Number System Concepts & Tricks

Concept & Tricks on Number System

Arithmetic is the science which deals with the relations of numbers to one another. It includes all the methods that are applicable to numbers.
Numbers are expressed by means of figures – 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0 ---- called digits. Out of these, 0 is called ***insignificant*** digit whereas the others are called ***significant*** digits.

Numbers

A group of figures, representing a number, is called a numeral. Numbers are divided into the following types.

Natural Numbers:

Numbers which we use for counting the objects are known as natural numbers. They are denoted by 'N'

N = {1,2,3,4,…….}

Whole Numbers:

When we include 'zero' in the natural numbers, it is known as whole numbers. They are denoted by 'W'.

W= {0,1,2,3,4,5,………}

Prime Numbers:

A number other than 1 id called a prime number if it is divisible only by 1 and itself.

To test whether a given number is prime number or not

If you want to test whether any number is a prime number or not, take an integer larger than the approximate square root of that number. Let it be 'x'. test the divisibility of the given number by every prime number less than 'x'. if it not divisible by any of them then it is prime number; otherwise it is a composite number (other than prime).

Example: Is 349 a prime number?

Solution:

The square root of 349 is approximate 19. The prime numbers less than 19 are 2, 3, 5, 7, 11, 13, 17.

Clearly, 349 is not divisible by any of them. Therefore, 349 is a prime number.

Composite Numbers:

A number, other than 1, which is not a prime number is called a composite number.

e.g. 4, 6, 8, 9, 12, 14 ...... and so on

Even Number:

The number which is divisible by 2 is known as an even number.

e.g. 2, 4, 8, 12, 24, 28 ...... and so on

It is also of the form 2n {where n = whole number}

Odd Number:

The number which is not divisible by 2 is known as an odd number.

e.g. 3, 9, 11, 17, 19 ...... and so on

Consecutive Number:

A series of numbers in which each is greater than that which precedes it be 1 is called a series of consecutive numbers.

e.g. 6, 7, 8 or 13, 14, 15, 16 or, 101, 102, 103, 104

Integers:

The set of numbers which consists of whole numbers and negative numbers is known as a set of integers it is denoted by 1.

e.g. I = {-4,-3,-2,-1,0,1,2,3,….}

Rational Number:

When the numbers are written in fraction, they are known as rational numbers. They are denoted by Q.

e.g. $\frac{1}{2}, \frac{3}{4}, \frac{8}{9}$ are called rational numbers.

Or, the numbers which can be written in the form $\frac{a}{b}$ {where a and b are integers and b $\neq$ 0} are called rational numbers.

Irrational Numbers:
The numbers which cannot be written in the form of p/q are known as irrational numbers (where p and q are integers and q $\neq$ 0).
Real Numbers:
Real numbers include both rational as well as irrational numbers.
Rules of Simplification
(i) In simplifying an expression, first of all, vinculum or bar must be removed. For example, we have known that – 8 – 10 = -18

But, $-\overline{8-10}$ = - (-2) = 2
(ii) After removing the bar, the brackets must be removed, strictly in the order (), {} and [].
(iii) After removing the brackets, we must use the following operations strictly in the order given below. (a) of (b) division (c) multiplication (d) addition and (e) subtraction.
Note: The rule is also known as the rule of **'VBODMAS'** where V, B, O, D, M, A and S stand for Vinculum, Brackets, Of, Division, Multiplication, Addition and Subtraction respectively.

Example: Simplify $1 \div \frac{3}{7} of (6 + 8 \times \overline{3-2}) + \left[\frac{1}{5} \div \frac{7}{25} - \left\{\frac{3}{7} + \frac{8}{14}\right\}\right]$
Solution:

$$1 \div \frac{3}{7} of (6 + 8 \times 1) + \left[\frac{1}{5} \div \frac{7}{25} - \left\{\frac{3}{7} + \frac{8}{14}\right\}\right]$$

$$= 1 \div \frac{3}{7} of (6 + 8) + \left[\frac{1}{5} \times \frac{25}{7} - 1\right]$$

$$= 1 \div \frac{3}{7} of 14 + \left[\frac{5}{7} - 1\right] = 1 \div 6 + \left[-\frac{2}{7}\right]$$

$$= \frac{1}{6} - \frac{2}{7} = \frac{7-12}{42} = -\frac{5}{42}$$

Ascending or Descending Order in Rational Numbers
Rule 1: When the numerator and the denominator of the fractions increase by a constant value, the last fraction is the biggest.
Example: Which of the following fractions is the greatest?

$\frac{3}{4}, \frac{4}{5}$ and $\frac{5}{6}$

Solution:
We see that the numerators as well as denominators of the above fraction increase by 1, so the last fraction, i.e. $\frac{5}{6}$ is the greatest fraction.
Rule 2: The fraction whose numerator after cross-multiplication given the greater value is greater.

Example: Which is greater : $\frac{5}{8} or \frac{9}{14}$?
Solution:

Students generally solve these questions by changing the fractions into decimal values or by equating the denominators. But, we suggest a better method for getting the answer more quickly.

Step 1: Cross –multiply the two given fractions.

$$\frac{5}{8} \quad \frac{9}{14}$$

We have, 5 × 14 = 70 and 8 ×9 =72

Step II. As 72 is greater than 70 and the numerator involved with the greater value is 9, the fraction $\frac{9}{14}$ is the greater of the two.

Example: Which is greater: $\frac{4}{15} or \frac{6}{23}$?

Solution:

Step I: 4 ×23 > 15 ×6

Step II: As the greater value has the numerator 4 involved with it, $\frac{4}{15}$ is greater.

You can see how quickly this method works. After a good practise, you won't need to calculate before answering the question.

The arrangement of fractions into the ascending or descending order becomes easier now. Choose two fractions at a time. See which one is greater. This way you may get a quick arrangement of fractions.

Note: Sometimes, when the values are smaller (i.e., less than 10), the conventional method, i.e., changing the values into decimals or equating the denominators after getting LCM, will prove more convenient for some of you.

Example: Arrange the following in ascending order.

$\frac{3}{7}, \frac{4}{5}, \frac{7}{9}, \frac{1}{2}$ and $\frac{3}{5}$

Solution: Method I

The LCM of 7,5,9,2,5, is 630.

Now, to equate the denominators, we divide the LCM by the denominators and multiply the quotient by the respective numerators.

Like for $\frac{3}{7}$, 630 ÷ 7 = 90, so, multiply 3 by 90.

Thus, the fractions change to $\frac{270}{630}, \frac{504}{630}, \frac{490}{630}, \frac{315}{630}$ and $\frac{378}{630}$

The fraction which has larger numerator is naturally larger. So,

$$\frac{504}{630} > \frac{490}{630} > \frac{378}{630} > \frac{315}{630} > \frac{270}{630}$$

$$or\ \frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

Method II:

Change the fractions into decimals like

$\frac{3}{7}$ = 0.428, $\frac{4}{5}$ = 0.8, $\frac{7}{9}$ = 0.777, $\frac{1}{2}$ = 0.5, $\frac{3}{5}$ = 0.6

Clearly,

$$\frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

Method III:
Rule of CM (cross-multiplication)
Step I: Take the first two fractions. Find the greater one by the rule of CM.

$\frac{3}{7} \quad \frac{4}{5}$

3 × 5< 7×4

$\therefore \frac{4}{5} > \frac{3}{7}$

Step II: Take the third fraction. Apply CM with the third fraction and the larger value obtained in a step I.

$\frac{4}{5} \quad \frac{7}{9}$

4 × 9 > 5 × 7

$\therefore \frac{4}{5} > \frac{7}{9}$

Now we see that $\frac{7}{9}$ can lie after $\frac{3}{7}$ or between $\frac{4}{5}$ and $\frac{3}{7}$.

Therefore, we apply CM with $\frac{3}{7}$ and $\frac{7}{9}$ see that $\frac{7}{9} > \frac{3}{7}$.

$\therefore \frac{4}{5} > \frac{7}{9} > \frac{3}{7}$

Step III: Take the next fraction. Apply CM with $\frac{3}{7}$ and $\frac{1}{2}$ and see that $\frac{1}{2} > \frac{3}{7}$. Next, we apply CM with $\frac{7}{9}$ and $\frac{1}{2}$ and see that $\frac{7}{9} > \frac{1}{2}$.
Therefore,

$$\therefore \frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

Step IV: With similar applications, we get the final result as:

$\therefore \frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$

Note: This rule has some disadvantages also. But if you act fast, it gives faster results. Don't reject this method at once. This can prove to be a better method for you.

Basic Formula of Number System:

1. Sum of all the first n natural numbers = $\frac{n(n+1)}{2}$

For example: 1+ 2 +3 +…..+105= $\frac{105(105+1)}{2} = 5565$

2. Sum of first n odd numbers = n^2

For example 1+3+5+7= 4^2 =16(as there are four odd numbers)

3. Sum of first n even numbers = n (n+1)

For example : 2+4+6+8+….+100 (or 50^{th} even number) = 50×(50+1)= 2550

4. Sum of squares of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$

For example: $1^2+2^2+3^2+.....10^2=\frac{10(10+1)(2\times10+1)}{6}$

$\frac{10\times11\times21}{6}=385$

5. Sum of cubes of first n naturals numbers = $\left[\frac{n(n+1)}{2}\right]^2$

For example:

$1^3+2^3+.....6^3=\left[\frac{6(6+1)}{2}\right]^2=(21)^2=441$

Example:

(1) What is the total of all the even numbers from 1 to 400?

Solution:

From 1 to 400, there are 400 numbers. So, there are 400/2= 200 even numbers.
Hence, sum = 200(200+1) = 40200 (From Rule III)

(2) What is the total of all the even numbers from 1 to 361?

Solution:

From 1 to 361, there are 361, there are 361 numbers; so there are $\frac{360-1}{2}=\frac{360}{2}=180$ even numbers. Thus, sum = 180(180+1)=32580

(3) What is the total of all the odd numbers from 1 to 180?

Solution:

Therefore are 180/2 = 90 odd numbers between the given range. So, the sum = $(90)^2=8100$

(4) What is the total of all the odd numbers from 1 to 51?

Solution

There are $\frac{51+1}{2}=26$ odd numbers between the given range. So, the sum = $(26)^2=676$

(5) Find the of all the odd numbers from 20 to 101.

Solution:

The required sum = Sum of all the odd numbers from 1 to 101.
Sum of all the odd numbers from 1 to 20
= Sum of first 51 odd numbers – Sum of first 10 odd numbers
= $(51)^2-(10)^2=2601-100=2501$

Miscellaneous

1. In a division sum, we have four quantities – **Dividend, Divisor, Quotient and Remainder**. These are connected by the relation.

Dividend = (Divisor × Quotient) + Remainder

2. When the division is exact, the remainder is zero (0). In this case, the above relation becomes

Dividend = Divisor × Quotient

Example: 1: The quotient arising from the divisor of 24446 by certain numbers is 79 and the remainder is 35; what is the divisor?

Solution:

Divisor × Quotient = Dividend - Remainder
79×Divisor = 24446 -35 =24411
Divisor = 24411 ÷ 79 = 309.

Example: 2: A number when divided by 12 leaves a remainder 7. What remainder will be obtained by dividing the same number by 7?

Solution:

We see that in the above example, the first divisor 12 is not a multiple of the second divisor 7. Now, we take the two numbers 139 and 151, which when divided by 12, leave 7 as the remainder. But when we divide the above two numbers by 7, we get the respective remainder as 6 and 4. Thus, we conclude that the question is wrong.