Classroom

ESE Mains Achiever's Study Plan

Electronics & Communication Engineering

Networks Part-1

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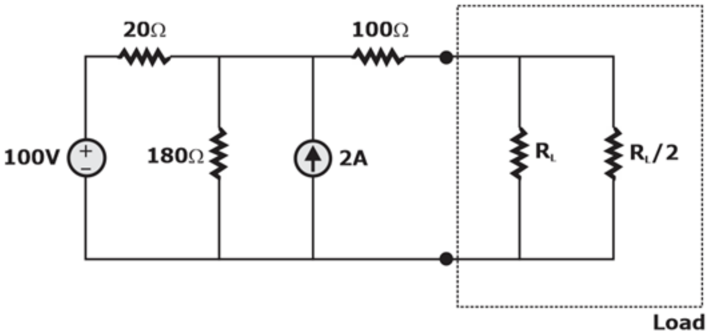
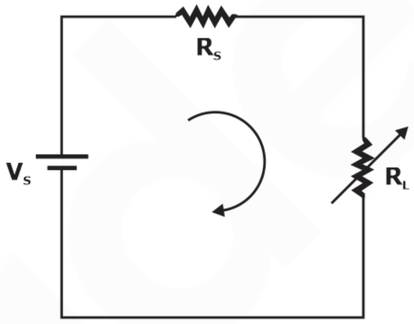
1. Define and prove maximum power transfer theorem also find power dissipated by load resistor R_L in maximum power transfer case.



Sol. Maximum power transfer theorem states that maximum power is transferred from source to load if load impedance is equal to the source impedance (or impedance seen across load terminal)

Prove of MPT:

Consider a circuit as shown below:



Consider a voltage source V_S having impedance R_S and having load impedance R_L.

Current (I) in circuit $= \frac{V_s}{R_s + R_L}$

Power dissipated in load resistance

$(R_L) = I^2 R_L$

$P = I^2 R_L$

For finding maximum power find

$$\frac{dP}{dR_L} = 0$$

So, $\frac{dP}{dR_L} = \frac{d}{dR_L} I^2 R_L = \frac{d}{dR_L} \frac{V_s^2}{(R_s + R_L)^2} R_L$

$$= V_s^2 \left[\frac{d}{dR_L} \frac{R_L}{(R_s + R_L)^2} \right]$$

$$= V_s^2\left[\frac{(R_s+R_L)^2 \times 1 - R_L\left(2(R_s+R_L)\right)}{(R_s+R_L)^2}\right]$$

$$= V_s^2\,\frac{(R_s+R_L)\left[(R_s+R_L)-2R_L\right]}{(R_s+R_L)^2} = 0$$

So, ($R_L = R_s$)

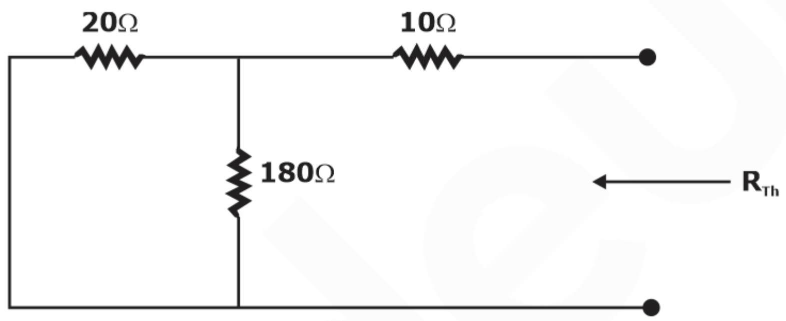
Which is condition of maximum power transfer theorem.

Solution of numerical:

Net load resistance:

$$R_L' = R_L \,||\, \frac{R_L}{2} = \frac{1}{3}R_L$$

Thevenin's resistance across load resistance:

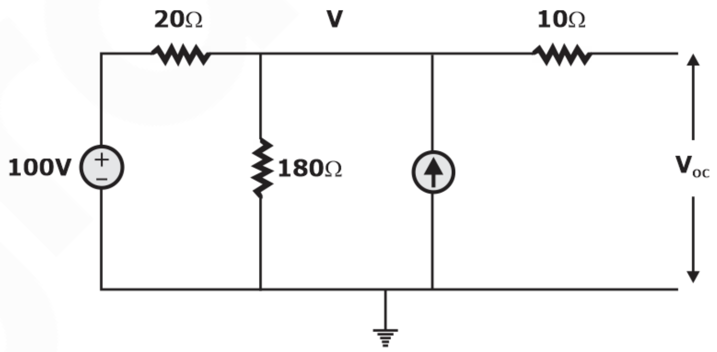


Replace the independent sources with internal impedance.

R_{Th} = (20 || 180) + 10 Ω

R_{Th} = 18 + 10 = 28 Ω

Open circuit voltage across R_L



By nodal analysis

$$\frac{V-100}{20} + \frac{V-0}{180} = 2$$

$$V\left(\frac{1}{20} + \frac{1}{180}\right) = 2 + \frac{100}{20} = 7$$

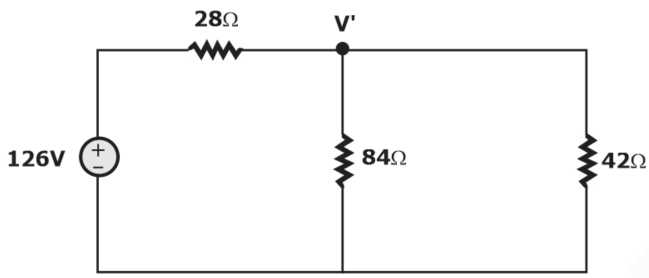
V = 18 × 7 = 126 V

$V = V_{OC}$ (as current in 10 Ω resistor is zero)

By MPT, R_{Th} = load resistance $R_L' = \frac{1}{3}R_L$

$R_L = 3 \times R_{Th} = 28 \times 3 = 84\ \Omega$

So, equivalent circuit will be :



By nodal analysis:

$$\frac{V'-126}{28} + \frac{V'-0}{84} + \frac{V'-0}{42} = 0$$

$$V'\left(\frac{1}{28} + \frac{1}{84} + \frac{1}{42}\right) = \frac{126}{28}$$

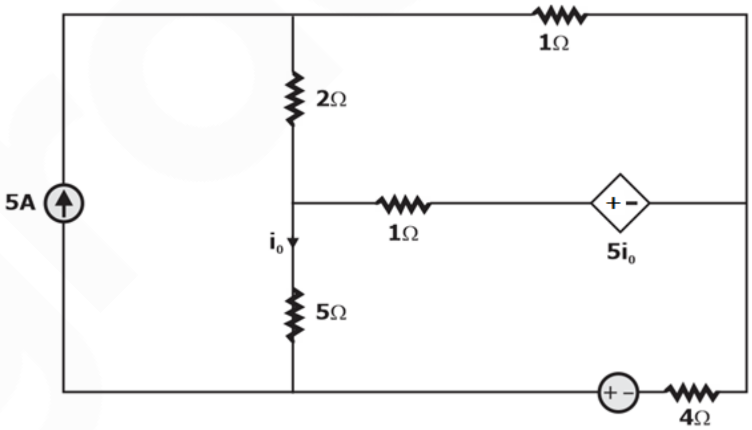
= 63 V

Power dissipated in

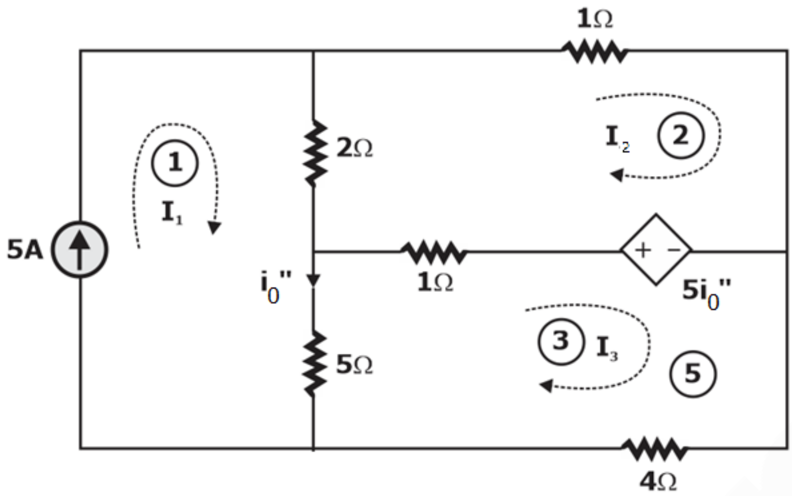
$$R_L = \frac{(V')^2}{R_L} = \frac{63^2}{84} = 47.25 \text{ watt}$$

The power dissipated in load resistor of R_L is 47.25 W

2. State superposition theorem. Find i_0 in the circuit using superposition theorem.



Sol. superposition theorem states that in a circuit having more than one independent sources the response in any branch can be calculated by algebraic sum of individual responses of each source and replacing all other source with their internal impedance. Consider 5A current source:



Applying loop equations:

In loop (1):

$I_1 = 5A$ ....(i)

In loop (2)

$-2I_1 + 4I_2 - I_3 - 5i_0' = 0$ ....(ii)

$\therefore i'_0 = I_1 - I_3$ ....(iii)

By (ii) & (iii)

$-2I_1 + 4I_2 - I_3 - 5(I_1 - I_3) = 0$

$-7I_1 + 4I_2 - 6I_3 = 0$ ....(iv)

In loop (3)

$-5I_1 - I_2 + 10I_3 + 5i'_0 = 0$

$-I_2 + 5I_3 = 0$

By (i), (iv) & (v)

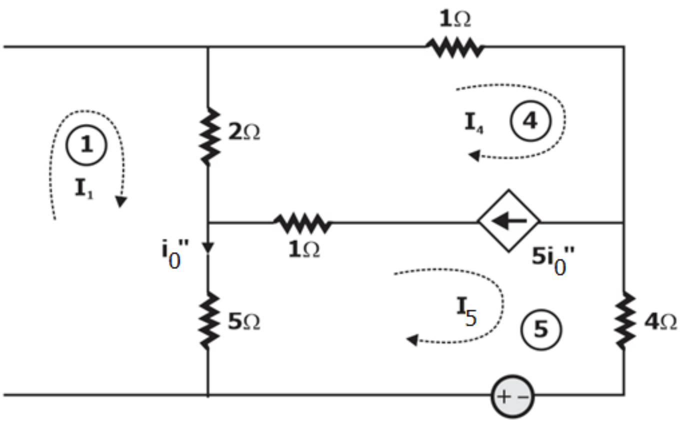
$I_1 = 5A$

$I_2 = 12.5A$

$I_3 = 2.5A$

$i'_0 = I_1 - I_3 = 2.5A$ ....(vi)

Considering voltage source of 20 V:



In loop 4

$4I_4 - I_5 - 5i''_0 = 0$ ....(vii)

$4I_4 - I_5 - 5(-I_5) = 0$ ($\therefore I''_0 = -I_5$)

$4I_4 + I_5 = 0$.............(viiii)

In loop 5:

$-I_4 + 10I_5 + 5i''_0 - 20 = 0$

$-I_4 + 5I_5 = 20$ ......(ix)

By (viii) & (ix)

$$I_4 = \frac{-10}{3}A$$

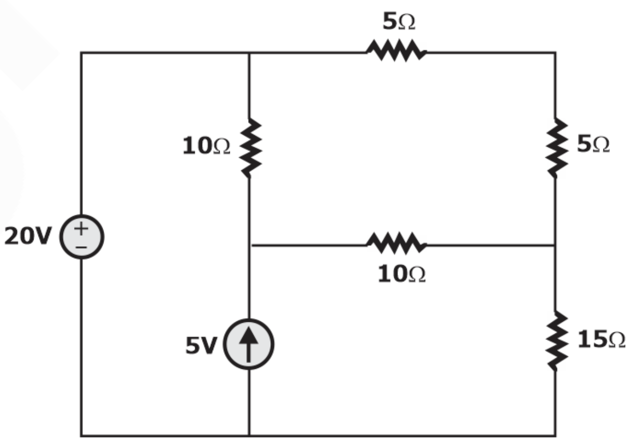
$$I_5 = \frac{10}{3}A$$

$$i''_0 = -I_5 = \frac{-10}{3} = -3.33A$$

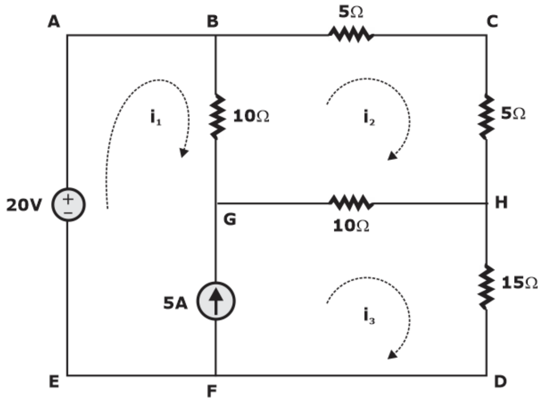
$\therefore$ response $i_0 = i'_0 + i''_0 = 2.5 - 3.33$

$$= -\frac{5}{6} = -0.833A$$

3. Determine current in various branches of circuit shown below using mesh analysis



Sol. In the above given circuit taking currents as



∵ Current source exists in loop so super mesh exits

Applying loop equation in ABGHDFE

$-20 + 10(i_1 - i_2) + 10(i_3 - i_2) + 15 i_3 = 0$

$10i_1 - 20i_2 + 25i_3 = 20$ .... (i)

Applying loop equation in GBCH

$10(i_2 - i_1) + (5 + 5)i_2 + 10(i_2 - i_3) = 0$

$-10i_1 + 30i_2 - 10i_3 = 0$ .... (ii)

Loop equation can't be applied EABGF & GHDF as it is super mesh

By KCL at node G :

$i_3 - i_1 = 5$

$-i_1 + i_3 = 5$ .... (iii)

Writing equation in matrix form:

$$\begin{bmatrix} 10 & -20 & 25 \\ -10 & 30 & -10 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 5 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 10 & -20 & 25 \\ -10 & 30 & -10 \\ -1 & 0 & 1 \end{bmatrix} \text{Det}(\Delta) = 1050$$

$$i_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta_1 = \begin{bmatrix} 20 & -20 & 25 \\ 0 & 30 & -10 \\ 5 & 0 & 1 \end{bmatrix} \Rightarrow \text{Det}|\Delta_1| = -2150$$

$$\Delta_2 = \begin{bmatrix} 10 & 20 & 25 \\ -10 & 0 & -10 \\ -1 & 5 & 1 \end{bmatrix} \Rightarrow \text{Det}|\Delta_2| = -350$$

$$\Delta_3 = \begin{bmatrix} 10 & 20 & 20 \\ -10 & 30 & 0 \\ -1 & 0 & 5 \end{bmatrix} \Rightarrow \text{Det}|\Delta_3| = 3100$$

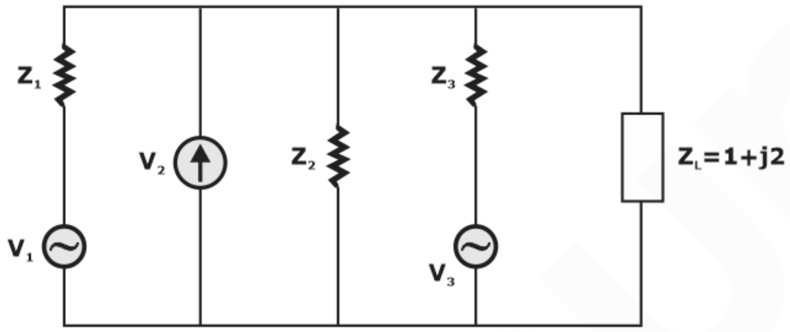
Current in different branches are

$$i_1 = \frac{\Delta_1}{\Delta} = -2.047A$$

$$i_2 = \frac{\Delta_2}{\Delta} = -0.33A$$

$$i_3 = \frac{\Delta_3}{\Delta} = 2.95A$$

4. Explain Millman's theorem. In the circuit given below, find the net response using Millman's theorem i.e. find current through Z_L.



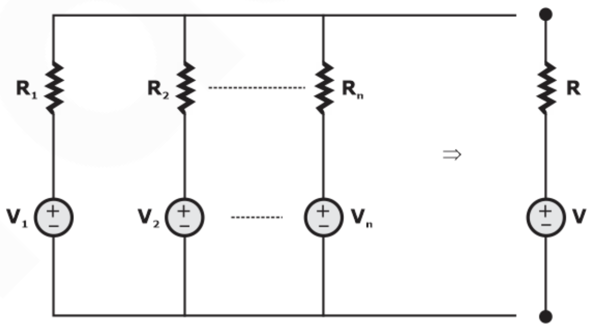
$V_1 = 5\angle 0°$, $Z_1 = 1.5\angle 0°\ \Omega$

$I_2 = 2\angle 0°$, $Z_2 = 5\ \Omega$

$V_3 = 10\angle 45°$, $Z_3 = 10\ \Omega$

Sol. Millman's Theorem:

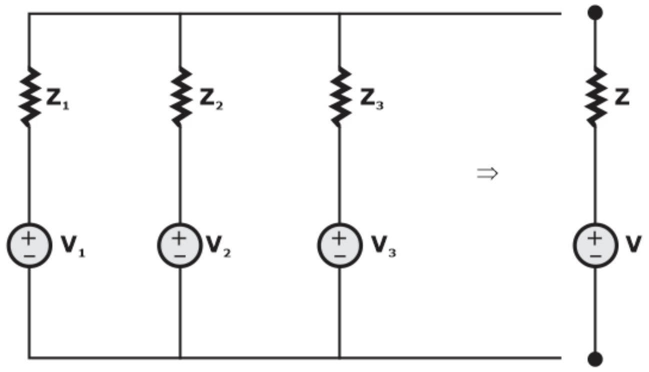
Millmans theorem states that in any network having independent voltage source having internal resistances and connected parallel, the entire combination above can be replaced by single voltage source V in series with resistance R where



$$V = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \ldots\ldots + \frac{V_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{R_n}}$$

$$R = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{R_n}\right)}$$

Rearranging the circuit given in question by source transformation



$V_2 = I_2 Z_2 = 2 \times 5 = 10\angle 0° \text{ V}$

$$V = \frac{V_1G_1 + V_2G_2 + V_3G_3}{G_1 + G_2 + G_3} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$V = \frac{\frac{5}{1.5} + \frac{10}{5} + \frac{10\angle 45°}{10}}{\left(\frac{1}{1.5} + \frac{1}{5} + \frac{1}{10}\right)}$$

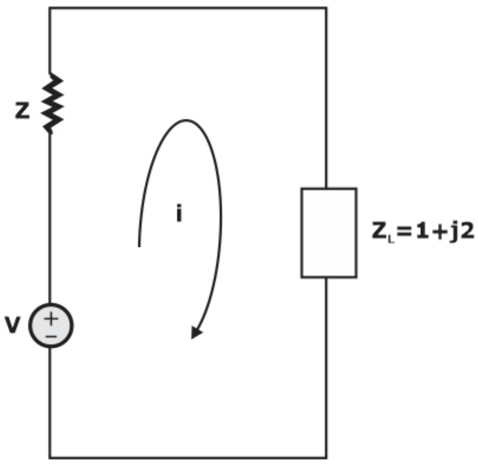
$$= \frac{6.04 + j0.707}{0.967}$$

$= 6.246 + j0.73$

$$Z = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\left(\frac{1}{1.5} + \frac{1}{5} + \frac{1}{10}\right)}$$

$$= \frac{30}{29} = 1.03\Omega$$

So net reduced circuit



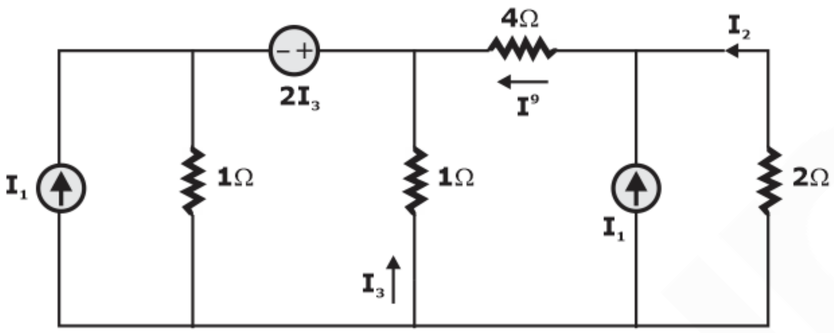
$$i = \frac{V}{z + z_L} = \frac{6.246 + j0.73}{(1.03 + 1 + j2)}$$

$i = 2.215 \angle -35.92°A$

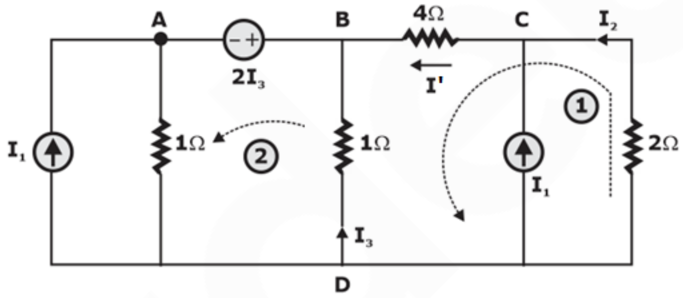
The current through load

$Z_L = 2.215 \angle -37.92°A$

5. For the network shown below determine ratio of current I_1/I_2.



Sol.



Applying KCL at node C,

$I_1 + I_2 = I'$ ……………… (i)

KVL in node 1:

$2I_2 + 4I' - I_3 = 0$

$2I_2 + 4(I_1 + I_2) - I_3 = 0$

$4I_1 + 6I_2 - I_3 = 0$ ……………… (ii)

KVL in node 2:

$-1\ (I_1 + I' + I_3) - 2I_3 - I_3 = 0$

$I_1 + I_3 + I_1 + I_2 + 2I_3 + I_3 = 0$

$2I_1 + I_2 + 4I_3 = 0$ ……………… (iii)

Equation (ii) × 4 and adding to (iii)

$4I_1 + 6I_2 - 4I_3 + 2I_1 + I_2 + 4I_3 = 0$

$6I_1 + 7I_2 = 0$

$\frac{I_1}{I_2} = \frac{-7}{6}$ is the required ratio:

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