Application Of Derivative

DERIVATIVE AS RATE MEASURE: Let $y = f(x)$ be a relation between two variable x and y. if δx is the small change in x and δy in y , then $\frac{\delta x}{\delta y}$ represent the average rate of change in x with respect to y in the interval $(x, x+\delta x)$.

Taking limit as $\delta y \to 0$ then the average rate of change $\frac{\delta x}{\delta y}$ become $\frac{\delta y}{\delta x}$ which is called instantaneous rate of change of y with respect to x.

VELOCITY AND ACCELERATION: if s is the distance moved by a particle in time t ; $s = f(t)$ then the velocity v and the acceleration a of the particle at any instant t are given by

$v = \frac{ds}{dt}$ and $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v\frac{dv}{ds}$

Velocity and acceleration both the positive in direction of s increasing with time t.

ANGULAR VELOCITY AND ACCELERATION: Let P be the position of the moving point one the curve at time t ; $\angle POX = \theta$ where OX is the initial line and o be the pole**. Then**

$angular\ velocity = \frac{d\theta}{dt}$ and $angular\ acceleration = \frac{d^2\theta}{dt^2}$

Both are positive in the direction of θ increasing with time.

APPROXIMATIONS: for the small value of δx we can take $\frac{\delta x}{\delta y} = \frac{dy}{dx}$ and therefore, $\delta x = \left(\frac{dy}{dx}\right)\delta y$

For the approximation calculation we can use the results $f(a+h) = f(a) + hf'(a)$ where h is very small compare to a.

This result enable us to find the value of $f(x)$ in the neighbourhood of a.

ERROR ESTIMATION:

(i) **Absolute Error**: in x is δx, in $f(x)$ is $\delta(f(x))$ i.e. $f'(x)\delta x$

(ii) **Relative Error** in x is $\frac{\delta x}{x}$ and $f(x)$ is $\frac{\delta(f(x))}{f(x)}$ i.e. $\frac{f'(x)}{f(x)}.\delta x$

(iii) **Percentage Error** in x is $\frac{\delta x}{x} \times 100$ and in $f(x)$ is $\frac{\delta(f(x))}{f(x)} \times 100$ i.e. $\frac{f'(x)}{f(x)}.\delta x \times 100$

LAW OF EXPONENTIAL GROWTH

The growth is some variable y with respect to x is said to be exponential if the rate of change in y proportional to itself

$$\frac{dy}{dx}\alpha y \Rightarrow \frac{dy}{dx} = \lambda y$$

$$\Rightarrow \frac{dy}{y} = \lambda dx$$

$$\Rightarrow \ln y = \lambda x + \ln c$$

$$\Rightarrow y = Ce^{\lambda x}$$

ROLLE'S THEORM

Statement : if a function $f(x)$ such that

(i) $f(x)$ is continuous in the close interval $[a,b]$

(ii) $f(x)$ is differential at every point in the open interval (a,b)

(iii) $f(a) = f(b)$, then there exist at least one value of x , say c, where $a < c < b$, such that $f'(c) = 0$

Alternative statement of the Rolle's Theorem

Let $f(x)$ be a real valued function defined and continuous on $[a,b]$, differentiable in (a,b) and $f(a) = f(b)$ then for some $\theta, 0 < \theta < 1$, $f'(a+\theta h) = 0$ where $h = b - a$

Verification

Let $f(x) = 3 - 4x + x^2 \, on [1,3]$

(i) $f(x)$, being polynomial is continuous and differentiable every where on the interval $[1,3]$

(ii) $f(1) = f(3) = 0$

$f'(x) = -4 + 2x = 0$

$\Rightarrow x = 2$

$\Rightarrow x \in (1,3)$

Hence rolle's theorem vailed for $f(x)$

GEOMETRICAL SIGNIFICANCE:

The theorem says that if

(i) The function has continuous graph in between $[a,b]$ (condition of continuity).

(ii) The graph of function has unique tangent (not vertical) in the interval except possibly at the end point. (condition of differentiability)

(iii) The value of function at the end points are equal; i.g. line joining the point $A(a, f(a))$ and $B(b, f(b))$ is parallel to x-axis.

Then there exist of at least one point between $[a,b]$ at which tangents is parallel to x-axis where $f'(x)=0$

ALGEBRIC SIGNIFICANCE

Let $f(x)$ is the polynomial function with its zeros a and b. i.e. $f(a)=f(b)=0$ since polynomial is continuous every where and differentiable as well , all the three condition of role's theorem satisfied in $[a,b]$.

Therefore there exist $a<c<b$ such that $f'(c)=0$

LAGRANGE'S MEAN VALUE THEOREM

Statement: let $f(x)$ be a real valued function such that

(i) $f(x)$ is continuous on $[a,b]$

(ii) $f(x)$ is differentiable in $[a,b]$

Then there exist at least one c such that $f'(c)=\dfrac{f(a)-f(b)}{b-a}$

Geometrical Significance $f'(c)=\dfrac{f(a)-f(b)}{b-a}$

Tangent at $Q(c,f(c))$ is parallel to AR.

CAUCHY'S MEAN VALUE THEOREM:

Statement : Let $f(x)$ and $g(x)$ be two real valued function define on $[a,b]$ such that

(i) Both continuous on $[a,b]$

(ii) Both Differentiable in (a,b)

(iii) $g'(x)\neq 0$ at any point in (a,b)

Then , there exist at least one $'c'; a<c<b$ such that $\dfrac{f(a)-f(b)}{g(a)-g(b)}=\dfrac{f'(c)}{g'(c)}, a<c<b$