Binomial Theorem

THE FACTORIAL FUNCTION

For $n \in N$ factorial on n, denoted by $n!$ is defined by $n! = n(n-1)(n-2).....3.2.1$

And the value of $0! = 1$

THE NOTATION ^nC_r

if $0 < r < n, n, r \in N, {}^nC_r = \dfrac{n!}{(n-r)!r!}$

i. $^nC_0 = {}^nC_n = 1$ and $^nC_r = 0$ for $r > n$

ii. $^nC_{n-r} = {}^nC_r$

iii. $^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

iv. $^nC_r = {}^nC_s$ if r = s or r + s = n

v. $^nC_r = \dfrac{n(n-1)(n-2)....(n-r-1)}{r!}$

SOME IMPORTANT DEDUCTION OF ^nC_r

i. $r.{}^nC_r = n.{}^{n-1}C_{r-1}$

ii. $r^2\,{}^nC_r = n(n-1)\,{}^{n-2}C_{r-2} - n\,{}^{n-1}C_{r-1}$

iii. $\dfrac{1}{r+1}\,{}^nC_r = \dfrac{1}{n+1}\,{}^{n+1}C_{r+1}$

GREATEST VALUE OF ^nC_r

Condition 1st: when n is even then ^nC_r will greatest at $r = \dfrac{n}{2}$

Condition 2nd: when n is odd then ^nC_r will greatest at $r = \dfrac{n+1}{2}$ $or\, r = \dfrac{n-1}{2}$

THE BINOMIAL THEOREM (FOR A POSITIVE INTEGER INDEX)

If n is positive integer index and $x \,\&\, y \in C$ then

$$(x+y)^n = {}^nC_0x^n + {}^nC_1x^{n-1}y + {}^nC_2x^{n-2}y^2 + ... + {}^nC_rx^{n-r}y^r + ... + {}^nC_ny^n$$

Here $^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_n$ are called binomial Coefficients.

SOME IMPORTANT POINT TO REMEMBER

For binomial expression $(x+y)^n$, where n is a positive integer.

i.The number of the term in the expansion is $(n+1)$
ii.In the-each term of the expansion, the sum of the exponents is n.
iii.The binomial coefficient from the beginning and the end are equal ${}^nC_r = {}^nC_{n-r}$

GENERAL TERM IN THE EXPANSION OF $(x+y)^n$

In the binomial expansion of $(x+y)^n, (r+1)^{th}$ term denote by T_{r+1} is given by
$T_{r+1} = {}^nC_r x^{n-r} y^r$

SOME IMPORTANT RESULTS OF BINOMIAL COEFFICIENT

The expansion $C_0 + C_1x + C_2x^2 + ... + C_nx^n = (1+x)^n$ can be written as $\sum_{r=0}^{n} {}^nC_r x^r = (1+x)^n$
and $\sum_{r=0}^{n} {}^nC_r = 2^n$

SUM OF THE BINOMIAL COEFFICIENTS

$C_0 + C_1x + C_2x^2 + ... + C_nx^n = (1+x)^n$

Putting $x=1$ then $C_0 + C_1 + C_2 + ... + C_n = 2^n$

Putting $x=-1$ then $C_0 - C_1 + C_2 - C_3...... = 0$

$\therefore C_0 + C_2 + C_4 + ... = C_1 + C_3 + C_5 + ... = 2^{n-1}$

******* TO FIND THE SUM OF COEFFICIENT IN $(X+Y)^n$ OR $(ax+by+cz+...)^n$**

Put all $x, y, z............$ etc equal to 1. For example sum of all coefficient in the expansion of $(2x-3y+5z)^n$ is $(2-3+5)^n = 4^n$

MULTINOMIAL THEOREM

The general term in the expansion of $(x_1 + x_2 + ... + x_m)^n, n \in N$ is given by

$$\frac{n!}{P_1!P_2!....P_m!} x_1^{p_1} x_2^{p_2} ....x_m^{p_m}, p_1 + p_2 + p_m = n$$

Thus, the number of the term in the expansion of $(x+y+z+u+...+m^{th}\ term)^n = {}^{n+m-1}C_n$

GENERAL TERM

i.In the expansion of $(1+x)^n$, the $(r+1)^{th}$ general term

$$T_{r+1} = \frac{n(n-1)(n-2)....(n-r+1)}{r!} x^r$$

ii. If n is a positive integer, then the expansion of $(1+x)^{-n}$

$$T_{r+1} = (-1)^r \, {}^{n+r-1}C_r x^r$$

iii. If n is a positive integer, then the expansion of $(1-x)^{-n}$

$$T_{r+1} = {}^{n+r-1}C_r x^r$$

SOME IMPORTANT EXPANSION

i. $(1-x)^{-1} = 1 + x + x^2 + x^3 + ... + x^r + ...$

ii. $(1+x)^{-1} = 1 - x + x^2 - x^3 + ... + (-1)^r x^r + ...$

iii. $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + ... + (r+1)x^r + ...$

iv. $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + ... + (-1)^r (r+1)x^r + ...$

GREATEST TERM IN THE EXPANSION OF $(1+x)^n$ **:** $T_r <=> T_{r+1}$ according to as

$\left| \frac{r}{(n-r+1)x} <=> 1, \right|$ taking equality sign then $r = \frac{|n+1||x|}{|x|+1}$

We should find the value of r by the equality then we can find the maximum and minimum value by applying the value of r.

**** In the expansion of** $(1+x)^n$ **,** $|x| < 1,$ **another more useful result assuming** $|T_{r+1}|$ **as the greatest term is** T_{r+1} **if** $\left|\frac{r}{n-r+1}\right| \le |x| \le \left|\frac{r+1}{n-r}\right|$

Here n can be any index rational, positive integer or negative integer.