

# Concepts & Tricks on Number System

**Quantitative Aptitude** deals mainly with the different topics in Arithmetic, which is the science which deals with the relations of numbers to one another. It includes all the methods that are applicable to numbers.

Numbers are expressed by means of figures – 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0 ---- called digits. Out of these, 0 is called **insignificant** digit whereas the others are called **significant** digits.

**Numbers**

A group of figures, representing a number, is called a numeral. Numbers are divided into the following types.

**Natural Numbers:**

Numbers which we use for counting the objects are known as natural numbers. They are denoted by 'N'

$$N = \{1, 2, 3, 4, \dots\}$$

**Whole Numbers:**

When we include 'zero' in the natural numbers, it is known as whole numbers. They are denoted by 'W'.

$$W = \{0, 1, 2, 3, 4, 5, \dots\}$$

**Prime Numbers:**

A number other than 1 is called a prime number if it is divisible only by 1 and itself.

**To test whether a given number is prime number or not**

If you want to test whether any number is a prime number or not, take an integer larger than the approximate square root of that number. Let it be 'x'. test the divisibility of the given number by every prime number less than 'x'. if it not divisible by any of them then it is prime number; otherwise it is a composite number (other than prime).

**Example:** Is 349 a prime number?

**Solution:**

The square root of 349 is approximate 19. The prime numbers less than 19 are 2, 3, 5, 7, 11, 13, 17.

Clearly, 349 is not divisible by any of them. Therefore, 349 is a prime number.

**Composite Numbers:**

A number, other than 1, which is not a prime number is called a composite number.

**e.g.** 4, 6, 8, 9, 12, 14 ..... and so on

**Even Number:**

The number which is divisible by 2 is known as an even number.

**e.g.** 2, 4, 8, 12, 24, 28 ..... and so on

It is also of the form  $2n$  {where  $n =$  whole number}

**Odd Number:**

The number which is not divisible by 2 is known as an odd number.

**e.g.** 3, 9, 11, 17, 19 ..... and so on

**Consecutive Number:**

A series of numbers in which each is greater than that which precedes it by 1 is called a series of consecutive numbers.

**e.g.** 6, 7, 8 or 13, 14, 15, 16 or, 101, 102, 103, 104

**Integers:**

The set of numbers which consists of whole numbers and negative numbers is known as a set of integers it is denoted by  $\mathbb{Z}$ .

**e.g.**  $\mathbb{Z} = \{-4, -3, -2, -1, 0, 1, 2, 3, \dots\}$

**Rational Number:**

When the numbers are written in fraction, they are known as rational numbers. They are denoted by  $\mathbb{Q}$ .

**e.g.**  $\frac{1}{2}, \frac{3}{4}, \frac{8}{9}$  are called rational numbers.

Or, the numbers which can be written in the form  $\frac{a}{b}$  {where  $a$  and  $b$  are integers and  $b \neq 0$ } are called rational numbers.

**Irrational Numbers:**

The numbers which cannot be written in the form of  $p/q$  are known as irrational numbers (where  $p$  and  $q$  are integers and  $q \neq 0$ ).

**Real Numbers:**

Real numbers include both rational as well as irrational numbers.

**Rules of Simplification**

(i) In simplifying an expression, first of all, vinculum or bar must be removed. For example, we have known that  $-8 - 10 = -18$

But,  $\overline{-8 - 10} = -(-2) = 2$

(ii) After removing the bar, the brackets must be removed, strictly in the order  $()$ ,  $\{\}$  and  $[\ ]$ .

(iii) After removing the brackets, we must use the following operations strictly in the order given below. (a) of (b) division (c) multiplication (d) addition and (e) subtraction.

**Note:** The rule is also known as the rule of '**VBODMAS**' where V, B, O, D, M, A and S stand for Vinculum, Brackets, Of, Division, Multiplication, Addition and Subtraction respectively.

$$1 \div \frac{3}{7} \text{ of } (6 + 8 \times \overline{3 - 2}) + \left[ \frac{1}{5} \div \frac{7}{25} - \left\{ \frac{3}{7} + \frac{8}{14} \right\} \right]$$

**Example:** Simplify

**Solution:**

$$\begin{aligned} & 1 \div \frac{3}{7} \text{ of } (6 + 8 \times 1) + \left[ \frac{1}{5} \div \frac{7}{25} - \left\{ \frac{3}{7} + \frac{8}{14} \right\} \right] \\ & = 1 \div \frac{3}{7} \text{ of } (6 + 8) + \left[ \frac{1}{5} \times \frac{25}{7} - 1 \right] \\ & = 1 \div \frac{3}{7} \text{ of } 14 + \left[ \frac{5}{7} - 1 \right] = 1 \div 6 + \left[ -\frac{2}{7} \right] \\ & = \frac{1}{6} - \frac{2}{7} = \frac{7 - 12}{42} = -\frac{5}{42} \end{aligned}$$

**Ascending or Descending Order in Rational Numbers**

**Rule 1:** When the numerator and the denominator of the fractions increase by a constant value, the last fraction is the biggest.

**Example:** Which of the following fractions is the greatest?

$$\frac{3}{4}, \frac{4}{5} \text{ and } \frac{5}{6}$$

**Solution:**

We see that the numerators as well as denominators of the above fraction increase by 1,

so the last fraction, i.e.  $\frac{5}{6}$  is the greatest fraction.

**Rule 2:** The fraction whose numerator after cross-multiplication given the greater value is greater.

$$\frac{5}{8} \text{ or } \frac{9}{14} ?$$

**Example:** Which is greater :

**Solution:**

Students generally solve these questions by changing the fractions into decimal values or by equating the denominators. But, we suggest a better method for getting the answer more quickly.

**Step 1:** Cross –multiply the two given fractions.

$$\begin{array}{ccc} \frac{5}{8} & \begin{array}{l} \longleftarrow \\ \longrightarrow \end{array} & \frac{9}{14} \\ \frac{8}{8} & \begin{array}{l} \longleftarrow \\ \longrightarrow \end{array} & \frac{14}{14} \end{array}$$

We have,  $5 \times 14 = 70$  and  $8 \times 9 = 72$

**Step II.** As 72 is greater than 70 and the numerator involved with the greater

value is 9, the fraction  $\frac{9}{14}$  is the greater of the two.

**Example: Which is greater:**  $\frac{4}{15}$  or  $\frac{6}{23}$ ?

**Solution:**

**Step I:**  $4 \times 23 > 15 \times 6$

**Step II:** As the greater value has the numerator 4 involved with it,  $\frac{4}{15}$  is greater.

You can see how quickly this method works. After a good practise, you won't need to calculate before answering the question.

The arrangement of fractions into the ascending or descending order becomes easier now. Choose two fractions at a time. See which one is greater. This way you may get a quick arrangement of fractions.

**Note: Sometimes, when the values are smaller (i.e., less than 10), the conventional method, i.e., changing the values into decimals or equating the denominators after getting LCM, will prove more convenient for some of you.**

**Example:** Arrange the following in ascending order.

$$\frac{3}{7}, \frac{4}{5}, \frac{7}{9}, \frac{1}{2} \text{ and } \frac{3}{5}$$

**Solution: Method I**

The LCM of 7,5,9,2,5, is 630.

Now, to equate the denominators, we divide the LCM by the denominators and multiply the quotient by the respective numerators.

Like for  $\frac{3}{7}$ ,  $630 \div 7 = 90$ , so, multiply 3 by 90.

$$\frac{270}{630}, \frac{504}{630}, \frac{490}{630}, \frac{315}{630} \text{ and } \frac{378}{630}$$

Thus, the fractions change to  $\frac{270}{630}, \frac{504}{630}, \frac{490}{630}, \frac{315}{630}$  and  $\frac{378}{630}$ . The fraction which has larger numerator is naturally larger. So,

$$\frac{504}{630} > \frac{490}{630} > \frac{378}{630} > \frac{315}{630} > \frac{270}{630}$$

$$\text{or } \frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

**Method II:**

Change the fractions into decimals like

$$\frac{3}{7} = 0.428, \frac{4}{5} = 0.8, \frac{7}{9} = 0.777, \frac{1}{2} = 0.5, \frac{3}{5} = 0.6$$

Clearly,

$$\frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

**Method III:**

**Rule of CM (cross-multiplication)**

**Step I:** Take the first two fractions. Find the greater one by the rule of CM.

$$\frac{3}{7} \quad \frac{4}{5}$$


$$3 \times 5 < 7 \times 4$$

$$\therefore \frac{4}{5} > \frac{3}{7}$$

**Step II:** Take the third fraction. Apply CM with the third fraction and the larger value obtained in a step I.

$$\frac{4}{5} \begin{array}{l} \swarrow \quad \searrow \\ \quad \quad \quad \end{array} \frac{7}{9}$$

$$4 \times 9 > 5 \times 7$$

$$\therefore \frac{4}{5} > \frac{7}{9}$$

Now we see that  $\frac{7}{9}$  can lie after  $\frac{3}{7}$  or between  $\frac{4}{5}$  and  $\frac{3}{7}$ .

Therefore, we apply CM with  $\frac{3}{7}$  and  $\frac{7}{9}$  see that  $\frac{7}{9} > \frac{3}{7}$ .

$$\therefore \frac{4}{5} > \frac{7}{9} > \frac{3}{7}$$

**Step III:** Take the next fraction. Apply CM with  $\frac{3}{7}$  and  $\frac{1}{2}$  and see that  $\frac{1}{2} > \frac{3}{7}$ . Next, we

apply CM with  $\frac{7}{9}$  and  $\frac{1}{2}$  and see that  $\frac{7}{9} > \frac{1}{2}$ .

Therefore,

$$\therefore \frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

**Step IV:** With similar applications, we get the final result as:

$$\therefore \frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

**Note:** This rule has some disadvantages also. But if you act fast, it gives faster results. Don't reject this method at once. This can prove to be a better method for you.