gradeup

GATE 2020
Civil Engineering

Mega Mock Challenge
(02 Jan-03 Jan 2020)

Questions & Solutions

1. **Direction:** In the given question, four words are given of which two are most nearly the same or opposite in meaning. Find the two words and indicate your answer by marking the option which represents the correct combination.

A) Diligent B) Adorable
C) Meticulous D) Prominent

A. B-D B. A-C
C. A-B D. A-D

Ans. B

Sol. The meanings of the words are:
Diligent: having or showing care and conscientiousness in one's work or duties.
Adorable: inspiring great affection or delight.
Meticulous: showing great attention to detail; very careful and precise.
Prominent: important; famous.
Hence, **option B** is the correct answer.

2. **Direction:** A statement with one blank is given below. Choose the set of words from the given options which can be used to fill the given blank.
Despite almost ubiquitous scepticism, the electoral bonds have prevailed and, that too, almost solely ________ rhetorical claims of "transparency of political funding system," "clean money," and "donor's anonymity."
i. with the backing of the ruling government's
ii. based on the endorsement derived from the political party at power's
iii. backed by the political party at power's

A. Only i B. Only ii
C. Only iii D. Both i and ii

Ans. D

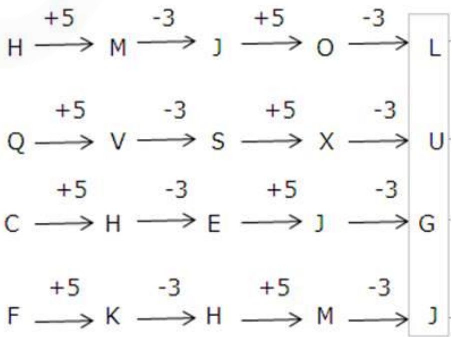
Sol. The given sentence talks about the prevailing nature of 'electoral bonds' in spite of concerns and doubts regarding the same. The sentence goes on to explain that this is occurring because of rhetorical claims by someone. From the options it is clear that the ruling part is responsible for these 'rhetorical claims'.
Option i – 'backing' means help or support and has been used in conjunction with the correct tense format of the sentence.
Option ii – 'endorsement' also means help or support and it tallies with the sentence structure.
Option iii – although 'backed' has been used it is in the incorrect tense form. This makes it incorrect.
Thus, option D is the correct answer.

3. Which letter—cluster will replace the question mark (?) in the following series?
HQCF, MVHK, JSEH, OXLM, ?

A. FTRD B. LUGJ
C. MKOP D. SWQ

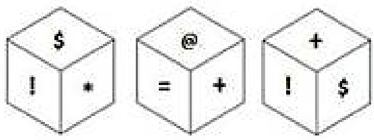
Ans. B

Sol. Pattern is-



Hence, the correct answer is option B.

4. Three different positions of the same dice are shown. Which symbol will be on the face opposite to the one having '*'?



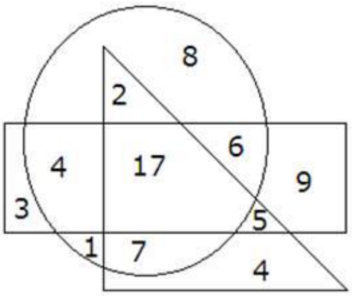
A. + B. !
C. $ D. @

Ans. A

Sol. Pick out the dices in which one symbol is common , after that arrange them in ACW or CW direction.

In II and III '+' is common
+ = @
* + ! $
Interchange the missing symbol '*' with repeated symbol '+'
Hence, option (A) is the correct answer.

5. In the following diagram, the triangle represents 'Dentists', the circle represents 'Professors' and the rectangle represents 'Doctors'. The numbers in different segments show the number of persons.

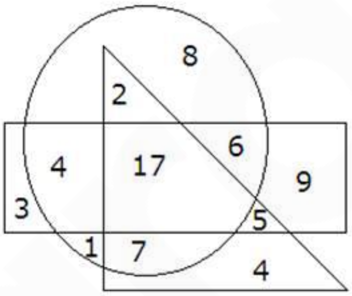


How many professors are dentists but not doctors?

A. 17 B. 9
C. 15 D. 13

Ans. B

Sol. Given diagram is-



circle represents Professors
rectangle represents Doctors
triangle represents Dentists
No. of professors who are dentists but not doctors=2+7=9
Hence, the correct answer is option B.

6. In the following question, some statements followed by some conclusions are given. Taking the given statements to be true even if they seem to be at variance from commonly known facts, read all the conclusions and then decide which of the given conclusions logically follows the given statements.

Statement:
Parents must understand that their child cannot attain excellence on his own. He needs their support. They must thus be open to help him at various steps rather than merely setting high expectations.

Conclusion:
I. Ideal students are not born ideal or perfect. They are nurtured to become ideal by their educators. The environment at home has a great impact on the way a student performs in school.
II. The life of an ideal student may seem tough from a distance. However, it is actually much more sorted as compared to those who procrastinate and do not give complete attention to their studies.

A. If only conclusion I follows
B. If only conclusion II follows
C. If both I and II conclusion follow
D. If neither I nor II conclusion follows

Ans. A

Sol. Conclusion I follows, based on the given statement a major component in the making of an Ideal student is described that it takes efforts not only from the students but also from the educators(Teachers and Parents)
Conclusion II is a correct statement that is the hard work and struggle that it takes to become an ideal student but it cannot be the conclusion of the given statement.

7. **Direction:** Each question below is followed by two statements I and II. You have to determine whether the data given in the statement is sufficient for answering the question. You should use the data and your knowledge of Mathematics to choose the best possible answer.

A man deposited Rs. 'x' in bank which gives simple interest at the rate of 8% p.a. Find the value of 'x'.

Statement I: After 3 years, amount received by him is Rs. (x + 672).

Statement II: Interest earned by him after 3 years is 24% of the amount deposited by him.

A. If the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question.

B. If the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question.

C. If the data either in Statement I or in Statement II alone are sufficient to answer the question.

D. If the data in both Statements I and II together are necessary to answer the question.

Ans. A

Sol. Statement I:

Simple interest earned by him

$= x + 672 - x = Rs.672$

So, $672 = \frac{x \times 8 \times 3}{100}$

$x = Rs.2800$

So, statement I alone is sufficient to answer the question.

Statement II:

We have to calculate principal(x) but we are not given interest since it is also in form of x. Hence, there are 2 unknowns.

Statement II alone is not sufficient to answer the question.

Thus, the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question.

So option (A) is the correct answer.

8. The given pie chart shows the breakup of total number of the employees of a company working in different offices (A, B, C, D and E).

Total no. of employees = 2400

What is the number of offices in which the number of employees of the company is between 350 and 650?

A. 3 B. 4

C. 2 D. 1

Ans. A

Sol. Total no. of Employees (360°) = 2400

No. of employees in office A(126°)

$= \frac{2400}{360} \times 126 = 840$

No. of employees in office B(18°)

$= \frac{2400}{360} \times 18 = 120$

No. of employees in office C(54°)

$= \frac{2400}{360} \times 54 = 360$

No. of employees in office D(90°)

$= \frac{2400}{360} \times 90 = 600$

No. of employees in office E(72°)

$= \frac{2400}{360} \times 72 = 480$

Number of offices in which the number of employees of the company is between 350 and 650 = 3

9. Find the numbers a, b, c between 2 and 18 such that

I. their sum is 25,

II. the numbers 2, a, b are consecutive terms of an A.P. and

III. The numbers b, c, 18 are consecutive terms of a G.P.

A. a=5, b=8, c=12

B. a=7, b=8, c=12

C. a=5, b=9, c=11

D. a=7, b=5, c=11

Ans. A

Sol. We have a + b + c = 25 ...(i)

2, a, b are in A.P. ⇒ 2a = 2 + b ...(ii)

b, c, 18 are in G.P. ⇒ 18b = c2 ...(iii)

Substituting for a and b in (1), using relations (2) and (3), we get

$\Rightarrow 1 + \frac{b}{2} + \frac{c^2}{18} + c = 25$

⇒ $c^2 + 12c - 288 = 0$

⇒ (c − 12) (c + 24) = 0

⇒ C= 12 or c= -24

Since the numbers lie between 2 & 18,

We take c=12

⇒ a + b = 13

⇒ a + 2a -2 = 13

⇒ b=8, a=5

10. **Statements:**

All lions are ducks.

No duck is a horse.

All horses are fruits.

Conclusions:

I. No lion is a horse.

II. Some fruits are horses.

III. Some ducks are lions.

IV. Some lions are horses.

A. Only either I or II and III & IV follow

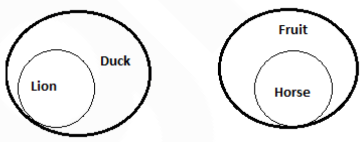
B. Only either I or IV and both II and III follow

C. Only either I or IV and II follow

D. Only Conclusion I & II and III follow

Ans. D

Sol.



We use elimination to find an exception to the generality of the question. Thus we prove they are not implied. The diagram above satisfy all the above statement but contradict with the conclusion (iv). Since we found an exception, the conclusion is not true in every case. Thus it is not implied.

We can draw many scenarios that satisfy the statements using Venn diagram & check for the validity of the conclusions.

Conclusions (i), (ii), (iii) hold good for every case so they are implied.

11. All queens and kings are removed from a deck of playing cards. Ace will be considered as 1 and jack will be considered as 0. You took out 4 cards. The probability that all cards will be in order (order is – 1234,0123,2345,6789….) from the same deck is M. What is (M x 10^5)?

A. 0.982 B. 0.700

C. 0.643 D. 0.500

Ans. A

Sol. Total cards = 52 − 4 − 4 = 44 (cards)

→ You took out 4 cards

∴ (1) Let order be 0, 1, 2, 3

So, P (order - 0, 1, 2, 3) = $\frac{4}{44} \times \frac{1}{43} \times \frac{1}{42} \times \frac{1}{41} = x$

∴ Lost possible set – (7, 8, 9, 10) – 8 such sets are possible

∴ P (Total) = 8 × P(order – 0, 1, 2, 3) = 8x

⇒ 8x × 10^5 = 0.982

12. The divergence of the vector field

$\vec{V} = (x^2 + y)\hat{i} + (z - 2xy)\hat{j} + (xy)\hat{k}$ at (1,1,1) is

A. 1 B. -1

C. 0 D. 2

Ans. C

Sol.

$\nabla \cdot \vec{V} = \frac{\partial}{\partial x}(x^2 + y) + \frac{\partial}{\partial y}(z - 2xy) + \frac{\partial}{\partial z}(xy)$

$= 2x - 2y$

$= 0 \; at \; (1,1,1)$

13. Using trapezoidal rule for the table given below

x :	4	4.2	4.4	4.6	4.8	5.0	5.2
Ln x:	1.39	1.44	1.48	1.53	1.57	1.61	1.65

Find the value of Integral

I = $\int_4^{5.2} \ln x . dx$

A. 1.83 B. 1.93
C. 1.64 D. 0.98

Ans. A

Sol. Width (h) =0.2

$$\int_4^{5.2} lnxdx = \frac{h}{2}[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$$

$$\int_4^{5.2} lnxdx = \frac{0.2}{2}[(1.39 + 1.65) + 2(1.44 + 1.48 + 1.53 + 1.57 + 1.61)]$$

$$= 1.83$$

14. Which of the following statements is incorrect?

A. Light weight concrete is a special type of concrete having density 2300kg/m3

B. High density concrete is used for atomic power plants.

C. Slump Value of self consolidated concrete lies between 650-750.

D. Fatty acids or alcohols in the concrete produce air entrained concrete.

Ans. A

Sol. Light weight concrete has density lesser than 1930 kg/m3. Other statements are correct.

15. Which of the following is the most appropriate theory for ductile material among following?

A. Maximum distortion energy theory

B. Max strain theory

C. Max shear stress theory

D. Maximum strain energy theory

Ans. A

Sol. The most appropriate theory for ductile material is 'maximum distortion energy theory' as this theory is in perfect agreement with the case of pure shear. According to this theory, maximum shear strain energy in a body should be less than or equal to maximum shear strain energy under uniaxial loading.

16. What is the anchorage length of a tie bar, which is bent through 135° round a bar of diameter 12mm?

A. 72 mm B. 96 mm
C. 48 mm D. 192mm

Ans. A

Sol. For secondary reinforcement such as stirrups in beams and transverse ties in a column, complete development length and anchorage shall be deemed to have been provided when the bar is bent through an angle of 135° and is continued beyond the end of the curve for a length of at least 6 diameters.

So, 6φ = 6 x 12 = 72 mm

17. A sample of soil weighs 153 grams. Its clay fraction weighs 34% of the total weight. If it's liquid limit is 62% and the Plastic limit is 27%. Classify the soil.

A. MH B. CL
C. CH D. Organic Clay

Ans. C

Sol. Plasticity Index, IP = LL- PL = 62 – 27 = 35%

I_p of A-line

I_p = 0.73(LL-20) = 30.66

Therefore Clay

Also LL>50.

Therefore highly compressible.

18. A canal and a river runs parallel at an average of 76m apart. The elevation of water level in river is at +311.2m and in the canal is at + 320.04m. A stratum of sand intersects both the river and canal below the water level. The sand layer is 1.80m thick, and is sandwiched between layers of impervious clay with the area as $24m^2$. The seepage loss in the canal is if the permeability of sand is 2 x 10^{-3} ft/sec.

A. 1.7 x 10^{-3} m^3/s

B. 2.7 x 10^{-3} m^3/s

C. 1.6 x 10^{-3} m^3/s

D. 2.8 x 10^{-3} m^3/s

Ans. A

Sol. $q = k\frac{\Delta h}{L}A$

k = 2 x 10^{-3} x 0.3048 = 0.61 x 10^{-3} m/sec

$$q = \frac{0.61 \times 10^{-3} \times (320.04 - 311.2) \times 24}{76} =$$

1.7 x 10^{-3} m^3/s

19. Match List I and List II and select the correct answer using the codes given below the lists.

List I	List II
P. Reservoir Routing	1. Uses equation of continuity and St.Venant's equation
Q. Channel Routing	2. Uses equation of continuity
R. Hydraulic routing	3. Unique function of both flood inflow and outflow
S. Hydrologic routing	4. Uses Modified puls method

A. P-4 Q-1 R-3 S-2
B. P-3 Q-4 R-1 S-3
C. P-4 Q-3 R-1 S-2
D. P-3 Q-1 R-2 S-4

Ans. C

Sol. The equation of continuity used in all **hydrologic routing** as the primary equation states that the difference between the inflow and outflow rate is equal to the rate of change of storage, i.e.

$$I - Q = \frac{dS}{dt}$$

I – inflow; Q – outflow; S – storage

The modified puls routing method is probably most often applied to **reservoir routing** considering the finite difference form of the continuity equation.

Hydraulic routing combines the continuity equation with a very simplified form of the St. Venant equations.

In **Channel routing**, the storage is a function of both inflow and outflow.

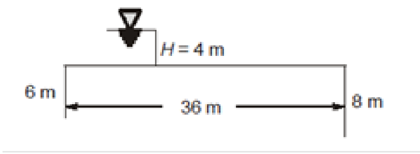
20. A weir consists of a 36 m long horizontal floor with two sheet piles of 6 m and 8 m depth at the upstream and downstream end of the floor, respectively. Under an impounded depth of 4 m, the residual head at centre is ______ m.

A. 2.12 B. 3.15
C. 5.56 D. 1.1

Ans. A

Sol. Creep length, L = 36 + (2 x 6) + (2 x 8)

= 64 m



At the mid-point of floor, creep length

= (6 + 6 + 18) = 30 m

Loss in seepage head at centre,

$$H_L = \frac{4 \times 30}{64} = 1.875m$$

Residual seepage head at centre = H – H_L = 4.0 – 1.875 = 2.125 m

21. Match **List I** with **List-II** and select the correct answer.

	List-I (Control Structures)		List-II (Functions)
P	Canal Drop	1.	Control of flow Depth
Q	Canal Escape	2.	Control of Bed Grade
R	Canal Cross Regulation	3.	Control of full supply level
S	Canal Outlet	4.	Control of discharge

A. P-2 Q-3 R-4 S-1
B. P-3 Q-2 R-1 S-4
C. P-3 Q-2 R-4 S-1
D. P-2 Q-3 R-1 S-4

Ans. D

Sol.

Canal Drop – Control of Bed Grade
Canal Escape – Control of full supply level
Canal Cross Regulation – Control of flow depth
Canal Outlet – Control of disharge

22. In a tidal model, the horizontal scale ratio is $\frac{1}{750}$ and the vertical scale is $\frac{1}{75}$. The model period (in minutes), corresponding to a prototype period of 18 hours, would be

A. 11.24
B. 12.47
C. 14.96
D. None of these

Ans. B

Sol. $L_r = \frac{1}{750}$ and $h_r = \frac{1}{75}$

From Froude's law, $\frac{V_r}{\sqrt{h_r}} = 1$

$\Rightarrow$ Time ratio, $T_r = \frac{L_r}{\sqrt{h_r}} = \frac{1/750}{\sqrt{1/75}}$

$= 0.01155 \quad \left(\because V_r \frac{L_r}{T_r}\right)$

$\Rightarrow$ Model period, $T_m = T_p \times T_r$

$= (18 \times 60 \times 60) \times 0.01155 = 748.24\ s$

$\therefore\ T_m = 12.47$ minutes

23. For a steady incompressible laminar flow through a circular pipe, the velocity distribution is.

A. Linear with zero value at the surface and maximum value at the centre.

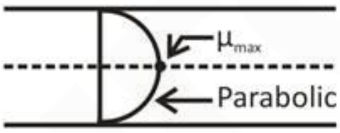
B. Linear with zero value at the centre and maximum value at the surface.

C. Parabolic with zero value at the surface and maximum value at the centre

D. Parabolic with zero value at the centre and maximum value at the surface.

Ans. C

Sol.



Velocity variation $V = \frac{1}{4\mu}\left[\frac{-\partial P}{\partial x}\right](R^2 - r^2)$

24. Elevation and temperature for places A, B and C are tabulated below :

Place	Elevation (m)	Temperature (°C)
A	45	21
B	350	18.5
C	500	14

Based on above data, for dry air lapse rate of place A to B and place B to C respectively are

A. Sub-adiabatic and Sub-adiabatic

B. Super-adiabatic and Sub-adiabatic

C. Sub-adiabatic and Super-adiabatic

D. Super-adiabatic and Super-adiabatic

Ans. C

Sol. From place A to B, Ambient lapse rate =

$\frac{21 - 18.5}{350 - 45} \times 1000 = 8.196 < 9.8°$ C/km

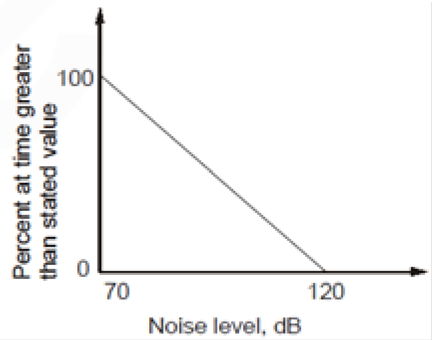
Hence the condition is sub-adiabatic

From place B to C, Ambient Lapse rate =

$\frac{18.5 - 14}{500 - 350} \times 1000 = 30\ °C/km > 9.8\ °C/km$

So, the condition is super-adiabatic.

25. At a certain location, the cumulative noise power distribution curve is given below:



The value of L_{50} (in dB) is equal to

A. 95 B. 90

C. 85 D. 80

Ans. A

Sol. L_{50} is the sound pressure level in dB which is exceeded for 50% of the gauging time.

Now, slope of the given curve

$= \frac{100 - 0}{120 - 70} = \frac{100}{50} = 2$

$\because$ Curve is a straight line, hence slope is constant.

$\Rightarrow 2 = \frac{50 - 0}{L_{50} - 70}$

$\Rightarrow L_{50} - 70 = 25$

$\Rightarrow L_{50} = 95 dB$

26. A line of levels was run from a benchmark of RL 51.540 and ended on a benchmark of RL 63.200. The sum of backsights and foresights were 86.755 m and 72.725 m respectively. The closing error of the work was ___ m.
A. 2.86 B. 2.37
C. 2.46 D. 2.29

Ans. B

Sol.

Measured value = ΣB.S – ΣF.S

= 86.755 – 72.725

= 14.03 m

True value = Last R.L – First R.L

= 63.2 – 51.54

= 11.66 m

Error = Measured value – true value

= 14.03 – 11.66 = 2.37 m

27. A road section of length 2km scale 16 cm on a vertical photograph. The focal length of the camera is 320mm. If the terrain is fairly level, then the flying height will be __________km.
A. 2.0 B. 4.0
C. 1.6 D. 2.4

Ans. B

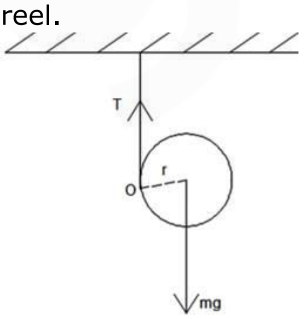
Sol.

$$Scale = \frac{f}{H}$$

$$\frac{16}{2000 \times 100} = \frac{32}{H}$$

$$H = 400000\ mm = 4\ km$$

28. A steel reel of mass m & radius r & radius of gyration k is rolling down from rest. From its one end, a thread wounded on it is held on the ceiling as shown. Find linear acceleration on the reel.



A. $\frac{gr}{r^2+k^2}$ B. $\frac{gr^2}{r^2+k^2}$
C. $\frac{gr}{r^2+k}$ D. $\frac{gr}{r+k^2}$

Ans. B

Sol. Area moment of inertia=I=mr^2+mk^2
Generated torque about point O=T=Iα= mα(r^2+ k^2),
Moment about point O=M=T=mgr.
So, mgr = mα(r^2+ k^2)
Or, $\alpha=\frac{gr}{r^2+k^2}$
$a= \alpha r=\frac{gr^2}{r^2+k^2}$

29. A steel wire of G = 85 GPa is to be proportioned such that the maximum shearing stress is 80 MPa for an angle of twist of 90°. The length to diameter ratio is _____

Sol. Angle of twist = 90° = $90\times\frac{\pi}{180}$rad

$$\frac{\tau_{max}}{R}=\frac{G\theta}{L}$$

$$\frac{\tau_{max}}{d/2}=\frac{G\theta}{L}$$

$$\frac{L}{d}=\frac{G\theta}{2\tau_{max}}$$

$$\frac{L}{d}=\frac{85\times10^3\times90\times\frac{\pi}{180}}{2\times80}$$

$$\frac{L}{d}=834.48$$

30. A 10m high retaining wall retains dry sand. Initially the soil is in loose state with void ratio of 0.5, γ_d = 17.4 kN/m^3 and φ= 30°. Subsequently, the soil is compacted and filled to the same height and now its new void ratio is 0.4, γ_d = 18.4 kN/m^3 and φ= 35°. According to Rankine's earth pressure theory, the ratio of initial passive pressure to final passive pressure is _____.

Sol. Initial passive pressure (for c=0 soil)

$\gamma = \gamma_d$ = 17.4 kN/m^3, φ= 30°

$P_{p1} = K_p\gamma H$ and $K_p = \left(\frac{1+\sin\phi}{1-\sin\phi}\right) = 3$

P_{p1} = 3 × 17.4 × 10 = 522 kN/m^2

Final passive pressure

$\gamma = \gamma_d$ = 18.4 kN/m^3, φ= 35°

$P_{p2} = K_p\gamma H$ and $K_p = \left(\frac{1+\sin\phi}{1-\sin\phi}\right)$ = 3.69

P_{p2} = 3.69 × 18.4 × 10 = 678.96 kN/m^2

Therefore, $\frac{P_{P1}}{P_{P2}} = \frac{522}{678.96} = 0.768$

31. A centrifugal pump needs 1000 W of power when operating at 2000 rpm. If the speed of pump is increased to 4000 rpm, then power requirement is ______ kW.

Sol. By similarity laws,

$$\frac{P_1}{\gamma D_1^5 N_1^3} = \frac{P_2}{\gamma D_2^5 N_2^3}$$

For a centrifugal pump, $D_1 = D_2$

$$\therefore \quad \frac{P_1}{N_1^3} = \frac{P_2}{N_2^3}$$

$$\Rightarrow \quad \frac{1000}{(2000)^3} = \frac{P_2}{(4000)^3}$$

$$\Rightarrow \quad 2^3 \times 1000 = P_2$$

$$\therefore \quad P_2 = 8000W = 8kW$$

32. The turning radius for subsonic aircraft having wheel base of 17.50m, thread of main loading gear is 6.62, turning speed of 45kmph and coefficient of friction between tyre and pavement equal to 0.13 is ________m. (up to 2 decimal places)

Sol. The turning radius is the maximum of:

1. $R = \frac{V^2}{125f} = \frac{45^2}{125\times0.13}$ = 124.615m

2. From Horonjeff's equation

$= \frac{0.388W^2}{\frac{T}{2}-S}$

$S = 6 + \frac{6.62}{2} = 9.31$

T = 22.5m

$R = \frac{0.388\times17.50^2}{\frac{22.5}{2}-9.31}$

R = 61.25m

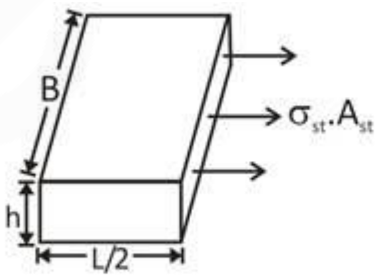
3. Absolute minimum turning radius for subsonic aircraft regardless of speed = 120m.

Radius of turning = 124.615m

33. A 3.5m reinforced concrete slab having thickness 20 cm and f = 1.5 with bars of diameter 1.0 cm at 0.3m spacing.

Given that allowable working stress in steel in tension is 1200 kg/cm^2 and unit weight of concrete is 2400 kg/m^3. The spacing between contraction joints in this case is _______ m [rounded to two decimal places]

Sol.



A_{st} = number of bars × $\frac{\pi}{4}d^2$

$$A_{st} = \frac{3.5}{0.3}\times\frac{\pi}{4}\times1^2 = 9.16cm^2$$

$$\sigma_{st}A_{st} = B\times\frac{L}{2}\times h\times\gamma\times f$$

$$1200\times9.16 = 3.5\times\frac{L}{2}\times\frac{20}{100}\times2400\times1.5$$

∴ spacing between contraction joint

$$L_C = \frac{1200\times9.16\times200}{3.5\times20\times2700\times1.5}$$

L_C = 8.72m

34. A speed study is conducted to have a knowledge about the time mean speed and space mean speed. The data collected is as below-

Speed range (m/s)	2-5	6-9	10-13	14-17
Frequency (q_i)	10	14	0	9

The ratio of space mean speed to time mean speed is _________ [correct to three decimal places]

Sol.

Speed Range	Average speed	q_i	$V_i\, q_i$	$\frac{q_i}{V_i}$
2-5	3.5	10	35	2.857
6-9	7.5	14	105	1.867
10-13	11.5	0	0	0
14-17	15.5	9	139.5	0.581
		$\Sigma q_i = 33$	$\Sigma V_i q_i = 279.5$	$\Sigma \frac{q_i}{V_i} = 5.305$

Time mean speed

$$= \frac{\sum V_i q_i}{\sum q_i} = \frac{279.5}{33} = 8.47 m/sec$$

Space mean speed

$$= \frac{\sum q_i}{\sum \frac{q_i}{V_i}} = \frac{33}{5.305} = 6.22 m/sec.$$

Hence

$$\frac{\text{Space mean speed}}{\text{Time mean speed}} = \frac{6.22}{8.47} = 0.734$$

35. The average and saturation flows on cross roads A and B during design period are as follows-

	Average flow	Saturation flow
Road A	350 pcu/hr	1120 pcu/hr
Road B	210 pcu/hr	900 pcu/hr

The total lost time for the signal design to be considered as 16 seconds. The green time for road B is __________ seconds.

Sol.

$$y_a = \frac{q_a}{S_a} = \frac{350}{1120} = 0.3125 \qquad y_b = \frac{q_b}{S_b} = \frac{210}{900} = 0.2333$$

$Y = y_a + y_b = 0.3125 + 0.2333 = 0.5458$

L = 16 seconds

$$C_o = \frac{1.5L+5}{1-Y} = \frac{1.5\times16+5}{1-0.5458} = 63.85 \text{ seconds}$$

$$G_b = \frac{y_a}{Y}(c_o - c) = \frac{0.2333}{0.5458}(63.85-16) = 20.45 \text{ seconds}$$

36. The solution curve of the differential equation $x\frac{dy}{dx} = y + 2x^3$ passes through the point (1,0). Then among the points given below, the curve also passes through:

A. (-1, 0) B. (0, -1)
C. (2, 10) D. (-2, 6)

Ans. A

Sol. The differential equation can be written as:

$$\frac{dy}{dx} = \frac{y}{x} + 2x^2$$

$$\frac{dy}{dx} - \frac{y}{x} = 2x^2$$

which is a linear equation in 'y'

$$IF = e^{-\int \frac{1}{x}dx} = e^{-\log x} = -\frac{1}{x}$$

Thus, the solution of the equation will be:

$$y\ x\ IF = \int 2x^2 x\ IF\ dx$$

$$-\frac{y}{x} = -\int 2\ x\ dx$$

$$\frac{y}{x} = x^2 + C$$

Putting (1,0) we get $C = -1$

Thus,

$$\frac{y}{x} = x^2 - 1$$

Clearly only (-1,0) satisfies the above equation.

37. Find the Laplace transform of the function f(t) given as

$$f(t) = (t-2)^2$$

A. $\frac{4}{s} - \frac{4}{s^2} + \frac{2}{s^3}$, s>0

B. $\frac{4}{s} - \frac{4}{s^2} + \frac{4}{s^3}$, s>0

C. $\frac{4}{s} - \frac{2}{s^2} + \frac{2}{s^3}$, s>0

D. $\frac{2}{s} - \frac{4}{s^2} + \frac{2}{s^3}$, s>0

Ans. A

Sol. Given L $[(t-2)^2]$

$$= \lim_{T\to\infty} \int_0^T (t-2)^2 e^{-st}\,dt$$

Using integration by parts with u' $= e^{-st}$ and V = $(t-2)^2$ we will find,

$$\int_0^T (t-2)^2 e^{-st}\,dt = -\left[\frac{(t-2)^2 e^{-st}}{s}\right]_0^T + \frac{2}{s}\int_0^T (t-2)e^{-st}\,dt$$

$= \frac{4}{s} - \frac{(T-2)^2 e^{-sT}}{s} +$
$\frac{2}{s}\int_0^T (t-2)e^{-st}\,dt$

thus,

$\lim_{T\to\infty}\int_0^T (t-2)^2 e^{-st}dt = \frac{4}{s} + \frac{2}{s}\lim_{T\to\infty}\int_0^T (t-2)e^{-st}dt$

Using by parts with u' = e^{-st} and v =t-2 we find

$\int_0^T (t-2)e^{-st}dt = [-\frac{(t-2)e^{-st}}{s} - \frac{1}{s^2}e^{-st}]_0^T$

Let T → ∞ in the above expression we will get

$\lim_{T\to\infty}\int_0^T (t-2)e^{-st}dt = -\frac{2}{s} + \frac{1}{s^2}, s > 0$

Hence,

F(s) = $\frac{4}{s} + \frac{2}{s}(-\frac{2}{s} + \frac{1}{s^2}) = \frac{4}{s} - \frac{4}{s^2} + \frac{2}{s^3}$, s>0

38. The state of stress of an element is given as follows.
The value of normal stress are $\sigma_x = 30MPa$, $\sigma_y = -10MPa$ and major principal stress $\sigma_1 = 50MPa$ .Then the value of shear stress is.

A. 30 MPa B. 25.20 MPa
C. 45.90 MPa D. 34.64 MPa

Ans. D

Sol. We know

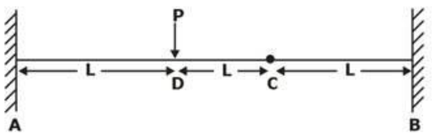
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$50 = \frac{30 + (-10)}{2} + \sqrt{\left(\frac{30-(-10)}{2}\right)^2 + (\tau_{xy})^2}$$

$$(\tau_{xy}) = \sqrt{40^2 - 20^2}$$

$$(\tau_{xy}) = 34.64MPa$$

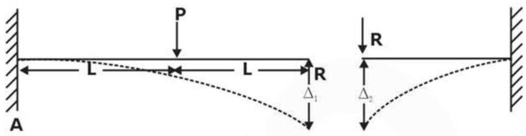
39. Determine the vertical reaction at support A



A. $\frac{5P}{18}$ B. $\frac{3P}{18}$
C. $\frac{13P}{18}$ D. $\frac{15P}{18}$

Ans. C

Sol.



Displacement at C will be equal for both sections

$D_1 = D_2$

$$\frac{PL^2}{2EI} \times L + \frac{PL^3}{3EI} - \frac{R(2L)^3}{3EI} = \frac{R \times (L)^3}{3EI}$$

$$\frac{5PL^3}{6EI} = \frac{9RL^3}{3EI}$$

R = 5P/18

R_A + R = P

R_A = 13P/18

40. A two hinged semicircular arch of uniform square cross section having side of 80mm, spanning 20m, is subjected to rise in temperature of 30°. Find horizontal thrust produced at supports. Take E = 2 x 10^5 N/mm^2 and α = 12 x 10^{-6} per °C.

A. 3.129kN B. 9.387kN
C. 37.58kN D. 18.29kN

Ans. A

Sol. For two hinged arch,

H = (4EIαT) / πR^2

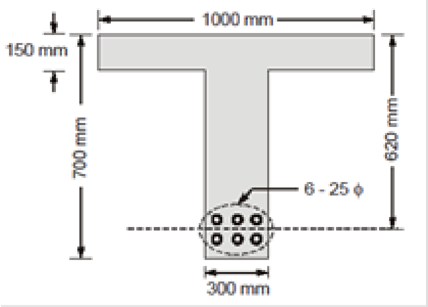
Here, I= d^4 / 12 = $(80)^4$ / 12

H = (4 x 2 x 10^5 x $(80)^4$ x 12 x 10^{-6} x 30) / (12 x π x $(10000)^4$)

H = 3.129 kN

41. An isolated simply supported T-beam, constructed using M20 concrete and Fe250 steel, having effective span of 9 m and cross-sectional dimensions as shown in the figure is subjected to a working moment of 270 kNm. If neutral axis lies in the web, then the depth of neutral axis is ________ mm.

[The beam is designed using WSM]



A. 157.62 B. 167.62
C. 177.62 D. 187.62

Ans. D

Sol. Effective span, L_0 = 9000 mm
Width of flange, b = 1000 mm
Width of web, b_w = 300 mm
Effective flange width,

$$b_f = \frac{L_0}{\frac{L_0}{b}+4} + b_w$$

$$= \frac{9000}{9+4} + 300$$

= 992.31 mm < (1000 mm)
Modular ratio (For M20 concrete) m = 13.33
Area of tension reinforcement,

$$A_{st} = 6 \times \pi \times \frac{25^2}{4} = 2945.25\ mm^2$$

Effective depth, d = 620 mm
Depth of flange, b_w = 300 mm
Df = 150 mm
Let the depth of neutral axis is x_a.
The neutral axis is located in the web
$\therefore x_a > D_f$
Equating area-moment about the Neutral axis for compression and tension side.

$$\Rightarrow 992.3 \times 150 \times (x_a - 75) + \frac{(x_a - 150)^2}{2} \times 300$$

$$= 13.33 \times 2945.25 \times (620 - x_a)$$

$$\Rightarrow 150(x_a)^2 + 143105.1825x_a - 32129688.15$$

x_a = 187.62 mm
So, the depth of neutral axis = 187.62 mm

42. A cylindrical specimen of 30 mm dia and 80 mm length is prepared from dry soil. If specimen has moisture content 10% and air void 15%. Find weight of dry soil (in grams) required if specific gravity = 2.7.

A. 113 B. 102
C. 110 D. 65

Ans. B

Sol. vol. of soil sample $= \frac{\Pi}{4} \times 0^2 \times 4$

$$= \frac{\Pi}{4} \times 3^2 \times 8 = 56.54\ cc$$

Water content

$$(w) = \frac{ww}{ws} = 10\% = 0.1$$

$$\eta a = 0.15 = \frac{Vair}{vol.\ of\ soil} = \frac{Va}{V}$$

$$Va = 0.15 \times 56.54 = 8.48\ cc$$

Now

$$w = \frac{ww}{ws} = \frac{vw \times yw}{vs \times ys} = \frac{vw}{G\,Vs}$$

$$\frac{vw}{vs} = gw$$

$$\Rightarrow \frac{vw}{vs} = 2.7 \times 0.1 = 0.27$$

$$vw + vs = 56.54 - 8.48$$

$$vw + vs = 48.06\ cc$$

$$0.27vs + vs = 48.06\ cc$$

$$1.27\ vs = 48.06\ cc$$

$$vs = 37.84\ cc$$

$$\therefore Vs = \frac{ws}{GTw}$$

$$\Rightarrow ws = Vs \times GTw = 37.84 \times 2.7 \times 1$$

$$= 102.17\ g$$

43. CU triaxial tests conducted on specimens of a saturated clay soil gave the following results

Test	Cell Pressure (kN/m^2)	Deviator Stress (kN/m^2)	Pore Water Pressure (kN/m^2)
1.	140	120	70
2.	280	200	140

The effective stress parameters c' and φ' will be

A. 15.95, 12.84°
B. 13.67, 21.32°
C. 16.83, 10.24°
D. 11.32, 23.54°

Ans. B

Sol. From Mohr's circle

$\sigma'_1 = \sigma_3 \tan^2 (45 + \frac{\Phi'}{2}) + 2c' \tan (45 + \frac{\Phi'}{2})$ .... (i)

For test (1)

σ_3 = 140 kPa, σ_d = 120 kPa, u= 70 kPa

Thus, $\sigma'_3 = \sigma_3$ - u = 140 – 70 = 70 kPa

$\sigma'_1 = \sigma_1$ - u = (140+120) - 70 = 190 kPa

For test (2)

σ_3 = 280 kPa, σ_d = 200 kPa, u= 140 kPa

Thus, $\sigma'_3 = \sigma_3$ - u = 280 – 140 = 140 kPa

$\sigma'_1 = \sigma_1$ - u = (280+200) - 140 = 340 kPa

From equation (i)

$190 = 70 \tan^2 (45 + \frac{\Phi'}{2}) + 2c' \tan (45 + \frac{\Phi'}{2})$ .... (ii)

$340 = 140 \tan^2 (45 + \frac{\Phi'}{2}) + 2c' \tan (45 + \frac{\Phi'}{2})$ ....(iii)

On solving the above equations, we get

$\tan^2 (45 + \frac{\Phi'}{2}) = 2.143$

Or, φ' = 21.32° and c'= 13.67

44. A square footing of having size 2.5 m rests on a dense sand which has angle of internal friction ø = 40 °. The depth of the foundation being 1.5 m. The unit weight of the sand above the water table is 17kN/m^3 and the saturated unit weight of soil is 22 kN/m^3. Values of bearing capacity parameters for ø = 40 ° are N_q= 81.3, N_γ=100.4

If the water table rests at the footing level, the ultimate bearing capacity of the soil is

A. 3162 kN/m^2 B. 3297 kN/m^2

C. 2998 kN/m^2 D. 3335 kN/m^2

Ans. B

Sol. When the water table is at the footing level, the soil below the footing i.e. in zone 2 will be submerged while the soil above the footing i.e. in zone 1 will be unaffected.

So $\gamma_q = \gamma_t$ = 17 kN/m^3 and $\gamma_s = \gamma'$ = γ_{sat} – 9.81 = 12.19 kN/m^3

Now $q_u = 1.3cN_c + \gamma_q D_f N_q + 0.4\gamma_s B N_\gamma$

= 0 + 17×1.5×81.3 + 0.4×12.19×2.5×100.4

= 2073.15 + 1223.876 = 3297.026 kN/m^2

45. Find the load capacity of a group of 9 piles, arranged into a 3x3 square pattern, if the center to center distance of each pile is 900mm and the adhesion factor is 0.9. The diameter of all the nine piles is 300m and the length of each pile is 8m. The soil type is medium stiff clay having an undrained compressive strength of 100 kN/m^2.

A. 1336KN B. 1450KN

C. 1300kn D. 1475KN

Ans. A

Sol. Given,

Diameter of the pile = D = 300mm = 0.3m

Length of the pile = 8m

Adhesion factor α= 0.9

C= $\frac{qu}{2}$ = 50KN/m^2

B = 2x0.9 + 0.3 = 2.1m

Ultimate load of group pile acting together,

$Q_{g1(U)} = q_p A_g + c(4BL)$

$=cN_cB^2 + c(4BL)$

= (50X9) X2.1^2 + 50X4X2.1X8 = 5344.5KN

Ultimate load of pile with each pile acting individually

$Q_{g2\,(u)} = NQ_u$

$= 9(cN_cA_p + \alpha c\,(\pi DL))$

= 9(50x9x(πx0.3^2/4) + 0.9x50xπx0.3x8)

=3339.9KN

As the ultimate load for individual pile failure case is less than the pile in group load, safe load is given by
Q_a = 3339.9/FOS
=3339.9/2.5
=1336 kN

46. A rectangular channel of width 4m is carrying a discharge of 36m^3/sec. If the depth of flow of one end of hydraulic jump is 1.2m then what is the power dissipated in the jump?
A. 255.77 KW B. 298.27 KW
C. 176.58 KW D. 198.27 KW

Ans. C

Sol. Froude number for rectangular channel

$F_1^2 = \frac{q^2}{gY_1^3}$, here q = Q/B = 36/4 = 9

$F_1^2 = \frac{9^2}{9.81 \times 1.2^3} = 4.78$

Depth of flow of another end of hydraulic jump

$\frac{Y_2}{Y_1} = \frac{1}{2}\left[\left(\sqrt{1+8F_1^2}\right)-1\right]$

$\frac{Y_2}{1.2} = \frac{1}{2}\left[\left(\sqrt{1+8 \times 4.78}\right)-1\right]$

Y_2 = 3.16m

Energy dissipation

$E_L = \frac{(Y_2-Y_1)^3}{4Y_1Y_2} = \frac{(3.16-1.2)^3}{4 \times 1.2 \times 3.16} = 0.5$ m

Power dissipated in jump

$P_L = \gamma Q E_L = 9.81 \times 36 \times 0.5 =$ 176.58 KW

47. Find the settling velocity of a discrete particle in water under conditions when Reynold's number is 112 . The diameter and specific gravity of the particle is 5×10^{-2} cm and 2.65, respectively. Water temperature is 20^0 Celsius.
A. .22 m/sec B. .11 m/sec
C. .45 m/sec D. .24 m/sec

Ans. C

Sol. When Reynold's number is greater than 1 and less than 100000 the flow is in transition.

$V_s = \sqrt{\frac{4(G-1)gd}{3 \times C_d}}$

$C_d = \frac{24}{R_e} + \frac{3}{\sqrt{R_e}} + .34 =$ .84

Hence, $V_s = .11\ m/s$....i

Or

$V_s = \frac{418(G-1)d}{1} \times \frac{3T+70}{100}$ (d is in mm and T in Degree Celsius)

$V_s = .448\ m/s$....ii

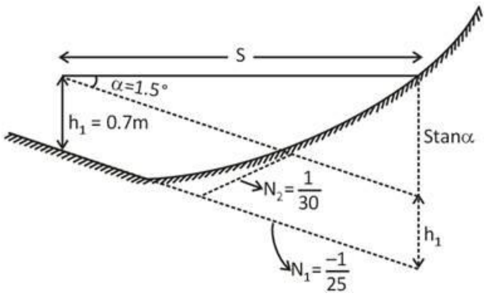
Maximum of I and ii, there for Vs is .448 m/s

48. If a 8^0 curve track diverges from a main curve of 5^0 in an opposite direction in the layout of a B.G. yard calculate the super-elevation if maxi mum speed permitted on the main line is 50 kmph.
A. 1.69 cm B. 1.99 cm
C. 1.15 cm D. 2.31 cm

Ans. B

Sol. Equilibrium cant required for 45 kmph speed, from the equation
e = GV^2 / 1.27R
Where,
G = 1.676 m for B.G
V = 40 kmph
R = 1720/5
Hence, e = (1.676×50×50/1.27) × 5/1720
= 9.59 cm
For broad gauge the cant deficiency permitted for main line is 7.6 cm
So the cant for main track = 9.59 – 7.6 = 1.99 cm
Therefore the cant to be provided for branch track = -1.99 cm
It is a negative cant of 1.99 cm

49. A valley curve is as shown below



If the stopping sight distance is 100m, the length of the valley curve for the headlight distance criteria is ________ m. [rounded to one decimal place]

Sol. Assume length of curve L_v > SSD

$$S = 100m;\ N = |N_2 - N_1| = \frac{1}{30} - \left(\frac{-1}{25}\right) = \frac{11}{150}$$

$$L = \frac{NS^2}{2h_1 + 2S\tan\alpha} = \frac{\frac{11}{150}\times 100^2}{2\times 0.7 + 2\times 100\times \tan(1.5^\circ)} = 110.488m$$

L = 110.5m > (SSD = 100 m)

Hence L = 110.5m

50. Determine the theoretical oxygen demand in mg/l of fructose solution which is present in quantity 600 mg/l of water.

A. 192 B. 180

C. 640 D. 384

Ans. C

Sol. Mol Wt. of Fructose ($C_6H_{12}O_6$) = 180 gms

Reaction

$C_6H_{12}O_6$ + $6O_2$Ã $6CO_2$ + $6H_2O$

1 mole of fructose requires 6 moles of O_2 = 192 gms.

Oxygen demand,

$$\frac{600}{180} \times 192 = 640 \text{ mg/l}$$

51. Match list I with list II and select the correct answer using the codes given below:

LIST I	LIST II
A. Correction for sag	1. Tacheometer
B. Least count 30'	2. Aerial photograph
C. overlap	3. base line
D. additive constant	4. prismatic compass

A. A-4, B-3, C-2, D-1

B. A-3, B-4, C-2, D-1

C. A-1, B-2, C-3, D-4

D. A-3, B-4, C-1, D-2

Ans. B

Sol. In a triangulation, baseline is measured very precisely.

- The sag correction is used for baseline measurement.
- Temperature correction is applied by measuring the temperature correction at least at three places.
- Finally, slope correction is also needed on steep slopes.
- The overlap of the photo are maintained to ensure complete coverage of the area.
- The overlap in the direction of light is known as longitudinal or forward overlap (about 60%).

The overlap between adjacent flights is known as side overlap (about 30%). It is used to :

i) Orient point so as to form a continuous flight slip.

ii) View the photograph by stereoscope.

52. Determine the distance between instrument station X and the staff station Y when staff readings are 1.231m, 0.823m & 0.350m. RL. of line of collimation is 200.150m and vertical angle is (- 4°45'). Also determine the RL. Of 'Y'. Take K=100 and C =0.8.

A. 88.3m, 206.657m

B. 88.3m, 191.997 m

C. 87.4m, 191.997 m

D. 87.4m, 206.657m

Ans. B

Sol. Horizontal distance (For angle of depression)

D = KS $\cos^2\theta$ + C $\cos\theta$

Here, S = (1.231 - 0.350) = 0.881m

D = [100 x 0.881 x $\cos^2$(4°45')] + [0.8 x cos (4°45')]

= 88.3m

Vertical distance, V = [(KS sin2θ)/2] + C sinθ

= [(100 x 0.881 x 0.165) / 2] + (0.8 x 0.083)

=7.33m

Rl. of Y = RL. of line of collimation – V - Staff reading

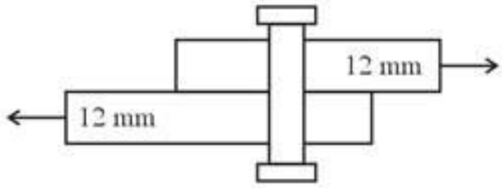
= 200.150 - 7.33 - 0.823

= 191.997m

53. Calculate strength of a 20 mm diameter bolt of grade 4.6 for the following:

The main plate to be joined are 12

mm thick with lap joint (Assume end distance e=33mm and pitch p=50 mm& thread intercept shear plane)



A. 45.26 kN B. 96 kN
C. 101 kN D. 83 kN

Ans. A

Sol. Shank diameter of bolt d=20 mm

Diameter of bolt hole d_0=20+2=22 mm

For grade 4.6 bolt, f_{ub}=400 MPa

t=12mm, e=33mm & p=50mm

When thread intercepts the shear plane

n_n=1 & n_s=0 for lap connection in case of one bolt.

Design strength of one bolt,

V_{db}=Lesser of V_{dsb} or V_{dpb}

Design shear strength of one bolt,

V_{dsb}=

$\frac{f_{ub}}{\sqrt{3}\Upsilon_{mb}}(n_nA_{nb}+n_sA_{sb}) = \frac{400}{\sqrt{3*1.25}}(1*1*0.78*\frac{\pi}{4}*20^2+0)$

=45.26 kN

Design bearing strength of one bolt,

V_{dpb}=2.5k.b.d.t.f_{ub}/Υ_{mb}

k_b is a bearing factor is lesser of $\frac{e}{3d_o} = \frac{33}{3*22} = 0.50$

$\frac{p}{3d_o} - 0.25 = \frac{50}{3*22}$-0.25=0.507

$\frac{Fub}{fu}$=$\frac{400}{410} = 0.97\sim1.0$

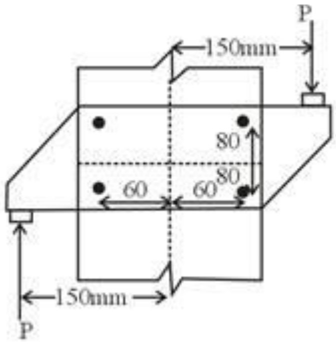
k_b=0.50

V_{dpb}=2.5k.b.d.t.f_{ub}/Υ_{mb}

=2.5*0.5*20*12*400/1.25=96 kN

Design strength of one bolt ,V_{db}=Lesser of V_{dsb} or V_{dpb}= 45.26 kN

54. A bracket has been attached to flange of a column as shown in the figure. The maximum resultant force in the bolt is



A. P/4 B. P/2
C. 3P/4 D. P

Ans. C

Sol. F_a=$\frac{P}{4} - \frac{P}{4}$ =0

F_m=$\frac{Per}{\sum r^2}$

r =$\sqrt{80^2 + 60^2}$=100

F_m=$\frac{Per}{\sum r^2}$==$\frac{P*300*100}{4*100*100}$=3P/4

F_{Rmax}=$\sqrt{Fa^2 + Fm^2 + 2FaFm\cos\theta}$

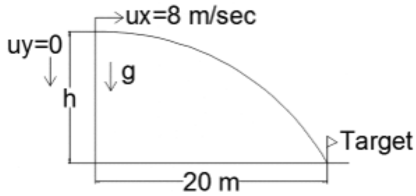
F_R=F_m=3P/4

55. A projectile is fired horizontally at 8m/s from a point of h meter height above and 20 m away from the target. What is the value of h required so that the projectile hits the target?

A. 30.25 B. 31.75
C. 30.75 D. 31.25

Ans. D

Sol. At initial position, u_x= 8m/s



S_x=20m

So, S_x= u_xt [t=time taken to hit the target]

Or, t=$\frac{20}{8}$ sec=2.5 sec

There is no initial velocity in Y direction

So,S_y=u_yt+$\frac{1}{2}$ X g X t^2 [u_y=0]

So, S_y=h= $\frac{1}{2}$ X 10 X 2.5^2 =31.25 m

56. The given integral $\int_0^{\frac{\pi}{k}}\int_x^{\frac{\pi}{k}}\frac{\sin y}{y}dy\ dx$ evaluates to $\frac{1}{2}$ for some $k \geq 1$. Then the value of k is:

Sol. Changing the order of integration we get:

$$\int_0^{\frac{\pi}{k}}\int_0^{y}\frac{\sin y}{y}dx\ dy$$

$$= \int_0^{\frac{\pi}{k}}\frac{\sin y}{y}X\ [x]_0^y\ dy$$

$$= \int_0^{\frac{\pi}{k}}\sin y\ dy$$

$$= [-\cos y]_0^{\frac{\pi}{k}} = -\cos\frac{\pi}{k} + 1$$

$$-\cos\frac{\pi}{k} + 1 = \frac{1}{2}$$

$$giving\ \frac{\pi}{k} = \cos^{-1}0.5$$

$$\frac{\pi}{k} = \frac{\pi}{3}$$

$$thus, k = 3$$

57. The line integral of the vector field F= 5xzi +(3x²+2y)j + x²zk along a path from (0,0,0) to (1,1,1) parametrized by (t,t²,t)is __________.

Sol. $F = 5xz\bar{i} + (3x^2 + 2y)\bar{j} + x^2z = \bar{k}$

$x = t; y = t^2; z = t$

⇒dx =dt; dy= 2tdt & dz=dt

∴ The line integral of the vector field is

$$\int_c \bar{F}.\bar{dr} = \int 5xzdx + (3x^2 + 2y)dy + (x^2z)dz$$

$$\int_0^1 (5t^2dt + 10t^3dt + t^3dt)$$

$$\int_0^1 (5t^2dt + 11t^3dt)$$

$$= 5\left[t^3/3\right]_0^1 + 11\left[t^4/4\right]_0^1$$

= 5(1/3) + 11(1/4)

$$= 5/3 + 11/4 = \frac{20+33}{12} = \frac{53}{12} = 4.4167$$

58. X and Y are two continuous random variable with joint distribution:

$$f(x,y) = \begin{cases} cx+1, & y \geq 0\ \&\ x+y<1 \\ 0, & otherwise \end{cases}$$

The value of the constant c is:

Sol. For a joint probability distribution, we must have:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\ dx\ dy\ = 1$$

$$\int_0^1\int_0^{1-x}(cx+1)\ dx\ dy\ = 1$$

$$\int_0^1(cx+1)(1-x)\ dx\ = 1$$

$$giving\ \frac{1}{2} + \frac{c}{6} = 1$$

Thus: $c = 3$

59. From the given data, the maximum resources load in any week will be _________weeks.

Activity	Start week	End week	Resources needed per week
A	1st	7th	12
B	1st	10th	16
C	8st	13th	13
D	9st	15th	10

Sol. For 1st to 7th week both activity A & B will progress, hence, resource needed = 12+16 = 28

For 8th week both activity B & C will progress, hence, resource needed =16 + 13 = 29

For 9th to 13th week activities B, C & D will be in progress, hence, resource needed = 16+13+10 = 39

For 11^{th} to 13^{th} week both activity C & D will be in progress, hence, resource needed = 13+10 = 26
Therefore, Maximum resources needed = 39

60. A rigid wheel 1.25 m in diameter is to be provided with a thin steel tyre. If the stress in the steel type is not to exceed 140 MN/m^2, the minimum temperature to which the type is to be raised so that it can be fitted over the wheel is ______°C. Take E = 200 GN/m^2; ∝ = 12 × 10^{-6}/°C.

Sol. Let D = Diameter of rigid wheel
D = least diameter of steel tyre

$$\text{Strain in tyre} = \frac{D-d}{d}$$

$$\text{Strain in tyre} = \left(\frac{D-d}{d}\right)E = \sigma$$

$$\therefore \frac{D}{d} - 1 = \frac{\sigma}{E}$$

$$\frac{D}{d} - 1 = \frac{140\times10^6}{200\times10^9} = 0.0007$$

$$\frac{D}{d} = 1.0007$$

Let the steel be subjected to a temperature rise of T°C.
πD = πd + πd · ∝T

$$\frac{\pi D}{\pi d} = 1 + \propto T$$

$$\frac{D}{d} = 1 + \propto T$$

1.0007 = 1 + ∝T

$$T = \frac{0.0007}{12\times10^{-6}} = 58.33^\circ C$$

61. A simply supported beam of span 8 meter having a rectangular cross section of 500mm*250mm.The beam is prestressed with force 1500KN at 50mm eccentricity below the neutral axis .What is the position in mm below the neutral axis at centre of span where tensile stress is zero? Considering the self weight of material of beam is 26KN/m.

Sol. Moment due to self weight at centre of span = wl^2/8 = 26* 8^2/8 = 208 KNm
The tensile stress f_t = P/A – M*y/I + P*e*y/I
= (1500*10^3)/(500*250) - (208 * 10^6 *y/(250*500^3/12) + (1500*10^3 * 50*y / (250*500^3/12)
= 12 – 0.0798y + 0.0288y
Equate the equation to zero ;
y =235.29mm

62. A standard penetration test was conducted in a soil having saturated density 18 kN/m^3, ground water table was found at a depth of 4.5 m. If the N value was determined as 28 at the depth of 5 m, then the corrected N value of the soil is

Sol. $\bar{\sigma} = 4.5 \times \gamma_{sat} + 0.5 \times \gamma'$
$= 4.5 \times 18 + 0.5 \times (18 - 9.81)$
$= 85.095$
Hence overburden correction is required.

$$N_1 = N_0 \times \frac{350}{\bar{\sigma} + 70}$$

$$= 28 \times \frac{350}{85.095 + 70}$$

$= 63.187$
Applying dilatancy correction,

$$N_2 = 15 + \frac{1}{2}(N_1 - 15)$$

$$N_2 = 15 + \frac{1}{2}(63.22 - 15)$$

$$N_2 = 39.11$$

63. Rainfall over a basin in three consecutive hours are 3.5 cm, 4 cm and 3 cm respectively. The infiltration loss can be estimated using the following Horton equation
f = 0.9 + 3.8 e^{-2t}
where f is infiltration in cm/hr and t = time in hour from start of rainfall.
Assuming negligible surface retention and evaporation losses, the surface run off from the basin is ________ cm [Rounded to two decimal places]

Sol. Given,

Rainfall intensity for 1st hour $\frac{3.5}{1}$ = 3.5 cm/hour

Rainfall intensity for 2nd hour = $\frac{4}{1}$ = 4 cm/ hour

Rainfall intensity for 3rd hour = $\frac{3}{1}$ = 3 cm/ hour

Calculate time when Horton curve cuts hyetograph.

Assuming that curve cuts the hyetograph before t = 1 hour

$0.9 + 3.8\, e^{-2t} = 3.5$

t = 0.1897 hours

Hence till t = 0.1018 hours, there will be no run off

Total infiltration F(t) = 3.5 × 0.1897

$$+ \int_{0.1897}^{3} \left(0.9+3.8e^{-2t}\right)dt$$

$$F(t) = 0.664 + \int_{0.1897}^{3} 0.9dt + \int_{0.1897}^{3} 3.8e^{-2t}dt$$

$$F(t) = 0.664 + 0.9(3-0.1897) + 3.8\int_{0.1897}^{3} e^{-2t}.dt$$

F(t) = 0.664 + 2.53 + 3.8 × 0.3409

F(t) = 4.489cm

Hence Runoff = Total precipitation - Infiltration

= (3.5 + 4 + 3) - 4.489

Runoff = 6.01cm

64. A tank 1.5 m high stands of a trolley and is full of water. It has an orifice of diameter 0.1 m at 0.3 from the bottom of the tank. If the orifice is suddenly opened, the propelling force on the trolley is _________N. [Take coefficient of discharge of the orifice as 0.60]

Sol. Discharge from the orifice=

$$C_d a\sqrt{2gH}$$

$$= 0.60 \times \frac{\pi}{4}(0.1)^2 \times \sqrt{2\times 9.81\times 1.2}$$

$$= 0.023 m^3/s$$

Velocity of the jet issuing from the orifice

$$= \frac{Q}{a} = \frac{0.023\times 4}{\pi\times(0.1)^2} = 2.93\, m/s$$

Propelling force, F = ρ QV

= 1000 x 0.023 x 2.93

= 67.39 N

65. A completely mix process is designed with the following data:

Flow of sewage (Q_0) =10000 m^3/day,

BOD after PST=150mg/l,

Effluent BOD= 5mg/l

Y=.5 Kg/kg

K_d=.05 per day

MLSS = 3000 mg/l.

Under flow concentration (X_u) = 10000 mg/l for secondary sedimentation tank, calculate the sludge wasted (in cumec per day). (Consider sludge age =8 days)

Sol. $We\ know, Q_w X_u = \frac{VX}{\theta}$,....i

$$VX = \frac{Q_0(S_0-S)\theta\times Y}{1+.05\theta}$$ ii

$$V\times 3000 = \frac{10000\times(150-5)\times 8\times .5}{1+.05\times 8}$$

$$V = 1380.95\, m^3$$

$$by\ putting\ value\ of\ V\ in\ eq.\,i, Q_w X_u = \frac{1380.95\times 10^3\times 3000\times 10^{-6}}{8}$$

$$= 517.88\frac{kg}{day}$$

$$Q_w = \frac{517.88}{10^4\times 10^{-6}}\times 10^{-3} = 51.786\, m^3/day$$

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