown in below table.

$H(X) = 2.36$ b/symbol

$L = 2.38$ b/symbol

$\eta = 0.99$



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