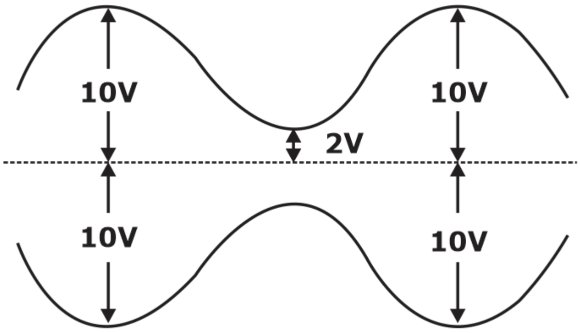
Classroom

ESE Mains Achiever's Study Plan

Electronics & Communication Engineering

Communication System Part-1

1. For the AM envelope shown below, determine



(a) Peak amplitude of the upper and lower side band frequencies.

(b) Peak amplitude of the carrier.

(c) Peak change in the amplitude of the envelope.

(d) Modulation coefficient.

(e) Percentage modulation.

(f) Power in the sidebands and the total power transmitted

Sol. From the envelope shown

V_{max} = 10 ......(1)

V_{min} = 2V ...(2)

But $V_{max} = A_c(1 + \mu)$ ...(3)

$V_{min} = A_c(1 - \mu)$ ...(4)

From (1), (2), (3), (4)

$$\frac{1+\mu}{1-\mu} = \frac{10}{2} = 5$$

$$\Rightarrow \mu = \frac{2}{3} = 0.67$$

$\Rightarrow A_c = 6$ V

(a) Peak amplitude of lower and upper sideband

$$= \frac{A_c\mu}{2} = \frac{6 \times \frac{2}{3}}{2} = 2V$$

(b) Peak amplitude of Carrier = Ac = 6 V

(c) Peak change in amplitude of envelope

$$= \frac{V_{max} - V_{min}}{2}$$

$$= \frac{10-2}{2} = 4V$$

(d) Modulation coefficient $= \mu = \frac{2}{3} = 0.67$

(e) Percentage modulating efficiency

$$= \frac{P_c \frac{\mu^2}{2}}{P_c\left(1+\frac{\mu^2}{2}\right)}$$

$$= \frac{\mu^2}{\mu^2+2} \times 100\%$$

$$= \frac{\left(\frac{2}{3}\right)^2}{\left(\frac{2}{3}\right)^2+2} \times 100\%$$

= 18.18%

(f) Sideband power

$$P_{SB} = \frac{P_c\mu^2}{2} = \frac{\left(\frac{Ac^2}{2}\right)\times\mu^2}{2}$$

$$= \frac{\frac{6^2}{2}}{2} \times \left(\frac{2}{3}\right)^2$$

= 4W

Total Power,

$$P_T = P_c\left(1+\frac{\mu^2}{2}\right)$$

$$= \frac{A_c^2}{2}\left(1+\frac{\mu^2}{2}\right)$$

= 22 W

2. The stationary process X(t) is passed through an LTI system and the output process is denoted by Y(t). Find the output autocorrelation function and the cross correlation function between the input and the output processes in each of the following cases.

(i) A delay system with delay Δ

(ii) A system with $h(t) = \frac{1}{t}$

(iii) A system described by the differential equation

$$\frac{d}{dt}Y(t) + Y(t) = \frac{d}{dt}X(t) - X(t)$$

(iv) A finite time average defined by the input – output relation

$$Y(t) = \frac{1}{2T}\int_{t-T}^{t+T} X(\tau)$$

Where T is a constant

Sol. (i) $R_{XY}(\tau) = R_x * \delta(-\tau - \Delta) = R_x(\tau) * \delta(\tau + \Delta)$

$= e^{-\alpha|\tau|} * \delta(t + \Delta) = e^{-\alpha|\tau+\Delta|}$

$R_Y(\tau) = R_{XY}(\tau) * \delta(-\tau - \Delta)$

$= e^{-\alpha|\tau+\Delta|} * \delta(\tau - \Delta) = e^{-\alpha|\tau|}$

(ii) $R_{XY}(\tau) = e^{-\alpha|\tau|} * \left(-\frac{1}{\tau}\right) = \int_{-\infty}^{\infty} \frac{e^{-\alpha|V|}}{t - V} dV$

$R_Y(\tau) = R_{XY}(\tau) * \frac{1}{\tau} = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{e^{-\alpha|V|}}{S - V}\frac{1}{\tau - S} dsdV$

The case of R_Y (τ) can be simplified as follows. Notice that $R_Y(T) = F^{-1}[S_Y(f)]$ where $S_Y(f) = S_x\,|H(f)|^2$. In our case, $S_x(f) = \frac{2\alpha}{\alpha^2 + 4\pi^2 f^2}$ and $|H(f)|^2 = \pi^2 \,\mathrm{sgn}^2(f)$. Since Sx (f) does not contain any impulse at the origin (f = 0) for which $|H(f)|^2 = 0$ we obtain $R_Y(\tau) = F^{-1}[S_Y(f)] = \pi^2 e^{-\alpha|T|}$

(iii) The system's transfer function is

$$H(f) = \frac{-1 + j2\pi f}{1 + j2\pi f}$$

Hence

$$S_{XY}(f) = S_x(f)H^*(f) = \frac{2\alpha}{\alpha^2 + 4\pi^2 f^2} + \frac{-1 - 2\pi f}{1 - j2\pi f}$$

$$= \frac{4\alpha}{1 - \alpha^2}\frac{1}{1 - j2\pi f} + \frac{\alpha - 1}{1 + \alpha}\frac{\alpha}{\alpha - 1}\frac{1}{\alpha - j2\pi f}$$

Thus, $R_{XY}(\tau) = F^{-1}\left[S_{XY}(f)\right]$

$$\frac{4\alpha}{1 - \alpha^2} e^{\tau} u_{-1}(-\tau) + \frac{\alpha - 1}{1 + \alpha} e^{-\alpha\tau} u_{-1}(\tau) + \frac{1 + \alpha}{\alpha - 1}$$

$$e^{\alpha\tau} u_{-1}(-\tau)$$

For the output power spectral density we have

$$S_Y(f) = S_x(f)\left|H(f)\right|^2 = S_x(f)\frac{1 + 4\pi^2 f^2}{1 + 4\pi^2 f^2} = S_x(f)$$

Hence, $R_Y(\tau) = F^{-1}\left[S_{XY}(f)\right] = e^{-\alpha|\tau|}$

(iv) The impulse response of the system is hence,

$$R_{XY}(\tau) = e^{-\alpha|\tau|} * \frac{1}{2T}\pi\left(\frac{-\tau}{2T}\right) = e^{-\alpha|\tau|} * \frac{1}{2T}\pi\left(\frac{\tau}{2T}\right)$$

$$= \frac{1}{2T}\int_{\tau - T}^{\tau - T} e^{-\alpha|V|} dV$$

If $\tau \geq T$, then

$$R_{XY}(\tau) = -\frac{1}{2T\alpha} e^{-\alpha v}\Big|_{\tau-T}^{\tau-T} = \frac{1}{2T\alpha}\left(e^{-\alpha(\tau-T)} - e^{-\alpha(\tau+T)}\right)$$

If $0 \leq \tau < T$, then

$$R_{XY}(\tau) = \frac{1}{2T}\int_{\tau-T}^{0} e^{\alpha v} dv + \frac{1}{2T}\int_{0}^{\tau+T} e^{-\alpha v} dv$$

$$= \frac{1}{2T\alpha}\left(2 - e^{\alpha(\tau-T)} - e^{\alpha(\tau+T)}\right)$$

The autocorrelation of the output is given by

$$R_Y(\tau) = e^{-\alpha|\tau|} * \frac{1}{2T}\pi\left(\frac{\tau}{2T}\right) * \frac{1}{2T}\pi\left(\frac{\tau}{2T}\right)$$

$$= e^{-\alpha|\tau|} * \frac{1}{2T} A\left(\frac{\tau}{2T}\right)$$

$$= \frac{1}{2T}\int_{-2T}^{2T}\left(1 - \frac{|x|}{2T}\right) e^{-\alpha|\tau-x|} dx$$

If $\tau \geq 2T$, then

$$R_Y(\tau) = \frac{e^{-\alpha\tau}}{2T\alpha^2}\left[e^{2\alpha T} + e^{-2\alpha T} - 2\right]$$

If $0 \leq T < 2T$, then

$$R_Y(\tau) = \frac{e^{-2\alpha T}}{4T^2\alpha^2}\left[e^{-\alpha\tau} + e^{\alpha\tau}\right] +$$

$$\frac{1}{T\alpha} - \frac{\tau}{2T^2\alpha^2} - 2\frac{e^{\alpha\tau}}{4T^2\alpha^2}$$

3. Consider the random process $X(t) = A\cos(\omega_0 t + \varphi)$ where A and $\omega 0$ are constants and φ is random variable distributed on $[-\pi, \pi]$. Check whether X(t) is Ergodic?

Sol. To check the Ergodicity time average and statistical average have to be calculated. Time average

$$\langle X(t)\rangle = \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{2T}\int_{-T}^{T} X(t) dt$$

$$= \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{2T}\int_{-T}^{T} A\cos(\omega_o t + \phi) dt = 0$$

$$\langle X(t)\rangle = \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{2T}\int_{-T}^{T} X^2(t) dt == \frac{A^2}{2}$$

$$\langle X(t)X(t+\tau)\rangle = \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{2T}\int_{-T}^{T} X(t)X(t+\tau) dt$$

$$= \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{2T}\int_{-T}^{T} A\cos(\omega_0 t + \phi)$$

$$= A\cos\left(\omega_0\left(t+\tau\right)+\phi\right)dt$$

$$= \underset{T\to\infty}{Lt}\frac{1}{2T}\int_{-\pi}^{\pi}\cos\left(\omega_0 t+\phi\right)$$

$$\cos\left(\omega_0\left(t+\tau\right)+\phi\right)dt = \frac{A^2}{2}\cos\left(\omega\tau\right)$$

Statistical average

$$E\left[X\left(t\right)\right] = \int_{-\pi}^{\pi} A\cos\left(\omega_0 t+\phi\right)d\phi = 0$$

$$E\left[X^2\left(t\right)\right] = \int_{-\pi}^{\pi} A^2\cos^2\left(\omega_0 t+\phi\right)d\phi = \frac{A^2}{2}$$

And autocorrelation function

$$R_X\left(\tau\right) = E\left\{X\left(t_1\right)X\left(t_2\right)\right\}$$

$$= \frac{A^2}{2}E\left[\cos\omega_0\left(t_1-t_2\right)+\cos\left(\omega t_1+\omega t_2+2\phi\right)\right]$$

$$= \frac{A^2}{2}E\left[\cos\omega_0\left(t_1-t_2\right)\right]+0$$

$$= \frac{A^2}{2}\cos\omega_0\tau$$

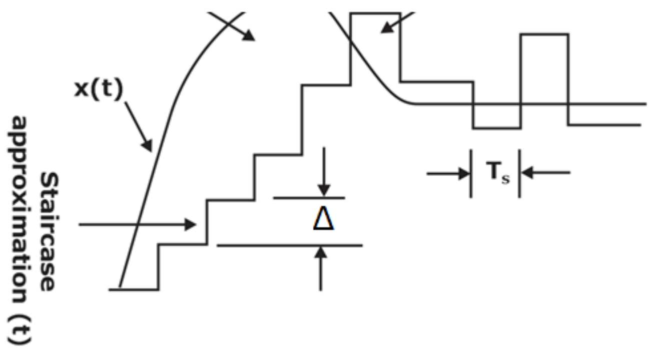
4. Explain the different types of quantization errors in Delta modulator? How these errors can be removed?

Sol. Delta modulation systems are subject to two types of quantization error

(i) Slope overload distortion

(ii) Granular noise.

Delta modulation is subject to rate of rise over load problems whenever the input changes too rapidly for the stepped wave form to follow it. If the input signal level remains constant, the reconstructed Delta modulation waveform exhibits a hunting behaviour known as idling noise. This idling noise is a square wave at one half the clock rate. If the clock rate is much greater than twice the highest frequency in the input signal, most of the idling noise can be filtered out at the receiver.



I) Slope Overload: Slope overload distortion occurs when the analog input signal changes at a faster rate than the DAC can maintain it, the slope of the analog signal is greater than the delta modulator can maintain.

In general, when the slope of stair case is less than (or) equal to modulating signal, the slope overloading occurs.

Increasing the clock frequency reduces the probability of slope overload occurring.

General method to reduce the slope overload is to increase the magnitude of the size.

Assume the input for Delta modulation be f(t) = A cos ω_m t

$$\text{Slope} = \left|\frac{df(t)}{dt}\right|_{max} = A\omega_m = A2\pi f_m$$

If the step size used in the Delta modulation system is 'Δ', then the maximum (rate of rise) slope over load is $\frac{\Delta}{T_s}$

$$\frac{\Delta}{T_s} \geq \left|\frac{d(m(t))}{dt}\right|_{max}$$

$$\frac{\Delta}{T_s} \geq A2\pi f_m \Rightarrow \Delta \geq A2\pi f_m T_s$$

II) Granular noise:

In general, Granular noise occurs

1. When the original analog input signal has a relatively constant amplitude, the reconstructed signal has variations that were not present in the original signal.

This is called granular noise.

2. The granular noise is analogous to the quantization noise in a PCM system.

3. When the step size Δ is too large relative to the local slope characteristics of the input waveform x(t), there by causing the stair case approximation u(t) to hunt around a relatively flat segment of the input waveform Granular noise can be reduced by decreasing the step size so that the stair case approximation may become more closer to the modulating signal.

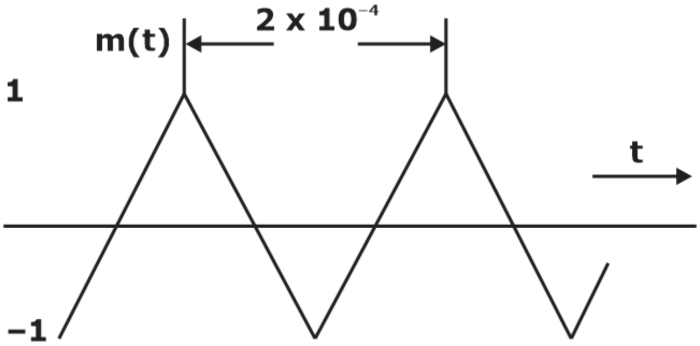
There is a need to have a large step size to accommodate a wide dynamic range, whereas small step size is required for the accurate representation of relatively low level signals.

$$\frac{\Delta}{T_s} < \left|\frac{dm(t)}{dt}\right|_{max} \Rightarrow \frac{\Delta}{T_s} < \Delta 2\pi f_m$$

$$\Rightarrow \Delta < A2\pi f_m T_s$$

Therefore, a granular noise can be removed by taking a small resolution of step size and a slope over load distortion can be removed by taking a large resolution.

5. (a) Estimate B_{FM} and B_{PM} for the modulating signal m(t) in fig. for $K_f = 2\pi \times 10^5$ and $K_p = 5\pi$. Assume the essential bandwidth of the periodic m(t) at the frequency of its third harmonic



(b) Repeat the problem if the amplitude of m(t) is doubled
[if m(t) is multiplied by 2].

Sol. (a) The peak amplitude of m(t) is unity. Hence, mp = 1 m(t) has the frequency f_m

$$f_m = \frac{1}{2 \times 10^{-4}} = 5\text{kHz}$$

The frequency of third harmonic m(t) will be B = 3 * 5 KHz = 15 KHz

For FM:

$$\Delta f = \frac{1}{2\pi} k_f m_p = \frac{1}{2\pi}\left(2\pi \times 10^5\right)(1) = 100\text{kHz}$$

And $B_{FM} = 2\left(\Delta f + B\right) = 230\text{kHz}$

For PM:

$$\Delta f = \frac{1}{2\pi} k_p \frac{\partial}{\partial t} m(t) = 50\text{kHz}$$

Hence, $B_{PM} = 2\left(\Delta f + B\right) = 130\text{kHz}$

(b) Doubling m(t) doubles its peak value. Hence, mp = 2. But its bandwidth is unchanged so that B = 15 kHz

For FM:

$$\Delta f = \frac{1}{2\pi} k_f m_p = \frac{1}{2\pi}\left(2\pi \times 10^5\right)(2) = 200\text{kHz}$$

And $B_{FM} = 2\left(\Delta f + B\right) = 430\text{kHz}$

For PM:

Doubling m(t) doubles its derivative so that now m_p = 2 and

$$\Delta f = \frac{1}{2\pi} k_p \frac{\partial}{\partial t} m(t) = 100\text{kHz}$$

and B_{PM} = 2(Δf + B) = 230 kHz

Observe that doubling the signal amplitude (doubling m(t)) roughly doubles frequency deviation Δf of both FM and PM waveforms.

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