## GATE 2020 <br> Electronics \& Communication Engineering

Mega Mock Challenge (02 Jan-03 Jan 2020)

## Questions \& Solutions

1. Direction: In the given question, four words are given of which two are most nearly the same or opposite in meaning. Find the two words and indicate your answer by marking the option which represents the correct combination.
A) Diligent
B) Adorable
C) Meticulous
D) Prominent
A. B-D
B. $\mathrm{A}-\mathrm{C}$
C. $A-B$
D. $A-D$

Ans. B
Sol. The meanings of the words are:
Diligent: having or showing care and conscientiousness in one's work or duties.
Adorable: inspiring great affection or delight.
Meticulous: showing great attention to detail; very careful and precise. Prominent: important; famous.
Hence, option B is the correct answer.
2. Direction: A statement with one blank is given below. Choose the set of words from the given options which can be used to fill the given blank. Despite almost ubiquitous scepticism, the electoral bonds have prevailed and, that too, almost solely $\qquad$ rhetorical claims of "transparency of political funding system," "clean money," and "donor's anonymity." i. with the backing of the ruling government's
ii. based on the endorsement derived from the political party at power's iii. backed by the political party at power's
A. Only i
B. Only ii
C. Only iii
D. Both i and ii

Ans. D
Sol. The given sentence talks about the prevailing nature of 'electoral bonds' in spite of concerns and doubts regarding the same. The sentence goes on to explain that this is occurring because of rhetorical claims
by someone. From the options it is clear that the ruling part is responsible for these 'rhetorical claims'.

Option i - 'backing' means help or support and has been used in conjunction with the correct tense format of the sentence.
Option ii - 'endorsement' also means help or support and it tallies with the sentence structure.
Option iii - although 'backed' has been used it is in the incorrect tense form. This makes it incorrect.
Thus, option D is the correct answer.
3. Which letter-cluster will replace the question mark (?) in the following series?
HQCF, MVHK, JSEH, OXLM, ?
A. FTRD
B. LUGJ
C. MKOP
D. SWQ

Ans. B
Sol. Pattern is-


Hence, the correct answer is option B.
4. Three different positions of the same dice are shown. Which symbol will be on the face opposite to the one having '*'?

A. +
B. !
C. $\$$
D. @

Ans. A
Sol. Pick out the dices in which one symbol is common, after that arrange them in ACW or CW direction.

In II and III ' + ' is common

+ = @
*     + ! \$

Interchange the missing symbol '*' with repeated symbol '+' Hence, option (A) is the correct answer.
5. In the following diagram, the triangle represents 'Dentists', the circle represents 'Professors' and the rectangle represents 'Doctors'. The numbers in different segments show the number of persons.


How many professors are dentists but not doctors?
A. 17
B. 9
C. 15
D. 13

Ans. B
Sol. Given diagram is-

circle represents Professors rectangle represents Doctors triangle represents Dentists No. of professors who are dentists but not doctors=2+7=9
Hence, the correct answer is option B.
6. In the following question, some statements followed by some conclusions are given. Taking the given statements to be true even if they seem to be at variance from commonly known facts, read all the
conclusions and then decide which of the given conclusions logically follows the given statements.

## Statement:

Parents must understand that their child cannot attain excellence on his own. He needs their support. They must thus be open to help him at various steps rather than merely setting high expectations.

## Conclusion:

I. Ideal students are not born ideal or perfect. They are nurtured to become ideal by their educators. The environment at home has a great impact on the way a student performs in school.
II. The life of an ideal student may seem tough from a distance. However, it is actually much more sorted as compared to those who procrastinate and do not give complete attention to their studies.
A. If only conclusion I follows
B. If only conclusion II follows
C. If both I and II conclusion follow
D. If neither I nor II conclusion follows

Ans. A
Sol. Conclusion I follows, based on the given statement a major component in the making of an Ideal student is described that it takes efforts not only from the students but also from the educators( Teachers and Parents)
Conclusion II is a correct statement that is the hard work and struggle that it takes to become an ideal student but it cannot be the conclusion of the given statement.
7. Direction: Each question below is followed by two statements I and II. You have to determine whether the data given in the statement is sufficient for answering the question. You should use the data and your knowledge of Mathematics to choose the best possible answer.

A man deposited Rs. ' $x$ ' in bank which gives simple interest at the rate of $8 \%$ p.a. Find the value of ' $x$ '.

Statement I: After 3 years, amount received by him is Rs. $(x+672)$.
Statement II: Interest earned by him after 3 years is $24 \%$ of the amount deposited by him.
A. If the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question.
B. If the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question.
C. If the data either in Statement I or
in Statement II alone are sufficient to answer the question.
D. If the data in both Statements I and II together are necessary to answer the question.
Ans. A
Sol. Statement I:
Simple interest earned by him
$=x+672-x=$ Rs. 672
So, $672=\frac{x \times 8 \times 3}{100}$
$x=$ Rs. 2800
So, statement I alone is sufficient to answer the question.
Statement II:
We have to calculate principal(x) but we are not given interest since it is also in form of $x$. Hence, there are 2 unknowns.
Statement II alone is not sufficient to answer the question.
Thus, the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question.
So option (A) is the correct answer.
8. The given pie chart shows the breakup of total number of the employees of a company working in different offices ( $A, B, C, D$ and $E$ ). Total no. of employees $=2400$


What is the number of offices in which the number of employees of the company is between 350 and 650 ?
A. 3
B. 4
C. 2
D. 1

Ans. A
Sol. Total no. of Employees $\left(360^{\circ}\right)=$ 2400
No. of employees in office $A\left(126^{\circ}\right)$
$=\frac{2400}{360} \times 126=840$
No. of employees in office $B\left(18^{\circ}\right)$
$=\frac{2400}{360} \times 18=120$
No. of employees in office $C\left(54^{\circ}\right)$
$=\frac{2400}{360} \times 54=360$
No. of employees in office $D\left(90^{\circ}\right)$
$=\frac{2400}{360} \times 90=600$
No. of employees in office $E\left(72^{\circ}\right)$
$=\frac{2400}{360} \times 72=480$
Number of offices in which the number of employees of the company is between 350 and $650=3$
9. Find the numbers $a, b, c$ between 2 and 18 such that
I. their sum is 25 ,
II. the numbers $2, a, b$ are
consecutive terms of an A.P. and
III. The numbers b, c, 18 are
consecutive terms of a G.P.
A. $a=5, b=8, c=12$
B. $a=7, b=8, c=12$
C. $a=5, b=9, c=11$
D. $a=7, b=5, c=11$

Ans. A
Sol. We have $a+b+c=25$
$2, a, b$ are in A.P. $\Rightarrow 2 a=2+b$
b, c, 18 are in G.P. $\Rightarrow 18 b=c 2$

Substituting for $a$ and $b$ in (1), using relations (2) and (3), we get
$\Rightarrow 1+\frac{b}{2}+\frac{c^{2}}{18}+c=25$
$\Rightarrow c^{2}+12 c-288=0$
$\Rightarrow(c-12)(c+24)=0$
$\Rightarrow C=12$ or $c=-24$
Since the numbers lie between 2 \&
18,
We take $\mathrm{c}=12$
$\Rightarrow \mathrm{a}+\mathrm{b}=13$
$\Rightarrow a+2 a-2=13$
$\Rightarrow b=8, a=5$
10. Statements:

All lions are ducks.
No duck is a horse.
All horses are fruits.

## Conclusions:

I. No lion is a horse.
II. Some fruits are horses.
III. Some ducks are lions.
IV. Some lions are horses.
A. Only either I or II and III \& IV follow
B. Only either I or IV and both II and

III follow
C. Only either I or IV and II follow
D. Only Conclusion I \& II and III follow
Ans. D
Sol.


We use elimination to find an exception to the generality of the question. Thus we prove they are not implied. The diagram above satisfy all the above statement but contradict with the conclusion (iv). Since we
found an exception, the conclusion is not true in every case. Thus it is not implied.
We can draw many scenarios that satisfy the statements using Venn diagram \& check for the validity of the conclusions.
Conclusions (i), (ii), (iii) hold good for every case so they are implied.
21. The minimal expression of function $f(A, B, C, D)$ is

A. $\overline{\mathrm{A}} \mathrm{D}+\overline{\mathrm{A}} \mathrm{B}+\mathrm{C}$
B. $\overline{\mathrm{A}} \mathrm{B}+\mathrm{AC}+\overline{\mathrm{A}} \mathrm{C}+\overline{\mathrm{A} D}$
C. $A+B+C+D$
D. $\overline{\mathrm{A}}+\overline{\mathrm{B}}+\overline{\mathrm{C}}+\overline{\mathrm{D}}$

Ans. A
Sol.


So, $f(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})=\overline{\mathrm{A}} \mathrm{D}+\overline{\mathrm{A}} \mathrm{B}+\mathrm{C}$
12. Consider an angle modulated signal with phase modulation as follows $x_{c}(t)=10 \cos \left(\omega_{c} t+3 \sin \omega_{m} t\right)$

Assume $f_{m}=1 \mathrm{kHz}$. Find the bandwidth (in kHz) when $f_{m}$ is doubled.
Sol.
$x_{P M}(t)=10 \cos \left(\omega_{c} t+k_{p} m(t)\right)=10 \cos \left(\omega_{c} t+3 \sin \omega_{m} t\right)$
Thus, $\mathrm{m}(\mathrm{t})=a_{m} \sin \omega_{m} t$
$x_{P M}(t)=10 \cos \left(\omega_{c} t+k_{p} a_{m} \sin \omega_{m} t\right)$

Modulation index $\beta=k_{p} a_{m}=3$
When $f_{m}=1 \mathrm{kHz}$,
$f_{B}=2(\beta+1) f_{m}=8 k H z$
When $f_{m}=2 \mathrm{kHz}, \beta=3_{\text {(as }} \beta_{\text {is }}$
independent of $f_{m}$ )
$f_{B}=2(\beta+1) 2=16 k H z$
13. If PSD of a real process $x(t)$ is shown in figure. Calculate the maximum value of auto correlation function.

A. $\frac{6000}{\pi}$
B. $\frac{5500}{\pi}$
C. $\frac{9000}{\pi}$
D. $\frac{4000}{\pi}$

Ans. D
Sol. Maximum
value $E\left[X^{2}(t)\right]=R_{x x}(0)$ (average power)
Average
power $P_{x x}=\frac{1}{2 \pi} \operatorname{area}\left(\delta_{x x}(\omega)\right)$
$=\frac{1}{2 \pi} \int_{-\infty}^{+\infty} \delta_{x x}(\omega) d \omega$
$=\frac{1}{\pi} \int_{0}^{+\infty} \delta_{x x}(\omega) d \omega$
$=\frac{1}{\pi}\left[\frac{1}{2} \times 2000 \times 4\right]$
$=\frac{4000}{\pi}$
14. Let $x(n)=\cos \left(2 n-\frac{\pi}{3}\right)$. Even component of $x(n)$ is $\operatorname{acos}(b n)$ then the value of $a \& b$ are respectively.
A. 0.5 and 2
B. -0.5 and 2
C. 0.5 and -2
D. -0.5 and -2

Ans. A
Sol. $X(n)=\cos \left(2 n-\frac{\pi}{3}\right)$
$X(-n)=\cos \left(-2 n-\frac{\pi}{3}\right)=\cos \left(2 n+\frac{\pi}{3}\right)$
Even component of $x(n)$ is given by
$X_{e}(n)=\frac{X(n)+X(n)}{2}$
$X_{e}(n)=\cos \left(2 n-\frac{\pi}{3}\right)+\cos \left(2 n+\frac{\pi}{3}\right)$
$=\frac{2 \cdot \cos (2 n) \cos \left(\frac{\pi}{3}\right)}{2}$
$=\cos \left(\frac{\pi}{3}\right) \cos (2 n)=0.5 \cos (2 n)$
Given $X_{e}(n)=a \cdot \cos (b n)$
So, $a=0.5 \& b=2$
15. The sum of $Z$-parameters of the two port network shown below is $\qquad$ ?


Sol. Apply kVL to inner and outer loops
$\mathrm{V}_{1}=4 \mathrm{I}_{1}-3 \mathrm{I}_{2}+\left(\mathrm{I}_{1}+\mathrm{I}_{2}\right)$
$=5 \mathrm{I}_{1}-2 \mathrm{I}_{2}$
$\mathrm{V}_{2}=\mathrm{I}_{2}+\left(\mathrm{I}_{1}+\mathrm{I}_{2}\right)=\mathrm{I}_{1}+2 \mathrm{I}_{2}$
$\left[\begin{array}{l}V_{1} \\ V_{2}\end{array}\right]=\left[\begin{array}{cc}5 & -2 \\ 1 & 2\end{array}\right]\left[\begin{array}{l}I_{1} \\ I_{2}\end{array}\right]$
The sum of the $Z$-parameters
$=Z_{11}+Z_{12}+Z_{21}+Z_{22}$
$=5-2+1+2=6$
16. If the transistors in Fig. have high values of $\beta$ and $a V_{B E}$ of 0.65 Volts, the current I flowing through $2 \mathrm{k} \Omega$ resistance is

A. $2 m A$
B. 6.5 mA
C. 1 mA
D. 10 mA

Ans. C
Sol.


For high values of $\beta$, current through base $I_{B}$ is almost zero.
$\therefore \mathrm{I}_{\mathrm{B}}=\mathrm{OA}$
By using voltage division principle at the bases
$\mathrm{v}_{2}=\mathrm{v}_{\mathrm{cc}} \times \frac{1.65}{1.65+1.85+6.5}$
$=10 \times \frac{1.65}{10}=1.65 \mathrm{~V}$
$\mathrm{V}_{\mathrm{E} 2}=\mathrm{V}_{2}-0.65 \mathrm{~V}$
$=1.65-0.65$
$\mathrm{V}_{\mathrm{E} 2}=1 \mathrm{~V}$
$I=\frac{V_{\mathrm{E} 2}}{1 \mathrm{k} \Omega}=\frac{1 \mathrm{~V}}{1 \mathrm{~K}}$
$\mathrm{I}=1 \mathrm{~mA}$
17. A two-port network is shown in figure. The parameter $h_{21}$, for this network can be given by


Sol. h - parameter equation to calculate $h_{21}$
$\mathrm{I}_{2}=\mathrm{h}_{21} \mathrm{I}_{1}+\mathrm{h}_{22} \mathrm{~V}_{2}$
$\mathrm{h}_{21}=\left.\frac{\mathrm{I}_{2}}{\mathrm{I}_{1}}\right|_{\mathrm{V}_{2}=0}$


The relevant circuit with $\mathrm{V}_{2}=0$ is shown in above figure.
Using current division rule,
$I_{2}=\frac{-I_{1}}{2}$
$\therefore h_{21}=\frac{I_{2}}{I_{1}}=\frac{-1}{2}$
18. If $\mathrm{A}_{3 \times 3}$ is any real matrix such that IAI $=24$, trace of ' $A$ ' $=9$ and one of the eigen values is ' 2 ' then trace of Adj A $=$ $\qquad$ ?
Sol. Let the another two eigen value of ' $A$ ' be $a, \beta$
Given that $(a+\beta+2)=9$
$a+\beta=7$
and 2.a. $\beta=24$
a. $\beta=12$

By solving (1) \& (2) we get $a, \beta=$ 4, 3
$\therefore$ The eigen value of ' $A$ ' are $2,3,4$ If $\lambda$ is eigen value of ' $A$ ' then $\frac{|A|}{\lambda}$ is and eigen value of adj.
Hence, the eigen value of Adj A are 6, 8, 12
$\therefore$ Trace of Adj A $=6+8+12=26$
19. The divergence of the vector field
$\vec{V}=\left(x^{2}+y\right) \widehat{i}+(z-2 x y) \widehat{j}+(x y)^{\widehat{k}}$
at $(1,1,1)$ is
A. 1
B. -1
C. 0
D. 2

Ans. C
Sol.

$$
\begin{aligned}
& \nabla \cdot \vec{V}=\frac{\partial}{\partial x}\left(x^{2}+y\right)+\frac{\partial}{\partial y}(z-2 x y)+\frac{\partial}{\partial z}(x y) \\
& =2 x-2 y \\
& =0 \text { at }(1,1,1)
\end{aligned}
$$

20. The mean value C of Cauchy's theorem for the functions $f(x)=\frac{1}{x}$ and $g(x)=\frac{1}{x^{2}}$ in the interval $[2,3]$ is
A. 2.4
B. 2.5
C. 2.6
D. 2.8

Ans. A
Sol. The conditions of Cauchy's theorem hold good for $f(x)$ and $g(x)$.
By Cauchy's theorem, there exists a value c such that
$\frac{f^{\prime}(c)}{g^{\prime}(c)}=\frac{f(3)-f(2)}{g(3)-g(2)}$
$\Rightarrow \frac{\left(\frac{-1}{\mathrm{C}^{2}}\right)}{\left(\frac{-2}{\mathrm{C}^{3}}\right)}=\frac{\left(\frac{1}{3}-\frac{1}{2}\right)}{\left(\frac{1}{9}-\frac{1}{4}\right)}$
$\Rightarrow \mathrm{c}=2.4$
21. The transfer characteristics of the circuit shown below?

A.

B.

C.

D.


Ans. B

Sol. Case I


When $\mathrm{V}_{\text {in }}>0$ (inverting terminal > non inverting terminal.)
So D1=ON; D2=OFF
Current through is 0 A ,
Hence $\mathrm{V}_{0}=0 \mathrm{~V}$
Case II.


When Vin < 0, (inverting terminal < non inverting terminal.)
D1=OFF \& D2=ON
$V_{A}=[-R / R] V_{\text {in }}=-V_{\text {in }}$
$V_{0}=[1+R / R] V_{A}=(2)\left(-V_{\text {in }}\right)$
$V_{0}=-2 V_{\text {in }}$
Hence $V_{0} / V_{\text {in }}=$ slope $=-2$
So option B is correct.
22. An NMOS transistor with threshold voltage $\mathrm{V}_{\mathrm{T}}=1 \mathrm{~V}, \mathrm{~m}_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}}=0.02 \mathrm{~A}$ $/ \mathrm{V}^{2}$ and the terminal voltages are $\mathrm{V}_{\mathrm{GS}}=1.2 \mathrm{~V}$ and $\mathrm{V}_{\mathrm{DS}}=0.1 \mathrm{~V}$.
The drain current of the NMOS transistor is $\qquad$ mA
Sol. $\mathrm{V}_{\mathrm{T}}($ Threshold voltage $)=1 \mathrm{~V}$
Conductance
parameter, $\mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}}\left(\frac{\mathrm{W}}{\mathrm{L}}\right)=0.002$
$\mathrm{A} / \mathrm{V}^{2}$
Gate to source voltage, $\mathrm{V}_{\mathrm{GS}}=1.2 \mathrm{~V}$
Drain to source voltage, $\mathrm{V}_{\mathrm{DS}}=0.1 \mathrm{~V}$
Since, $\mathrm{V}_{\mathrm{GS}}>\mathrm{V}_{\mathrm{T}} \Rightarrow$ NMOS transistor is ON
$\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{T}}=1.2-1=0.2$
$V_{D S}<V_{G S}-V_{T}$
Hence, NMOS transistor is in linear region.

Drain current,
$I_{D}=\mu_{n} C_{o x} \frac{W}{L}\left[\left(V_{G S}-V_{T}\right) V_{D S}-\frac{V_{D S}^{2}}{2}\right]$
$I_{D}=0.002\left[(0.2)(0.1)-\frac{(0.1)^{2}}{2}\right] A$
$I_{D}=30{ }^{\mu_{A}}$
23. For the given pole zero plot which one could correspond to an even function of time.
A.

B.

C.

D.


Ans. C
Sol. For a signal to be even it must be either two sided or finite duration. Therefore if $X(s)$ has poles and the ROC must have strip in s-plane.
For Plot A
$X(s)=\frac{A s}{(s+1)(s-1)}$
For even function
X $(\mathrm{s})=\mathrm{X}(-s)$
But
$X(-s)=\frac{A s}{(S-1)(S+1)}=-X(s)$
Therefore $X(A)$ is not even (it is odd) For Plot B:
We note that the ROC cannot be chosen to correspond to a two-sided function $\mathrm{X}(\mathrm{t})$ therefore signal is not even.
For plot C:

We get
$X(s)=\frac{A(s-j)(s+j)}{(s+1)(s-1)}=\frac{A\left(s^{2}+1\right)}{s^{2}-1}$
$X(-s)=\frac{A\left(s^{2}+1\right)}{s^{2}-1}=X(s)$
Hence, $x(t)$ is even provided ROC is chosen to be $-1<\operatorname{Re}(s)<2$
For Plot D:
We note that the ROC cannot be chosen to correspond to a two sided function $X(t)$.
Hence, it is not even.
24. An EM wave at wave length 500 nm traveling with speed of $2 \times 10^{8} \mathrm{~m} / \mathrm{s}$ in certain medium and enters another medium of refractive index $\frac{5}{4}$ times that of the first medium, then velocity of the second medium is $\qquad$ $\times$ $10^{8} \mathrm{~m} / \mathrm{s}$
Sol. $\mathrm{f}=\frac{\mathrm{C}}{\lambda}=\frac{2 \times 10^{8}}{500 \times 10^{-9}}=4 \times 10^{14} \mathrm{~Hz}$
$V_{P 1}=\frac{3 \times 10^{8}}{n_{2}}=\frac{3 \times 10^{8}}{\left(\frac{15}{8}\right)}=1.6 \times 10^{8} \mathrm{~m} / \mathrm{s}$
$\left(n_{2}=\frac{5}{4} n_{1}=\frac{5}{4}\left(\frac{3}{2}\right), n_{1}=\frac{3 \times 10^{8}}{2 \times 10^{8}}=\frac{3}{2}\right)$
25. Which of the following is correct ?
A. $(\bar{x}+y) \odot(x \oplus y)=\bar{x} y$
B. $(\bar{x}+y) \odot(x \oplus y)=x \bar{y}$
C. $(\bar{x}+y) \odot(x \oplus y)=x y$
D. $(\bar{x}+y) \odot(x \oplus y)=x+y$

## Ans. A

Sol.
$(\bar{x}+y) \odot(x \oplus y)=(\bar{x}+y)(\bar{x} y+x \bar{y})+\overline{(\bar{x}+y)}(x y+\bar{x} \bar{y})$
$=(\bar{x} y+0+\bar{x} y+0)+(0+0)$
$=\bar{x} y$
26. Consider the two diode circuit shown in the figure below:


The diodes $D_{1}$ and $D_{2}$ can be modelled as a constant voltage source of 0.7 V , when forward biased. The value of current flowing through diode $D_{1}$ is equal to $I_{D 1}$, then the value of current $I_{D 1}$ is equal to
$\qquad$ mA
Sol. Assuming both the diodes to be ON, thus the circuit will reduce to


The current $\mathrm{I}_{\mathrm{D} 2}$ will be equal to
$I_{D 2}=\frac{0-(-5)}{10 \mathrm{~K} \Omega}=\frac{5}{10} \times 10^{-3}=0.5 \mathrm{~mA}$
Now, the current I from the 5 V source will be equal to
$I=\frac{5-0.7}{5 \mathrm{~K} \Omega}=0.86 \mathrm{~mA}$
Current $\mathrm{I}_{\mathrm{D} 1}=\mathrm{I}-\mathrm{I}_{\mathrm{D} 1}$
$=(0.86-0.5) \times 10^{-3} \mathrm{~A}$
$=0.36 \mathrm{~mA}$
$\because I_{D 1}$ and $I_{D 2}$ both are positive, hence out assumption was correct.
$I_{D 1}=0.36 \mathrm{~mA}$
27. In the series voltage regulator circuit shown below $\mathrm{V}_{\mathrm{BE}}=0.7 \mathrm{~V}, \beta=50$, $\mathrm{V}_{\mathrm{z}}=8.3 \mathrm{~V}$. The output voltage $\mathrm{V}_{0}$ is
$\qquad$ volts.

A. 15
B. 11
C. 10
D. 12

Ans. A

Sol. In this circuit, zener diode works as a voltage regulator. So, the equivalent circuit is drawn as


From the circuit, we have
$\mathrm{V}_{1}=\mathrm{V}_{\mathrm{BE}}+\mathrm{V}_{\mathrm{z}}=0.7+8.3=9 \mathrm{~V}$
Using voltage divider rule, we get
$\mathrm{V}_{1}=\frac{30 \mathrm{k}}{20 \mathrm{k}+30 \mathrm{k}} \mathrm{V}_{0}$
$\mathrm{V}_{0}=\frac{5}{3} \times 9=15 \mathrm{~V}$
28. In the circuit shown in figure, voltage $V_{0}$ is


Sol. We solve this problem using principal of power conservation.


Using passive sign convention, we calculate power absorbed by each element
Element $\mathrm{e}_{1}: \mathrm{P}_{1}=6 \times 5=30 \mathrm{~W}$ (absorbed)
Element $\mathrm{e}_{2}: \mathrm{P}_{2}=3 \mathrm{~V}_{0}$ (absorbed)
Element $e_{3}: P_{3}=1 \times 14=14 \mathrm{~W}$
(absorbed)

Element $\mathrm{e}_{4}: \mathrm{P}_{4}=1 \times 14=14 \mathrm{~W}$
(absorbed)
15 V source: $\mathrm{P}_{5}=-15 \times 5=-75 \mathrm{~W}$
or, $\mathrm{P}_{5}=75 \mathrm{~W}$ (delivered)
Dependent source: $\mathrm{P}_{6}=-\left(5 \mathrm{I}_{0}\right) \times 2=$ -10 W
or, $\mathrm{P}_{6}=10 \mathrm{~W}$ (delivered)
Power absorbed = Power delivered
$30+3 \mathrm{~V}_{0}+14+14=75+10$
$3 \mathrm{~V}_{0}=27 \Rightarrow \mathrm{~V}_{0}=9 \mathrm{~V}$
29. If $(x)_{r}+\left(x^{2}\right)_{r+1}+\left(x^{3}\right)_{r+2}=(109)_{10}$,
where $x$ represents the minimum number of NOR gates required to implement a half adder, then the positive value of radix ' $r$ ' is $\qquad$ _.
Sol. Given that:
$(x)_{r}+\left(x^{2}\right)_{r+1}+\left(x^{3}\right)_{r+2}=(109)_{10}$
Where $\mathrm{x}=$ Number of NOR Gates required to implement a half adder $=$ 5
$(5)_{r}+\left(5^{2}\right)_{r+1}+\left(5^{3}\right)_{r+2}=(109)_{10}$
Decimal equivalent of each individual term is as follows:
$(5)_{r}=\left(r^{0} \times 5\right)_{10}=5$
$(25)_{r+1}=\left[2 \times(r+1)^{1}+5 \times(r+1)^{0}\right]_{10}=(2 r+7)_{10}$
$(125)_{r+2}=\left[1 \times(r+2)^{2}+2 \times(r+2)^{2}+5 \times(r+2)^{0}\right]_{10}=\left(r^{2}+6 r+13\right)_{10}$
Substituting all decimal values in the above equation,
$5+2 r+7+r^{2}+6 r+13=109$
$r^{2}+8 r+25=109$
$r^{2}+8 r-84=0$
$r^{2}-6 r+14 r-84=0$
$(r-6)(r+14)=0$
$r=6,-14$
Hence, the positive value of radix ' $r$ ' is 6.
30. The transfer function of a network can be written as $\frac{1+s}{1+0.5 s}$. The maximum phase angle occurs at a frequency of
$\qquad$ rad/sec.
A. 2
B. 5
C. 8
D. 16

Ans. A

Sol. Comparing with standard transfer function
$\frac{1+s \tau}{1+\alpha s \tau}$
Here, $\tau=1$
and $a \tau=0.5$
or $a=0.5$
$\because \alpha<1$
$\therefore$ Lead network
Maximum phase occurs at a
frequency of
$\omega_{m}=\frac{1}{\alpha \sqrt{\tau}}=\frac{1}{\alpha}=2 \mathrm{rad} / \mathrm{sec}$
31. An 8 bit unipolar successive approximation register type ADC is used to convert 6.1 volts to digital equivalent output. The reference voltage is +15 V . The output of ADC at the end of 4th clock pulse after the start of the conversion, is
A. 01010000
B. 01110000
C. 01100000
D. 01001000

Ans. C
Sol. $V_{R}=15 \mathrm{~V}$ size of $A D C=8$ bit ( $\mathrm{D}_{0}$ to $D_{7}$ )
$\mathrm{V}_{\mathrm{D} 7}=7.5 \mathrm{~V}, \mathrm{~V}_{\mathrm{D} 6}=3.75 \mathrm{~V}, \mathrm{~V}_{\mathrm{D} 5}=$ $1.875 \mathrm{~V}, \mathrm{~V}_{\mathrm{D} 4}=0.9375 \mathrm{~V}$

$\therefore$ Find output at the end of 4th clock is 01100000
The output at the starting of 5th clock is 01101000
32. The input impedance of shortcircuited transmission line of length $\frac{\lambda}{2}$
A. $\mathrm{jz} \mathrm{o}_{0}$
B. $\infty$
C. Zero
D. -jzo

Ans. C
Sol. For short circuited line
$Z_{\text {in }}=j Z_{0} \tan \beta 1$

$$
\begin{aligned}
& =j Z_{0} \tan \frac{2 \pi}{\lambda} \cdot \frac{\lambda}{2} \\
& =0
\end{aligned}
$$

33. A pulse $\mathrm{X}(\mathrm{t})=\operatorname{Arect}\left(\frac{t}{2 \tau}\right)\left[1-\left(\frac{t}{\tau}\right)^{2}\right]$

Where $\mathrm{A}>0, \mathrm{~T}>0$ are constants, is added to white noise. The impulse response matched to $X(\mathrm{t})$ and output signal to noise ratio (SNR) respectively are.
A.

$(S N R)_{0}=\frac{8 A^{2} \tau}{3 N_{0}}$
B.

$(S N R)_{0}=\frac{8 A^{2} \tau}{3 N_{0}}$
C.

$(S N R)_{0}=\frac{52 A^{2} \tau}{15 N_{0}}$
D.

$(S N R)_{0}=\frac{52 A^{2} \tau}{15 N_{0}}$
Ans. D

Sol. Maximum signal to Noise ratio of matched filter is $(S N R)_{0}=\frac{2 E}{N_{0}}$
$E=$ Energy of the signal
$X(t)=$ Arect $\left[\frac{t}{2 \tau}\right]\left[1-\left(\frac{t}{2 \tau}\right)^{2}\right]$
Energy of $X(t)$ is
$E=2 \int_{0}^{\tau}\left[A\left[1-\left(\frac{t}{\tau}\right)^{2}\right]\right]^{2} d t$
$E=2 A^{2} \int_{0}^{\tau}\left[1-\frac{t^{4}}{\tau^{4}}-\frac{2 t^{2}}{\tau^{2}}\right] d t=2 A^{2}\left[\tau+\frac{\tau}{5}-\frac{2 \tau}{3}\right]$
$E=2 A^{2}\left[\frac{15 \tau+3 \tau-5 \tau}{15}\right]$
$E=\frac{2 A^{2} \times 13 \tau}{15}=\frac{26 A^{2} \tau}{15}$
$(S N R)_{0}=\frac{2 E}{N_{0}}=\frac{52 A^{2} \tau}{15 N_{0}}$

34. $X$ and $Y$ are two continuous random variable with joint distribution:
$f(x, y)= \begin{cases}c x+1, & y \geq 0 \& x+y<1 \\ 0, & \text { otherwise }\end{cases}$
The value of the constant c is:
Sol. For a joint probability distribution, we must have:
$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) d x d y=1$
$\int_{0}^{1} \int_{0}^{1-x}(c x+1) d x d y=1$
$\int_{0}^{1}(c x+1)(1-x) d x=1$
giving $\frac{1}{2}+\frac{c}{6}=1$
Thus: $c=3$
35. A bipolar transistor has $\mathrm{I}_{\text {CBO }}=0.5$ mA , common base current gain $\mathrm{a}=$ 0.98 and emitter current $\mathrm{I}_{\mathrm{E}}=2 \mathrm{~mA}$. Then the trans conductance of the transistor is $\qquad$ m J .
(Assume $\mathrm{V}_{\mathrm{T}}=26 \mathrm{mV}$ and $\eta=1$ )
Sol. $\mathrm{I}_{\mathrm{E}}=2 \mathrm{~mA}$
Common base current gain,
$a=0.98$
$\mathrm{I}_{\text {сво }}=0.5 \mathrm{~mA}$
Collector current, $\mathrm{I}_{\mathrm{C}}=\mathrm{a} \mathrm{I}_{\mathrm{E}}+\mathrm{I}_{\mathrm{CBO}}$
$=0.98\left(2 \times 10^{-3}\right)+0.5 \times 10^{-3}$
$=2.46 \mathrm{~mA}$
Transconductance of the transistor,
$\mathrm{q}_{\mathrm{m}}=\frac{\mathrm{I}_{\mathrm{C}}}{\mathrm{V}_{\mathrm{T}}}=\frac{2.46 \times 10^{-3}}{26 \times 10^{-3}}$
$=94.62 \mathrm{mv}$
36. The Fourier series coefficient for the periodic signal below is

A. $\frac{A}{2 \pi n}\left(e^{-j\left(\frac{4 \pi n}{3}\right)}-1\right)$
B. $j \frac{A}{2 \pi n}\left(e^{-j\left(\frac{4 \pi n}{3}\right)}-1\right)$
C. $-j \frac{A}{2 \pi n}\left(e^{-j\left(\frac{4 \pi n}{3}\right)}-1\right)$
D. $\frac{-\mathrm{A}}{2 \pi n}\left(\mathrm{e}^{-\mathrm{j}\left(\frac{4 \pi n}{3}\right)}-1\right)$

Ans. B
Sol. We have $T=2 \pi$ and $\omega_{0}=\frac{2 \pi}{2 \pi}=1$, $x(t)= \begin{cases}A, & 0<t<\frac{4 \pi}{3} \\ 0, & \frac{4 \pi}{3}<t<2 \pi\end{cases}$
$c_{n}=\frac{1}{T} \int_{0}^{T} x(t) e^{-j \omega_{0} n t} d t$
$=\frac{1}{2 \pi} \int_{0}^{2 \pi} x(t) e^{-j n t} d t$
$=\frac{1}{2 \pi} \int_{0}^{4 \pi / 3} A e^{-j n t} d t=\frac{j A}{2 \pi n}\left[e^{-j\left(\frac{4 \pi n}{3}\right)}-1\right]$
37. In the circuit shown in figure below, what is the value of $\mathrm{R}_{\mathrm{L}}$ such that maximum power is transferred to the load?

A. $20 \Omega$
B. $18 \Omega$
C. $25 \Omega$
D. $16 \Omega$

Ans.
Sol. For maximum power to be transferred to the load, $R_{\mathrm{L}}=\mathrm{R}_{\mathrm{Th}}$


For calculating $\mathrm{R}_{\mathrm{Th}}$, independent sources are replaced by their internal resistance.
Therefore, the circuit reduces to


Connect the 1 V source across the ab terminals,

$\mathrm{R}_{\mathrm{Th}}=\mathrm{V} / \mathrm{I}^{\prime}=1 / \mathrm{I}^{\prime}$
Applying KVL in the loop.
$1-10 \mathrm{I}-10 \mathrm{I}=0$
$1-20 \mathrm{I}=0$
$\mathrm{I}=1 / 20=0.05 \mathrm{~A}$

Voltage across $10 \Omega$ resistor $=10 \mathrm{I}=$ 0.5 V .

Current through $40 \Omega$ resistor $=$ $0.5 / 40=0.0125$ Amp.
$\mathrm{I}^{\prime}=$ current through $10 \Omega$ resistor + current through $40 \Omega$ resistor
$\mathrm{I}^{\prime}=0.05+0.0125$
$I^{\prime}=0.0625 \mathrm{Amp}$
$\mathrm{R}_{\text {Th }}=1 / 0.0625=16 \Omega$
$R_{\text {Th }}=16 \Omega$
38. In a silicon $\mathrm{p}-\mathrm{n}$ junction, the variation of the hole concentration in the space charge region is shown in the figure below, where the hole concentration changes from $P_{p o}$ to $P_{n o}$ aver the space charge layer width


The hole diffusion constant in the space charge region is $6.5 \mathrm{~cm}^{2} / \mathrm{sec}$. Then the hole diffusion current density $J_{p}$ is $\qquad$ kA/cm ${ }^{2}$.
Sol. Depletion width in the p -side, $\mathrm{x}_{\mathrm{p}}=1$ $\mu \mathrm{m}$
Depletion width in the $n$-side, $x_{n}=$ $2^{\mu \mathrm{m}}$
Total depletion width, $\mathrm{W}=\mathrm{x}_{\mathrm{n}}+\mathrm{x}_{\mathrm{p}}=$ $3^{\mu m}$
The diffusion constant of holes, $D_{p}=$ $6.5 \mathrm{~cm}^{2} / \mathrm{sec}$
The diffusion current density of holes
is $J_{p}=-q D_{p} \frac{d p}{d x}$
The concentration gradient across the space charge region is $\frac{d p}{d x}$
$\frac{d p}{d x}=-\left[\frac{P_{p o}-P_{n o}}{W}\right]=-\left[\frac{10^{18}-2.25 \times 10^{15}}{3 \times 10^{-4}}\right]$
$\frac{d p}{d x}=-3.33 \times 10^{21} \mathrm{~cm}^{-4}$
The diffusion current density of holes $J_{p}$ is
$J_{p}=-q D_{p} \frac{d p}{d x}$
$=-1.6 \times 10^{-19} \times 6.5 \times(-3.33 \times$
$10^{21}$ )
$\mathrm{J}_{\mathrm{P}}=3.46 \mathrm{kA} / \mathrm{cm}^{2}$
39. Considered two signals, $X(n)=\{1,2$, $4\}, h(n)=\{1,1,1,1\}$ also $Y(n)=$ $X(n) * h(n)$.
Then the value of $\sum_{k=1}^{3} Y(k)$ ?
Sol.
(n(n)
$X(n)=\{1,2,4\}$
$h(n)=\{1,1,1,1\}$
$Y(\mathrm{n})=\mathrm{X}(\mathrm{n}) * \mathrm{~h}(\mathrm{n})=\{1,3,7,7,6,4\}$
$\sum_{K=1}^{3} Y(K)=7+7+6=20$
40. An analog signal of bandwidth 20 kHz is sampled at a rate of 40 kHz , and quantized into 16 levels. The resultant digital signal is transmitted using M-ary PSK with raised cosine pulse(with $a=0.3$ ). A channel with a 110 kHz bandwidth is available to transmit the data. Then find the smallest acceptable value of number of phase angles.
A. 3
B. 4
C. 2
D. none of the above

Ans. B
Sol. 16 levels $=2^{4}$ levels
Therefore, $\mathrm{n}=4$ bits/sample
Bit rate
$=40000 \frac{\text { samples }}{\text { sec }} X \frac{4 \text { bits }}{\text { sample }}=160 \mathrm{kbps}$
For M-ary PSK signalling the pulse rate would be
$\frac{R \mathrm{bits} / \mathrm{sec}}{\log _{2} M \text { bits } / \mathrm{symbol}}=\frac{R}{\log _{2} M}$ symbols $/ \mathrm{sec}$
$2 f_{B}=\frac{(1+\alpha) \cdot R}{\log _{2} M} \leq 110 \mathrm{kHz}$
$\log _{2} M \geq \frac{1.3 \times 160}{110}=1.89$
$\therefore$ The smallest acceptable value of M is 4
41. A unity-feedback servo-driven instrument has an open loop transfer function given as $\frac{K}{s(s+2)}$. It is given that the peak overshoot of the system on step input is $35.1 \%$. Find the steady state error to an input (1 + 4t) $u(t)$.
A. 0.70
B. 0.60
C. 0.80
D. 0.50

Ans. C
Sol. Consider, $G(s)=\frac{K}{s(s+2)}$


Closed loop transfer function is given by,
$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)}=\frac{K}{s^{2}+2 s+K}$
Comparing it this with the general second order transfer function
$T(s)=\frac{\omega_{n}^{2}}{s^{2}+2 \zeta \omega_{n} s+\omega_{n}^{2}}$
We get, $\omega_{n}=\sqrt{K}$ and
$2 \zeta \omega_{n}=2 \Rightarrow \zeta=\frac{1}{\sqrt{K}}$
Peak overshoot is given by,
$M_{p}=e^{-\pi \frac{\zeta}{\sqrt{1-\zeta^{2}}}}=0.351$
$\Rightarrow-\pi \sqrt{\frac{1}{K-1}}=\ln 0.351$
$\Rightarrow K=1+\left(\frac{\pi}{\ln 0.351}\right)^{2} \cong 10$
Since, its an LTI system thus error due to sum of input is equal to sum of errors due to single inputs.
Error due to $u(t)$ is:
$K_{p}=\lim _{s \rightarrow 0} G(s)=\infty \Rightarrow e_{s s}=\frac{1}{1+K_{p}}=0$
Error due to $4 \mathrm{tu}(\mathrm{t})$ is:
$K_{v}=\lim _{s \rightarrow 0} s G(s)=\lim _{s \rightarrow 0} \frac{10}{s+2}=5 \Rightarrow e_{s s}=\frac{4}{K_{v}}=\frac{4}{5}=0.8$
Total steady state error is $E_{s s}=0.8$
42. Find the Laplace transform of the function $f(t)$ given as
$f(t)=(t-2)^{2}$
A. $\frac{4}{s}-\frac{4}{s^{2}}+\frac{2}{s^{3}}, s>0$
B. $\frac{4}{s}-\frac{4}{s^{2}}+\frac{4}{s^{3}}, s>0$
C. $\frac{4}{s}-\frac{2}{s^{2}}+\frac{2}{s^{3}}, s>0$
D. $\frac{2}{s}-\frac{4}{s^{2}}+\frac{2}{s^{3}}, s>0$

Ans. A
Sol. Given $L\left[(t-2)^{2}\right]$
$=\lim _{T \rightarrow \infty} \int_{0}^{T}(t-2)^{2} e^{-s t} d t$
Using integration by parts with $\mathrm{u}^{\prime}$
$=e^{-s t}$ and $\mathrm{V}=(t-2)^{2}$ we will
find,
$\int_{0}^{T}(t-2)^{2} e^{-s t} d t=-\left[\frac{(t-2)^{2} e^{-s t}}{s}\right]_{0}^{T}+$
$\frac{2}{s} \int_{0}^{T}(t-2) e^{-s t} d t$
$=\frac{4}{s}-\frac{(T-2)^{2} e^{-s T}}{s}+\frac{2}{s} \int_{0}^{T}(t-2) e^{-s t} d t$
thus,
$\lim _{T \rightarrow \infty} \int_{0}^{T}(t-2)^{2} e^{-s t} d t=\frac{4}{s}+\frac{2}{s} \lim _{T \rightarrow \infty} \int_{0}^{T}(t-2) e^{-s t} d t$
Using by parts with $u^{\prime}=e^{-s t}$ and $v$ $=t-2$ we find
$\int_{0}^{T}(t-2) e^{-s t} d t=\left[-\frac{(t-2) e^{-s t}}{s}-\frac{1}{s^{2}} e^{-s t}\right]_{0}^{T}$
Let $\mathrm{T} \rightarrow \infty$ in the above expression
we will get
$\operatorname{Lim}_{T \rightarrow \infty} \int_{0}^{T}(t-2) e^{-s t} d t=-\frac{2}{s}+\frac{1}{s^{2}}, s>0$
Hence,
$F(s)=\frac{4}{s}+\frac{2}{s}\left(-\frac{2}{s}+\frac{1}{s^{2}}\right)=\frac{4}{s}-\frac{4}{s^{2}}+\frac{2}{s^{3}}$ , $s>0$
43. An LTI system has the impulse response shown below.


If the system is excited by an input $x(t)=\delta(t-1)+\delta(t-3)$, then output $y(t)$ will be
A.

B.

C.

D.


Ans. D
Sol. Output of the system is

$$
\begin{aligned}
& \mathrm{Y}(\mathrm{t})=\mathrm{h}(\mathrm{t}) * \mathrm{x}(\mathrm{t}) \\
& =\mathrm{h}(\mathrm{t}) *[\delta(\mathrm{t}-1)+\delta(\mathrm{t}-3)] \\
& =\mathrm{h}(\mathrm{t}) * \delta(\mathrm{t}-1)+\mathrm{h}(\mathrm{t}) * \delta(\mathrm{t}-3) \\
& =\mathrm{h}(\mathrm{t}-1)+\mathrm{h}(\mathrm{t}-3)
\end{aligned}
$$

As we know that, $x(t) * \delta\left(t-t_{0}\right)=x$ ( $\mathrm{t}-\mathrm{t}_{0}$ )

Signal $h(t-1)$ and $h(t-3)$ are obtained by shifting $h(t)$ to the right by 1 unit and 3 unit respectively as shown below



Now, adding above two we get the output

44. Calculate the time constant, T (in sec) of the circuit shown below?


Sol. Calculation of open circuit voltage across capacitor,

$V_{O C}=2 i_{x}=2\left(\frac{10-V_{O C}}{2}\right)$
$2 \mathrm{~V}_{\text {oc }}=10$
$\mathrm{V}_{\mathrm{oc}}=5 \mathrm{~V}$
Calculation of $\mathrm{I}_{\mathrm{sc}}$ :


$$
\begin{equation*}
I_{S C}=\frac{2 i_{x}}{4}=\frac{i_{x}}{2} \tag{1}
\end{equation*}
$$

writing KVL in outer loop, $10-2 \mathrm{i}_{\mathrm{x}}-4 \mathrm{I}_{\mathrm{sc}}=0$

## ESE 2020 Prelims | Live Analysis Join us on 5 Jan @ 05:30 PM

By putting the value of $i_{x}$ from
equation (1)
$10-2 \times 2 \mathrm{I}_{\mathrm{sc}}-4 \mathrm{I}_{\mathrm{sc}}=0$
$I_{S C}=\frac{10}{8}=1.25$
Resistance seen by capacitance is
$\mathrm{R}_{\mathrm{Th}}=\frac{\mathrm{V}_{\mathrm{oc}}}{\mathrm{I}_{\mathrm{SC}}}=\frac{5}{1.25}=4 \Omega$
Time constant, $\mathrm{T}=\mathrm{R}_{\mathrm{Th}} \times \mathrm{C}=4 \times 1$
$=4 \mathrm{sec}$
45. Find the output voltage of the following circuit assume the ideal opamp behavior $\qquad$ V .


Sol.


By virtual short circuit $\mathrm{V}_{\mathrm{c}}=1 \mathrm{~V}$
$v_{A}=V_{o} \times \frac{1}{1+7}=\frac{V_{o}}{8}$
(1)

Applying KCL at node $A$
$\frac{V_{A}-1}{1 k \Omega}+\frac{V_{A}-V_{B}}{2 k \Omega}=0$
$\mathrm{V}_{\mathrm{B}}=3 \mathrm{~V}_{\mathrm{A}}-2$
From eq (1)
$v_{B}=\frac{3 V_{o}}{8}-2$
Applying KCL at node C
$\frac{V_{C}-V_{B}}{4}+\frac{V_{C}-V_{o}}{8}=0$
$3 V_{C}-2 V_{B}-V_{0}=0$
$\mathrm{V}_{\mathrm{C}}=1 \mathrm{~V}$
$V_{B}=\frac{3 V_{o}}{8}-2 \quad$ (Fromeq (2))
$3 \times 1-2\left[\frac{3 V_{o}}{8}-2\right]-V_{o}=0$
$V_{o}=4 \mathrm{~V}$
46. The value of integral $I=\frac{i}{\pi_{f}} \oint \frac{\cos \pi z}{z^{2}-1}$, where $C$ is the square with vertices at $\pm i, 2 \pm i$ is $\qquad$
A. -1
B. -2
C. -3
D. -4

Ans. A
Sol. $\frac{1}{2 \pi_{f}} \oint \frac{\cos \pi z}{(z-1)(z+1)} d z=$ sum of
Residue
$\frac{1}{\pi_{f}} \oint \frac{\cos \pi z}{(z+1)} \times \frac{1}{(z-1)} d z=2 \times$ sum of Re sidue
At $Z=1$, Residue of
$f(z)=\lim _{z \rightarrow 1} \frac{\cos \pi z}{z+1}=\frac{\cos \pi}{2}=-0.5$
$I=2 \times$ Residue $=2 \times(0.5)$
$=-1$

47. An enhancement type NMOS transistor has threshold voltage of 0.8 V , process transconductance parameter $\mu_{n} C_{o x}=20 \mu \mathrm{~A} / \mathrm{V}^{2}$ and channel length modulation parameter $\lambda=\mathrm{O}, \mathrm{V}_{\mathrm{G}}=2.8 \mathrm{~V}$, drain Voltage ( $\mathrm{V}_{\mathrm{D}}$ ) $=5 \mathrm{~V}$, Source Voltage $\mathrm{V}_{\mathrm{S}}=1 \mathrm{~V}$ and drain current $I_{D}=0.24 \mathrm{~m}$. Then the ratio (W/L) is
A. 10
B. 14
C. 24
D. 34

Ans. C
Sol. Given, threshold voltage,
$\mathrm{V}_{\mathrm{T}}=0.8 \mathrm{~V}$
Process trans conductance parameter,
$\mu_{n} C_{0 x}=20 \mu \mathrm{~A} / \mathrm{V}^{2}$
Gate Voltage, $\mathrm{V}_{\mathrm{G}}=2.8 \mathrm{~V}$
Drain voltage, $\mathrm{V}_{\mathrm{D}}=5 \mathrm{~V}$
Source voltage, $\mathrm{V}_{\mathrm{S}}=1 \mathrm{~V}$
Drain current, $\mathrm{I}_{\mathrm{D}}=0.24 \mathrm{~mA}$
$V_{G S}=V_{G}-V_{S}=2.8-1=1.8 \mathrm{~V}$
$V_{D S}=V_{D}-V_{S}=5-1=4 V$
$V_{G S}-V_{T}=1.8-0.8=1 \mathrm{~V}$
Clearly, $\mathrm{V}_{\mathrm{DS}}>\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{T}}$
Hence, the MOSFET in saturation mode
$I_{D}=\frac{1}{2} \mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}} \frac{\mathrm{W}}{\mathrm{L}}\left(\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{T}}\right)^{2}$
$0.24 \times 10^{-3}=\frac{1}{2} \times 20 \times 10^{-6 \times}$
$\frac{\mathrm{W}}{\mathrm{L}}(1)^{2}$
$\frac{W}{L}=24$
48. The magnetic field intensity in a region is $\vec{H}=2 x \widehat{a}_{x}-3 y \widehat{a}_{y}+5 z \widehat{a}_{z}$, the current density in $A / m^{2}$ at a point $(1,2,3)$ is
Sol. Given that:
$\vec{H}=2 x \widehat{a}_{x}-3 y \widehat{a}_{y}+5 z \widehat{a}_{z}$
From Maxwell's equation, $\nabla \times \vec{H}=\vec{J}$

49. The loop transfer function of the system $^{G(s) H(s)}=\frac{k}{s(s+18)\left(s^{2}+18 s+81\right)}$.
The root locus plot of the system has
A. Three real breakaway points
B. No breakaway point
C. One real and one complex breakaway point
D. Only one breakaway point

Ans. A
Sol. $1+G(s) H(s)=0$
$=>1+\frac{k}{s(s+18)\left(s^{2}+18 s+81\right)}=0$
$\Rightarrow s(s+18)\left(s^{2}+18 s+81\right)+k=0$
$=>k=-s(s+18)\left(s^{2}+18 s+81\right)$

For breakaway points, $\frac{d k}{d s}=0$
$=>$
$s(s+18)(2 s+18)+s\left(s^{2}+18 s+81\right)+(s+18)\left(s^{2}+18 s+81\right)=0$
$=>$
$(2 s+18)\left(s^{2}+18 s\right)+\left(s^{2}+18 s+81\right)(s+s+18)=0$
$=>$
$(2 s+18)\left(s^{2}+18 s\right)+\left(s^{2}+18 s+81\right)(2 s+18)=0$
$=>(2 s+18)\left(2 s^{2}+36 s+81\right)=0$
Therefore, by solving we get, S=-9, -2.64, -15.37
So, there are three real breakaway points.
50. Consider the NMOS transistor as shown in figure. The MOSFET has parameters $\frac{\mu_{n} C_{o x} W}{2 L}=0.5 \mathrm{~mA} / V^{2}$
, $\mathrm{V}_{\mathrm{T}}=2 \mathrm{~V}$ and $\lambda=0$. The transistor is used to amplify the small signal $\mathrm{V}_{\text {in }}$ as shown in the figure. If the value of signal $\mathrm{V}_{\text {in }}=3 \sin (w t) \mathrm{mV}$, then the value of output signal $\mathrm{V}_{0}(\mathrm{t})$ is equal to :-

A. $-30 \sin (\omega t) V$
B. $-15 \cos (\omega t) V$
C. $-15 \cos (\omega t) \mathrm{mV}$
D. $-30 \sin (\omega t) \mathrm{mV}$

Ans. D
Sol. First applying the D.C. analysis, we have


Now, assuming MOS to be in
saturation region
$I_{D}=\frac{\mu_{n} C_{o x} W}{2 L}\left(V_{G S}-V_{T}\right)^{2}=0.5 \times 10^{-3}(3-2)^{2}$
$\mathrm{I}_{\mathrm{D}}=0.5 \times 10^{-3} \mathrm{~mA}$
$V_{D S}=10-10 \times 10^{+3} \times 0.5 \times 10^{-3}=$
$10-5=5 \mathrm{~V}$
$\mathrm{V}_{\mathrm{DS}}>\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{T}}$
5>3-2
Hence, our assumption was true.
Now, $g_{m}=2 \sqrt{\frac{\mu_{n} C_{o x} W}{2 L} . I_{D}}$
$=2 \sqrt{0.5 \times 0.5 \times 10^{-6}}$
$\mathrm{g}_{\mathrm{m}}=1 \mathrm{~mA} / \mathrm{V}$
Now, drawing the small signal
equivalent circuit, we get

$V_{0}=-g_{\mathrm{m}} \mathrm{V}_{\mathrm{gs}} \mathrm{R}_{\mathrm{D}}$
$V_{0}=-\left(g_{m} R_{D}\right) V_{\text {in }}=-\left[1 \times 10^{-3} \times 10 \times\right.$
$\left.10^{3}\right] \times 3 \sin \omega t \times 10^{-3}$
$V_{0}=-30 \sin \omega t \mathrm{mV}$
51. The given integral
$\int_{0}^{\frac{\pi}{k}} \int_{x}^{\frac{\pi}{k}} \frac{\sin y}{y} d y d x$ evaluates to $\frac{1}{2}$
for some $k \geq 1$. Then the value of $k$ is:
Sol. Changing the order of integration we get:

$$
\begin{aligned}
& \int_{0}^{\frac{\pi}{k}} \int_{0}^{y} \frac{\sin y}{y} d x d y \\
& =\int_{0}^{\frac{\pi}{k}} \frac{\sin y}{y} X[x]_{0}^{y} d y \\
& =\int_{0}^{\frac{\pi}{k}} \sin y d y \\
& =[-\cos y]_{0}^{\frac{\pi}{k}}=-\cos \frac{\pi}{k}+1 \\
& -\cos \frac{\pi}{k}+1=\frac{1}{2} \\
& \text { giving } \frac{\pi}{k}=\cos ^{-1} 0.5 \\
& \frac{\pi}{k}=\frac{\pi}{3} \\
& \text { thus, } k=3
\end{aligned}
$$

52. A binary PAM communication system employs rectangular pulse of duration $T_{b}$ and amplitudes $\pm A$ to transmit digital transmission information at a rate $R_{b}=10^{5} \mathrm{bps}$. If power-spectral density of AWGN is $N_{0} / 2$ where $N_{0}=$ $10^{-2} \mathrm{~W} / \mathrm{Hz}$, determine the value of A that is required to achieve $a$ probability of error $\mathrm{P}_{\mathrm{e}}=10^{-6}$
Assume $\mathrm{Q}(4.5) \approx 10^{-6}$
A. 100.62 V
B. 200.62 V
C. 300.62 V
D. 400.62 V

## Ans. A

Sol. $\mathrm{R}_{\mathrm{b}}=10^{5} \mathrm{bps}, \mathrm{N}_{\mathrm{o}}=10^{-2}(\mathrm{~W} / \mathrm{Hz}) \mathrm{Pe}=$ $10^{-6}$
$P e=Q\left(\sqrt{\frac{E d}{2 N_{o}}}\right)$


$\mathrm{S}_{1}(\mathrm{t})-\mathrm{S}_{2}(\mathrm{t})=2 \mathrm{~A}, 0<\mathrm{t}<\mathrm{Tb}$
$E_{d}=\int_{0}^{T b} 4 A^{2} d t=4 A^{2} T_{b}=\frac{4 A^{2}}{R_{b}}$
$P_{e}=Q\left(\sqrt{\frac{4 \mathrm{~A}^{2}}{10^{5} \times 2 \times 10^{-2}}}\right)=10^{-6}$
$Q\left(\sqrt{\frac{A^{2}}{500}}\right)=10^{-6}$
Using given data $\sqrt{\frac{\mathrm{A}^{2}}{500}}=4.5$
$\Rightarrow A=100.62 \mathrm{~V}$
53. The cut off wavelength of wave guide is given as 80 mm . find the length (in km ) of waveguide to ensure signal of 5 GHz emerging out of waveguide is delayed by $10 \mu$ sec with respect to the signal that is propogating in free space outside the waveguide is
$\qquad$ km .

Sol. $f c=\frac{C}{\lambda}=\frac{3 \times 10^{10}}{80 \times 10^{-1}}=3.75 \times 10^{+9} \mathrm{~Hz}$
$=3.75 \mathrm{GHz}$
$T_{\text {delay }}=\frac{l}{V g}-\frac{l}{C}=l\left(\frac{1}{V g}-\frac{1}{C}\right)$
Where ' $I$ ' is the length of the waveguide and Vg is the group velocity of signal through the waveguide, $C$ is velocity of wave outside the waveguide.
$V g=c \sqrt{1-\left(\frac{f c}{f}\right)^{2}}=3 \times 10^{8} \sqrt{1-\left(\frac{3.75}{5}\right)^{2}}$
$\mathrm{V}_{\mathrm{g}}=1.98 \times 10^{8} \mathrm{~m} / \mathrm{s}$
$\therefore \mathrm{T}_{\text {delay }}=\mathrm{I}\left[\frac{1}{1.98 \times 10^{8}}-\frac{1}{3 \times 10^{8}}\right]$
$10 \times 10^{-6}=1\left[1.717 \times 10^{-9}\right]$
$l=5.823 \mathrm{~km}$
54. A memory is mapped to 8085 microprocessor. The memory map is B00F to FCDE H. The number of bytes stored in memory are $\qquad$ $)_{10}$.
Sol. Starting address $=$ B00F H Ending address $=$ FCDE H Total number of bytes $=$ Total number address
$=(\mathrm{FCDE}-\mathrm{BOOF}+1)_{\mathrm{H}}$
$=4 C C F+1=4 C D 0 H=(19664)_{10}$
55. The normalized radiation intensity of an antenna in spherical coordinates is given by
$U=\sin \theta \sin \varphi, 0 \leq \theta \leq п, 0 \leq \varphi \leq п$
The directivity of antenna is given by
$\qquad$ -.
Sol. $U=\sin \theta=\sin \phi$,
$|U|_{\max }=1$ for $\theta=\phi=\frac{\pi}{2}$
$P_{r a d}=\iint_{\theta} U \sin \theta d \theta d \phi$
$=\int_{\theta=0}^{\pi} \int_{\phi=0}^{\pi} \sin ^{2} \theta \sin \phi d \theta d \phi$
$=\int_{\theta=0}^{\pi} \sin ^{2} \theta d \theta \int_{\phi=0}^{\pi} \sin \phi d \phi=\left(\frac{\pi}{2}\right) 2=\pi$
$D=\frac{4 \pi U_{\max }}{P_{\mathrm{rad}}}=\frac{4 \pi .1}{\pi}=4$
56. In a lightly depend p-type semiconductor, the corresponding accepter doing concentration to get minimum conducting is $\ldots 10^{10} \mathrm{~cm}^{-3}$.
(Assume intrinsic carries concentration is
$\left.n_{i}=2.5 \quad 10^{10} / \mathrm{cm}^{3} \& m_{p}=0.4 m_{n}\right)$
Sol. $n_{i}=2.5 \times 10^{10} / \mathrm{cm}^{3}$
$\mu_{p}=0.4 \mu_{n}$
$\frac{\mu_{p}}{\mu_{n}}=0.4$
We know that electrical neutrality
condition $N_{D}+P=N_{A}+n$
for lightly doped p-type
semiconductor
$\mathrm{N}_{\mathrm{D}}=\mathrm{O}$
$\mathrm{N}_{\mathrm{A}}=\mathrm{P}-\mathrm{n}$
$N_{A}=P-\frac{n_{i}^{2}}{P}$
$N_{A}=\sqrt[n_{i}]{\sqrt{\mu_{n}}}-\frac{n_{i}^{2}}{n_{i} \sqrt{\frac{\mu_{n}}{\mu_{\rho}}}}$
$\left[\because\right.$ For minimum conductivity, $\left.\mathrm{p}=\mathrm{n}_{1} \sqrt{\frac{\mu_{n}}{\mu_{p}}}\right]$
$N_{A}=n_{i}\left[\sqrt{\frac{\mu_{n}}{\mu_{p}}}-\sqrt{\frac{\mu_{p}}{\mu_{n}}}\right]$
$=2.5 \times 10^{10}\left[\sqrt{2.5}-\sqrt{\frac{1}{2.5}}\right]$
$=2.372 \times 10^{10} / \mathrm{cm}^{3}$
57. The Nyquist plot for a stable open loop system is shown below:


For $K>\frac{3}{2}$, how many closed loop poles of a unity feedback system are in the right half of $s$-plane

Sol. Given that open loop system is stable, so we have no pole of open loop in right half of s-plane, i.e. $\mathrm{P}=$ 0

The intersection point of Nyquist plot with negative real axis
is $\left(-\frac{2}{3} K, 0\right)$ and $K>\frac{3}{2}$
Now, for the given range,
$K>\frac{3}{2}$
Or $\frac{2}{3} K>1$
Or $-\frac{2}{3} K>-1$
The intersection point will be in left of the critical point, so $(-1+j 0)$ point will lie in the small loop in Nyquist plot. Therefore, the Nyquist plot encircles the critical point $(-1+j 0)$ two times in clockwise direction, i.e. $\mathrm{N}=-2$
So, we obtain the number of closed loop poles in right half plane as
$N=P-Z$
Or -2 = $0-Z$
Or $Z=2$
Hence, the closed loop system has two poles in the right half of s-plane.
58. The switch $S$ in the circuit of figure has been closed for a long time and is opened at $t=0$. The current $\mathrm{i}(\mathrm{t})$ for t $>0$ is

A. $i(t)=(15+1.5 t) e^{-5 t} A$
B. $i(t)=1.5 \mathrm{e}^{-5 t} \mathrm{~A}$
C. $i(t)=7.5 \mathrm{e}^{-10 t} \mathrm{~A}$
D. $i(t)=(7.5+1.5 t) e^{-5 t} A$

Ans. B
Sol. For t < 0:

$\mathrm{i}\left(0^{-}\right)=\frac{10+5}{5+5}=\frac{15}{10}=1.5 \mathrm{~A}$
$-5+5 i\left(0^{-}\right)-\mathrm{V}_{\mathrm{C}}\left(\mathrm{O}^{-}\right)=0(\mathrm{KVL}$ in
the right sided mesh)
$V_{C}\left(0^{-}\right)=-5+5(1.5)=2.5 \mathrm{~V}$
For $t>0$, the circuit becomes as an RLC series network.


Characteristic equation,
$s^{2}+\frac{R}{L} s+\frac{1}{L C}=0$
$s^{2}+\frac{10}{1} s+\frac{1}{1 \times 40 \times 10^{-3}}=0 \quad(R=5+5=10 \Omega)$
$s^{2}+10 s+25=0$
Roots, $(s+5)^{2}=0 \Rightarrow s_{1}=-5$,
$\mathrm{S}_{2}=-5$
$\alpha=\frac{10}{2}=5, \omega_{0}=5$
$a=\omega_{0}$ (Critically damped)
So, $i(t)=(A+B t) e^{-a t}$
$i(t)=(A+B t) e^{-5 t}$
Now, we obtain $A$ and $B$ using initial conditions
$I\left(0^{-}\right)=i\left(0^{+}\right)=1.5=(A+0) e^{0}=A$
$\mathrm{A}=1.5$
At $t=0^{+}$,


## ESE 2020 Prelims | Live Analysis Join us on 5 Jan @ 05:30 PM

Writing KVL at $\mathrm{t}=0^{+}$
$-5+\mathrm{L} \frac{\mathrm{di}\left(0^{+}\right)}{\mathrm{dt}}+5 \mathrm{i}\left(0^{+}\right)-\mathrm{V}_{\mathrm{C}}\left(0^{+}\right)=0$
$-5+\mathrm{L} \frac{\mathrm{di}\left(0^{+}\right)}{\mathrm{dt}}+10 \times 1.5-2.5=0$
$L \frac{\mathrm{di}\left(0^{+}\right)}{\mathrm{dt}}=-7.5$
Differentiating equation (1)
$\frac{\mathrm{di}(\mathrm{t})}{\mathrm{dt}}=(\mathrm{A}+\mathrm{Bt})\left(-5 \mathrm{e}^{-5 \mathrm{t}}\right)+B \mathrm{e}^{-5 \mathrm{t}}$
$\frac{\mathrm{di}\left(0^{+}\right)}{\mathrm{dt}} \mathrm{A}(-5)+\mathrm{B}=-7.5$
$B=-7.5+5(A)=-7.5+5(1.5)=$ 0

So $i(t)=1.5 e^{-5 t} \mathrm{~A}$
59. The solution curve of the differential equation $x \frac{d y}{d x}=y+2 x^{3}$ passes through the point $(1,0)$. Then among the points given below, the curve also passes through:
A. $(-1,0)$
B. $(0,-1)$
C. $(2,10)$
D. $(-2,6)$

Ans. A
Sol. The differential equation can be written as:
$\frac{d y}{d x}=\frac{y}{x}+2 x^{2}$
$\frac{d y}{d x}-\frac{y}{x}=2 x^{2}$
which is a linear equation in ' $y$ '
$I F=e^{-\int \frac{1}{x} d x}=e^{-\log x}=-\frac{1}{x}$
Thus, the solution of the equation will be:
$y x I F=\int 2 x^{2} x I F d x$
$-\frac{y}{x}=-\int 2 x d x$
$\frac{y}{x}=x^{2}+C$
Putting $(1,0)$ we get $C=-1$
Thus,
$\frac{y}{x}=x^{2}-1$
Clearly only ( $-1,0$ ) satisfies the above equation.
60. Consider the channel
$\mathrm{P}(\mathrm{Y} / \mathrm{X})=\left[\begin{array}{lll}0.6 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6\end{array}\right]$
$\mathrm{P}\left(\mathrm{X}_{1}\right)=1 / 8 \mathrm{P}\left(\mathrm{X}_{2}\right)=1 / 8 \mathrm{P}\left(\mathrm{X}_{3}\right)=6 / 8$
Calculate information carried by the channel.
Sol. We have
$P(X, Y)=\left[\begin{array}{ccc}0.6 / 8 & 0.2 / 8 & 0.2 / 8 \\ 0.2 / 8 & 0.6 / 8 & 0.2 / 8 \\ 0.2 \times 6 / 8 & 0.2 \times 6 / 8 & 0.6 \times 6 / 8\end{array}\right]$
Now $H=\sum_{k=1}^{N} P_{k} \log _{2}\left(\frac{1}{P_{k}}\right)$ bits/message
$H(Y)=\frac{1}{4} \log _{2} 4+\frac{1}{4} \log _{2} 4+\frac{1}{2} \log _{2} 2$
$=\frac{3}{2}$
$H(X, Y)=4\left(\frac{0.2}{8} \log _{2} \frac{8}{0.2}\right)+2\left(\frac{0.6}{8} \log _{2} \frac{8}{0.6}\right)+2\left(\frac{0.2 \times 6}{8} \log _{2} \frac{8}{1.2}\right)$
$+\frac{0.6 \times 6}{8} \log _{2} \frac{8}{3.6}$
$=2.432$
$H(X)=\frac{1}{8} \log _{2} 8+\frac{1}{8} \log _{2} 8+\frac{6}{8} \log _{2} \frac{8}{6}$
$=1.0613$
Now mutual information is given by
$I(X, Y)=H(X)+H(Y)-H(X, Y)$
$=1.0613+1.5-2.432$
$=0.1293$
61. The characteristic equation of the system is
C. $E=s^{5}-2 s^{4}-2 s^{3}+4 s^{2}+s-2=0$

The number of symmetric poles located in right half of s-plane is
A. 3
B. 1
C. 4
D. 2

Ans.
Sol. C.E $=s^{5}-2 s^{4}-2 s^{3}+4 s^{2}+s-2=$ 0

| $S^{5}$ | 1 | -2 | 1 |
| :--- | :---: | :---: | :---: |
| $S^{4}$ | -2 | 4 | -2 |
| $S^{3}$ | $0(-1)$ | $0(1)$ | 0 |
| $S^{2}$ | +2 | -2 | 0 |
| $S^{1}$ | $0(4)$ | 0 | 0 |
| $S^{0}$ | -2 |  |  |

$A E_{1}=-2 s^{4}+4 s^{2}-2=0$
$\frac{d}{d s} A E_{1}=-8 s^{3}+8 s \Rightarrow$ take 8 common $\Rightarrow-1,1$
$A E_{2}=2 s^{2}-2=0$
$\frac{d}{d s} A E_{2}=4 s$
3 signs changes in the first columns
$\Rightarrow A E_{1}=-2 s^{4}+4 s^{2}-2=0$
Or, $s^{4}-2 s^{2}+1=0$
Or, $\left(s^{2}-1\right)^{2}=0$

$\therefore \Rightarrow$ There are two symmetric poles are located in RHP.
$\Rightarrow 1$ non symmetric pole in RHP.
Total 3 poles in RHP.
62. All queens and kings are removed from a deck of playing cards. Ace will be considered as 1 and jack will be considered as 0 . You took out 4 cards. The probability that all cards will be in order (order is 1234,0123,2345,6789....) from the same deck is M . What is $\left(\mathrm{M} \times 10^{5}\right)$ ?
A. 0.982
B. 0.700
C. 0.643
D. 0.500

Ans. A
Sol. Total cards $=52-4-4=44$
(cards)
$\rightarrow$ You took out 4 cards
$\therefore$ (1) Let order be $0,1,2,3$
So, $\quad P($ order $-0,1,2,3)=\frac{4}{44} \times \frac{1}{43} \times \frac{1}{42} \times \frac{1}{41}=x$
$\therefore$ Lost possible set $-(7,8,9,10)-8$ such sets are possible
$\therefore \mathrm{P}($ Total $)=8 \times \mathrm{P}($ order $-0,1,2$,
3) $=8 x$
$\Rightarrow 8 \mathrm{x} \times 10^{5}=0.982$
63. Consider the counter circuit shown in figure below:


Then the modulus of the given counter is $\qquad$ .(Assume $Q_{3}$ be MSB \& $Q_{0}$ be LSB)
Sol. Let the initial contents of the flipflops $\left(\mathrm{Q}_{3} \mathrm{Q}_{2} \mathrm{Q}_{1} \mathrm{Q}_{0}\right)$ be 0000
To reset the counter, output Y must be equal to 1 .
The expression for the output $Y$ is given as

$$
\begin{aligned}
\mathrm{Y} & =\overline{\overline{\mathrm{Q}_{1} \mathrm{O}_{3}}} \cdot \overline{\mathrm{Q}_{2} \mathrm{Q}_{3}} \\
& =\overline{\mathrm{Q}_{1} \mathrm{Q}_{3}}+\overline{\overline{\mathrm{Q}_{2} \mathrm{Q}_{3}}}=\mathrm{Q}_{1} \mathrm{Q}_{3}+\mathrm{Q}_{2} \mathrm{Q}_{3} \\
& =\mathrm{Q}_{3}\left(\mathrm{Q}_{1}+\mathrm{Q}_{2}\right)
\end{aligned}
$$

So, from the table below,

| S.No. | $\mathbf{Q}_{\mathbf{3}}$ | $\mathbf{Q}_{\mathbf{2}}$ | $\mathbf{Q}_{\mathbf{1}}$ | $\mathbf{Q}_{\mathbf{0}}$ | $\mathbf{Y}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 1 | 1 | 0 |
| 4 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 | 0 |
| 6 | 0 | 1 | 1 | 0 | 0 |
| 7 | 0 | 1 | 1 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 | 0 |
| 10 | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |

Now, the output $Y$ becomes 1, hence the combination $\left(\mathrm{Q}_{3} \mathrm{Q}_{2} \mathrm{Q}_{1} \mathrm{Q}_{0}\right)$ becomes 1010.

The decimal equivalent of 1010 is 10 . Hence the modulus of the given counter is 10 .
64. A uniform plane wave with $\mathrm{E}=a_{x} E_{x}$ propagates in a lossless medium $\left(\varepsilon_{r}=4, \mu_{r}=1, \sigma=0\right)$ in the +z direction. Consider $E_{x}$ is sinusoidal with a frequency $100(\mathrm{MHz})$ and has a maximum value of $+10^{-4}$ $(\mathrm{V} / \mathrm{m})$ at $\mathrm{t}=0$ and $\mathrm{z}=1 / 8 \mathrm{~m}$. Instantaneous expression for H is
$\qquad$ .
A.
$\mathrm{H}(\mathrm{z}, \mathrm{t})=a_{x} \frac{10^{-4}}{60 \pi} \cos \left(2 \pi 10^{8} t+\frac{4 \pi}{3}\left(\mathrm{z}-\frac{1}{8}\right)\right) \mathrm{A} / \mathrm{m}$
B.
$\mathrm{H}(\mathrm{z}, \mathrm{t})=a_{y} \frac{10^{-4}}{60 \pi} \cos \left(2 \pi 10^{8} t-\frac{4 \pi}{3}\left(z-\frac{1}{8}\right)\right) \mathrm{A} / \mathrm{m}$
C.
$\mathrm{H}(\mathrm{z}, \mathrm{t})=a_{z} \frac{10^{-4}}{60 \pi} \cos \left(2 \pi 10^{8} t-\frac{4 \pi}{3}\left(z-\frac{1}{8}\right)\right) \mathrm{A} / \mathrm{m}$
D.
$\mathrm{H}(\mathrm{z}, \mathrm{t})=a_{x} \frac{10^{-4}}{60 \pi} \cos \left(2 \pi 10^{8} t-\frac{4 \pi}{3}\left(z+\frac{1}{8}\right)\right) \mathrm{A} / \mathrm{m}$

Ans. B
Sol. AS we know, standard equation of Plane wave is :
$\mathrm{E}(\mathrm{z}, \mathrm{t})=\mathrm{E} \cos (\omega t-\beta z+\varphi)$
So , here we need to calculate $\beta$, wave number.
$\beta=\omega \sqrt{\varepsilon \mu}=\frac{\omega}{c} \sqrt{\mu_{r} \varepsilon_{r}}$
$f=100 \mathrm{MHz}=10^{8} \mathrm{~Hz}$
$\beta=\frac{4 \pi}{3}$
Therefore, $\mathrm{E}(\mathrm{z}, \mathrm{t})=a_{x} E_{x}=$
$a_{x} 10^{-4} \cos \left(2 \pi 10^{8} t-\beta z+\varphi\right)$
$=a_{x} 10^{-4} \cos \left(2 \pi 10^{8} t-\frac{4 \pi}{3} z+\varphi\right)$
Since $E_{x}$ equals $+10^{-4}$ when the argument of the cosine function equals zero, i.e.
$2 \pi 10^{8} t-\frac{4 \pi}{3} z+\varphi=0$ when $t=0$,
$z=1 / 8$
$\varphi=\frac{\pi}{6} \mathrm{rad}$
So
$\mathrm{E}(\mathrm{z}, \mathrm{t})=a_{x} 10^{-4} \cos \left(2 \pi 10^{8} t-\frac{4 \pi}{3} z+\frac{\pi}{6}\right)$
Now, instantaneous expression for H
is , $\mathrm{H}=a_{y} H_{y}=a_{y} \frac{E_{x}}{\eta}$
Where, $\eta=\sqrt{\frac{\mu}{\varepsilon}}=\frac{\eta_{0}}{\sqrt{\varepsilon_{r}}}=60 \pi(\Omega)$
Hence,
$\mathrm{H}(\mathrm{z}, \mathrm{t})=a_{y} \frac{10^{-4}}{60 \pi} \cos \left(2 \pi 10^{8} t-\frac{4 \pi}{3} z+\frac{\pi}{6}\right)$
A/m
$\mathrm{H}(\mathrm{z}, \mathrm{t})=a_{y} \frac{10^{-4}}{60 \pi} \cos \left(2 \pi 10^{8} t-\frac{4 \pi}{3}\left(z-\frac{1}{8}\right)\right.$
) $A / m$
65. For on $n$-channel $E-M O S F F T$ used in
the circuit shown below, the threshold voltage, $\mathrm{V}_{\mathrm{T}}=1 \mathrm{~V}$, the channel length modulation parameter
$I=0$ and $\mu_{n} \operatorname{Cox} \frac{W}{L}=0.3 \mathrm{~mA} / \mathrm{V}^{2}$.
Then the output voltage $\mathrm{V}_{0}$ is

A. -3.35 V
B. 3.35 V
C. -4.68 V
D. 4.68 V

## Ans. B

Sol. Given,
Threshold voltage, $\mathrm{V}_{\mathrm{T}}=1 \mathrm{~V}$
$\mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}} \frac{\mathrm{W}}{\mathrm{L}}=0.3 \mathrm{~mA} / \mathrm{v}^{2}$
As drain and gate are connected together MOSFET is in saturation region.
Drain current,
$\mathrm{I}_{\mathrm{D}}=\frac{5-\mathrm{V}_{0}}{2 \mathrm{k} \Omega}$
But,
$I_{D}=\frac{1}{2}\left(0.3 \times 10^{-3}\right)\left(V_{G S}-1\right)^{2}$
Equating equation (i) \& (ii)
$\frac{5-\mathrm{V}_{0}}{2 \mathrm{k} \Omega}=\frac{1}{2}\left(0.3 \times 10^{-3}\right)\left(\mathrm{V}_{\mathrm{GS}}-1\right)^{2}$
Since, $V_{G S}=0$
$\frac{5-\mathrm{V}_{0}}{2 \mathrm{k} \Omega}=\frac{1}{2}\left(0.3 \times 10^{-3}\right)\left(\mathrm{V}_{0}-1\right)^{2}$
$5-\mathrm{V}_{0}=0.3\left(\mathrm{~V}_{0}-1\right)^{2}$
$5-\mathrm{V}_{0}=0.3\left(\mathrm{~V}_{0}^{2}-2 \mathrm{~V}_{0}+1\right)$
$5-\mathrm{V}_{0}=0.3^{\mathrm{V}_{0}^{2}}-0.6 \mathrm{~V}_{0}+0.3$
$V_{0}=\frac{-4 \pm \sqrt{16+4(3)(47)}}{6} V$
Since, $\mathrm{V}_{0}$ can be only +Ve
$V_{0}=\frac{-4+\sqrt{580}}{6}=3.35 \mathrm{~V}$

# Next Mega Mock Challenge 

 ISRO 2019-20 ME/EC/CS08 Jan(12 PM) to 09 Jan(12 PM)
Register Now

GATE 2020 ME/EC/CS/EE/CE 15 Jan(12 PM) to 16 Jan(12 PM)

## Stay Connected

