

NDA 2020: Solution

1. Ans. C.

Maximum number of points of intersection of n non-overlapping circles

$$= 2 \times {}^n C_2$$

Here, n = 5

Hence, Maximum number of points of intersection of 5 non-overlapping

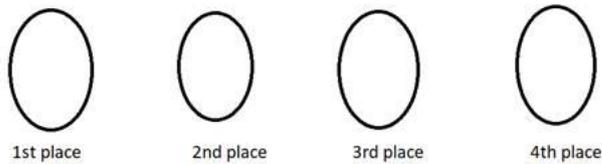
$$\text{circles} = 2 \times {}^5 C_2 = 2 \times \frac{5 \times 4 \times 3!}{2! \times 3!} = 20$$

2. Ans. B.

Consider word : ABLE

Vowels : A , E

Consonants : B , L



If vowels can take only even place then first fix vowels at even places.

Then consonants can be arranged in two ways:

1 st place	3 rd place
B	L
L	B

Also , these vowels can also be arranged in two ways.

$$\text{Hence, Required Number of ways} = 2 \times 2 = 4$$

3. Ans. C.

$$A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \text{ and } B = \begin{bmatrix} x \\ y \\ z \end{bmatrix},$$



$$AB = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax + hy + gz \\ hx + by + fz \\ gx + fy + cz \end{bmatrix}$$

4. Ans. D.

$$\begin{vmatrix} i & i^2 & i^3 \\ i^4 & i^6 & i^8 \\ i^9 & i^{12} & i^{15} \end{vmatrix}$$

We know that $i^2 = -1$

$$\Rightarrow \begin{vmatrix} i & -1 & -i \\ 1 & -1 & 1 \\ i & 1 & -i \end{vmatrix}$$

Operating $C_1 \rightarrow C_1 + C_2$

$$\Rightarrow \begin{vmatrix} i-1 & -1 & -i \\ 0 & -1 & 1 \\ i+1 & 1 & -i \end{vmatrix}$$

Operating $C_3 \rightarrow C_2 + C_3$

$$\Rightarrow \begin{vmatrix} i-1 & -1 & -1-i \\ 0 & -1 & 0 \\ i+1 & 1 & 1-i \end{vmatrix}$$

Expanding this determinant along R_2

$$\Rightarrow -1 \left| (i-1)(1-i) - (i+1)(-1-i) \right|$$

$$\Rightarrow -1 \left| i+1-1+i+i-1+1+i \right| = -4i$$

5. Ans. C.

$$S = \{1, 2, 3, \dots\}$$

A relation R on $S \times S$ is defined by xRy if $\log_a x > \log_a y$ when $a = \frac{1}{2}$.

Reflexive :

$$\log_{\frac{1}{2}}(x) \text{ is not greater than } \log_{\frac{1}{2}}(x)$$



$\Rightarrow (x, x) \notin R$ for any $x \in S$

$\Rightarrow R$ is not Reflexive.

Symmetric :

Let (x, y) belongs to relation R .

$$\Rightarrow \log_{\frac{1}{2}}(x) > \log_{\frac{1}{2}}(y)$$

$$\Rightarrow \log_{\frac{1}{2}}(y) \text{ can not greater than } \log_{\frac{1}{2}}(x)$$

$$\Rightarrow (y, x) \notin R$$

$\Rightarrow R$ is not symmetric.

Transitive :

Let (x, y) and (y, z) belongs to relation R .

$$\Rightarrow \log_{\frac{1}{2}}(x) > \log_{\frac{1}{2}}(y) \dots\dots\dots(1)$$

$$\Rightarrow \log_{\frac{1}{2}}(y) > \log_{\frac{1}{2}}(z) \dots\dots\dots(2)$$

From (1) and (2)

$$\Rightarrow \log_{\frac{1}{2}}(x) > \log_{\frac{1}{2}}(z)$$

$\Rightarrow (x, z)$ belongs to R .

\Rightarrow Hence, R is transitive.

6. Ans. D.

Given : $1.5 \leq x \leq 4.5$

$$\Rightarrow \frac{3}{2} \leq x \leq \frac{9}{2}$$

$$\Rightarrow \frac{3}{2} \leq x \quad \text{and} \quad x \leq \frac{9}{2}$$

$$\Rightarrow 2x - 3 \geq 0 \quad \text{and} \quad 2x - 9 \leq 0$$





$$\Rightarrow (2x - 3)(2x - 9) \leq 0$$

7. Ans. B.

It is given that $AB = C$

$$\Rightarrow \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+2y-y \\ 4x-x+y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

We know that two matrix are equal when corresponding elements of both matrix are equal.

$$\Rightarrow 2x + y = 3 \dots\dots\dots(1)$$

$$\Rightarrow 3x + y = 2 \dots\dots\dots(2)$$

Subtract (1) from (2)

$$\Rightarrow x = -1$$

Put the value of x in (1)

$$\Rightarrow y = 5$$

Hence, $A = \begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 4 & 5 \\ -2 & -6 \end{vmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 4 & 5 \\ -2 & -6 \end{vmatrix} = -24 + 10 = -14$$

8. Ans. C.

$$x = \log_c(ab), y = \log_a(bc), z = \log_b(ca)$$

$$\Rightarrow x = \frac{\log ab}{\log c}$$

$$\Rightarrow x+1 = \frac{\log ab}{\log c} + 1$$

$$\Rightarrow x+1 = \frac{\log ab + \log c}{\log c} = \frac{\log abc}{\log c}$$

$$\Rightarrow \frac{1}{x+1} = \frac{\log c}{\log abc} \dots\dots(1)$$

Similarly,

$$\Rightarrow \frac{1}{y+1} = \frac{\log a}{\log abc} \dots\dots\dots(2)$$

$$\Rightarrow \frac{1}{z+1} = \frac{\log b}{\log abc} \dots\dots\dots(3)$$

Adding (1) , (2) and (3)

$$\Rightarrow (1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1} = \frac{\log a}{\log abc} + \frac{\log b}{\log abc} + \frac{\log c}{\log abc}$$

$$\Rightarrow (1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1} = \frac{\log a + \log b + \log c}{\log abc}$$

$$\Rightarrow (1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1} = \frac{\log abc}{\log abc} = 1$$

9. Ans. A.

Consider $\frac{1}{10} \log_5 1024 - \log_5 10 + \frac{1}{5} \log_5 3125$

$$\Rightarrow \log_5 (1024)^{\frac{1}{10}} - \log_5 (10) + \log_5 (3125)^{\frac{1}{5}}$$

$$\Rightarrow \log_5 (2)^{10 \times \frac{1}{10}} - \log_5 (10) + \log_5 (5)^{5 \times \frac{1}{5}}$$



$$\Rightarrow \log_5(2) - \log_5(10) + \log_5(5) = \log_5\left(\frac{2 \times 5}{10}\right) = \log_5(1) = 0$$

10. Ans. C.

$$(1101101 + 1011011)_2 = (11001000)_2$$

$$\text{Now } (11001000)_2 = (1 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0)_{10}$$

$$\Rightarrow (128 + 64 + 8)_{10} = (200)_{10}$$

11. Ans. D.

If matrix $\begin{bmatrix} 0 & k & 4 \\ -k & 0 & -5 \\ -k & k & -1 \end{bmatrix}$ is singular then

$$\Rightarrow \begin{vmatrix} 0 & k & 4 \\ -k & 0 & -5 \\ -k & k & -1 \end{vmatrix} = 0$$

$$\Rightarrow -k(k - 5k) + 4(-k^2) = 0$$

$$\Rightarrow 4k^2 - 4k^2 = 0$$

$$\Rightarrow 0 = 0$$

As value of determinant will always be zero irrespective of value of k.

So, k can take infinite number of values.

12. Ans. C.

$$\text{If } C(20, n+2) = C(20, n-2) \dots \dots \dots (1)$$

We know that if ${}^nC_x = {}^nC_y$

$$\Rightarrow n = x + y$$

From (1)

$$\Rightarrow 20 = n+2 + n-2$$

$$\Rightarrow 20 = 2n$$

$\Rightarrow n = 10$

13. Ans. B.

Given $(1 + 2x - x^2)^6 = a_0 + a_1x + a_2x^2 + \dots + a_{12}x^{12}$

Put $x = -1$

$(1 + 2(-1) - (-1)^2)^6 = a_0 + a_1(-1) + a_2(-1)^2 + \dots + a_{12}(-1)^{12}$

$a_0 - a_1 + a_2 - a_3 + a_4 - \dots + a_{12} = (1 + 2(-1) - (-1)^2)^6 = 64$

14. Ans. A.

Consider $\left(\frac{2}{x^2} - \sqrt{x}\right)^{10}$

We know that $T_{r+1} = {}^{10}C_r \left(\frac{2}{x^2}\right)^{10-r} (-\sqrt{x})^r$

$T_{r+1} = {}^{-10}C_r (2)^{10-r} \left(\frac{1}{x^{20-2r}}\right) (x)^{\frac{r}{2}} = {}^{-10}C_r (2)^{10-r} \left(\frac{1}{x^{20-2r}}\right) (x)^{\frac{r}{2}} = {}^{-10}C_r (2)^{10-r} x^{-20+2r+\frac{r}{2}} = {}^{-10}C_r (2)^{10-r} x^{-20+\frac{5r}{2}}$

In order to find independent term of x:

$\Rightarrow \frac{5r}{2} = 20$

$\Rightarrow r = 8$

Now , $T_{8+1} = {}^{10}C_8 (2)^2 = \frac{10 \times 9 \times 8!}{8! \times (10-8)!} \times (2)^2 = 180$

15. Ans. C.

A square matrix with complex entries is called Hermitian matrix if its conjugate transpose is equal to the original matrix.

Consider $A = \begin{bmatrix} 1-i & i \\ -i & 1-i \end{bmatrix}$

$\bar{A} = \begin{bmatrix} 1+i & -i \\ i & 1+i \end{bmatrix}$



$$\overline{A^T} = \begin{bmatrix} 1+i & i \\ -i & 1+i \end{bmatrix} \dots\dots\dots(1)$$

As, $A \neq \overline{A^T}$

Hence, A is not Hermitian matrix.

A square matrix with complex entries is called skew-Hermitian matrix if its conjugate transpose is equal to the negative of the original matrix.

$$-A = \begin{bmatrix} -1+i & -i \\ i & -1+i \end{bmatrix} \dots\dots\dots(2)$$

From (1) and (2)

As, $-A \neq \overline{A^T}$

Hence, A is not skew-Hermitian matrix.

Now, consider $(\overline{A})^T + A = \begin{bmatrix} 1+i & i \\ -i & 1+i \end{bmatrix} + \begin{bmatrix} 1-i & i \\ -i & 1-i \end{bmatrix} = \begin{bmatrix} 2 & 2i \\ -2i & 2 \end{bmatrix}$

Let $(\overline{A})^T + A = B$

$$B = \begin{bmatrix} 2 & 2i \\ -2i & 2 \end{bmatrix}$$

$$\Rightarrow \overline{B} = \begin{bmatrix} 2 & -2i \\ 2i & 2 \end{bmatrix}$$

$$\Rightarrow \overline{B^T} = \begin{bmatrix} 2 & 2i \\ -2i & 2 \end{bmatrix}$$

As, $B = \overline{B^T}$

Hence, $(\overline{A})^T + A$ is Hermitian matrix.

16. Ans. A.

Let $z = \frac{1 - i\sqrt{3}}{1 + i\sqrt{3}}$

$$z = \frac{1 - i\sqrt{3}}{1 + i\sqrt{3}} = \frac{1 - i\sqrt{3}}{1 + i\sqrt{3}} \times \frac{1 - i\sqrt{3}}{1 - i\sqrt{3}} = \frac{(1 - i\sqrt{3})^2}{1 + 3} = \frac{1 + 3i^2 - 2\sqrt{3}i}{4} = \frac{-2 - 2\sqrt{3}i}{4} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

Clearly, z lies in 3rd quadrant.

arg(z) =

$$180^\circ + \tan^{-1}\left(\frac{b}{a}\right) = 180^\circ + \tan^{-1}\left(\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right) = 180^\circ + \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = 180^\circ + \tan^{-1}(\sqrt{3}) = 180^\circ + 60^\circ = 240^\circ$$

17. Ans. D.

Consider following two relations:

$$\alpha + \beta = \alpha^2 + \beta^2 \dots\dots\dots(1)$$

and

$$\alpha\beta = \alpha^2\beta^2\dots\dots\dots(2)$$

First of all we will find values of α and β using hit and trial such that both above mentioned relations are satisfied.

Case 1: $\alpha = 1, \beta = 0$

Sum of roots = 1

Product of roots = 0

Equation:

$$x^2 - (\text{Sum of roots})x + (\text{Product of roots}) = 0$$

$$\Rightarrow x^2 - x = 0$$

Case 2: $\alpha = 0, \beta = 0$

Sum of roots = 0

Product of roots = 0

Equation:

$$x^2 - (\text{Sum of roots})x + (\text{Product of roots}) = 0$$



$$\Rightarrow x^2 = 0$$

Case 3: Case:2 : $\alpha = 1, \beta = 1$

Sum of roots = 2

Product of roots = 1

Equation:

$$x^2 - (\text{Sum of roots})x + (\text{Product of roots}) = 0$$

$$\Rightarrow x^2 - 2x + 1 = 0$$

Case 4: $\alpha = \omega, \beta = \omega^2$

Sum of roots = $\omega + \omega^2 = -1$

Product of roots = $\omega \cdot \omega^2 = \omega^3 = 1$

Equation:

$$x^2 - (\text{Sum of roots})x + (\text{Product of roots}) = 0$$

$$\Rightarrow x^2 - x + 1 = 0$$

So total number of possible equations = 4

18. Ans. B.

$\cot \alpha$ and $\cot \beta$ are the roots of the equation $x^2 - 3x + 2 = 0$

$$\cot \alpha + \cot \beta = \frac{-(-3)}{1} = 3$$

$$\cot \alpha \cdot \cot \beta = \frac{2}{1} = 2$$

$$\Rightarrow \cot(\alpha + \beta) = \frac{\cot \alpha \cdot \cot \beta - 1}{\cot \alpha + \cot \beta} = \frac{2 - 1}{3} = \frac{1}{3}$$

19. Ans. C.

Given p^2, q^2 and r^2 (where $p, q, r > 0$) are in GP



$$\Rightarrow \frac{q^2}{p^2} = \frac{r^2}{q^2}$$

$$\Rightarrow \frac{q}{p} = \frac{r}{q} \dots\dots\dots(1)$$

$\Rightarrow p, q$ and r are in GP.

From (1)

$$\Rightarrow \frac{q}{p} = \frac{r}{q}$$

Taking logarithm

$$\Rightarrow \log\left(\frac{q}{p}\right) = \log\left(\frac{r}{q}\right)$$

$$\Rightarrow \log q - \log p = \log r - \log q$$

$$\Rightarrow 2\log q = \log p + \log r$$

$\Rightarrow \ln p, \ln q$ and $\ln r$ are in AP.

20. Ans. C.

A is a matrix of order 3×5 and B is a matrix of order 5×3

Then order of $AB = 3 \times 3$

B is a matrix of order 5×3 and A is a matrix of order 3×5

Then order of $BA = 5 \times 5$

21. Ans. C.

If $\sin x + \sin y = \cos y - \cos x$

$$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = 2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\Rightarrow \tan\left(\frac{x-y}{2}\right) = 1$$



22. Ans. D.

$$\sin(\alpha + 1^\circ) + \cos(\beta + 1^\circ) =$$

$$\sin(15^\circ + 1^\circ) + \cos(15^\circ + 1^\circ) = \sin 16^\circ + \cos 16^\circ = \sqrt{2} \left[\frac{1}{\sqrt{2}} \sin 16^\circ + \frac{1}{\sqrt{2}} \cos 16^\circ \right] = \sqrt{2} [\cos 45^\circ \sin 16^\circ + \sin 45^\circ \cos 16^\circ]$$

$$= \sqrt{2} [\sin 61^\circ] =$$

$$\sqrt{2} [\sin 61^\circ] = \sqrt{2} [\sin(60^\circ + 1^\circ)] = \sqrt{2} [\sin 60^\circ \cos 1^\circ + \cos 60^\circ \sin 1^\circ]$$

$$= \sqrt{2} \left[\frac{\sqrt{3}}{2} \cos 1^\circ + \frac{1}{2} \sin 1^\circ \right] = \frac{\sqrt{2}}{2} [\sqrt{3} \cos 1^\circ + \sin 1^\circ]$$

$$= \frac{1}{\sqrt{2}} (\sqrt{3} \cos 1^\circ + \sin 1^\circ)$$

23. Ans. D.

$$\sin 7\alpha - \cos 7\beta = \sin 105^\circ - \cos 105^\circ = \sin(90^\circ + 15^\circ) - \cos(90^\circ + 15^\circ) = \cos 15^\circ + \sin 15^\circ$$

Now, $\sin 15^\circ + \cos 15^\circ = \sin 15^\circ + \cos(90^\circ - 75^\circ) = \sin 15^\circ + \sin 75^\circ$

We know that $\sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$

Hence, $\sin 15^\circ + \sin 75^\circ = 2 \sin \left(\frac{15^\circ + 75^\circ}{2} \right) \cos \left(\frac{15^\circ - 75^\circ}{2} \right) = 2 \sin 45^\circ \cos 30^\circ =$

$$2 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{\sqrt{2}}$$

24. Ans. D.

$$\sin \alpha + \cos \beta = \sin 15^\circ + \cos 15^\circ = \sin 15^\circ + \cos(90^\circ - 75^\circ) = \sin 15^\circ + \sin 75^\circ$$

We know that $\sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$



Hence, $\sin 15^\circ + \sin 75^\circ = 2 \sin\left(\frac{15^\circ + 75^\circ}{2}\right) \cos\left(\frac{15^\circ - 75^\circ}{2}\right) = 2 \sin 45^\circ \cos 30^\circ =$
 $2 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{\sqrt{2}}$

25. Ans. C.

$$t_{10} = \sin^{10} 45^\circ + \cos^{10} 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^{10} + \left(\frac{1}{\sqrt{2}}\right)^{10} = 2 \times \frac{1}{2^5} = \frac{1}{2^4} = \frac{1}{16}$$

26. Ans. B.

Consider $t_1^2 - t_2 = (\sin \theta + \cos \theta)^2 - (\sin^2 \theta + \cos^2 \theta)$

$$\Rightarrow 1 + 2 \sin \theta \cos \theta - 1$$

$$\Rightarrow \sin 2\theta$$

27. Ans. A.

Consider $t_n = \sin^n \theta + \cos^n \theta$.

Now, $\frac{t_3 - t_5}{t_5 - t_7} = \frac{\sin^3 \theta + \cos^3 \theta - \sin^5 \theta - \cos^5 \theta}{\sin^5 \theta + \cos^5 \theta - \sin^7 \theta - \cos^7 \theta}$

$$\Rightarrow \frac{\sin^3 \theta (1 - \sin^2 \theta) + \cos^3 \theta (1 - \cos^2 \theta)}{\sin^5 \theta (1 - \sin^2 \theta) + \cos^5 \theta (1 - \cos^2 \theta)}$$

$$\Rightarrow \frac{\sin^3 \theta (\cos^2 \theta) + \cos^3 \theta (\sin^2 \theta)}{\sin^5 \theta (\cos^2 \theta) + \cos^5 \theta (\sin^2 \theta)}$$

$$\Rightarrow \frac{\sin^2 \theta \cos^2 \theta (\sin \theta + \cos \theta)}{\sin^2 \theta \cos^2 \theta (\sin^3 \theta + \cos^3 \theta)}$$

$$\Rightarrow \frac{t_1}{t_3}$$

28. Ans. B.

$p \tan x = q \tan y$

$$\Rightarrow \frac{p}{q} = \frac{\tan y}{\tan x}$$



$$\Rightarrow \frac{p^2}{q^2} = \frac{\tan^2 x}{\tan^2 y} \dots\dots\dots(1)$$

Now, $a \sin^2 x + b \cos^2 x = c \dots\dots\dots(2)$

From (2)

$$\Rightarrow a \sin^2 x + b(1 - \sin^2 x) = c$$

$$\Rightarrow a \sin^2 x + b - b \sin^2 x = c$$

$$\Rightarrow \sin^2 x(a - b) = c - b$$

$$\Rightarrow \sin^2 x = \frac{c - b}{a - b}$$

From (2)

$$\Rightarrow a(1 - \cos^2 x) + b \cos^2 x = c$$

$$\Rightarrow a - a \cos^2 x + b \cos^2 x = c$$

$$\cos^2 x = \frac{c - a}{b - a}$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{c - b}{a - b}}{\frac{c - a}{b - a}} = \frac{c - b}{a - c} \dots\dots\dots(3)$$

Hence,

Consider $b \sin^2 y + a \cos^2 y = d \dots\dots\dots(4)$

From (4)

$$\Rightarrow b \sin^2 y + a(1 - \sin^2 y) = d$$

$$\Rightarrow b \sin^2 y + a - a \sin^2 y = d$$

$$\Rightarrow \sin^2 y(b - a) = d - a$$

$$\Rightarrow \sin^2 y = \frac{d - a}{b - a}$$

From (4)



$$\Rightarrow b(1 - \cos^2 y) + a \cos^2 y = d$$

$$\Rightarrow b - b \cos^2 y + a \cos^2 y = d$$

$$\cos^2 y = \frac{d - b}{a - b}$$

Divide (2) by (3)

$$\Rightarrow \frac{d - a}{b - d} = \tan^2 y \dots\dots\dots(5)$$

From (3) and (5)

$$\Rightarrow \frac{p^2}{q^2} = \frac{\tan^2 x}{\tan^2 y} = \frac{(a - d)(c - a)}{(b - c)(d - b)}$$

29. Ans. C.

Consider $b \sin^2 y + a \cos^2 y = d \dots\dots\dots(1)$

From (1)

$$\Rightarrow b \sin^2 y + a(1 - \sin^2 y) = d$$

$$\Rightarrow b \sin^2 y + a - a \sin^2 y = d$$

$$\Rightarrow \sin^2 y (b - a) = d - a$$

$$\Rightarrow \sin^2 y = \frac{d - a}{b - a} \dots\dots\dots(2)$$

From (1)

$$\Rightarrow b(1 - \cos^2 y) + a \cos^2 y = d$$

$$\Rightarrow b - b \cos^2 y + a \cos^2 y = d$$

$$\cos^2 y = \frac{d - b}{a - b} \dots\dots\dots(3)$$

Divide (2) by (3),

$$\Rightarrow \frac{d - a}{b - d} = \tan^2 y$$



30. Ans. A.

$$a \sin^2 x + b \cos^2 x = c \dots\dots\dots(1)$$

From (1)

$$\Rightarrow a \sin^2 x + b(1 - \sin^2 x) = c$$

$$\Rightarrow a \sin^2 x + b - b \sin^2 x = c$$

$$\Rightarrow \sin^2 x (a - b) = c - b$$

$$\Rightarrow \sin^2 x = \frac{c - b}{a - b}$$

From (1)

$$\Rightarrow a(1 - \cos^2 x) + b \cos^2 x = c$$

$$\Rightarrow a - a \cos^2 x + b \cos^2 x = c$$

$$\cos^2 x = \frac{c - a}{b - a}$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{c - b}{a - b}}{\frac{c - a}{b - a}} = \frac{c - b}{a - c}$$

Hence,

31. Ans. B.

We know that intercept form of the equation of plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

in question it is given that the value of intercept of z axis is c=5 and xy axis is parallel so it means x=0,y=0

$$\text{Therefore } \frac{0}{a} + \frac{0}{b} + \frac{z}{5} = 1$$

$$z = 5$$

32. Ans. C.

The eight compartments of the coordinate planes divide the space.

Eight compartments which are known as eight octants and since each

of the coordinates of a point in them may be positive or negative, there are

$2^3 = 8$ points whose coordinate have the same numerical values.

33. Ans. D.

There given direction ratio are $a+b, b+a, c+a$

Now, Direction ratios are

$$\frac{a+b}{\sqrt{(a+b)^2 + (b+a)^2 + (c+a)^2}}, \frac{b+a}{\sqrt{(a+b)^2 + (b+a)^2 + (c+a)^2}}, \frac{c+a}{\sqrt{(a+b)^2 + (b+a)^2 + (c+a)^2}}$$

according to question sum of square of direction cosines

$$\left[\frac{a+b}{\sqrt{(a+b)^2 + (b+a)^2 + (c+a)^2}} + \frac{b+a}{\sqrt{(a+b)^2 + (b+a)^2 + (c+a)^2}} + \frac{c+a}{\sqrt{(a+b)^2 + (b+a)^2 + (c+a)^2}} \right]^2$$

$$= \frac{(a+b)^2}{(a+b)^2 + (b+a)^2 + (c+a)^2} + \frac{(b+a)^2}{(a+b)^2 + (b+a)^2 + (c+a)^2} + \frac{(c+a)^2}{(a+b)^2 + (b+a)^2 + (c+a)^2}$$

$$= \frac{(a+b)^2 + (b+a)^2 + (c+a)^2}{(a+b)^2 + (b+a)^2 + (c+a)^2} = 1$$

We know that the sum of the square of its direction cosine are 1.

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

34. Ans. B.

The coordinate of general point on $\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0} = \lambda$

i.e $x=\lambda, y=0, z=0$

Let the coordinate of L be $(\lambda, 0, 0)$

Direction ratio of PL are

$$\lambda-2, -3, -4$$

Direction of given line are 1,0,0

Since PL is perpendicular to the given line

$$\therefore 1(\lambda-2)+0(-3)+0(-4)=0$$

$$\lambda=2, L(2,0,0)$$

$$PL = \sqrt{(2-2)^2 + (0+3)^2 + (0+4)^2} = \sqrt{25} = 5 \text{ units}$$

35. Ans. D.

Let 'r' be the radius of the sphere in the given plane.

Hence r can be defined by-



$$r = \frac{ax_1 + by_2 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$$

$$r = \frac{6 + 6 + 6 - 4}{\sqrt{6^2 + 3^2 + 2^2}}$$

$$r = \frac{14}{\sqrt{49}}$$

$$r = 2 \text{ units}$$

So, $diameter = 2 \times radius = 4 \text{ units}$

36. Ans. D.

We know that Mean = Sum of all observation / Total number of observation

It is given that a data of n observation has mean 2M so this can be written as,

$$2M = \frac{\text{Sum of all observation}}{n}$$

sum of all observation = 2Mn

While another data set 2n observation has mean M

So,

Sum of all observation = 2Mn

Then the mean of the combined data set of 3n observation will be

$$x = \frac{2Mn + 2Mn}{n + 2n}$$

$$x = \frac{4Mn}{3n}$$

$$x = \frac{4M}{3}$$

37. Ans. D.

Deviation of an observation from 2.5 will be defined as = 2.5 - (that particular observation)

Hence according to the given conditions we can evaluate the following two equations-

$$2.5n - \sum_{i=1}^n x_i = -50 \quad \dots i$$

$$3.5n - \sum_{i=1}^n x_i = 50 \quad \dots ii$$



After subtracting eqn. (i) from eqn. (ii)

$n=100$.

38. Ans. C.

Given that $P(A \cup B) = \frac{5}{6}$

$$P(A \cap B) = \frac{1}{3}$$

$$P(\bar{A}) = \frac{1}{2}$$

$$P(\bar{A}) = 1 - P(A)$$

$$P(A) = \frac{1}{2}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{5}{6} = \frac{1}{2} + P(B) - \frac{1}{3}$$

$$P(B) = \frac{2}{3}$$

$$P(A) P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$$

$$P(A) + P(B) = \frac{1}{2} + \frac{2}{3} = \frac{7}{6} > P(A \cup B) = \frac{5}{6} >$$

39. Ans. C.

When three dice are rolled then

let S be the sample and A be the event occurrence of one face having the number

$$n(S) = 6 \times 5 \times 4 = 120$$

$$\begin{aligned} n(A) &= n(1^{st} \text{ dice shown } 6) + n(2^{nd} \text{ dice shown } 6) + n(3^{rd} \text{ dice shown } 6) \\ &= (1 \times 5 \times 4) + (5 \times 1 \times 4) + (5 \times 4 \times 1) \\ &= 60 \end{aligned}$$

$$\text{Hence required probability} = \frac{n(A)}{n(S)} = \frac{60}{120} = \frac{1}{2}$$

40. Ans. B.

According to question arithmetic mean = 6

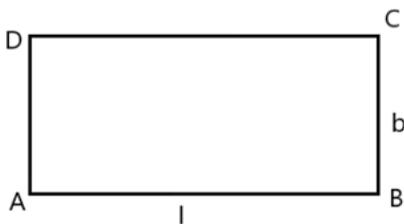


It means

$$\begin{aligned}\frac{4x+9(x-1)}{x+x-1} &= 6 \\ \Rightarrow \frac{4x+9x-9}{x+x-1} &= 6 \\ \Rightarrow \frac{13x-9}{2x-1} &= 6 \\ \Rightarrow 13x-9 &= 12x-6 \\ \Rightarrow x &= -6+9 \\ x &= 3\end{aligned}$$

41. Ans. D.

Here ABCD is rectangle which have



Length $AB=l$ and breadth $BC=b$

Let area is y

$$l + b = k$$

$$l = k - b \text{ (given)}$$

Now, area $y = lb = l(k-l) = kl - l^2$

For maximum area y ,

$$\frac{dy}{dl} = 0 \dots (i)$$

$$k - 2l = 0$$

$$l = \frac{k}{2}$$

When differentiating eqn (i) $\frac{d^2y}{dl^2} = -2 < 0$

Hence y has maximum value when $l = \frac{k}{2}$



$\Rightarrow b = \frac{k}{2} \therefore$ maximum area $\frac{k}{2} \cdot \frac{k}{2} = \frac{k^2}{4}$

42. Ans. A.

The given Differential equation $\ln\left(\frac{dy}{dx}\right) = x$

We know that $\{\ln(a)=b\}$

[then $a=e^b$]

So $\ln\left(\frac{dy}{dx}\right) = x$

$\frac{dy}{dx} = e^x \dots\dots(i)$

So, $dy = e^x dx \dots\dots(ii)$

Integrating equation (ii) on both side

$\int dy = e^x \int dx$

Therefore, we get $y = e^x + c$ [here c is constant of integration]

43. Ans. B.

From the equation of the function $y = \frac{1}{x-1}$ it is evident that here x is define for all real number except $x = 1$

For domain $x = \frac{y+1}{y}; y \neq 0$

Domain is all real number except $y = 0$

Now at $x = 0 \Rightarrow y = -1$, hence the graph intersects y-axis at (0, -1).

44. Ans. C.

According to the given question

We have $f(x) = \ln x$



So, differentiating on both side

We get $f'(x) = \frac{1}{x}$

Here $f'(x) > 0$ and it is an increasing function

Therefore, $f(x)$ increases in the interval $(0, \infty)$

Now, $f(x) = \tan x$

So, differentiating on both side

We get $f'(x) = \sec^2 x$

Here $f'(x) > 0$ and it is an increasing function

So, it also increases in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Hence both the given statements are true.

45. Ans. A.

We know that range of cosine function is $[-1, 1]$

Means $-1 \leq \cos \theta \leq 1$

Here $\theta = \left(A + \frac{\pi}{3}\right)$ where $A \in R$

Now,

$$-1 \leq \cos\left(A + \frac{\pi}{3}\right) \leq 1$$

Now multiplying by 3

Therefore $-3 \leq 3 \cos\left(A + \frac{\pi}{3}\right) \leq 3$

So there minimum value of $\cos\left(A + \frac{\pi}{3}\right) = -3$

46. Ans. B.

In the given differential Equation is $x(dx - dy) + y(dy - dx) = 0$



Now, $x dx - x dy + y dy - y dx = 0 \dots (i)$

$(x-y)dx = (x-y)dy \dots (ii)$

Integrating both side

$$\int (x-y) dx = \int (x-y) dy$$

Therefore,

$$\frac{x^2}{2} - xy = xy - \frac{y^2}{2}$$

$$\frac{x^2 + y^2}{2} = 2xy + c$$

$$x^2 + y^2 = 2xy + c$$

47. Ans. A.

As we know that differentiation of $\tan^{-1}x$ w.r.t x is $\frac{1}{1+x^2}$

$$\text{and } \cot^{-1}x = -\frac{1}{1+x^2}$$

$$\text{Now, } \frac{d(\tan^{-1}x)}{d(\cot^{-1}x)} = \frac{d(\tan^{-1}x)}{dx} \cdot \frac{dx}{d(\cot^{-1}x)}$$

$$= \left(\frac{1}{1+x^2}\right) \times -\left(\frac{1+x^2}{1}\right) = -1$$

48. Ans. A.

$$\text{Given } \lim_{x \rightarrow 0} \frac{3^x + 3^{-x} - 2}{x}$$

it is $\left(\frac{0}{0}\right)$ indeterminate form

$$\text{Given } \lim_{x \rightarrow 0} \frac{3^x + 3^{-x} - 2}{x}$$

it is $\left(\frac{0}{0}\right)$ indeterminate form



by using L'Hospital rule

Differentiating numerator and denominator

$$\text{so, } \frac{d}{dx} \left(\frac{3^x + 3^{-x} - 2}{x} \right) = \frac{(3^x - 3^{-x}) \ln 3}{1}$$

$$\text{Therefore } \lim_{x \rightarrow 0} \frac{(3^x - 3^{-x}) \ln 3}{1} = 0$$

49. Ans. C.

Now, we can write as $\sin x \cos x = \frac{1}{2} \sin 2x$

To find out maximum of the $\sin x \cos x$ differentiate it w.r.t x and then equate it to 0.

$$\frac{d}{dx} \left(\frac{1}{2} \sin 2x \right) = \frac{1}{2} \frac{d}{dx} \sin 2x = \frac{1}{2} 2 \cos 2x = 0$$

$$\cos 2x = 0$$

$$\cos 2x = \cos \frac{n\pi}{2}, \text{ where } n=1,3,5,\dots(\text{odd integer})$$

$$2x = \frac{n\pi}{2}$$

$$x = \frac{n\pi}{4}, \text{ where } n=1,3,5,\dots(\text{odd integer})$$

For maximum value $n=1,5,9,\dots$ because $3,7,\dots$ will result in minimum value.

$$\therefore \sin \frac{\pi}{4} \cos \frac{\pi}{4} = \frac{1}{2}$$

50. Ans. A.

The given function $f(x) = e^{-|x|}$

At first, we check continuity of $f(x)$ at $x=0$

Left hand limit:- In this case $x < 0 \therefore |x| = -x$

So function $f(x) = f(x) = e^{-(-x)} = e^x$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h)$$

$$= \lim_{h \rightarrow 0} e^{(0-h)} = \lim_{h \rightarrow 0} e^h = 1$$



Now, Right-hand limit

$$x > 0 \therefore |x| = x$$

So, $f(x) = e^{-|x|} = e^{-x}$

Now $\lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h)$

$$= \lim_{h \rightarrow 0} e^{-(0+h)} = e^0 = 1$$

Value of function $f(x) = e^{-|x|}$ at $x=0$

$$f(0) = e^{-|0|} = 1$$

So, LHL = RHL = $f(x) = 1$

Therefore $f(x) = e^{-|x|}$ at $x=0$ is continuous

Now we check $f(x) = e^{-|x|}$ is differentiable at $x=0$ or not

Value of function $f'(0) = 1$

RHD

$$\begin{aligned} f'(0) &= \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{-(0+h)} - 1}{h} = 0 \end{aligned}$$

So it is clear that $RHD \neq f'(0)$ and hence $f(x) = e^{-|x|}$ is not differentiable at $x=0$

51. Ans. B.

We know sine rule ,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

(1) We have,

$$b = c$$

$$\Rightarrow \sin B = \sin C$$



And, $A = 2B$

$$\Rightarrow \sin A = \sin 2B$$

$$\Rightarrow \sin(180 - (B + C)) = \sin 2B$$

$$\Rightarrow \sin(B + C) = 2 \sin B \cos B$$

$$\Rightarrow \sin B \cos C + \cos B \sin C = 2 \sin B \cos B$$

$$\Rightarrow \sin B \cos C + \cos B \sin B = 2 \sin B \cos B \quad [\because \sin B = \sin C]$$

$$\Rightarrow \sin B (\cos C + \cos B) = 2 \sin B \cos B$$

$$\Rightarrow \cos C + \cos B = 2 \cos B$$

$$\Rightarrow \cos C = \cos B$$

If $\sin B = \sin C$ and $\cos C = \cos B$

means, $B = C = 45^\circ$

Then $A = 90^\circ$. So, in this triangle there is no obtuse angle. so this triangle ABC is not obtuse angle triangle.

This statement is wrong.

(2) If $A = 40^\circ$ and $B = 65^\circ$

We know that $A + B + C = 180^\circ$

$$\Rightarrow C = 180 - 45 - 65 = 75^\circ$$

We know sine rule ,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{a}{\sin 40^\circ} = \frac{b}{\sin 65^\circ} = \frac{c}{\sin 75^\circ}$$



$$\Rightarrow \frac{a}{\sin 40^\circ} = \frac{c}{\sin 75^\circ}$$

$$\Rightarrow \frac{a}{c} = \sin 40^\circ \operatorname{cosec} 75^\circ$$

So, here $\frac{a}{c} \neq \sin 40^\circ \operatorname{cosec} 15^\circ$

Then this statement is right.

52. Ans. D.

(1) If $\triangle ABC$ is a right-angled triangle.

$$\angle A = 90^\circ \text{ and } \sin B = \frac{1}{3}$$

$$\text{then } \operatorname{cosec} C = \frac{BC}{BA} = \frac{3}{\sqrt{9-1}} = \frac{3}{8}$$

So, first statement is wrong.

(2) $b \cos B = c \cos C$

$$\Rightarrow \frac{b(a^2 + c^2 - b^2)}{2ac} = \frac{c(a^2 + b^2 - c^2)}{2ab}$$

$$\Rightarrow b^2(a^2 + c^2 - b^2) = c^2(a^2 + b^2 - c^2)$$

$$\Rightarrow b^2 a^2 + b^2 c^2 - b^4 = c^2 a^2 + c^2 b^2 - c^4$$

$$\Rightarrow b^2 a^2 - a^2 c^2 = b^4 - c^4$$

$$\Rightarrow a^2(b^2 - c^2) = (b^2)^2 - (c^2)^2$$

$$\Rightarrow a^2(b^2 - c^2) = (b^2 + c^2)(b^2 - c^2)$$

$$\Rightarrow a^2 = b^2 + c^2$$



So, this is right angled triangle. then statement-2 is wrong.

53. Ans. B.

We have,

$$\cos 48^\circ - \cos 12^\circ$$

$$\Rightarrow -2 \sin\left(\frac{48^\circ + 12^\circ}{2}\right) \sin\left(\frac{48^\circ - 12^\circ}{2}\right) \left[\because \cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right) \right]$$

$$\Rightarrow -2 \sin\left(\frac{60^\circ}{2}\right) \sin\left(\frac{36^\circ}{2}\right)$$

$$\Rightarrow -2 \sin 30^\circ \sin 18^\circ$$

$$\Rightarrow -2 \times \frac{1}{2} \times \left(\frac{\sqrt{5}-1}{4}\right)$$

$$\Rightarrow \frac{1-\sqrt{5}}{4}$$

54. Ans. B.

We know that

$$\cos A \cos 2A \cos 2^2 A \dots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$$

We have,

$$8 \cos 10^\circ \cos 20^\circ \cos 40^\circ$$

$$\Rightarrow 8 \times \frac{\sin(2^3 \times 10^\circ)}{2^3 \sin 10^\circ}$$

$$\Rightarrow 8 \times \frac{\sin 80^\circ}{8 \sin 10^\circ}$$

$$\Rightarrow \frac{\sin(90^\circ - 10^\circ)}{\sin 10^\circ}$$



$$\Rightarrow \frac{\cos 10^\circ}{\sin 10^\circ} = \cot 10^\circ \left[\because \sin(90^\circ - x) = \cos x \text{ \& } \frac{\cos x}{\sin x} = \cot x \right]$$

55. Ans. D.

We know that the limit of $\cos x$ is $[-1, 1]$.

We have,

$$y = 2 + \cos x$$

Then the limit of y is $[-1+2, 1+2] = [1, 3]$

So, the ordinate of $y = 2 + \cos x$ is $[1, 3]$.

56. Ans. A.

We know that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

and, $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

We have,

$$\sin 3x + \cos 3x + 4\sin^3 x - 3\sin x + 3\cos x - 4\cos^3 x$$

$$\Rightarrow 3\sin x - 4\sin^3 x + 4\cos^3 x - 3\cos x + 4\sin^3 x - 3\sin x + 3\cos x - 4\cos^3 x$$

$$\Rightarrow 0$$

57. Ans. D.

We have,

$$3\sin 2A = 2\sin 2B$$

$$\Rightarrow 6\sin A \cos A = 2\sin 2B$$

$$\Rightarrow \frac{3\sin A}{\cos A} = \frac{\sin 2B}{\cos^2 A}$$

$$\Rightarrow \tan A = \frac{1}{3} \frac{\tan 2B \cos 2B}{\cos^2 A}$$



$$\Rightarrow \tan A = \frac{1}{3} \frac{\tan 2B (3 \sin^2 A)}{\cos^2 A} \left[\cos 2B = 3 \sin^2 A \right]$$

$$\Rightarrow \tan A = \tan 2B \tan^2 A$$

$$\Rightarrow \tan A \tan 2B = 1$$

$$\Rightarrow \tan A = \cot 2B$$

$$\Rightarrow \tan A = \tan \left(\frac{\pi}{2} - 2B \right) \left[\because \cot x = \tan \left(\frac{\pi}{2} - x \right) \right]$$

$$\Rightarrow A = \frac{\pi}{2} - 2B$$

$$\Rightarrow A + 2B = \frac{\pi}{2}$$

58. Ans. B.

We have,

$$\tan \theta = \frac{\cos 17^\circ - \sin 17^\circ}{\cos 17^\circ + \sin 17^\circ}$$

$$\Rightarrow \tan \theta = \frac{\cos 17^\circ (1 - \tan 17^\circ)}{\cos 17^\circ (1 + \tan 17^\circ)}$$

$$\Rightarrow \tan \theta = \frac{1 - \tan 17^\circ}{1 + \tan 17^\circ}$$

$$\Rightarrow \tan \theta = \frac{\tan 45^\circ - \tan 17^\circ}{1 + \tan 45^\circ \tan 17^\circ} \left[\because \tan 45^\circ = 1 \right]$$

$$\Rightarrow \tan \theta = \tan (45^\circ - 17^\circ) \left[\because \tan (a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b} \right]$$

$$\Rightarrow \theta = 28^\circ$$

59. Ans. C.

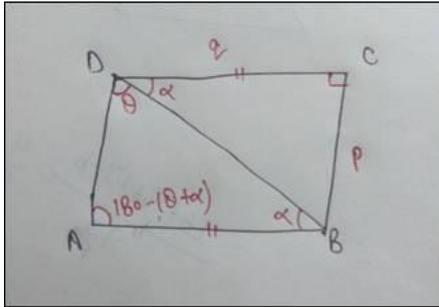


In $\triangle BCD$,

$$BD^2 = BC^2 + CD^2$$

$$\Rightarrow BD^2 = p^2 + q^2$$

$$\Rightarrow BD = \sqrt{p^2 + q^2}$$



In $\triangle ABD$,

$$\frac{AB}{\sin \theta} = \frac{BD}{\sin(180^\circ - (\theta + \alpha))} = \frac{AD}{\sin \alpha}$$

$$\Rightarrow \frac{AB}{\sin \theta} = \frac{BD}{\sin(\theta + \alpha)}$$

$$\Rightarrow AB = \frac{BD \sin \theta}{\sin(\theta + \alpha)}$$

$$\Rightarrow AB = \frac{BD^2 \sin \theta}{BD \sin(\theta + \alpha)}$$

$$\Rightarrow AB = \frac{BD^2 \sin \theta}{BD(\sin \theta \cos \alpha + \cos \theta \sin \alpha)}$$

$$\Rightarrow AB = \frac{(p^2 + q^2) \sin \theta}{\sqrt{p^2 + q^2} \left(\sin \theta \frac{q}{\sqrt{p^2 + q^2}} + \cos \theta \frac{p}{\sqrt{p^2 + q^2}} \right)}$$



$$\Rightarrow AB = \frac{(p^2 + q^2) \sin \theta}{q \sin \theta + p \cos \theta}$$

60. Ans. C.

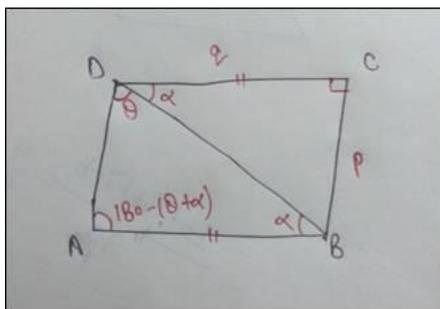
We know that AB and CD are parallel and BC is perpendicular to them .

We know sine rule ,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

In $\triangle ABD$,

$$\frac{AB}{\sin \theta} = \frac{BD}{\sin(180^\circ - (\theta + \alpha))} = \frac{AD}{\sin \alpha}$$



$$(1) \frac{AB}{\sin \theta} = \frac{AD}{\sin \alpha}$$

$$\Rightarrow AB \sin \alpha = AD \sin \theta$$

$$(2) \frac{AB}{\sin \theta} = \frac{BD}{\sin(180^\circ - (\theta + \alpha))}$$

$$\Rightarrow AB \sin(\theta + \alpha) = BD \sin \theta$$

61. Ans. A. B.

We have,

$$\tan^2 A = \frac{k-3}{3k-1}$$



$$\Rightarrow \tan A = \sqrt{\frac{k-3}{3k-1}}$$

The value of k from $\frac{1}{2}$ to 2 , $\tan A$ is imaginary.

And, the value from $\frac{1}{3}$ to 3 , $\tan A$ is also imaginary .

other than these values $\tan A$ is real.

62. Ans. B.

$$\text{We have, } \frac{\tan 3A}{\tan A} = k$$

$$\Rightarrow \frac{\sin 3A \cdot \cos A}{\cos 3A \cdot \sin A} = k$$

$$\Rightarrow \frac{\sin 3A \cdot \cos A + \cos 3A \cdot \sin A}{\sin 3A \cdot \cos A - \cos 3A \cdot \sin A} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{\sin(3A+A)}{\sin(3A-A)} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{\sin 4A}{\sin 2A} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{2 \sin 2A \cos 2A}{\sin 2A} = \frac{k+1}{k-1}$$

$$\Rightarrow 2 \cos 2A = \frac{k+1}{k-1}$$

$$\Rightarrow 2(1 - 2 \sin^2 A) = \frac{k+1}{k-1}$$

$$\Rightarrow 2 - 4 \sin^2 A = \frac{k+1}{k-1}$$

$$\Rightarrow 4 \sin^2 A = 2 - \frac{k+1}{k-1}$$



$$\Rightarrow 4 \sin^2 A = \frac{2k-2-k-1}{k+1} = \frac{k-3}{k+1}$$

$$\Rightarrow \sin^2 A = \frac{1}{4} \cdot \frac{k-3}{k-1}$$

And, $\cos^2 A = 1 - \frac{1}{4} \left(\frac{k-3}{k-1} \right)$

$$\Rightarrow \cos^2 A = \frac{4k-4-k+3}{k-1} = \frac{3k-1}{k-1}$$

$$\tan^2 A = \frac{\sin^2 A}{\cos^2 A} = \frac{\left(\frac{k-3}{k-1} \right)}{\left(\frac{3k-1}{k-1} \right)} = \frac{k-3}{3k-1}$$

So,

63. Ans. A.

We have,

If the number of elements belonging to neither X, nor Y, nor Z is equal to p.

Now,

$$n(\bar{X}) = p + b + c + 17$$

We have $n(Z) = 12 + 18 + 17 + c = 90$

$$c = 90 - 12 - 18 - 17 = 43$$

$$n(\bar{X}) = p + b + 43 + 17$$

$$\Rightarrow n(\bar{X}) = p + b + 60$$

64. Ans. D.

We have

$$n(X) = a + 16 + 12 + 18 = a + 46$$



$$n(Y) = 16 + b + 18 + 17 = 51 + b$$

$$n(Z) = 90$$

$$n(X \cap Y) = 16 + 18 = 34$$

$$n(Y \cap Z) = 17 + 18 = 35$$

$$n(X \cap Z) = 12 + 18 = 30$$

$$n(X \cap Y \cap Z) = 18$$

Now,

$$n(X) + n(Y) + n(Z) - n(X \cap Y) - n(Y \cap Z) - n(X \cap Z) + n(X \cap Y \cap Z)$$

$$\Rightarrow a + 46 + 51 + b + 90 - 34 - 35 - 30 + 18$$

$$\Rightarrow a + b + 106$$

65. Ans. C.

We have,

No of elements in Z = 90

And, No of elements in Y = $16 + 18 + 17 + b = 51 + b$

$$\text{So, } \frac{n(Y)}{n(Z)} = \frac{4}{5}$$

$$\Rightarrow \frac{51 + b}{90} = \frac{4}{5}$$

$$\Rightarrow 51 + b = 72$$

$$\Rightarrow b = 21$$

66. Ans. C.

We have the equations of a pair of opposite sides of a square .

$$3x - 4y - 5 = 0$$



$$3x - 4y + 15 = 0$$

As we can see that both lines slopes are equal. So, both lines are a parallel.

So, distance between parallel lines is side of a square.

We know that the distance between the parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$, is

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

We have $3x - 4y - 5 = 0$ and $3x - 4y + 15 = 0$, so distance

$$d = \frac{|-5 - 15|}{\sqrt{3^2 + 4^2}} = \frac{20}{5} = 4$$

So, area of a square = $(side)^2 = 4^2 = 16$

67. Ans. B.

We know that if m_1 and m_2 are the slopes of two lines, then the acute angle θ between them is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{2 - \sqrt{3} - 2 - \sqrt{3}}{1 + (2 - \sqrt{3})(2 + \sqrt{3})} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{-2\sqrt{3}}{1 + 4 - 3} \right|$$

$$\Rightarrow \tan \theta = \left| -\sqrt{3} \right| = \sqrt{3}$$

$$\Rightarrow \theta = 60^\circ$$

But, obtuse angle = $180^\circ - \theta$



$$= 180 - 60 = 120^\circ$$

68. Ans. C.

We know that the foot (h, k) of the perpendicular drawn from the point (x_1, y_1) on the line $ax + by + c = 0$ is given by

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = -\left(\frac{ax_1 + by_1 + c}{a^2 + b^2}\right)$$

We have,

The foot of the perpendicular drawn from the point $(0, k)$ to the line $3x - 4y - 5 = 0$ is $(3, 1)$.

$$\frac{3 - 0}{3} = \frac{1 - k}{-4} = -\left(\frac{3 \times 0 - 4 \times k - 5}{3^2 + (-4)^2}\right)$$

$$\Rightarrow 1 = \frac{k - 1}{4} = -\left(\frac{-4k - 5}{25}\right)$$

$$\Rightarrow k - 1 = 4$$

$$\Rightarrow k = 5$$

69. Ans. B.

We know that the triangle midpoint theorem, it says that the line segment connecting the mid points of two sides of a triangle is parallel to the third side and is congruent to one half of the third side.

So here DE is the parallel to BC and also $DE = \frac{1}{2}BC$.

Given $D(2, 5)$ and $E(5, 9)$.

$$DE = \sqrt{(5 - 2)^2 + (9 - 5)^2}$$

$$\Rightarrow DE = \sqrt{9 + 16}$$

$$\Rightarrow DE = 5$$



Then,

$$BC = 2 \times DE$$

$$\Rightarrow BC = 2 \times 5 = 10$$

70. Ans. C.

We know if the points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear, then

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

So, here the points are (a, b) , (c, d) and $(a-c, b-d)$, then

$$\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$$

$$\Rightarrow a(d-b+d) - b(c-a+c) + 1(bc - cd - ad + cd) = 0$$

$$\Rightarrow 2ad - ab - 2bc + ab + bc - ad = 0$$

$$\Rightarrow ad = bc$$

71. Ans. B.

We have,

the parabola $y^2 = x$.

We compare this with $y^2 = 4ax$. So, $a = \frac{1}{4}$

Let $O(0,0)$ be the vertex and $P(at^2, 2at)$ be the other end of the chord OP of the parabola $y^2 = x$. Then,

$$OP = \sqrt{(at^2 - 0)^2 + (2at - 0)^2}$$



$$\Rightarrow OP = at\sqrt{4+t^2}$$

Now, put $a = \frac{1}{4}$

Then, $OP = \frac{t}{4}\sqrt{4+t^2}$

Since OP makes angle θ with x-axis.

$$\tan \theta = \frac{2at - 0}{at^2 - 0}$$

$$\Rightarrow \tan \theta = \frac{2}{t}$$

$$\Rightarrow t = \frac{2}{\tan \theta}$$

Hence, $OP = \frac{2}{4 \tan \theta} \sqrt{4 + \left(\frac{2}{\tan \theta}\right)^2}$

$$\Rightarrow OP = \frac{2}{4 \tan \theta} \sqrt{4 + \frac{4}{\tan^2 \theta}}$$

$$\Rightarrow OP = \frac{2}{4 \tan \theta} \frac{2\sqrt{\tan^2 \theta + 1}}{\tan \theta}$$

$$\Rightarrow OP = \frac{\sqrt{\sec^2 \theta}}{\tan^2 \theta} \left[\because \tan^2 x + 1 = \sec^2 x \right]$$

$$\Rightarrow OP = \frac{\cos \theta}{\sin^2 \theta} \left[\because \sec x = \frac{1}{\cos x} \text{ \& } \tan x = \frac{\sin x}{\cos x} \right]$$

$$\Rightarrow OP = \cos \theta \cdot \sec^2 \theta$$

72. Ans. C.

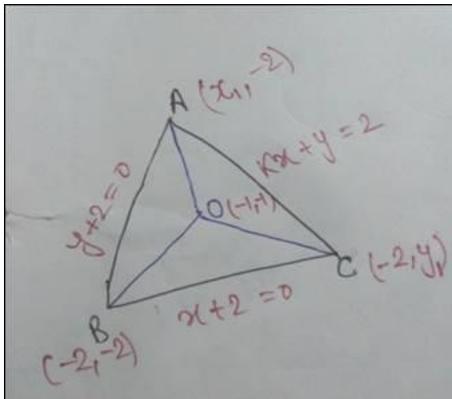
We have, the lines

$$x + 2 = 0$$



$$y + 2 = 0$$

$$kx + y + 2 = 0$$



∴ Sides of triangles are

$$x = -2$$

$$y = -2$$

$$kx + y = -2$$

So coordinates of triangles are

$$A(x_1, -2), B(-2, -2) \text{ and } C(-2, y_1)$$

We know that length from circumcentre to each coordinates are equal.
here circumcentre $O = (-1, -1)$

For circumcentre, $OA^2 = OB^2$

$$(x_1 + 1)^2 + (-2 + 1)^2 = (-2 + 1)^2 + (-2 + 1)^2$$

$$x_1 + 1 - 1 = 0$$

And, $OC^2 = OB^2$

$$(-2 + 1)^2 + (y_1 + 1)^2 = (-2 + 1)^2 + (-2 + 1)^2$$

$$y_1 + 1 = 1$$

$$y_1 = 0$$



So, $A(0, -2)$ and $C(-2, 0)$

$$\begin{aligned} \text{Then, AC equations} &= y + 2 = \frac{0 + 2}{-2 + 0}(x + 0) \\ &= x + y + 2 = 0 \end{aligned}$$

Then, compare with $kx + y + 2 = 0$ means $k = 1$

73. Ans. B.

We have,

$$25x^2 + 16y^2 = 400$$

$$\Rightarrow \frac{x^2}{16} + \frac{y^2}{25} = 1$$

We compare this equation with equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$$a^2 = 16 \text{ and } b^2 = 25$$

Eccentricity $(e) = \sqrt{1 - \frac{a^2}{b^2}}$ as $a < b$

$$e = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = 3/5$$

So, the coordinates of foci are $(0, \pm be)$.

i.e. $(0, 3)$ and $(-3, 0)$

Thus, Q and R are the foci of ellipse.

Since, the sum of the focal distances of a point on an ellipse is equal to its major axis.

$$\therefore PQ + QR = 2b = 2 \times 5 = 10$$

74. Ans. C.



Firstly we see that the point $(1, -1)$ is satisfies $3x + 2y - 5 = 0$ or not.

$$3 \times 1 + 2 \times (-1) - 5 = 3 - 2 - 5 = -4 \neq 0$$

So, the point $(1, -1)$ is belongs to other diagonal.

We know that the diagonals of square are perpendicular to each other then multiplications of slopes is equal to -1 .

$$m_1 \cdot m_2 = -1$$

Then, we calculate slope of $3x + 2y - 5 = 0$

$$\Rightarrow y = -\frac{3}{2}x + \frac{5}{2}$$

$$\Rightarrow m_1 = \frac{-3}{2}$$

Now, $m_1 \cdot m_2 = -1$

$$\Rightarrow m_2 = \frac{2}{3}$$

So, equation of other diagonal is $y = m_2x + c$

$$\Rightarrow y = \frac{2}{3}x + c$$

As we know that the point $(1, -1)$ satisfies $y = \frac{2}{3}x + c$

Then,
$$-1 = \frac{2}{3} \times 1 + c$$

$$\Rightarrow c = -1 - \frac{2}{3} = \frac{-5}{3}$$

So, the equation of diagonal = $y = \frac{2}{3}x - \frac{5}{3}$

$$\Rightarrow 2x - 3y = 5$$



75. Ans. C.

We have,

$$(x-2a)(x-2b)+(y-2c)(y-2d)=0$$

$$\Rightarrow x^2 - 2ax - 2bx + 4ab + y^2 - 2cy - 2dy + 4cd = 0$$

$$\Rightarrow x^2 + y^2 + 2(-a-b)x + 2(-c-d)y + 4ab + 4cd = 0$$

We know that the equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle having centre at

$$(-g, -f)$$

So we compare $x^2 + y^2 + 2(-a-b)x + 2(-c-d)y + 4ab + 4cd = 0$ with standard one.

Then, $g = (-a-b)$ and $f = (-c-d)$

Then centre is $(a+b, c+d)$.

76. Ans. A.

For any non-singular matrix A of order n , we have

$$|adj A| = |A|^{n-1} \text{ and } adj(adj A) = |A|^{n-2} A$$

Also, $adj(adj A) = |A|^{n-2} A$ is obtained by replacing A by $adj A$ in the relation $A(adj A) = |A|I_n = (adj A)A$.

\therefore For a 3×3 matrix, we have

$$|adj A| = |A|^2 \text{ and } adj(adj A) = |A|A$$

We replace A by $adj A$.

So, here $A(adj A) = (adj A)A$ is true. But $|adj A| = |A|$ is not true.



77. Ans. A.

We have,

$$(1+x)^9$$

We know

$$\text{that } (x+a)^n = {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + \dots + {}^nC_r x^{n-r} a^r + \dots + {}^nC_n x^0 a^n$$

Then,

$$(1+x)^9 = {}^9C_0 1^9 x^0 + {}^9C_1 (1)^{9-1} x^1 + \dots + {}^9C_5 1^{9-5} x^5 + {}^9C_6 1^{9-6} x^6 + {}^9C_7 1^{9-7} x^7 + {}^9C_8 1^{9-8} x^8 + {}^9C_9 1^0 x^9$$

$$(1+x)^9 = 1 + 9x + \dots + 126x^5 + 84x^6 + 36x^7 + 9x^8 + x^9$$

$$\text{So, sum of last five coefficient} = 126 + 84 + 36 + 9 + 1$$

$$= 256$$

78. Ans. A.

We have,

$$D = \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix}$$

$$\Rightarrow D = p(qr - p^2) - q(q^2 - pr) + r(qp - r^2)$$

$$\Rightarrow D = pqr - p^3 - q^3 + pqr + pqr - r^3$$

$$\Rightarrow D = 3pqr - (p^3 + q^3 + r^3)$$

Given that p, q and r be three distinct positive real numbers.

$$\text{Let } p = 1, q = 2, r = 3$$

$$\text{then } 1^3 + 2^3 + 3^3 = 36 \text{ and } 3 \times 1 \times 2 \times 3 = 18$$

$$\text{As } p^3 + q^3 + r^3 > 3pqr$$



So, $D < 0$

79. Ans. C.

Let $A = \{1, 2, 3, 4\}$

Then, Subsets of A ,

$$B = \Phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{2, 4\}, \{1, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}, \{1, 3, 4\}, \{1, 2, 3, 4\}$$

And we know that a subset B of a set A is called a proper subset of A if $A \neq B$.

Then, proper subset of A

$$= \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{2, 4\}, \{1, 4\}, \{3, 4\}, \{1, 2, 3\}, \{2, 3, 4\}, \{1, 3, 4\}$$

We know that whenever a set B is a subset of set A, we say that A is a superset of B or $A \supseteq B$

So, super set of $\{3\}$ in subset of B

$$== \{1\}, \{2\}, \{4\}, \{1, 2\}, \{2, 4\}, \{1, 4\}, \{1, 2, 4\}$$

80. Ans. B.

We have,

$$\frac{\cos \theta + i \sin \theta}{\cos \theta - i \sin \theta}$$

$$\Rightarrow \frac{\cos \theta + i \sin \theta}{\cos \theta - i \sin \theta} \times \frac{\cos \theta + i \sin \theta}{\cos \theta + i \sin \theta} \quad [\text{By rationalization}]$$

$$\Rightarrow \frac{(\cos \theta + i \sin \theta)^2}{\cos^2 \theta + \sin^2 \theta}$$

$$\Rightarrow \cos^2 \theta - \sin^2 \theta + 2i \sin \theta \cos \theta \quad [\because \cos^2 \theta + \sin^2 \theta = 1]$$



$$\Rightarrow \cos 2\theta + i \sin 2\theta \left[\because \cos^2 \theta - \sin^2 \theta = \cos 2\theta \text{ \& } 2 \sin \theta \cos \theta = \sin 2\theta \right]$$

So, modulus of $\cos 2\theta + i \sin 2\theta$

$$\Rightarrow \sqrt{\cos^2 2\theta + \sin^2 2\theta}$$

$$\Rightarrow 1$$

81. Ans. C.

The given function $\frac{3x^2 + 8 - 4k}{x}$

Integrating the above function

$$= \int \frac{3x^2 + 8 - 4k}{x} dx$$

$$= \int \left(3x + \frac{8 - 4k}{x} \right) dx$$

$$= 3 \frac{x^2}{2} + (8 - 4k) \ln x + c$$

For the solution to be rational $(8 - 4k) \ln x$ must be zero.

$$\text{Hence } 8 - 4k = 0 \rightarrow k = 2$$

82. Ans. B.

Given function $|x - 1|$ is a mod function.

So any value that will come out of it, would be a positive value or zero.

Hence the minimum value of $|x - 1|$ would be zero only.

83. Ans. A.

$$\text{let } I = \int \frac{dx}{x(x^n + 1)} = \int \frac{x^{n-1} dx}{x^n(x^n + 1)}$$

$$\text{let } x^n + 1 = t \Rightarrow x^{n-1} dx = dt$$



$$\begin{aligned} &= \frac{1}{n} \int \frac{dt}{(t-1)t} \\ &= \frac{1}{n} \int \left(\frac{1}{t-1} - \frac{1}{t} \right) dt \\ &= \frac{1}{n} \ln \left(\frac{t-1}{t} \right) + c \\ &= \frac{1}{n} \ln \left(\frac{x^n}{x^n+1} \right) + c \end{aligned}$$

84. Ans. B.

Given $x^m y^n = a^{m-n}$

Differentiating w.r.t. x

$$\begin{aligned} \frac{d}{dx} x^m y^n &= \frac{d}{dx} a^{m-n} \\ x^m n y^{n-1} \frac{dy}{dx} + m x^{m-1} y^n &= 0 \\ x^m n y^{n-1} \frac{dy}{dx} &= -m x^{m-1} y^n \\ \frac{dy}{dx} &= \frac{-m x^{m-1} y^n}{x^m n y^{n-1}} \\ \frac{dy}{dx} &= \frac{-m y}{n x} \end{aligned}$$

85. Ans. A.

Given that $f(x) = 2x - x^2$ and $f(x+2) + f(x-2) = ?$ at $x=0$

$$\begin{aligned} &f(x+2) + f(x-2) \\ \text{So} &= 2 \times (x+2) - (x+2)^2 + 2 \times (x-2) - (x-2)^2 \end{aligned}$$

Putting $x=0$

$$\begin{aligned} &= 2 \times (0+2) - (0+2)^2 + 2 \times (0-2) - (0-2)^2 \\ &= 2 \times 2 - 2^2 - 2 \times 2 - 2^2 \\ &= -8 \end{aligned}$$

86. Ans. C.

Given curves $y^2 = 2x$ and $y=x$

Solving both for points of intersection

Putting $y=x \Rightarrow x^2 = 2x$

$x=2$ and hence $y=2$ and $x=0, y=0$



the two points are (0,0) and (2,2)

now calculating the area enclosed between the two curves.

Enclosed area= area under parabola in the first quadrant - area under straight line in the first quadrant

$$\text{Enclosed area} = \int_0^2 (y_1 - y_2) dx$$

$$= \int_0^2 (\sqrt{2x} - x) dx$$

$$= \int_0^2 \sqrt{2x} dx - \int_0^2 x dx$$

$$= \sqrt{2} \left[\frac{x^{3/2}}{3/2} \right]_0^2 - \left[\frac{x^2}{2} \right]_0^2$$

$$= \sqrt{2} \times \frac{2}{3} (2^{3/2} - 0) - \frac{2^2}{2} - 0$$

$$= \frac{8}{3} - \frac{4}{2}$$

$$= \frac{16 - 12}{6}$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$

87. Ans. A.

As we know the range of the function $\cos x$ for any value of x is $[-1, 1]$

$$-1 \leq x \leq 1$$

As here in the question the given function is $f(x) = \cos^{-1}(x - 2)$

So the domain of $f(x)$ is

$$-1 \leq x - 2 \leq 1$$

$$1 \leq x \leq 3$$

So, the domain of the function $f(x) = \cos^{-1}(x - 2)$ is $[1, 3]$

88. Ans. C.

Given that $\int (e^{\log x} + \sin x) \cos x dx$



$$\begin{aligned} &= \int (x + \sin x) \cos x \, dx \\ &= \int (x \cos x + \sin x \cos x) \, dx \\ &= \int (x \cos x) \, dx + \int (\sin x \cos x) \, dx \\ &= (x \sin x + \cos x \cdot 1) + \int \left(\frac{2}{2} \sin x \cos x \right) \, dx + c \\ &= x \sin x + \cos x + \int \left(\frac{\sin 2x}{2} \right) \, dx + c \\ &= x \sin x + \cos x + \frac{1}{2} \left(\frac{-\cos 2x}{2} \right) + c \\ &= x \sin x + \cos x - \frac{\cos 2x}{4} + c \\ &= x \sin x + \cos x - \frac{1 - 2 \sin^2 x}{4} + c \\ &= x \sin x + \cos x - \frac{1}{4} + \frac{2 \sin^2 x}{4} + c \\ &= x \sin x + \cos x + \frac{\sin^2 x}{2} + \left(c - \frac{1}{4} \right) \\ &= x \sin x + \cos x + \frac{\sin^2 x}{2} + C \end{aligned}$$

89. Ans. B.

Given equation $dy = (1 + y^2) \, dx$

$$\frac{dy}{(1 + y^2)} = dx$$

Integrating both sides

$$\begin{aligned} \int \frac{dy}{(1 + y^2)} &= \int dx \\ \tan^{-1}(y) &= x + c \\ y &= \tan(x + c) \end{aligned}$$

90. Ans. B.

Given that the function $f(x) = \frac{\sin x}{x}$, is continuous at $x=0$

As we know the condition of function to be continuous is $\lim_{x \rightarrow a} f(x) = f(a)$

Hence

$$\begin{aligned} \lim_{x \rightarrow 0} f(0) &= f(0) \\ \lim_{x \rightarrow 0} \frac{\sin x}{x} &= f(0) \end{aligned}$$



Applying L Hospitals rule

$$\lim_{x \rightarrow 0} \frac{\cos x}{1} = f(0)$$
$$\frac{\cos 0^\circ}{1} = f(0)$$
$$1 = f(0)$$
$$f(0) = 1$$

91. Ans. C.

Given $y = 3x^2 + 2$

So value of y when $x=10 \Rightarrow$

$$y_1 = 3 \times 10^2 + 2$$
$$y_1 = 302$$

Similarly value of y when $x=10.1$

$$y_2 = 3 \times 10.1^2 + 2$$
$$y_2 = 3 \times 102.01 + 2$$
$$y_2 = 308.03$$

Hence $y_2 - y_1 = 308.03 - 302$

$$y_2 - y_1 = 6.03$$

92. Ans. B.

$$\int_0^{\pi/4} (\tan^3 x + \tan x) dx$$
$$= \int_0^{\pi/4} \tan x (\tan^2 x + 1) dx$$
$$= \int_0^{\pi/4} \tan x \cdot \sec^2 x dx$$
$$= \int_0^{\pi/4} \tan x \cdot \sec x \cdot \sec x dx$$

Assuming $\sec x = t$

So $\sec x \cdot \tan x dx = dt$

Putting it, we get $= \int t dt$

$$= \frac{t^2}{2}$$



Putting back $\sec x = t$

$$= \left[\frac{\sec^2 x}{2} \right]_0^{\pi/4}$$

$$= \frac{1}{2} [(\sqrt{2})^2 - 1]$$

$$= \frac{1}{2} [1]$$

$$= \frac{1}{2}$$

93. Ans. B.

Given that $p(x) = (4e)^{2x}$

Integrating both the sides

$$\int p(x) dx = \int (4e)^{2x} dx$$

$$\int p(x) dx = \int 4^{2x} \cdot e^{2x} dx$$

$$\int p(x) dx = 4^{2x} \cdot \int e^{2x} dx - \int \left(\frac{d}{dx} 4^{2x} \cdot \int e^{2x} dx \right) dx + c$$

$$\int p(x) dx = 4^{2x} \cdot \frac{e^{2x}}{2} - \int \left(4^{2x} \cdot 2 \cdot \ln 4 \cdot \frac{e^{2x}}{2} \right) dx + c$$

$$\int p(x) dx = \frac{p(x)}{2} - \ln 4 \cdot \int (4^{2x} \cdot e^{2x}) dx + c$$

$$\int p(x) dx = \frac{p(x)}{2} - \ln 4 \cdot \int p(x) dx + c$$

$$\int p(x) dx + \ln 4 \cdot \int p(x) dx = \frac{p(x)}{2} + c$$

$$\left(\int p(x) dx \right) (1 + \ln 4) = \frac{p(x)}{2} + c$$

$$\int p(x) dx = \frac{p(x)}{2(1 + \ln 4)} + c$$

$$\int p(x) dx = \frac{p(x)}{2(1 + 2 \ln 2)} + c$$

94. Ans. B.



Given $e^{\theta\varphi} = c + 4\theta\varphi$

And $\varphi = f(\theta)$, $\varphi d\theta = ?$

Taking log both sides $e^{\theta\varphi} = c + 4\theta\varphi$

$$\log(e^{\theta\varphi}) = \log(c + 4\theta\varphi)$$

$$\theta\varphi = \log(c + 4\theta\varphi)$$

$$\varphi + \theta \frac{d\varphi}{d\theta} = \frac{1}{c + 4\theta\varphi} \left(0 + 4\varphi + 4\theta \frac{d\varphi}{d\theta} \right)$$

$$\varphi + \theta \frac{d\varphi}{d\theta} = \frac{1}{e^{\theta\varphi}} \left(4\varphi + 4\theta \frac{d\varphi}{d\theta} \right)$$

$$\varphi + \theta \frac{d\varphi}{d\theta} = \frac{4\varphi}{e^{\theta\varphi}} + \frac{4\theta}{e^{\theta\varphi}} \frac{d\varphi}{d\theta}$$

$$\frac{d\varphi}{d\theta} \left(\theta - \frac{4\theta}{e^{\theta\varphi}} \right) = \frac{4\varphi}{e^{\theta\varphi}} - \varphi$$

$$\frac{d\varphi}{d\theta} \theta \left(\frac{e^{\theta\varphi} - 4}{e^{\theta\varphi}} \right) = \varphi \frac{4 - e^{\theta\varphi}}{e^{\theta\varphi}}$$

$$\frac{d\varphi}{d\theta} \theta \left(\frac{e^{\theta\varphi} - 4}{e^{\theta\varphi}} \right) = \varphi \times - \left(\frac{e^{\theta\varphi} - 4}{e^{\theta\varphi}} \right)$$

$$\frac{d\varphi}{d\theta} \theta = -\varphi$$

$$\theta d\varphi = -\varphi d\theta$$

$$\varphi d\theta = -\theta d\varphi$$

95. Ans. B.

Given $f(x) = 3x^2 - 5x + p$

Hence $f(0) = 0 - 0 + p$

$$f(0) = p$$

similarly $f(1) = 3 \cdot 1^2 - 5 + p$

$$f(1) = p - 2$$

it is given that $f(0)$ and $f(1)$ are opposite in sign.



Hence

$$p = -p + 2$$

$$2p = 2$$

$$p = 1$$

96. Ans. B.

Given that X bakes a cake with probability to fail to rise $= 0.02 = \frac{2}{100}$

And he bakes 50% Of cakes at the restaurant.

Similarly Y bakes a cake with probability to fail to rise $= 0.03 = \frac{3}{100}$

And he bakes 30% Of cakes at the restaurant.

Similarly X bakes a cake with probability to fail to rise $= 0.05 = \frac{5}{100}$

And he bakes 20% Of cakes at the restaurant.

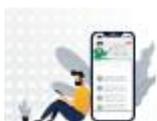
HENCE the required probability that X bakes a cake and it fails

$$\begin{aligned} \text{to rise} &= \frac{\frac{2}{100} \times \frac{50}{100}}{\frac{2}{100} \times \frac{50}{100} + \frac{3}{100} \times \frac{30}{100} + \frac{5}{100} \times \frac{20}{100}} \\ &= \frac{2 \times 50}{2 \times 50 + 3 \times 30 + 5 \times 20} \\ &= \frac{100}{100 + 90 + 100} \\ &= \frac{100}{290} \\ &= \frac{10}{29} \end{aligned}$$

97. Ans. D.

We have $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$, So

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.6 + 0.5 - 0.4 = 0.7 \end{aligned}$$



$$\begin{aligned} P(\overline{A \cup B}) &= P(B - A) = P(B) - P(A \cap B) \\ &= 0.5 - 0.4 = 0.1 \end{aligned}$$

$$\begin{aligned} P(\overline{A \cup \overline{B}}) &= P(A \cap B)' = 1 - P(A \cup B) \\ &= 1 - 0.7 = 0.4 \end{aligned}$$

98. Ans. D.

we have parameters $n = 10$ and $p = \frac{1}{5}$ for variable X.

Given that $Y = 10 - X$ or $X + Y = 10$

Since both the variable are linearly dependent so the parameters n remains unchanged.

And $P_x + P_y = 1 \Rightarrow P_y = 1 - \frac{1}{5} = \frac{4}{5}$

99. Ans. B.

Given that $\overline{X} = 65$, $\overline{Y} = 67$, $\sigma_x = 2.5$, $\sigma_y = 3.5$ and $r(X, Y) = 0.8$.

We know that equation of regression of Y on X $\Rightarrow Y - \overline{Y} = b_{YX}(X - \overline{X})$

Where $b_{YX} = r \cdot \frac{\sigma_x}{\sigma_y}$

So $b_{YX} = 0.8 \cdot \frac{2.5}{3.5}$

$$= \frac{28}{25}$$

Hence equation of regression of Y on X $\Rightarrow Y - \overline{Y} = b_{YX}(X - \overline{X})$



$$Y - 67 = \frac{28}{25}(X - 65)$$

$$Y = \frac{28}{25} \times X - \frac{28}{25} \times 65 + 67$$

$$Y = 1.12X - 72.8 + 67$$

$$Y = 1.12X - 5.8$$

100. Ans. C.

Given that there are 10 lottery tickets, from which 2 tickets are drawn simultaneously.

So the all possible sets of prime numbers = (2,3), (2,5), (2,7), (3,5), (3,7), (5,7)

Hence probability = $\frac{\text{number of favourable sets}}{\text{total number of sets}}$

$$= \frac{6}{{}^{10}C_2}$$

$$= \frac{6}{\frac{10 \times 9}{2}}$$

$$= \frac{6}{45}$$

$$= \frac{2}{15}$$

101. Ans. D.

The number of ways to select 3 persons from a group of 2 boys and 2 girls = ${}^4C_3 = 4$

The number of ways to select 2 boys and 1 girl

from a group of 2 boys and 2 girls = ${}^2C_2 \times {}^2C_1 = 2$

Hence Probability is given as = $\frac{2}{4} = \frac{1}{2}$

102. Ans. A.

The number of ways to select 4 gold coins out of 15 = ${}^{15}C_4$



The number of ways to select 4 counterfeit gold coins out of 6 = 6C_4

$$\text{Hence Probability} = \frac{{}^6C_4}{{}^{15}C_4} = \frac{1}{91}$$

103. Ans. A.

Let H represents the set of husband & W represents the set of wife.

Since the events are given to be independent = $P(H \cup W) = 0$

$$\begin{aligned} \text{Hence } P(H \cup W) &= P(H)P'(W) + P(W)P'(H) + P(W)P(H) \\ &= (1/7).(4/5) + (1/5).(6/7) + (1/7) + (1/5) \\ &= 11/35 \end{aligned}$$

104. Ans. B.

We know that

$$\text{Arithmetic Mean} = \frac{\text{Total sum of observations}}{\text{Number of observations}}$$

Total sum of observations = $40 \times 1000 = 4000$

$$\text{Correct Arithmetic Mean} = \frac{4000 + 53 - 83}{100} = 39.70$$

105. Ans. B.

As February will have 29 days in a Leap Year,

$$\text{Hence Number of Weeks} = \frac{29}{7} = 4 \text{ Weeks and 1 day}$$

This 1 day can be either of the days of the week,

Hence the probability that February of a leap year will have five Sundays = $\frac{1}{7}$

106. Ans. D.

We know that



$$\text{coefficient of variation} = \left(\frac{\text{Standard Deviation}}{\text{Mean}} \times 100 \right)$$

$$\text{Mean} = \frac{\sum x_i}{n} = \frac{20}{10} = 2$$

$$\text{Standard Deviation} = \frac{1}{n} \sqrt{n \sum x_i^2 - (\sum x_i)^2}$$

$$= \frac{1}{10} \sqrt{(10)(200) - (20)^2}$$

$$= 4$$

$$\text{Coefficient of variation} = \frac{4}{2} \times 100$$

$$= 200$$

107. Ans. B.

We know that Standard Deviation is given by $= \frac{1}{n} \sqrt{n \sum x_i^2 - (\sum x_i)^2}$

Where n is the number of observations.

$$\sum x_i = 0, \sum x_i^2 = 32$$

$$\text{Form } n = 8$$

$$= \frac{1}{8} \sqrt{(8)(32)}$$

$$= 2$$

(All the above values can be inferred from the given set of observations)

108. Ans. C.

Let A=35 h=10



Interval	Mid valve	Physics(fi)	u_i	$f_i u_i$
10-20	15	8	-2	-16
20-30	25	11	-1	-11
30-40	35	30	0	0
40-50	45	26	1	26
50-60	55	15	2	30
60-70	65	10	3	30

We know that Mean is defined as= $A+h \left[\frac{\sum f_i u_i}{\sum f_i} \right]$

$$= 35 + \frac{59}{100} \times 10 = 40.9$$

109. Ans. A.

The highest frequency of physics is in the interval 30-40

Mean of physics=39.4

Mean of maths=36.2

Mode of physics=30

Mode of math=38

Applying 2median=3Mean-mode

We get median of math=35.3

Median of physics=44.1

Hence median of physics is greater than median of math.

110. Ans. C.

If we calculate the difference between the students of Physics & Mathematics in each range we will find out that in the range 40-50 the difference is 11 which is highest among the all interval given.

111. Ans. A.

$$\lim_{x \rightarrow 0} \frac{\sin x \times \log(1-x)}{x^2}$$



The given limit can be break upon as-

$$= \left[\lim_{x \rightarrow 0} \frac{\sin x}{x} \right] \left[\lim_{x \rightarrow 0} \frac{\log(1-x)}{x} \right]$$

$$= (1)(-1) = -1$$

112. Ans. B.

We have $k \frac{dy}{dx} = \int \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{2}{3}} dx$

Differentiating both sides, we get,

$$\frac{d}{dx} \left[k \frac{dy}{dx} \right] = \frac{d}{dx} \left[\int \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{2}{3}} dx \right]$$

$$k \frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{2}{3}}$$

Cubing both sides, we get-

$$\left[k \frac{d^2y}{dx^2} \right]^3 = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^2$$

Hence Order=2 and Degree=3

113. Ans. C.

$$\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow k} \frac{x^3 - k^3}{x^2 - k^2},$$

$$\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x+1)(x^2+1)}{x-1} = 4$$

And- $\lim_{x \rightarrow k} \frac{x^3 - k^3}{x^2 - k^2} = \lim_{x \rightarrow k} \frac{(x-k)(x^2 + k^2 + xk)}{(x-k)(x+k)} = \frac{3}{2}k$

But it is given that-

$$\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow k} \frac{x^3 - k^3}{x^2 - k^2},$$



$$\frac{3}{2}k = 4$$

$$k = \frac{8}{3}$$

114. Ans. A.

Circumference of circle = $2\pi r$ (r is the radius of the circle)

Change in circumference = $2\pi \frac{dr}{dt}$ ($\frac{dr}{dt}$ is given as 0.7 cm/sec.)

$$= 4.4 \text{ cm/sec}$$

115. Ans. D.

$$\lim_{x \rightarrow 1} \frac{x + x^2 + x^3 - 3}{x - 1}$$

Applying L'Hospital Rule,

$$\lim_{x \rightarrow 1} \frac{1 + 2x + 3x^2}{1} = 6$$

116. Ans. B.

We know that if three vectors are coplanar then their Scalar Triple Product is 0.

Scalar Triple product is defined as

$$[abc] = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$

$$\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$$

$$\vec{c} = c_1 \hat{i} + c_2 \hat{j} + c_3 \hat{k}$$



$$[abc] = \begin{vmatrix} 2 & 1 & 0 \\ -3 & 2 & 1 \\ 1 & -3 & p \end{vmatrix} = 0$$

$$\text{Hence, } = 2(2p+3) - 1(-3p-1) = 7p+7$$

$$7p+7=0$$

$$P=-1$$

117. Ans. A.

We know that

$$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2$$

$$144 = (4)^2 |\vec{b}|^2$$

$$|\vec{b}| = \pm 3$$

118. Ans. C.

Let AB & BC be the consecutive sides of the given parallelogram.

Then the diagonals of the parallelogram is defined by $\vec{AC} = \vec{AB} - \vec{BC}$ & $\vec{AB} + \vec{BC}$

$$\vec{AB} - \vec{BC} = \hat{i} - 2\hat{j} + 8\hat{k}$$

$$\vec{AB} + \vec{BC} = 3\hat{i} - 6\hat{j} + 2\hat{k}$$

Their dot product is given as $= 31$

Hence magnitude = 31

119. Ans. C.

Let α , β and γ are the angles vector OA makes with the x,y and z axis respectively.

Then,



$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\frac{1}{2} + \frac{1}{4} + \cos^2 \gamma = 1 \quad [\alpha = 45^\circ, \beta = 60^\circ]$$

$$\cos^2 \gamma = \frac{1}{4}$$

$$\cos \gamma = \pm \frac{1}{2}$$

It is given that

$$\vec{OA} = |\vec{OA}|(\cos \alpha \hat{i} + \cos \beta \hat{j} + \cos \gamma \hat{k})$$

$$= 6\sqrt{2}\hat{j} + 6\hat{i} \pm 6\hat{k}$$

120. Ans. A.

\vec{a} is defined as-

$$\vec{a} = |\vec{a}| \cos \theta \hat{i} + |\vec{a}| \sin \theta \hat{j} \quad \text{where } \theta \text{ is the angle made by the vector with positive x-axis.}$$

$$= (1) \cos 30^\circ \hat{i} + (1) \sin 30^\circ \hat{j}$$

$$= \frac{\sqrt{3}\hat{i} + \hat{j}}{2}$$

